Part III

Derivation of Plaquette Boson-Fermion Model for Cuprates, using COntractor REnormalization

References

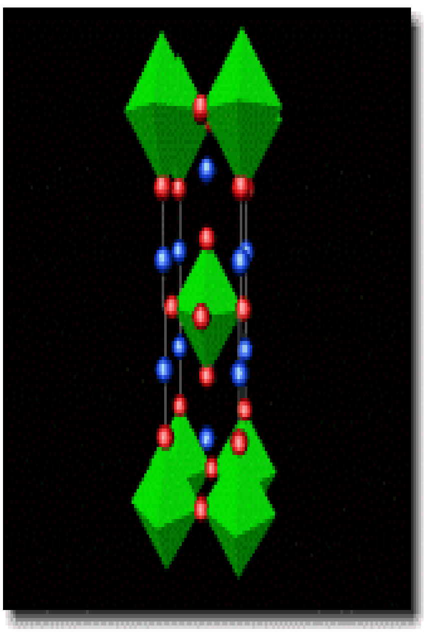
1. ``*1nteracting Electrons and Quantum Magnetism*", **A.Auerbach**, Springer-Verlag (NY).
2. ``*Quantum Magnetism Approaches to SCES*" , **A. Auerbach, F. Berutto and L. Capriotti** *"Field Theories in Low .. Eds. G. Morandi et. al. Springer-verlag (00), also* ***cond-mat/9801294)***
3. "*Projected SO(5) Hamiltonian for Cuprates and its Applications*" **A.Auerbach and E. Altman,** ***IJMPB 15, 2509 (01), also cond-mat/0009421.***
4. "*Plaquette Boson Fermion Model of Cuprates*" **E. Altman and A.Auerbach,** ***Phys. Rev. B (in press), also cond-mat/0108087***

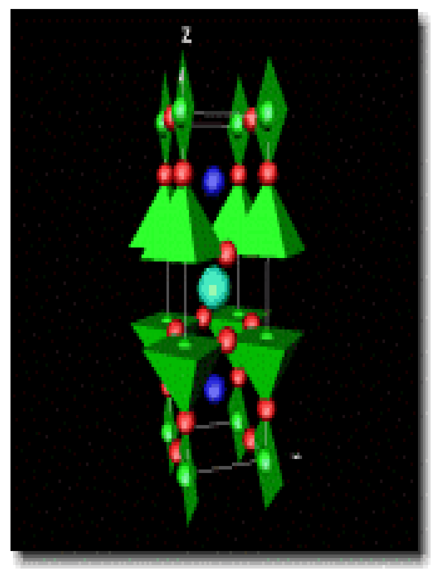
Review 1

Cuprate (High Tc) Superconductors

$La_{2-x}Sr_xCuO_4$

$YBa_2Cu_3O_7$





Tc = 40^0

Tc = 90^0

Bednorz &Muller ‘86

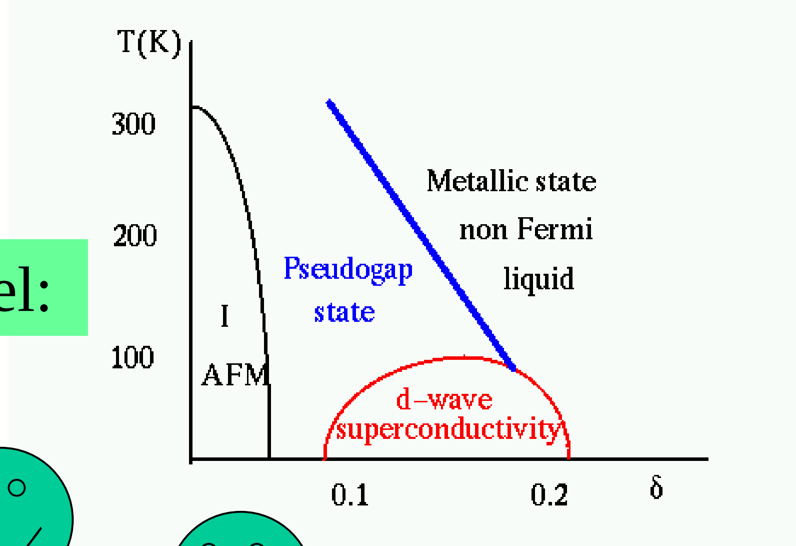
Wu et.al. ‘87

What is the Mechanism? (Non BCS?)

Review 2

High Tc Phemomenology: Summary of Problems

Hubbard Model:



unconventional SC

“Hi Tc”

spin fluctuations

d-wave superconductivity

Low Superfluid Density

Short Coherence Length

Non Fermi Liuid

Abnormal “Normal State”

Mysteries of ARPES

Weird Transport

Pseudogap temperature

T* versus doping

IN NEED OF A MODEL & ITS SOLUTION

Review 3

2D Hubbard Model : Previous Status

Undoped

Heisenberg model, Neel phase.

Single hole:

ground state momentum ($\pi/2,\pi/2$) , ε(k) .

Two holes::

Pair binding on small clusters (numerical).

Will they bind in large lattices?

Many holes:

Phase separation/stripes?

Superconductivity?

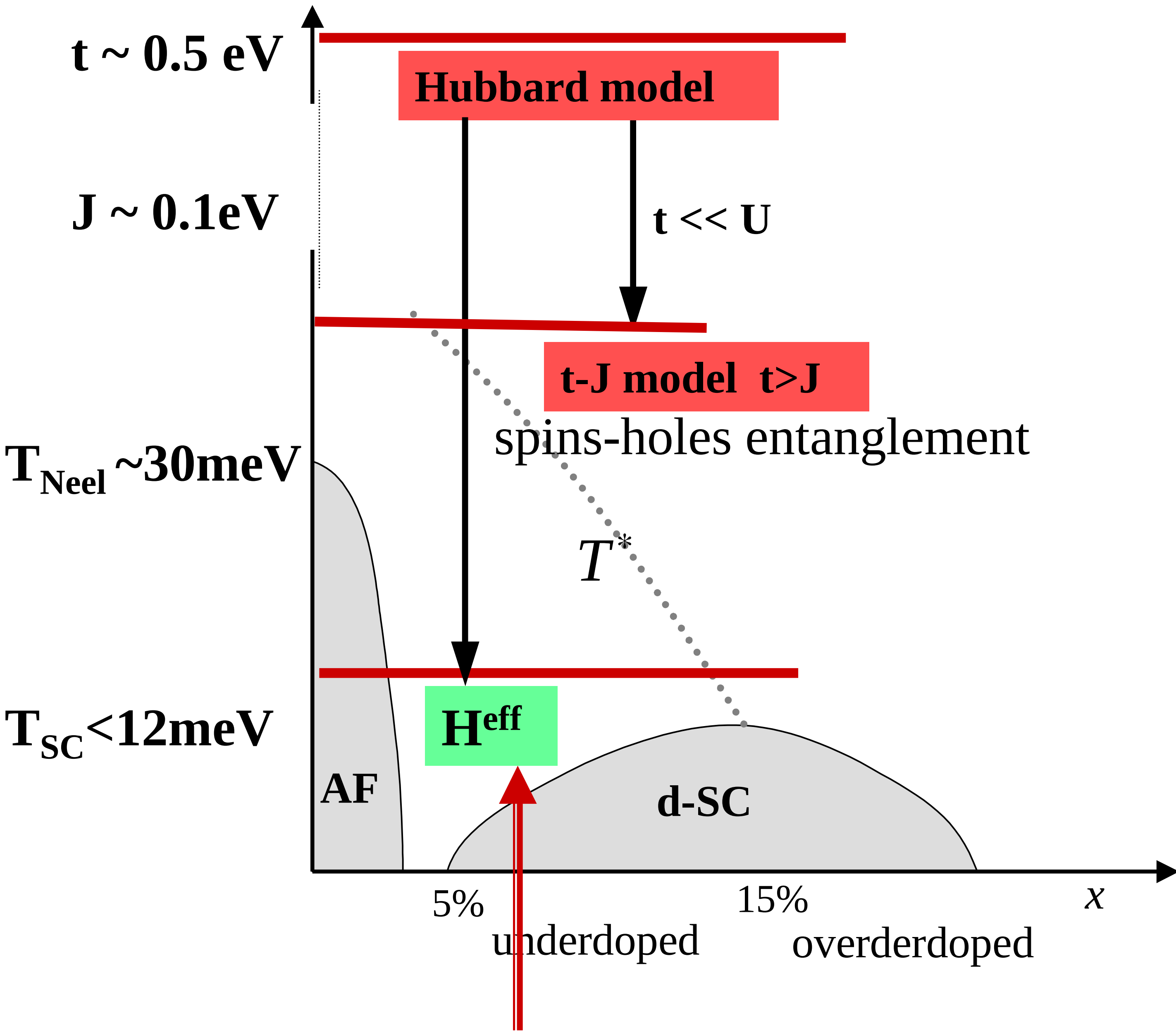
What destroys Neel order?

Conflicting mean field theories…..

t-J Model: spins and holes are strongly entangled on lattice scale.

We need a lower effective H!

Energy scales



What is the effective Hamiltonian for the Hubbard Model in this regime?

Can it describe the experimental phase diagram?

CORE

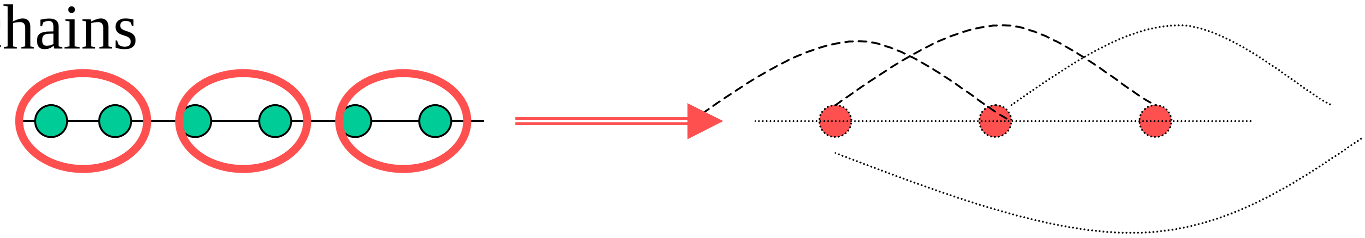
COntractor REnormalization

C.J. Morningstar & M. Weinstein PRD 54, '96

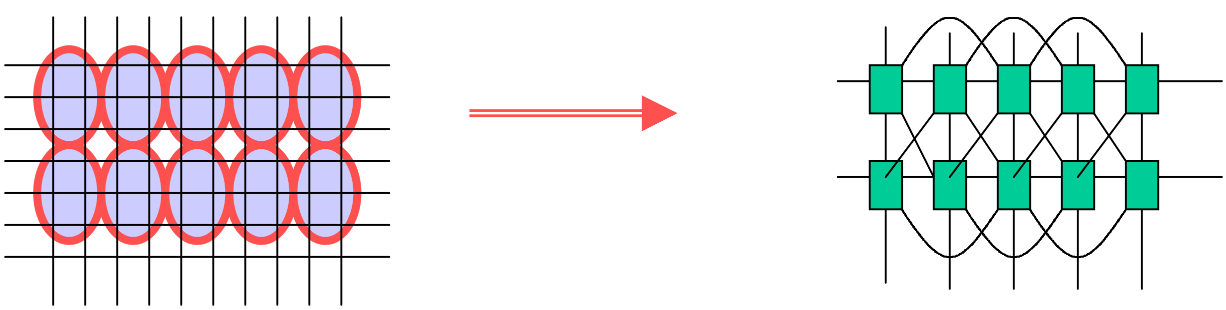
Discrete Real Space Renormalization

H_{ij} : Full H.S. $\Longrightarrow$ h^{eff}_{lk} : Reduced H.S.

chains



2D

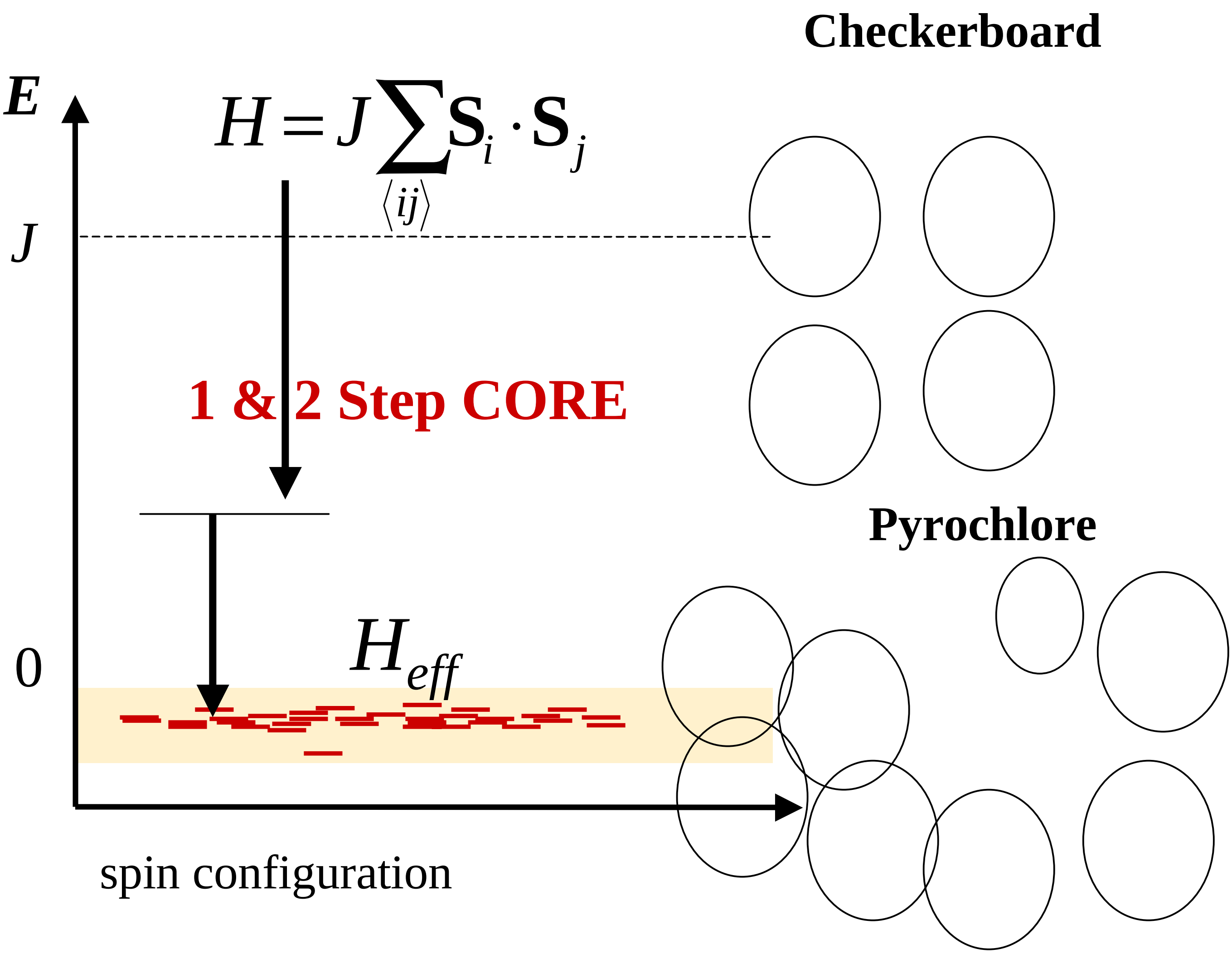


1 Step CORE:

Microscopic d.o.f. (electrons, spins)

–> local collective d.o.f. (hole pairs, singlets, triplets)

Highly frustrated magnets

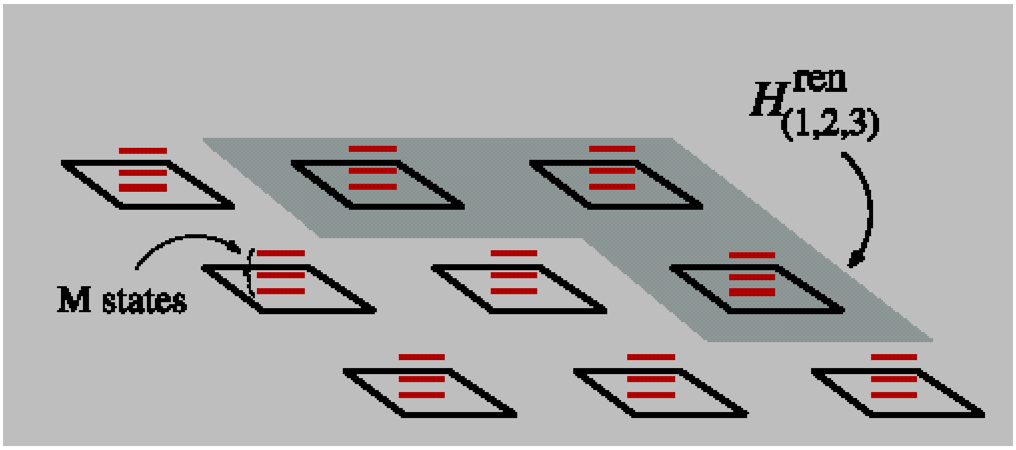


Deriving the low spectrum in local eigenstates representation

Erez Berg, Ehud Altman,AA, *preprint*.

The Scheme

Choose a connected cluster on N plaquettes (N=3)



1. Truncate each plaquette to M states.

2. The Reduced Hilbert has size M^N

$$\underline{\Lambda} = \{ |\alpha\_1,..\alpha\_N> \} << \Lambda$$

3. Diagonalize H on the cluster and obtain $\{ \varepsilon_n , \psi_n \}$ $n=1,,M^N$

4. Gram-Schmidt Orthogonalize

$$|\tilde{\psi}_n\rangle = \frac{1}{Z_n}\left(|\psi_n\rangle - \sum_{m<n} |\tilde{\psi}_m\rangle \langle \tilde{\psi}_m | \psi_n \rangle \right)$$

The Scheme—cont'd

5. Determine renormalized Hamiltonian

$$\mathcal{H}^{ren} \equiv \sum_{n}^{\mathcal{M}} \epsilon_n \,|\, \tilde{\psi}_n \rangle \, \langle \tilde{\psi}_n \,|$$

6. Connected Renormalized Hamiltonian

$$h_{i_1,\ldots,i_N} = H^{ren}_{\langle i_1,\ldots,i_N \rangle} - \sum_{\langle i_1,\ldots,i'_N \rangle} h_{i_1,\ldots,i'_N}$$

conn. subclus.

7. Cluster Expansion of Full Lattice

$$\mathcal{H}_{eff} = \sum_{i} h_i + \sum_{\langle ij \rangle} h_{ij} + \sum_{\langle ijk \rangle} h_{ijk} + \ldots$$

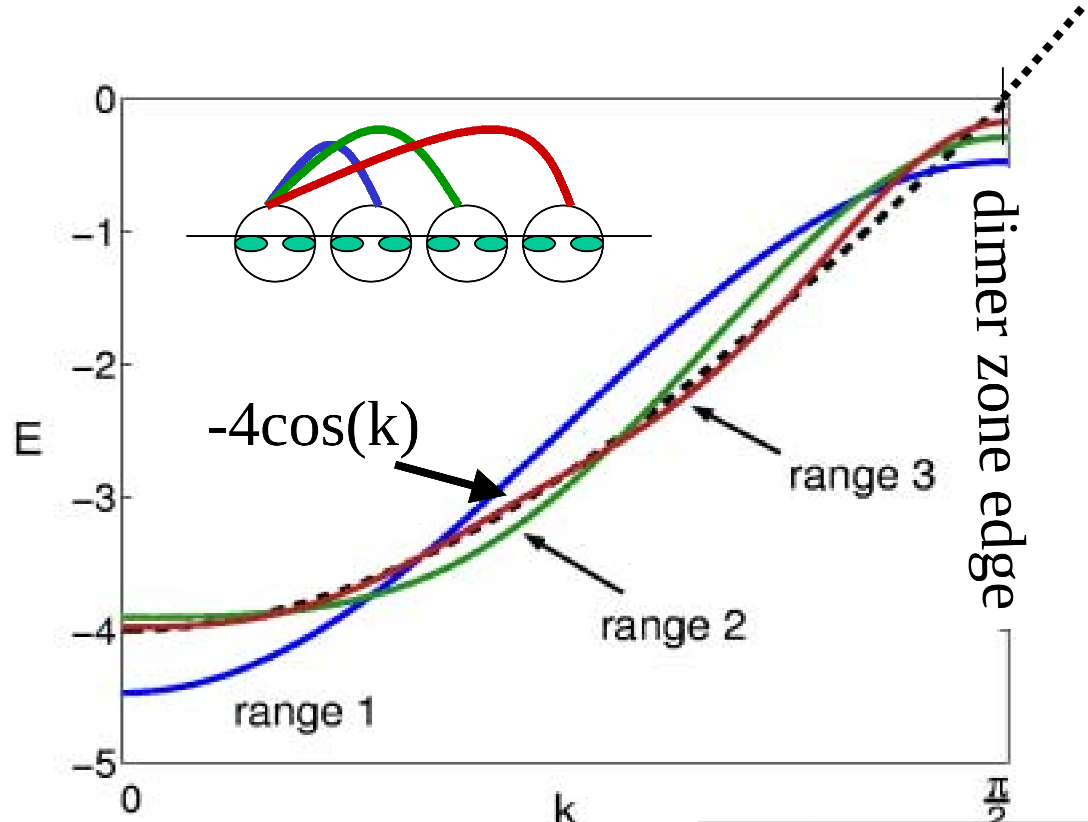
Range 1 Range 2 Range 3

6. **Truncate at finite range interactions**
OR
Pade approximate to infinite range.

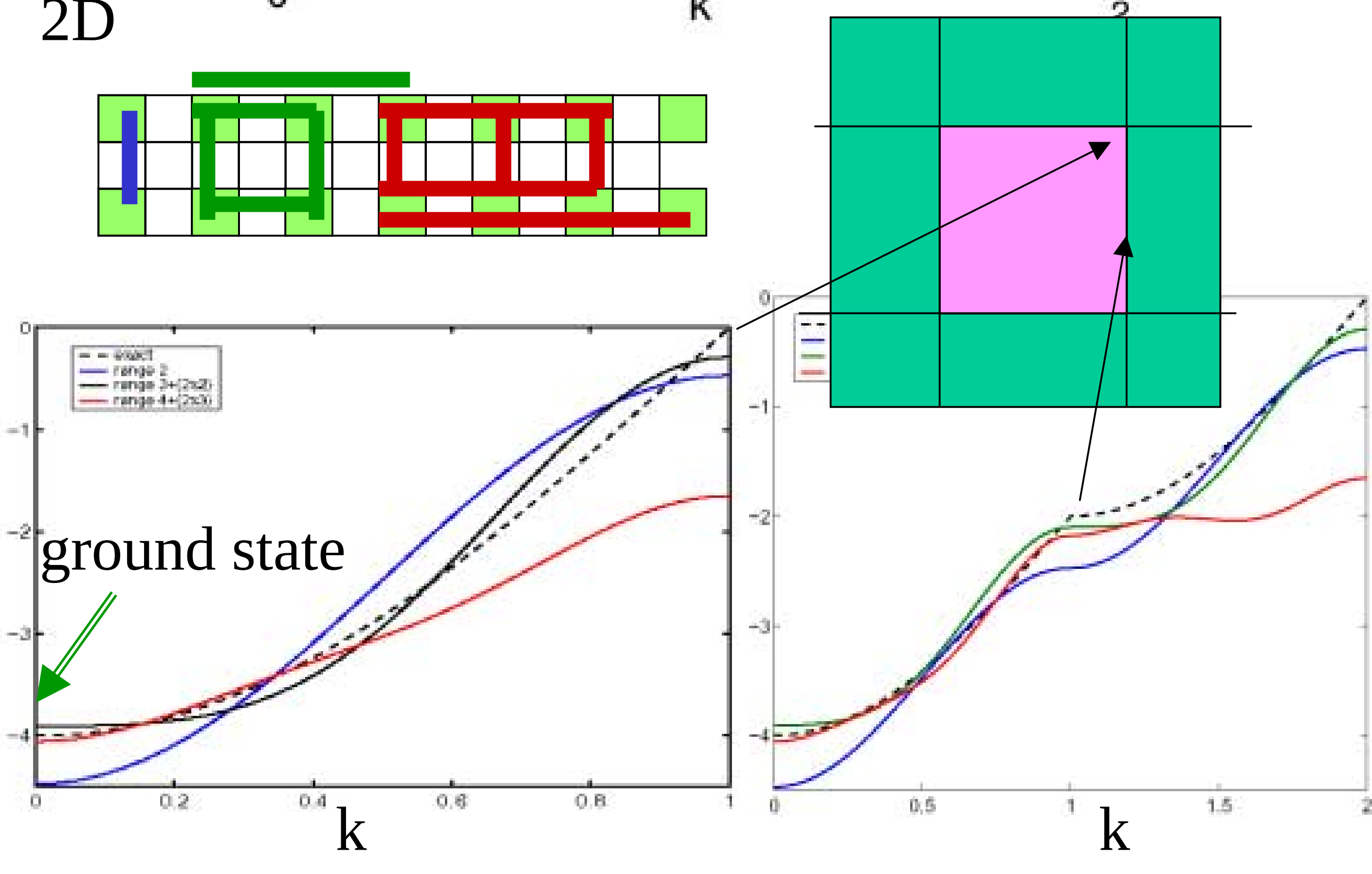
A similar cluster expansion exists for all observables

Test: Tight Binding models

1D



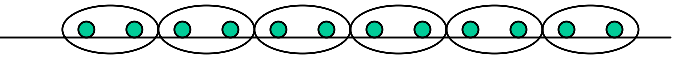
2D

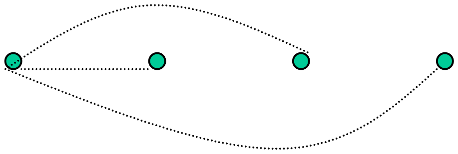


Test of CORE

S=1/2 Heisenberg chain

$$JS_i \cdot S_{i+1} \longrightarrow E_0 / N = \sum_r h_0^r$$





C.J. Morningstar&M. Weinstein hep-lat/000202

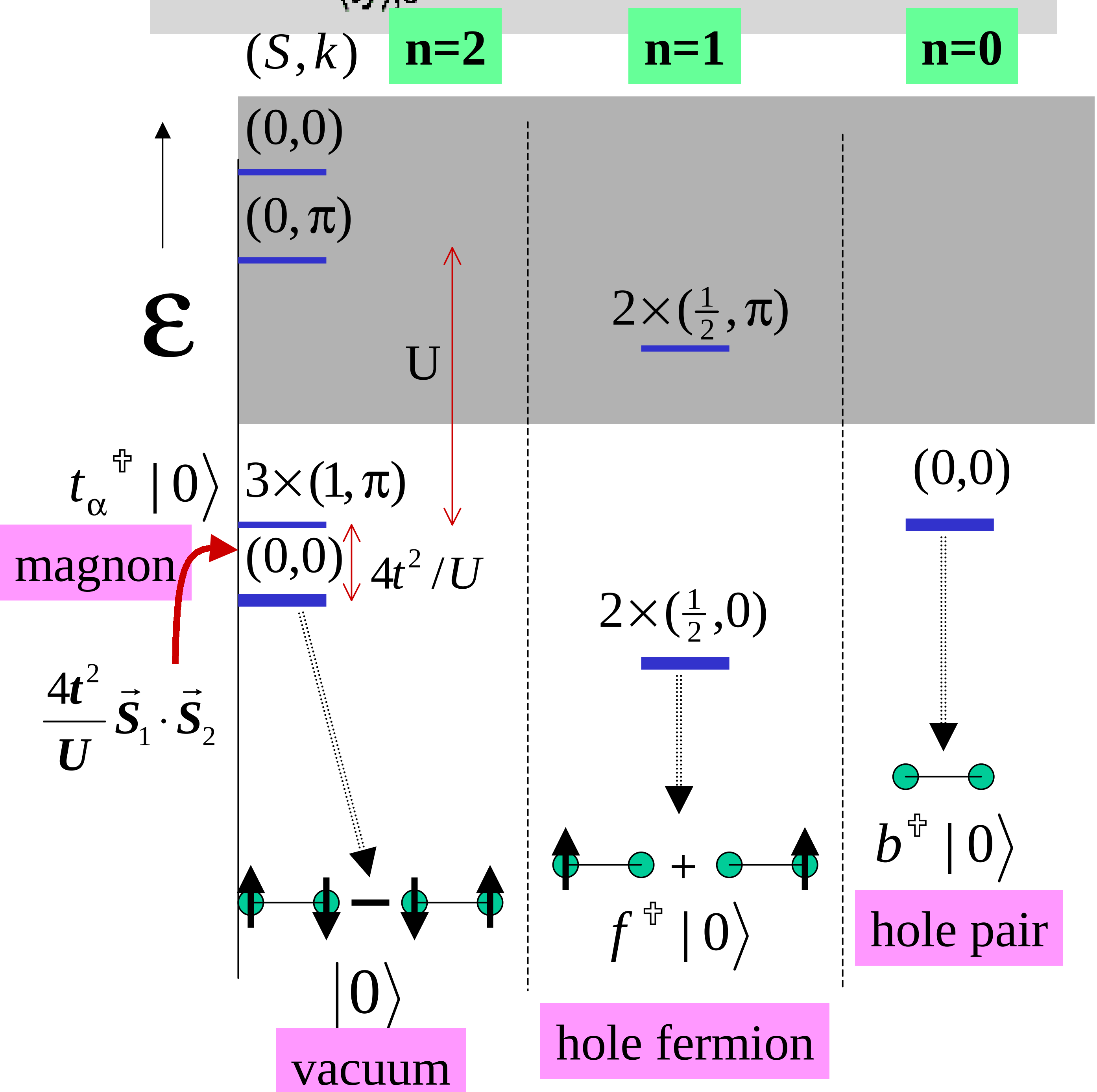
Table 1: Spin-1/2 HAF: Exact Energy Density $= -\ln(2) + 1/4 = -0.4431472$

Range (sites)	Energy Density CORE	Padé [N/M]	Energy Density
1 (2)	-0.3750000		
2 (4)	-0.4330127		
3 (6)	-0.4387759	[1/1]	-0.4428182
4 (8)	-0.4406777	[1/2]	-0.4431005
		[2/1]	-0.4431022
5 (10)	-0.44155130	[2/2]	-0.4431337
6 (12)	-0.44202771	[2/3]	-0.4431412
		[3/2]	-0.4431412

1. Quantum Hamiltonian Renormalization (no time retardation)
2. Non perturbative (in parameters of H)
3. Estimable truncation error
4. Efficient in memory usage: **stores interactions, not wavefunctions.**

The Hubbard Dimer

$$\mathcal{H} = -t \sum_{\langle ij \rangle, s}^{sl} \left(c_{is}^{\dagger} c_{js} + \mathrm{H.c} \right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow},$$

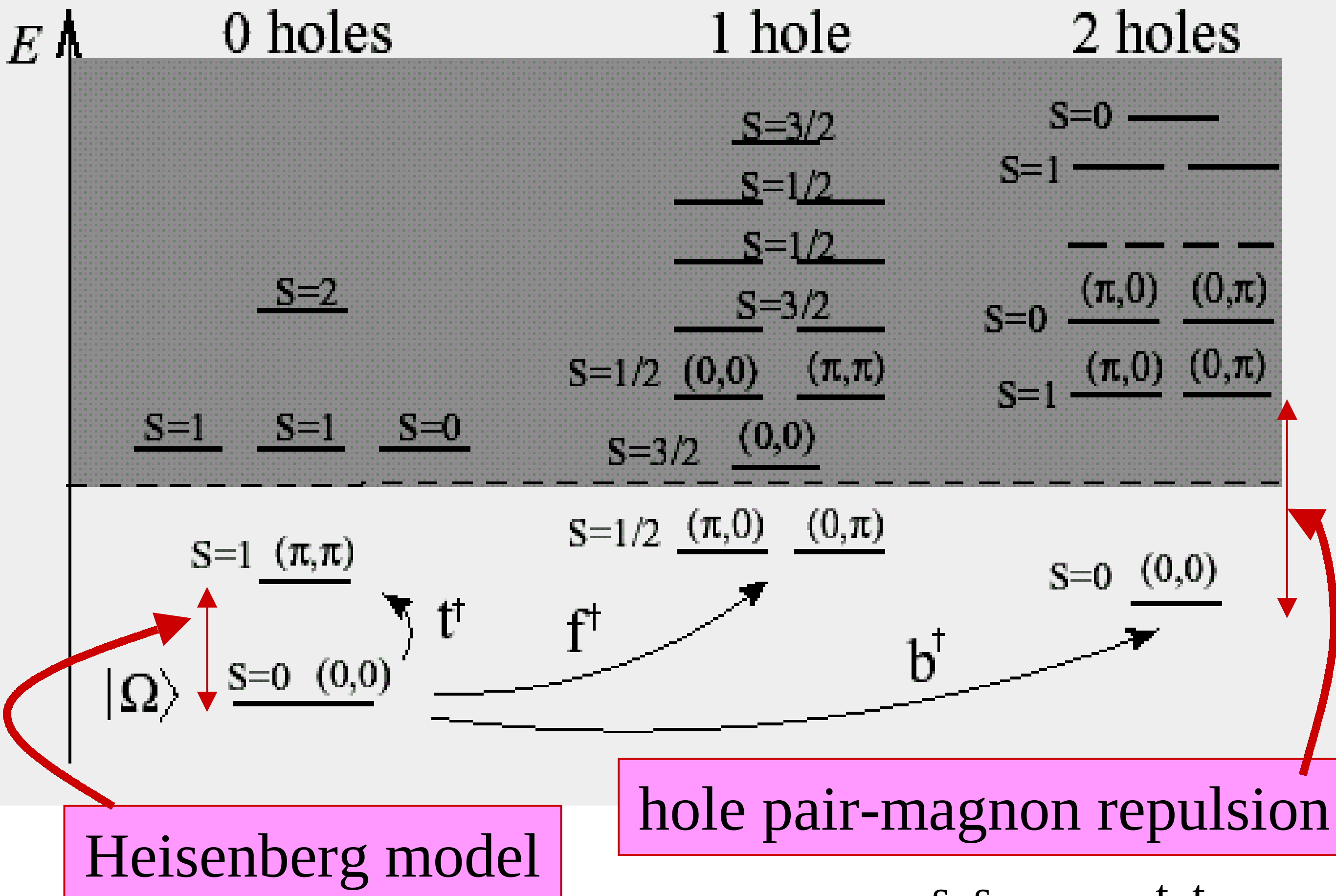


No pair binding, no d-wave hole pairs.

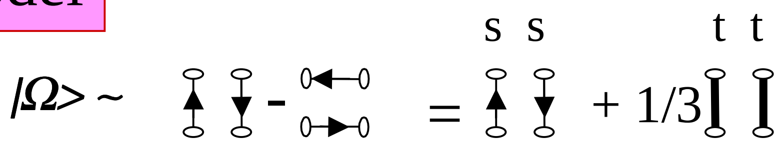
The secret of High Tc Superconductivity hides in the Hubbard Plaquette!

The Hubbard Plaquette

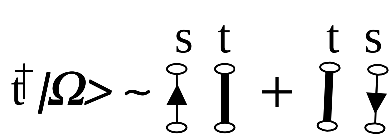
$$\mathcal{H} = -t \sum_{\langle ij \rangle, s}^{sl} \left(c^{\dagger}_{is} c_{js} + \mathrm{H.c} \right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow},$$



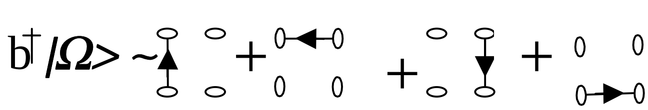
The Vacuum (RVB)



The (antiferro)magnon



The d-wave Hole Pair



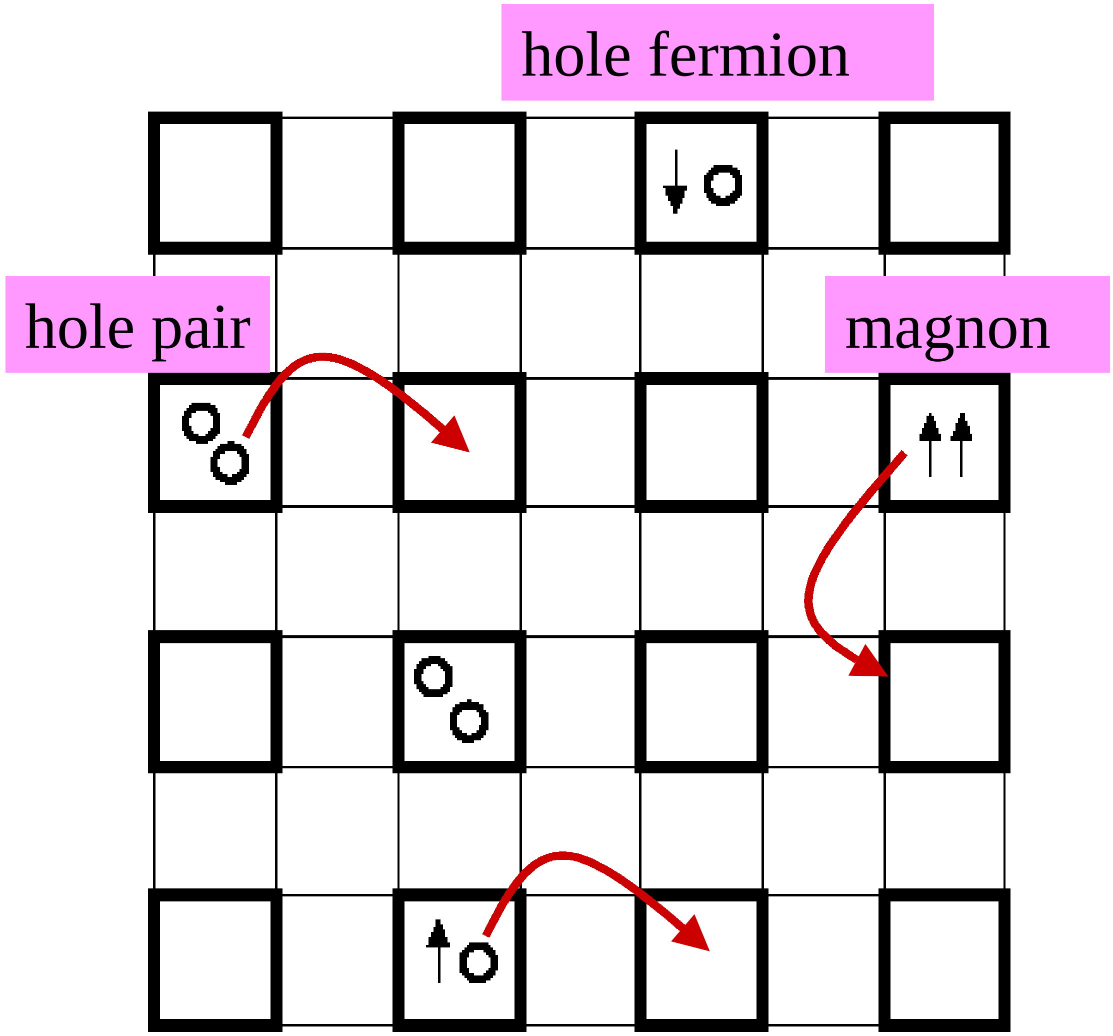
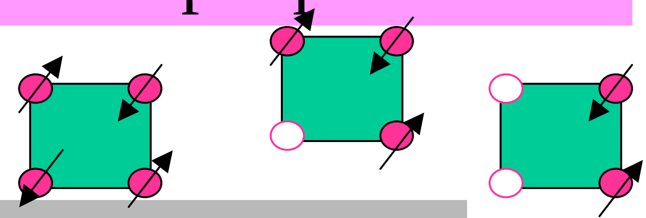
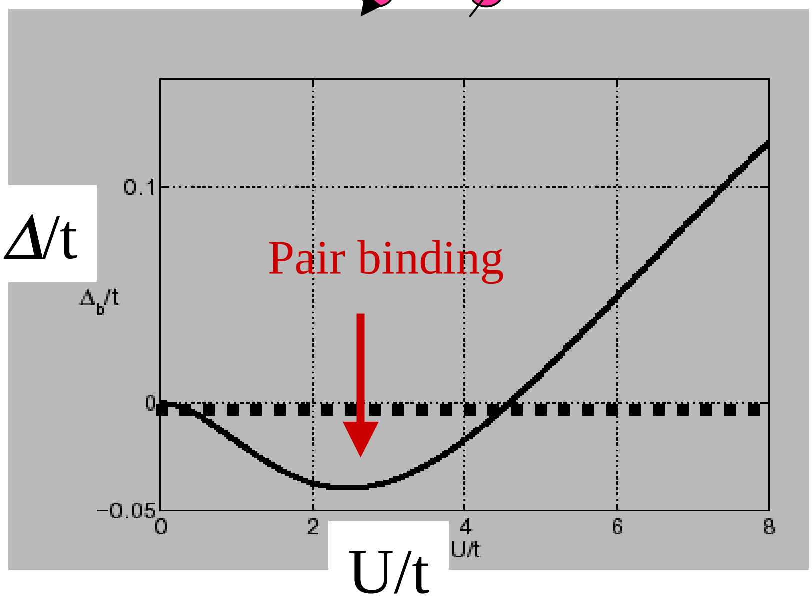


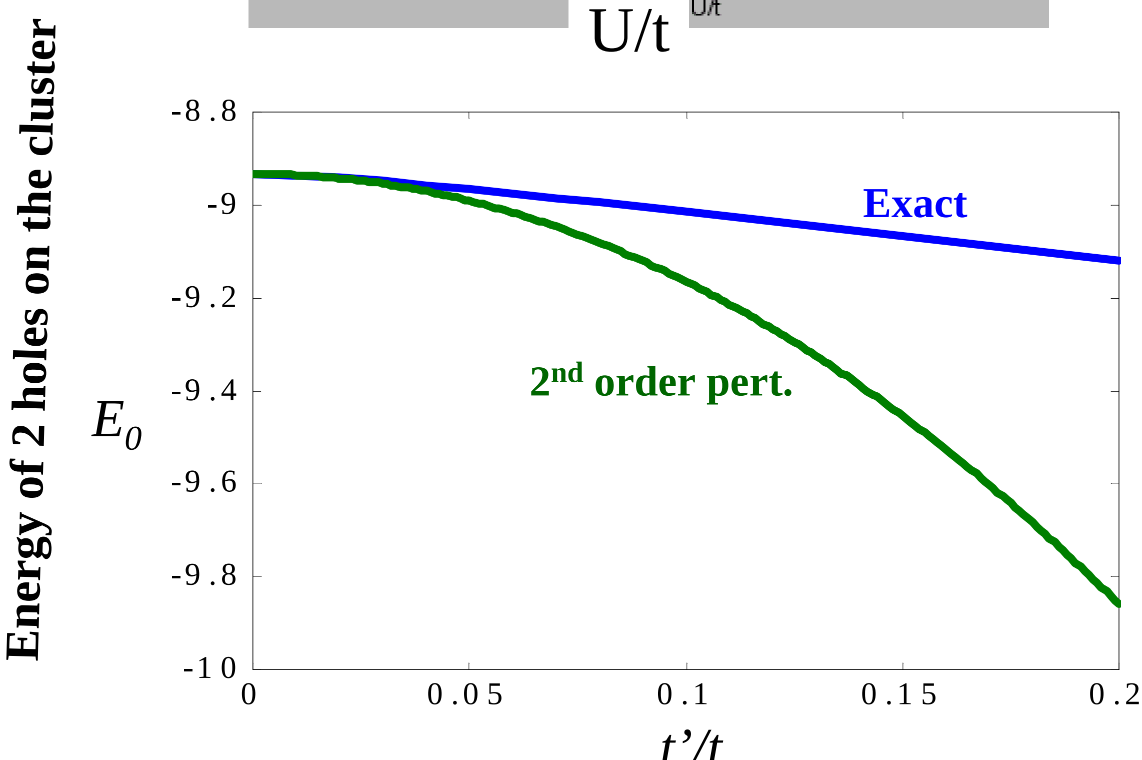
FIG. 1. **Local bosons and fermions on the plaquette lattice.** The singlet RVB vacua are depicted as solid squares. Holes are depicted by circles. The triplets, single hole and hole pairs Hubbard eigenstates define the degrees of freedom of the effective Plaquette Boson-Fermion Model. Interplaquette couplings are computed using Contractor Renormalization.

Pair binding on the plaquette

$$\Delta \equiv E_2 - 2E_3 + E_4$$







Pair binding survives weak interplaquette coupling

$t' << \Delta$ .Will it survive non perturbative coupling?

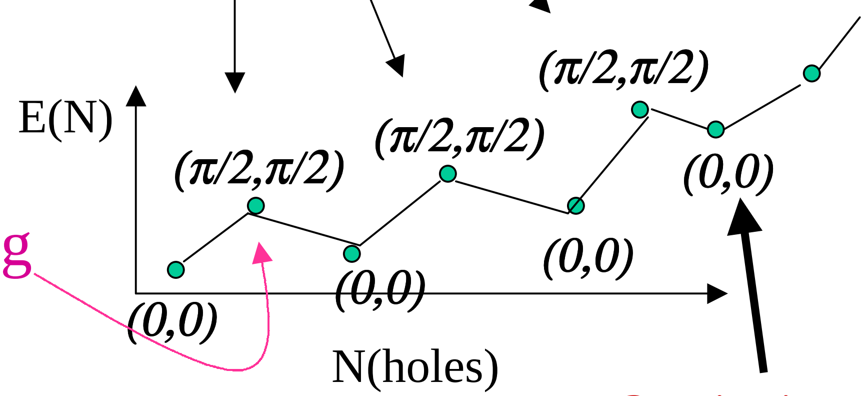
2 hole pairing

Static and dynamical properties of doped Hubbard clusters

E. Dagotto, A. Moreo, F. Ortolani,* D. Poilblanc,† and J. Riera‡

4×4

Fermion holes in nodal pockets

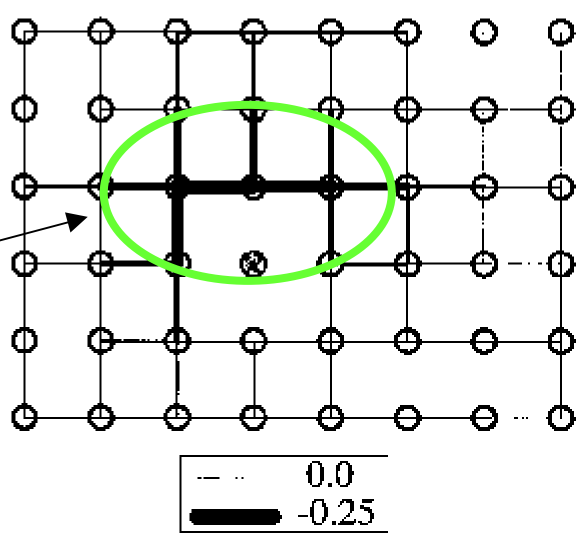


Pair binding

3 pairs in a condensate

Pair correlations in t-J model by DMRG (*White&Scalapino*)

6x8



short pair coherence length

CORE: The 4 Boson Model

hole pair

$$\mathcal{H}^b = (\epsilon_b - 2\mu)\sum_i b_i^\dagger b_i - J_b \sum_{\langle ij\rangle}\left(b_i^\dagger b_j + \text{H.c.}\right)$$

$$\mathcal{H}^t = \epsilon_t \sum_{i\alpha} t_{\alpha i}^\dagger t_{\alpha i} - \frac{J_t}{2}\sum_{\alpha\langle ij\rangle}(t_{\alpha i}^\dagger t_{\alpha j} + \text{H.c.})$$

magnons

$$- \frac{J_{tt}}{2}\sum_{\alpha\langle ij\rangle}(t_{\alpha i}^\dagger t_{\alpha j}^\dagger + \text{H.c.}).$$

Hard core bosons: $\sum_\alpha t_{\alpha i}^\dagger t_{\alpha i} + b_i^\dagger b_i = 1$

$$\mathcal{H}^{int} = V_b \sum_{\langle ij\rangle} n_{bi} n_{bj} + \sum_{\langle ij\rangle}\left[V_0 (t_i t_j)_0^\dagger (t_i t_j)_0 + V_1 (t_i t_j)_1^\dagger (t_i t_j)_1 + V_2 (t_i t_j)_2^\dagger (t_i t_j)_2\right]$$

$$- J_{bt}\sum_{\langle ij\rangle\alpha}(b_i^\dagger b_j t_{\alpha j}^\dagger t_{\alpha i} + \text{h.c.})$$

$$+ V_{bt}\sum_{\langle ij\rangle\alpha}(b_i^\dagger b_i t_{\alpha j}^\dagger t_{\alpha j} + b_j^\dagger b_j t_{\alpha i}^\dagger t_{\alpha i}),$$

Parameters table: cond-mat/01018087

Projected SO(5) Theory

S-C Zhang, J.P. Hu, E. Arrigoni, W. Hanke, AA, PRB60, (99)

$$H^{pSO(5)} = (\varepsilon_b - 2\mu)\sum_i b_i^\dagger b_i - J_b \sum (b_i^\dagger b_j + H.c)$$

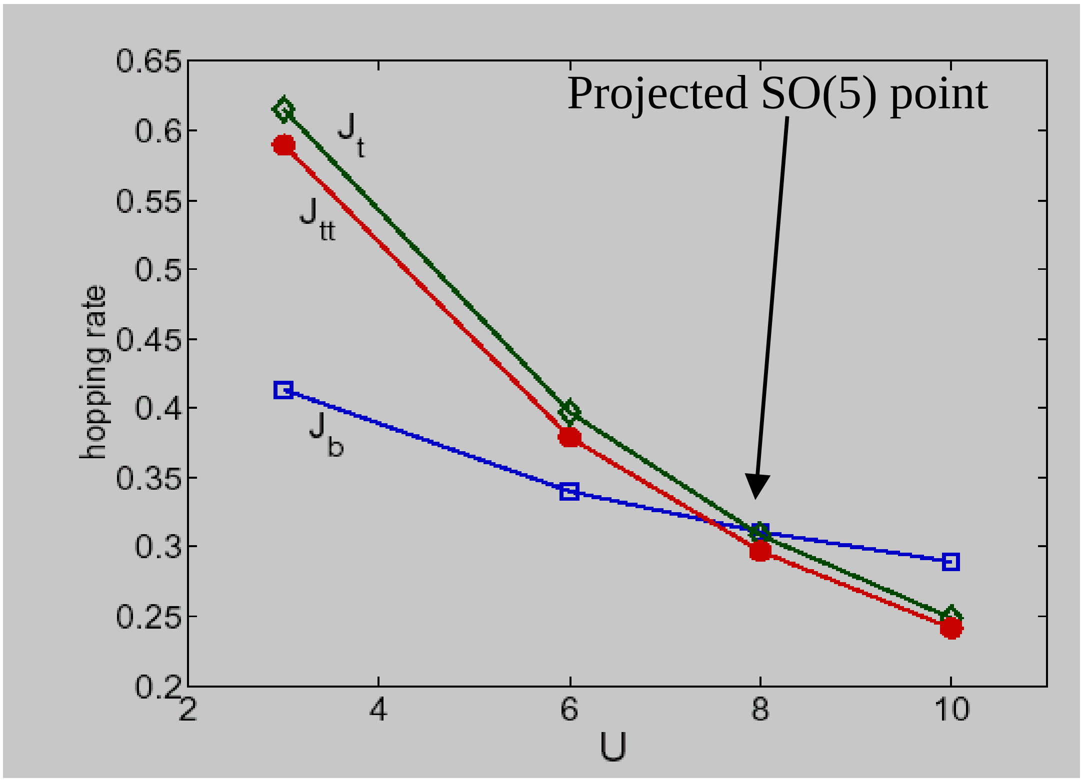
Hole Pair Hopping

$$+\varepsilon_s \sum_{i\alpha=1,2,3} t_{i\alpha}^\dagger t_{i\alpha} - \frac{J_t}{2}\sum_{ij}(t_{i\alpha}^\dagger + t_{i\alpha})(t_{j\alpha}^\dagger + t_{j\alpha}$$

continuum:

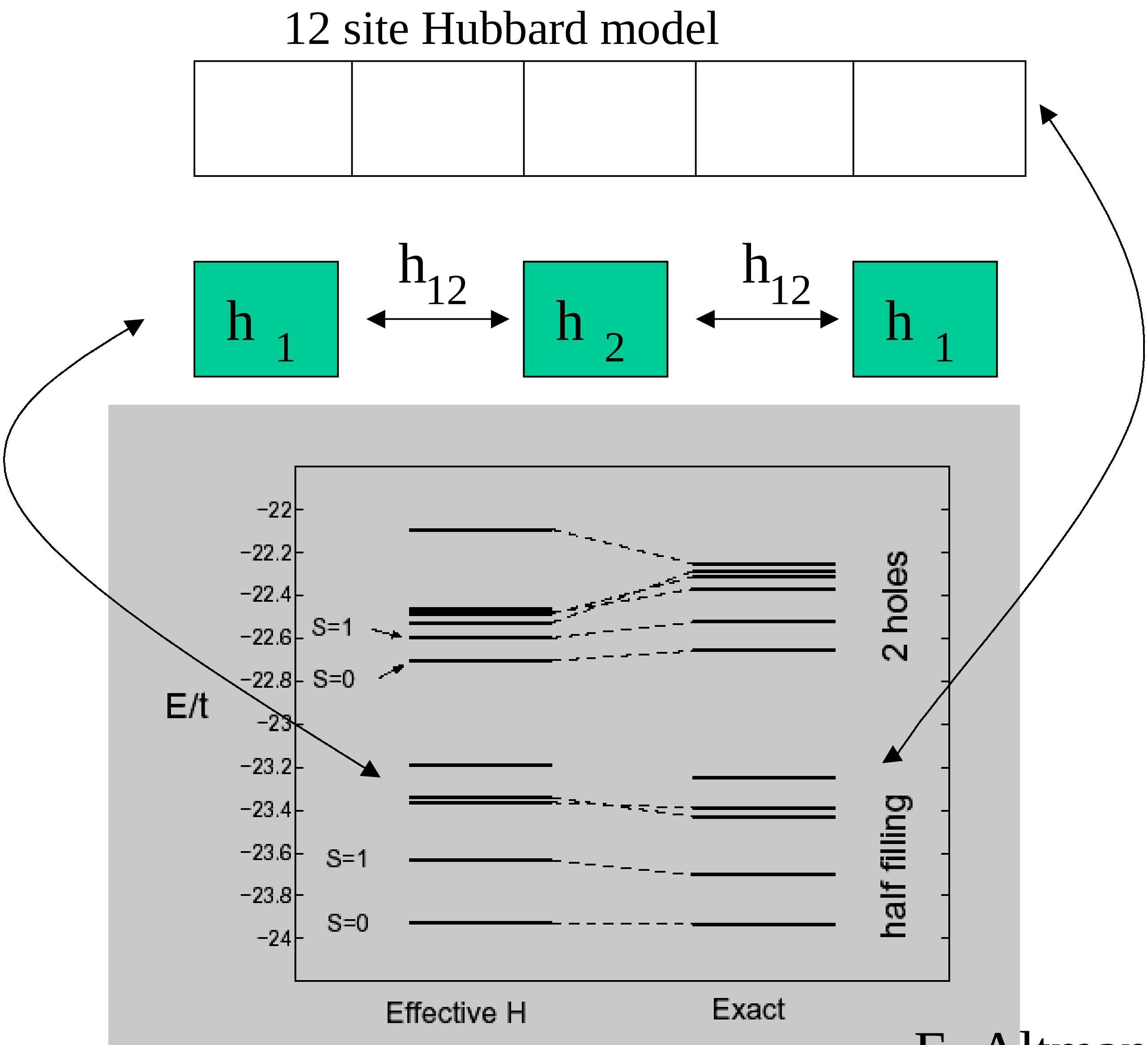
$$Z = \int D\phi_\alpha D\psi \exp\Big[-\int d\tau d^2 x\Big(\tfrac{1}{2m_s}\rho_s(\partial_\mu \phi_\alpha)^2 - a\phi_\alpha^2$$

$$+\psi^*(\partial_\tau - 2\mu)\psi + \tfrac{1}{2n_b}\rho_c |\nabla\psi|^2 + U\Big(|\psi|^2 + \sum_\alpha \phi_\alpha^2\Big)\Big)\Big]$$



S.Sachdev

Hubbard Model CORE: Range-2



E. Altman

	S=0	S=1
0 holes	330	7.7
2 holes	27	19.5

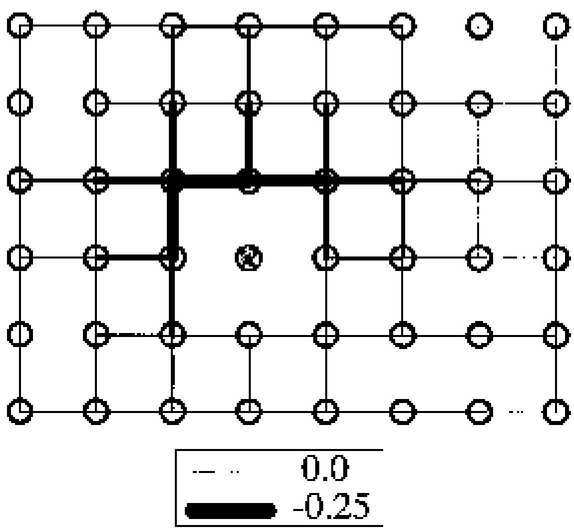
TABLE II. Convergence of the cluster expansion.

Coherence length for hole pairs and magnons is of order 1 plaquette

The Pairing Mechanism

Hole Pair coherence length is of order one plaquette

1. Rapid decay of CORE interactions from range 2 to 3.
2. Pair correlations in t-J model (White&Scalapino)



Pair correlations are short even without plaquette pair binding U/t > 4.5

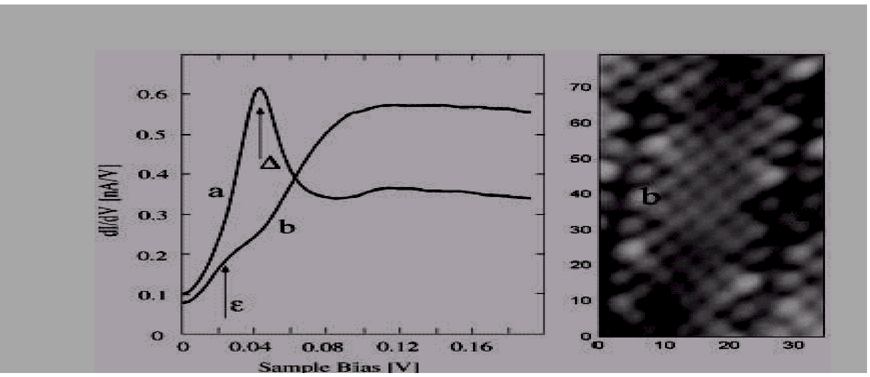
Weigmann, Lee, Wen:

Kinematic (QED) hole pairing in a quantum spin liquid

Superconductivity stabilized by large value of pair hopping $J_b \sim J_t$

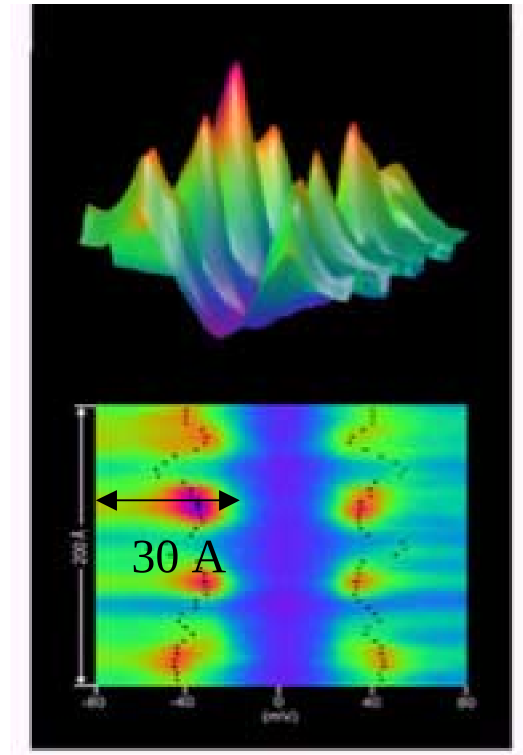
Short Coherence Length

Tunneling Microscopy



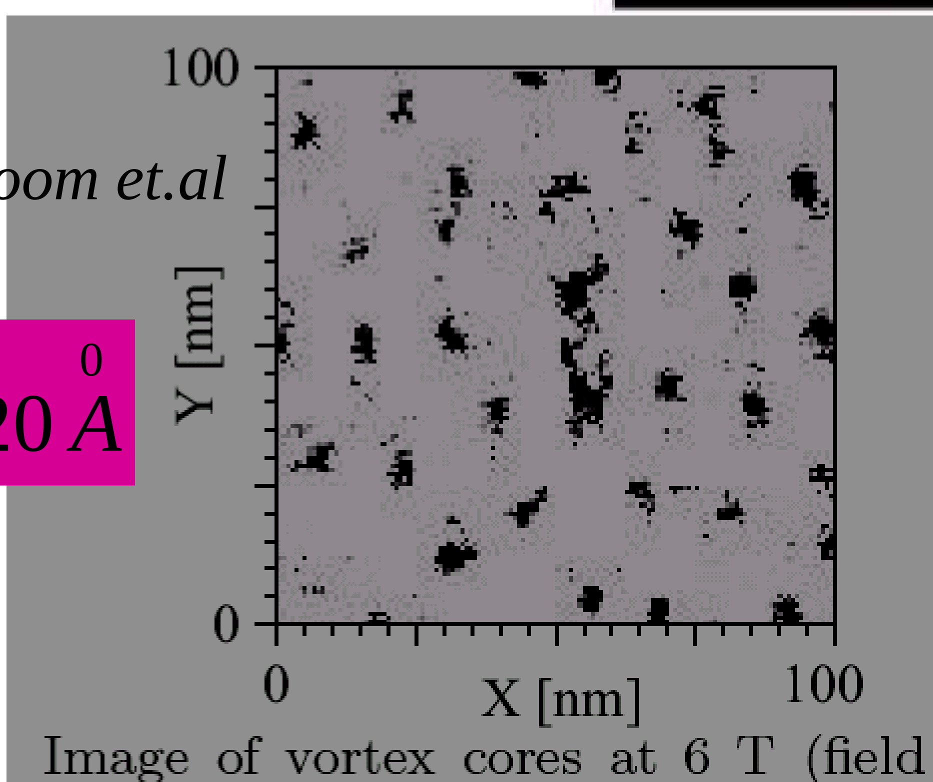
Howald et.al (Stanford)

Pan et.al (Berkeley)



Hoogenboom et.al

$$\xi \approx 20 \overset{0}{A}$$



Variational Theory

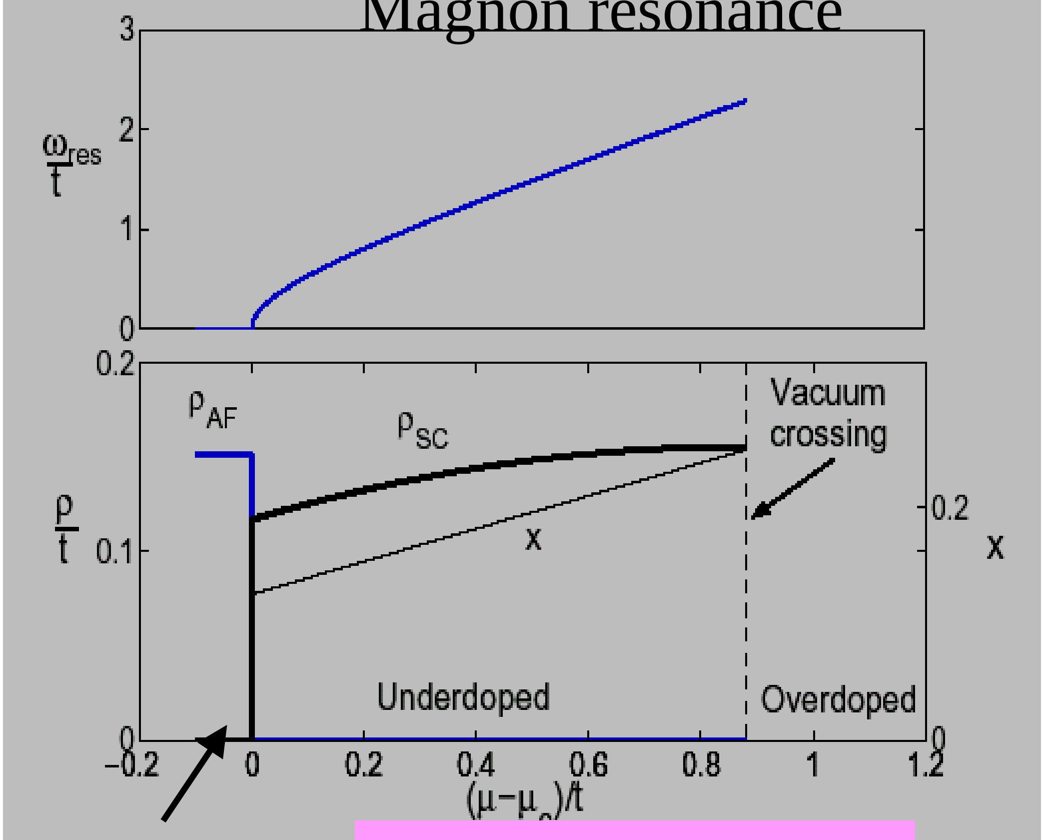
d-wave SC ground state $\Psi^{d-scF} \equiv \prod_{i}^{plaq} (\cos\theta + \sin\theta e^{i\varphi} b_i^\dagger) \,|\, \Omega \,\rangle$

Neel State $\Psi^{afm} = \prod_{i}^{plaq} (\cos\theta + \sin\theta m^\alpha \; t_{i\alpha}^\dagger) \,|\, \Omega \,\rangle$

$$\rho_{AF} = 2J_t \langle t \rangle^2$$
$$\rho_{SC} = 2J_b \langle b \rangle^2$$

Results (U/t=8)

Magnon resonance



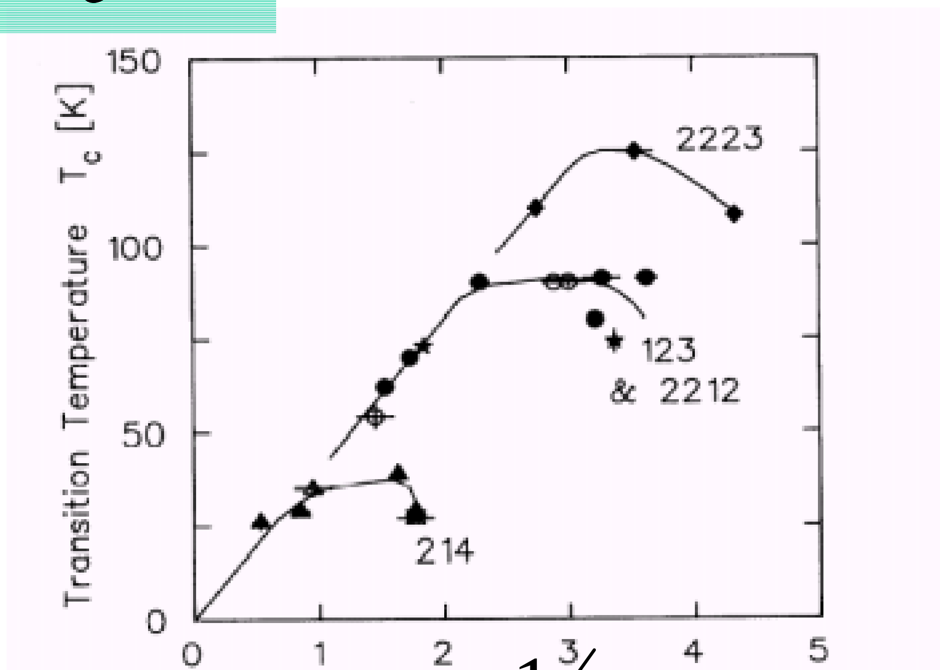
First order

Phase diagram

Experiments

$$\rho_c \approx T_c \propto \chi$$

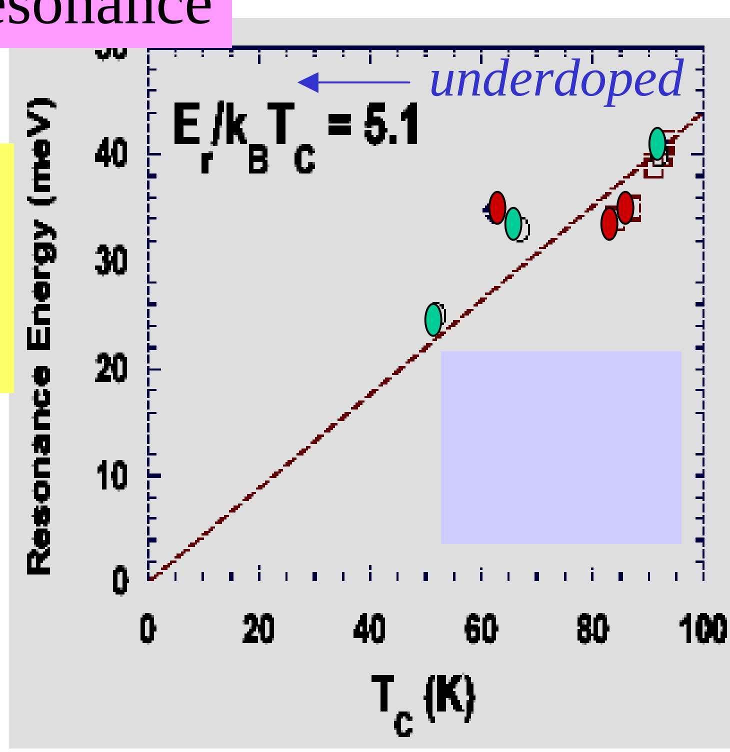
Uemura's Plot (89)



$$\rho_c \propto 1/\lambda^2 \longrightarrow$$

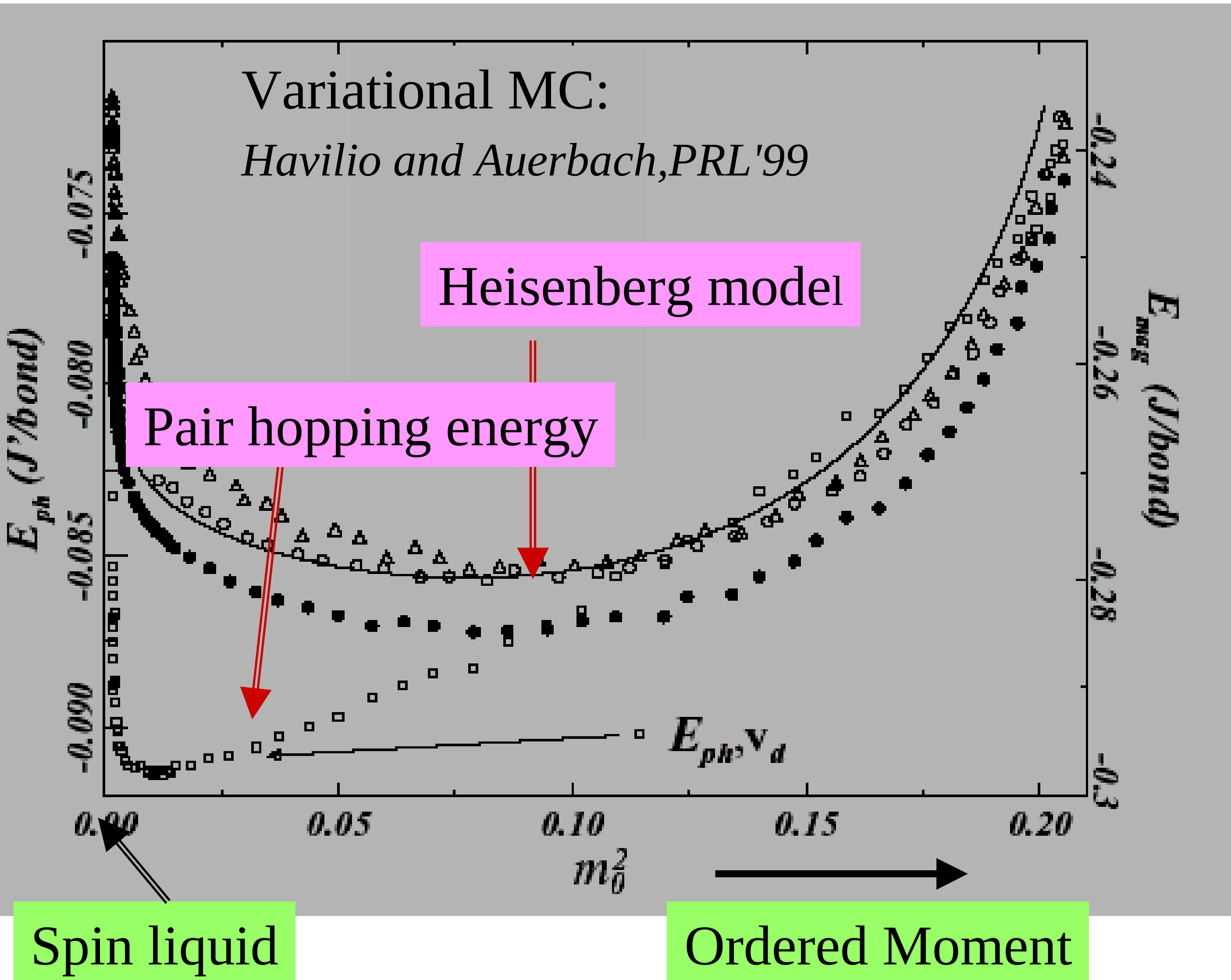
Antiferromagnetic resonance

resonance energy decreasing with doping



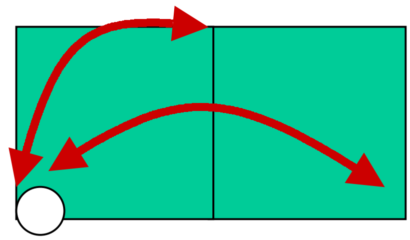
Role of Pair Hopping

1. $-J_b \sum b^{\dagger} b$ determines superfluid density, and T_c.
2. Competes with charge ordering.
3. Destroys antiferromagnetic order



Single hole dispersion

Semiclassics and numerics yield same sublattice hopping



$$\mathcal{H}^f = \sum_{\mathbf{k}s} (\epsilon^f_{\mathbf{k}} - \mu) f^\dagger_{\mathbf{k}s} f_{\mathbf{k}s},$$

$$\epsilon^f_{\mathbf{k}} = t'(\cos(k_x a) + \cos(k_x a))^2 + t''(\cos(k_x a) - \cos(k_y a))^2$$

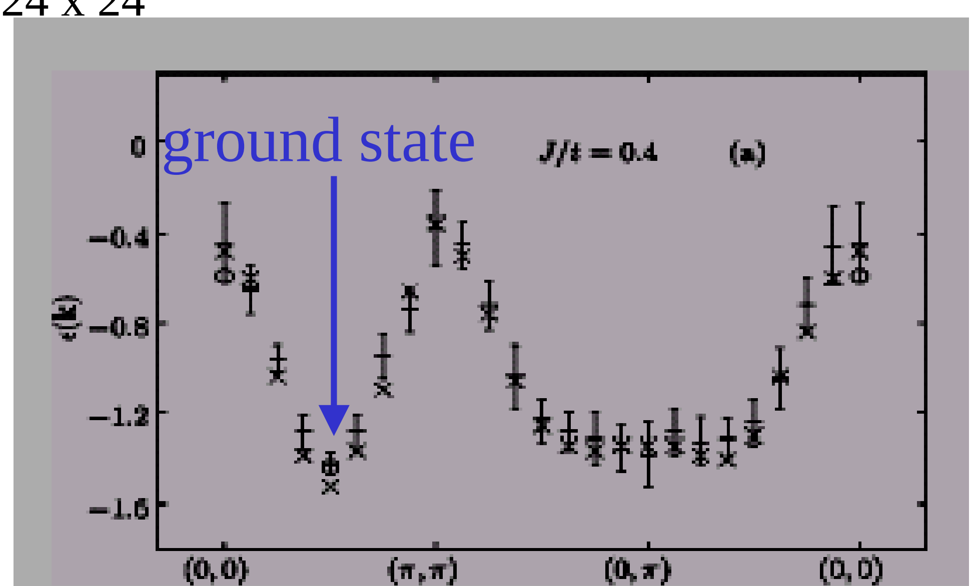
VOLUME 62, NUMBER 23

Single-hole dynamics in the t–J model on a square lattice

Michael Brunner, Fakher F. Assaad, and Alejandro Muramatsu

Quantum Monte-Carlo

24 x 24



t’’ ~ 0.1 t’
t’ ~ J

Hole Fermions

$$\mathcal{H}^f = \sum_{\mathbf{k}s} (\epsilon^f_{\mathbf{k}} - \mu) f^\dagger_{\mathbf{k}s} f_{\mathbf{k}s},$$

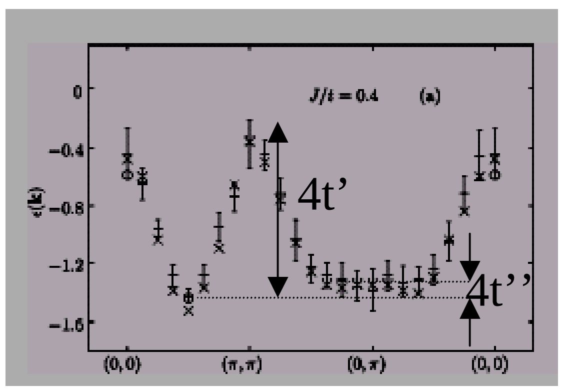
$$\epsilon^f_{\mathbf{k}} = t'(\cos(k_x a) + \cos(k_x a))^2 + t''(\cos(k_x a) - \cos(k_y a))^2$$

VOLUME 62, NUMBER 23

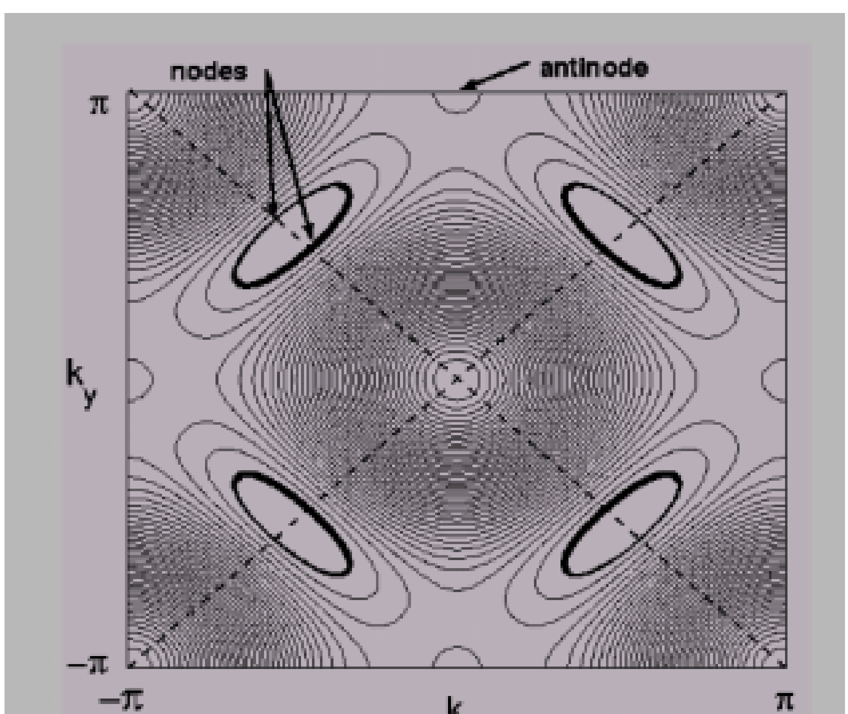
Single-hole dynamics in the *t-J* model on a square lattice

Michael Brunner, Fakher F. Assaad, and Alejandro Muramatsu

24 x 24



t'' ~ 0.1 t'
t' ~ J



Plaquette Boson Fermion Model

$$\mathcal{H}^{PBFM} = \mathcal{H}^{4b}[2\mu] + \mathcal{H}^{f}[\mu] + \mathcal{H}^{bf} + \mathcal{H}^{tf}$$

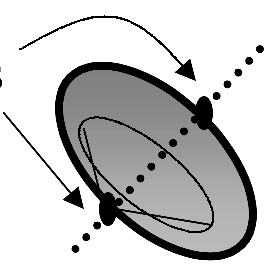
Andreev coupling

$$\mathcal{H}^{bf} = g_b \sum_{\mathbf{k},\mathbf{q}} \left(d_{\mathbf{k}+\mathbf{q}/2} b^{\dagger}_{\mathbf{q}} f_{\mathbf{k}\uparrow} f_{-\mathbf{k}+\mathbf{q}\downarrow} + \mathrm{H.c} \right)$$

1. Chemical equilibrium: $2n_b(2\mu, T) + n_f(\mu, T) = x$

2. Induced fermion gap (vanishes at Tc)

nodal points



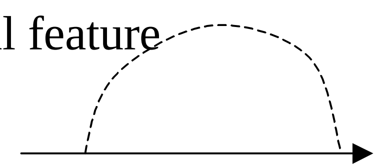
$$E_{\mathbf{k}} = \pm\sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + \Delta^2_{\mathbf{k}}}$$
$$\Delta^{sc}_{\mathbf{k}} = g_b d_{\mathbf{k}} \langle b \rangle.$$

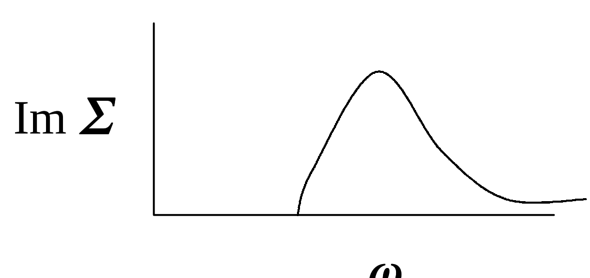
3. Dirac Fermions+QED *Franz&Tesanovic*

Fermion-magnon interaction

$$\mathcal{H}^{tf} = g_t \sum_{ms\mathbf{kq}} \left((t^{\dagger}_{m\mathbf{q}} + t_{-m-\mathbf{q}}) f^{\dagger}_{\mathbf{k}s} f_{\mathbf{k}+\mathbf{q}+\vec{\pi}s+m} + \mathrm{H.c} \right)$$

Dip-hump spectral feature

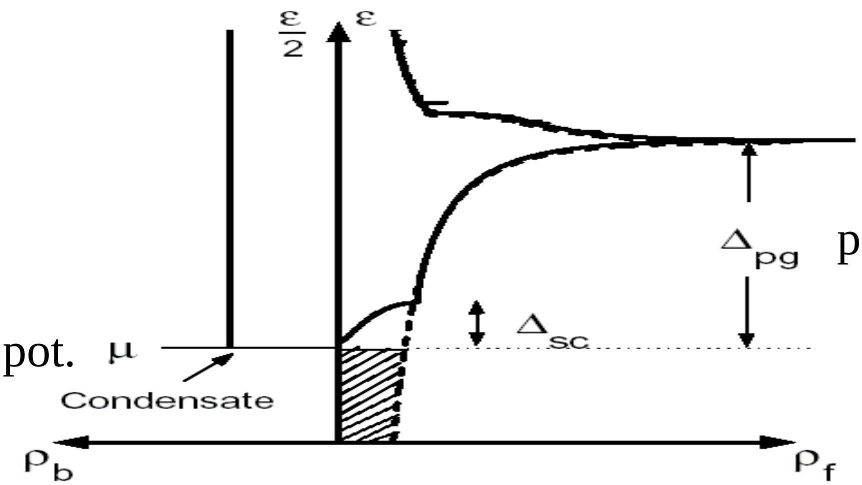




Eschrig&Norman, *Abanov&Chubukov*

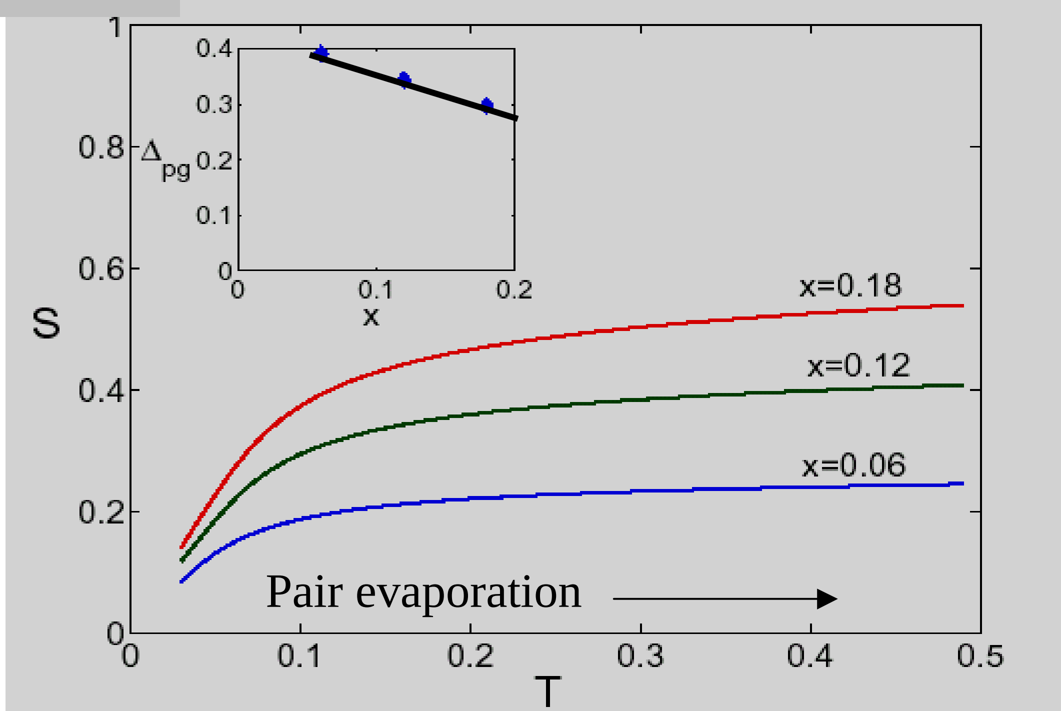
Boson fermion theormodynamics

Boson density of states Hole fermion density of states



$$\mu(x) - \mu(0) = \frac{x}{(2\kappa_b + \kappa_f)}$$

Hole entropy



Experimental Predictions

1. Hole spectral weight in Luttinger theorem violating momenta

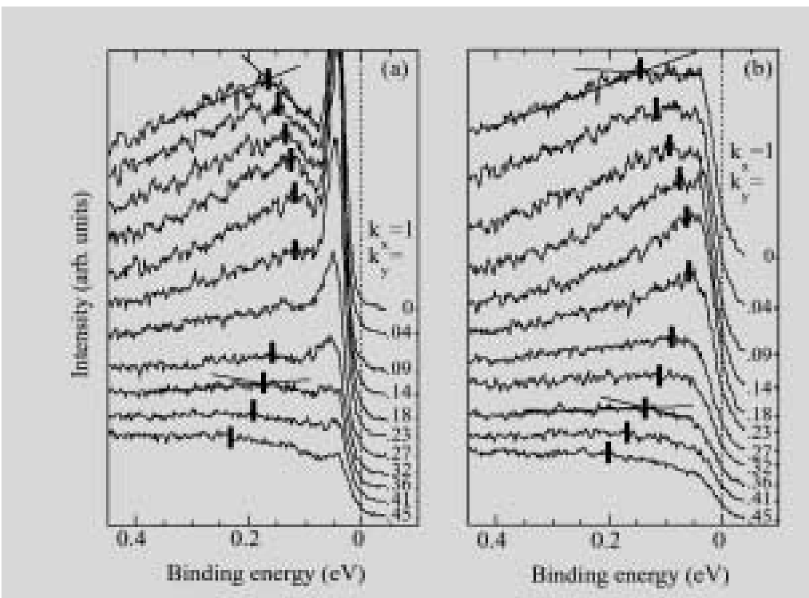
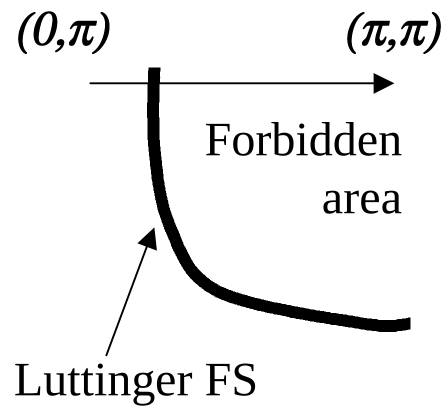
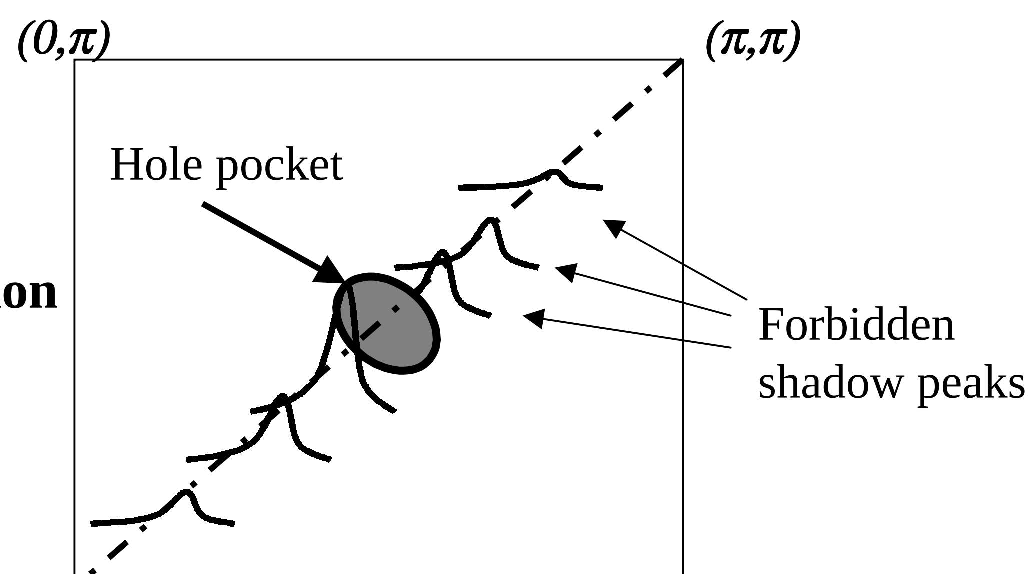


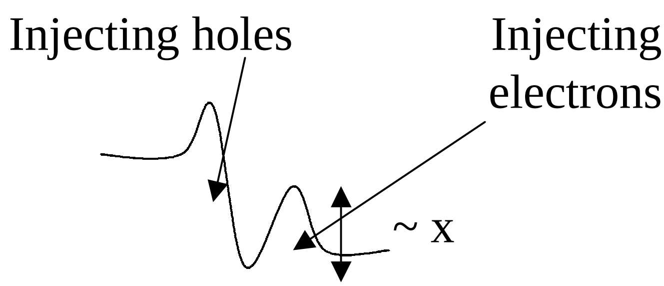
FIG. 2. Spectra along $(\pi,0) \to (\pi,\pi)$ in (a) the superconducting state (T = 60 K), and (b) the pseudogap state (T = 100 K) for an underdoped 75 K sample (curves are labeled in units of π/a). The thick vertical bar indicates the position of

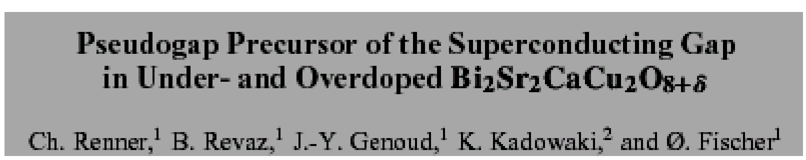




Predictions cont’d

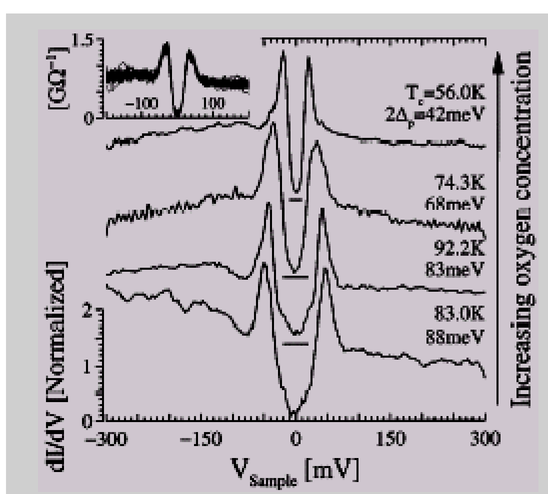
2. **Asymmetry** in tunneling conductance





Pseudogap Precursor of the Superconducting Gap in Under- and Overdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$

Ch. Renner,[1] B. Revaz,[1] J.-Y. Genoud,[1] K. Kadowaki,[2] and Ø. Fischer[1]



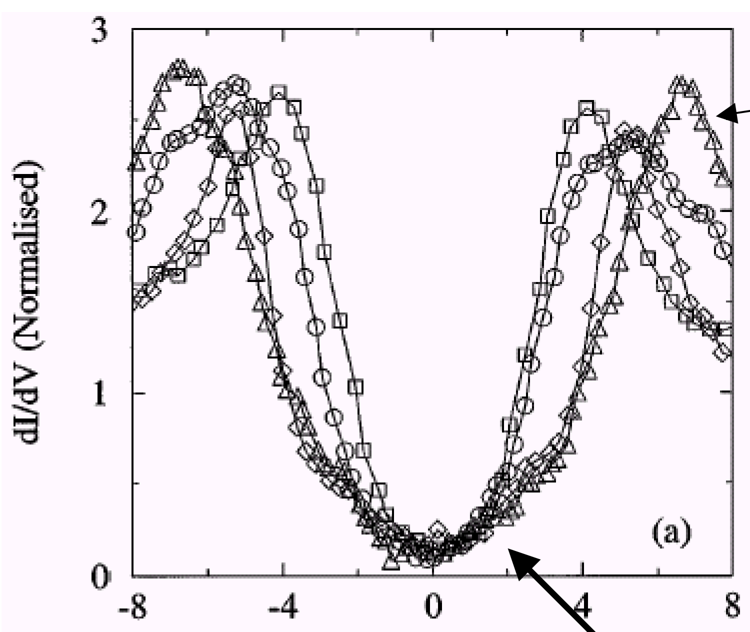
Predictions cont'd

Nodal transverse velocity

$$v_\perp \sim \sqrt{Tc} \sim \sqrt{\rho_{sc}}$$

Relationship between the Superconducting Energy Gap and the Critical Temperature in High-T_c Superconductors

Christos Panagopoulos[1] and Tao Xiang[1,2]



pseudogap

Pairing gap

Vacuum Crossing

underdoped **overderdoped**

$|\Omega\rangle$ Hole pair

Increase μ ⟹

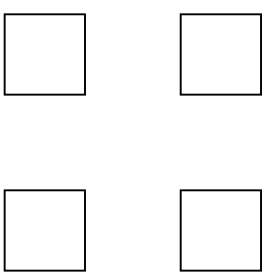
$|\Omega\rangle$ Hole pair

new boson

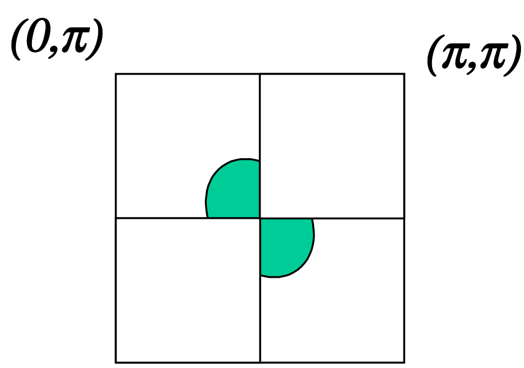
new vacuum

<u>Is there a phase transition?</u>

<u>Speculation:</u> *underdoped is plaquettized*

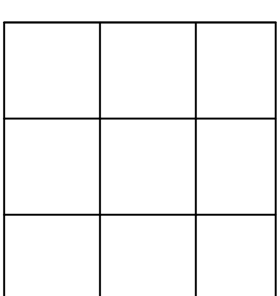


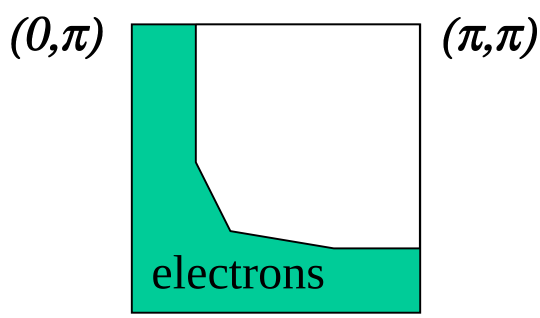
Plaquette bosons



Hole fermions

overdoped has full lattice symmetry





Summary

Hubbard model → PBFM using CORE

Underdoped phase:

d-wave superconductivity

pSO(5) phase diagram

Pseudogap in fermion spectrum

Future:

CORE with fermions (4x4clusters)

(*with S. Capponni, Toulouse***)**

PBFM: Transport and Thermodynamics, vortex cores, surface states, impurity states, etc.

Underdoped-overdoped transition

Extended Hubbard models