Hall Resistivity and Dephasing in the Quantum Hall Insulator

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The long-standing problem of the Hall resistivity ρ_{xy} in the Hall insulator phase is addressed using four-lead Chalker-Coddington networks. Electron interaction effects are introduced via a finite dephasing length. In the quantum coherent regime, we find that ρ_{xy} scales with the longitudinal resistivity ρ_{xx}, and they both *diverge* exponentially with the dephasing length. In the Ohmic limit (where the dephasing length is shorter than the Hall puddle size), ρ_{xy} remains quantized and independent of ρ_{xx}. This suggests a new experimental probe for dephasing processes. [S0031-9007(99)08448-3]

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The ideal quantum Hall (QH) effect can be defined as the *simultaneous* quantization of Hall conductance and vanishing longitudinal conductance:

$$G_{xy} = \nu e^2/h, \qquad G_{xx} = 0, \tag{1}$$

where ν, the filling factor, is some integer or odd denominator fraction. Consider a perfect QH sample with four Ohmic leads. By Kirchoff's laws, it is easy to see that, while the *measured* G_{xy}, G_{xx}, and R_{xx} depend on the lead resistances, the Hall resistance is independent of them and is precisely quantized at

$$R_{xy} = h/\nu e^2. \tag{2}$$

This robustness of R_{xy} generalizes to large circuits, as proven by Shimshoni and Auerbach [1] for the two dimensional Ohmic puddle network model. However, as we shall see below, the quantum interference between different tunnel junctions (absent for the Ohmic transport) spoils the quantization of R_{xy} [2].

Quantum interference also drives localization and related $T = 0$ QH-to-insulator transition [3,4]. This has been confirmed by explicit calculations of the localization length exponents, in quasiclassical approximation [5] and numerically [6,7] on Chalker-Coddington (CC) networks.

There is no consensus, however, about interference effects on the Hall resistivity; its value in the insulating phase is still controversial. Several theories expect it to be finite [8,9] $\rho_{xy}^{T=0} < \infty$, or even quantized [10,11] $\rho_{xy}^{T=0} = h/e^2$ ("semicircle law"). Experimental observations also vary: some data can be fit by the Drude form [12,13] $\rho_{xy} \propto B$, while others see a weaker magnetic field (B) dependence, with ρ_{xy} nearly quantized [14].

In contrast, Entin-Wohlman *et al.* [15] pointed out that $\rho_{xy}^{T=0}$ is not a self-averaging quantity in the insulator. Using a model of local hopping in an external magnetic field, they concluded that ρ_{xy} diverges in the limit $T \to 0$. Unfortunately, the applicability of this model to QH systems is somewhat questionable.

The purpose of this paper is to determine the low temperature Hall resistivity in the magnetic field driven insulating phase. Differing from a previous calculation [11], we directly compute the four-terminal resistances $R_{\alpha\beta}$, $\alpha, \beta = x, y$ for random CC networks using the Landauer-Büttiker quantum transport theory. Distributions of $R_{\alpha\beta}$ are obtained for different linear system sizes L and magnetic field parameters θ, defined below.

Our key result is that, at large L, the Hall resistance $R_{xy}(\theta, L)$ of the CC network changes from the asymptotically approaching quantized value $h/\nu e^2$ in the QH phase to an *exponential divergence* in the insulator [16]. Second, effects of inelastic scattering due to electron-electron and electron-phonon interactions are introduced via a phenomenological *dephasing length* l_φ [3,17] (for convenience, we measure all physical length in the units of typical puddle size l_V). For l_φ shorter than the puddle size (phase-incoherent limit), we map the chiral network to the Ohmic puddle network model [1], which has a quantized Hall resistance. In general, the macroscopic resistivity is approximately given by an Ohmic resistor network formed by elements of size $L = l_\varphi$ with the computed distributions of $R_{\alpha\beta}$. The Ohmic network problem is solved using percolation theory following Ambegaokar, Halperin, and Langer [18] (AHL). This approach, justified by the wide distribution of R_{xx}, yields the ensemble of elements which determine the Hall voltage. The macroscopic Hall resistivity ρ_{xy} is obtained as an average over this ensemble. We show that the divergence of both Hall and longitudinal macroscopic resistivities in the insulator is consistent with one length scale $\xi(\theta)$. We conclude with the suggestion that $\rho_{xy}(T)$ could serve as an experimental probe of the dephasing length.

Four-terminal CC network.—Consider a square CC network in Fig. 1, with corner puddles serving as external leads. Carriers, described at a given energy by complex amplitudes $\{A_p^\alpha\}$, where α, p are the edge and puddle labels, respectively, can propagate only in one direction around the puddles, as indicated by arrows. The corresponding currents are just $I_p^\alpha = |A_p^\alpha|^2$. Edge amplitudes across each tunnel junction pq are related by a unitary

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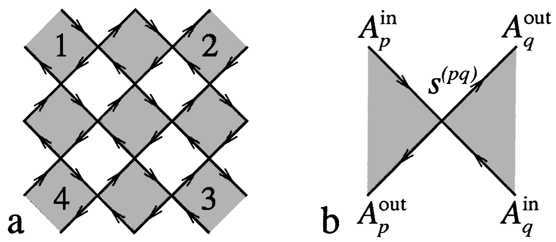

FIG. 1. (a) Four-terminal $L = 6$ CC network. QH puddles are shaded. Edge currents with amplitudes A_p^α propagate along the arrows. (b) The incoming and outgoing amplitudes at the tunnel junction between puddles p and q are related by the scattering matrix (3).

scattering matrix $s^{(pq)}$,

$$A_r^{\text{out}} = \sum_{r'=p,q} s_{rr'}^{(pq)} e^{i\phi_{r'}^{\text{in}}} A_{r'}^{\text{in}}, \qquad s^{(pq)} = \begin{pmatrix} \cos\theta_{pq} & \sin\theta_{pq} \\ -\sin\theta_{pq} & \cos\theta_{pq} \end{pmatrix}, \tag{3}$$

where $\theta_{pq} \in [\theta + \delta, \theta - \delta]$, and edge phases $\phi_r^\alpha \in [0, 2\pi)$ are independent random variables; we have chosen [7] $\delta = 0$ to reduce the statistical errors. The magnetic field parameter θ can be tuned across the QH-insulator transition at $\theta_c = \pi/4$. In the QH regime $\theta < \theta_c$ the neighboring leads are connected by highly transmitting edge states, while the insulating phase $\theta < \theta_c$ consists of weakly connected puddles without global edge states. Near the transition, $\theta - \theta_c \propto B - B_c$. Away from the transition, tunneling across saddle points yields asymptotically $\tan(\theta) \propto e^{-A(B-B_c)}$ [19].

The scattering relationships provide a set of linear equations for all A_p^α which depends on the incoming amplitudes A_i^{in} in the external leads $i = 1$–4. The global scattering matrix $S_{ij}[\theta_r, \phi_r^\alpha]$ relates the amplitudes at the external leads, $A_i^{\text{out}} = \sum_{j=1}^4 S_{ij} A_j^{\text{in}}$, so that the current transmission matrix $T_{ij} = |S_{ij}|^2$ defines the probability of scattering from incoming channel i to outgoing channel j. We assume that the external leads, also formed by an incompressible QH liquid, are in equilibrium, which implies that the chemical potentials of their edges are [20]

$$\mu_i^\alpha = (h/\nu e) I_i^\alpha. \tag{4}$$

The resistance tensor is now given by a compactified version of the Büttiker-Landauer formula [21]:

$$\tilde{R}_{\alpha\beta} = (h/\nu e^2) \Lambda_\alpha^t (1 + T) P [P(1 - T)P]^{-1} P \Lambda_\beta, \qquad \Lambda_x^t = (1, 0, -1, 0), \qquad \Lambda_y^t = (0, 1, 0, -1), \tag{5}$$

where the vectors Λ_α^t are transposed Λ_α, and the operator P projects out the zero eigenvector $(1, 1, 1, 1)$ of the matrix $1 - T$ (matrix S is unitary). It is easy to show that the field-antisymmetrized Hall resistance is $R_{xy} = (\tilde{R}_{xy} - \tilde{R}_{yx})/2$.

Numerically, we calculated the $L \times L$ transfer matrix relating the amplitudes on L open wires at the left and right sides of the sample (see Fig. 1). We used the transfer-matrix technique with intermediate orthogonalizations [22] to reduce numerical errors for large systems. Then the closed edges were "linked" by eliminating the corresponding pairs of incoming and outgoing amplitudes. This resulted in a reduced 4×4 transfer matrix, with which the full scattering matrix S_{ij} was computed. The numerical accuracy was controlled by checking the unitarity of S_{ij}, and by test runs at quadruple accuracy.

To obtain the distributions of R_{xx} and R_{xy} for different values of θ and the system size L, we repeated each calculation up to 10^6 times, taking $(\tilde{R}_{xx}, \tilde{R}_{yy})$ as two different members which have the same R_{xy}. In the insulating phase $\theta > \theta_c$, R_{xx} has a very wide, resembling log-normal, distribution over several decades, as generally expected in a localized insulator phase [15]. The Hall resistance R_{xy} is seen to have a much narrower distribution, but with mean and variance also increasing exponentially with the size of the system in the insulating phase.

Phase-incoherent network model.—In the opposite limit, we consider a network where inelastic mechanisms completely destroy quantum interference between different tunnel junctions. In this case, different tunneling events happen independently, and the incoming and outgoing currents are related at each junction,

$$I_r^{\text{out}} = \sum_{r'=p,q} |s_{rr'}^{pq}|^2 I_{r'}^{\text{in}}, \tag{6}$$

where the tunneling matrix elements $s^{(pq)}$ are given by Eq. (3). According to Eq. (4), local chemical potentials at each edge are proportional to the corresponding currents. Thus, the dissipative and Hall voltages at each junction are related to the tunneling current I_{pq} as

$$V_{pq}^{\text{dis}} = R_{pq} I_{pq}, \qquad V_{pq}^H = \text{sign}(B)\,(h/\nu e^2) I_{pq}, \tag{7}$$

where $R_{pq} = (h/\nu e^2) \cot^2\theta_{pq}$ is symmetric under the reversal of the magnetic field. These local relations reduce the network model to the Ohmic puddle network model of Ref. [1], for which $R_{xy}(l_\varphi \leq l_V) = (h/\nu e^2)$ for any realization of $\{\theta, \phi\}$. This model equally applies to integer and fractional QH regimes. Equations (7) also imply that the phase-incoherent network model has an exact current-voltage duality [2,10,11], which interchanges the QH and insulating regions and simultaneously inverts the resistance R_{pq} associated with each junction. This duality also inverts the macroscopic dissipative resistance of the system, determined, according to the AHL percolation argument [18], by the median of the distribution of R_{pq}.

To describe the regime of intermediate dephasing length, $l_V < l_\varphi < \infty$, where l_V is a typical puddle size, we use finite-size four-lead CC networks. In the simulation, the phase breaking occurs only at the leads, and we *define* $L = l_\varphi$. Square blocks of linear sizes l_φ or larger connect as Ohmic resistors, with the resistances $R_{\alpha\beta}(L = l_\varphi)$ chosen from the numerically determined distribution. Ignoring relatively small Hall voltages at this stage, we notice that R_{xx} is exponentially widely distributed. So, we can again follow AHL and insert all resistors in increasing order of R_{xx} until the percolation threshold is reached. Here, we assume a percolation threshold of 50%, applicable for self-dual lattices. Thus the last resistors to connect the percolating cluster (PC) have the median value R_{xx}^{med}. Since all higher resistors $R_{xx} > R_{xx}^{\mathrm{med}}$ transport negligibly little current, they can be discarded. By AHL, and subsequent numerical confirmations [18], the macroscopic resistivity of the network is simply $\rho_{xx} = R_{xx}^{\mathrm{med}}(l_\varphi)$.

The Hall resistivity ρ_{xy} can now be determined as the weighted average over the PC. Since, numerically, $|R_{xy}| \ll R_{xx}$, and we draw no current from the Hall leads, it is safe to assume that the Hall voltage fluctuations across the PC are averaged out by secondary local currents, and the macroscopic Hall resistivity can be estimated by the average $\rho_{xy}^{<\mathrm{med}} = \langle R_{xy}\rangle_{R_{xx}\le R^{\mathrm{med}}}$. As an upper bound, we also calculate the average Hall resistance of the necks of the PC given by $\rho_{xy}^{\mathrm{med}} = \langle R_{xy}\rangle_{R_{xx}\approx R^{\mathrm{med}}}$, which gives similar results as shown below.

Results.—In the localized regime, $\theta > \theta_c$, both ρ_{xx} [7] and ρ_{xy} diverge exponentially at large l_φ. Both resistivities fit very well [see Fig. 2 for the fit of $\rho_{xy}(L)$] to an exponential scaling function with finite-size correction:

$$\rho(L) \sim A_\theta L^\gamma \exp[L/\xi(\theta)], \tag{8}$$

where γ is a θ-independent exponent determined by the geometry of the system, and $\xi(\theta)$ is the localization length (both quantities are defined separately for ρ_{xx} and ρ_{xy}). As illustrated in the inset of Fig. 2, the divergence of both correlation lengths as $\theta \to \theta_c$ is consistent with $\xi(\theta) \sim (\theta_c - \theta)^{-7/3}$. This form for ξ_{xx} is in agreement with previous studies [6,7,23]. The fact that the correlation length for ρ_{xy} also diverges with the same exponent implies that the transition is actually governed by a single length scale $\xi(\theta)$, and asymptotically $\rho_{xy} \sim (\rho_{xx})^k \to \infty$ with $k \approx 0.32$–0.35 (Fig. 3). This divergence, one of the primary results of this paper, contradicts the expectations [8–11] of finite Hall resistivity in the insulator at $l_\varphi \to \infty$. Our results agree with the conclusion of Ref. [15], although here we have used a different model, applicable for transport in quantizing magnetic fields.

Figure 2 also shows ρ_{xy} in the metallic QH phase for $\theta > \theta_c$. In this phase, as expected, $\rho_{xy}(L)$ approaches the quantized value at large system sizes. We did not attempt to determine the corresponding correlation length since our geometry has narrow leads (see Fig. 1). We did, however, check that our results are not limited to systems with narrow leads by making a limited set of runs for a special self-dual network geometry, which remains identical under the interchange of QH and insulating regions and the replacement $\theta \to 2\theta_c - \theta$.

We also attempted to suppress the quantum interference by calculating the ensemble averaged $\langle T_{ij}\rangle$, which is formally equivalent to considering the same noninteracting system at very high temperatures. This averaging resulted in both Hall and longitudinal resistance of an insulating phase much smaller than the average quantum values. However, the specific values of these resistances differed significantly from the values obtained numerically for the phase-incoherent networks of identical geometry; particularly, the Hall resistance was not quantized, with the offset increasing into the insulating phase. This demonstrates that, at $T > 0$, without inelastic scattering quantum interference cannot be completely suppressed.

In the fractional Hall effect regime, electron-electron interaction effects are primarily threefold: (i) stabilization of fractional $\nu < 1$ QH phases in the puddles, (ii) renormalization of interpuddle electron tunneling rates, and (iii) dephasing of charge carriers by inelastic scattering. At low temperatures, the second effect is expected to be finite, since infrared divergence of the tunneling amplitudes is cut off by the finite size of the puddles. In this

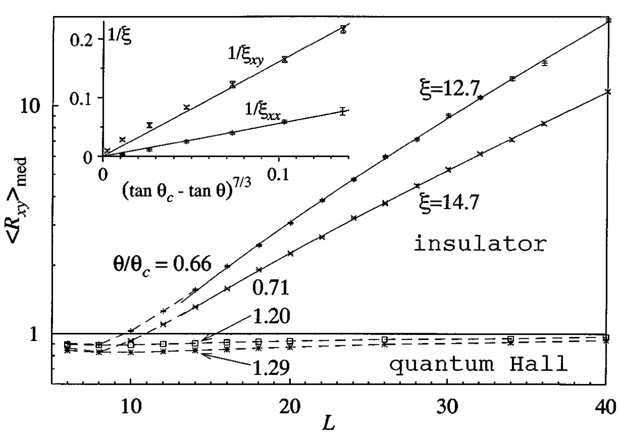

FIG. 2. QH to insulator transition in finite-size scaling. Solid lines are the best fits of Eq. (8), and dashed lines are guides to the eye. Inset: The resulting inverse correlation lengths are consistent with the critical exponent $\nu = 7/3$.

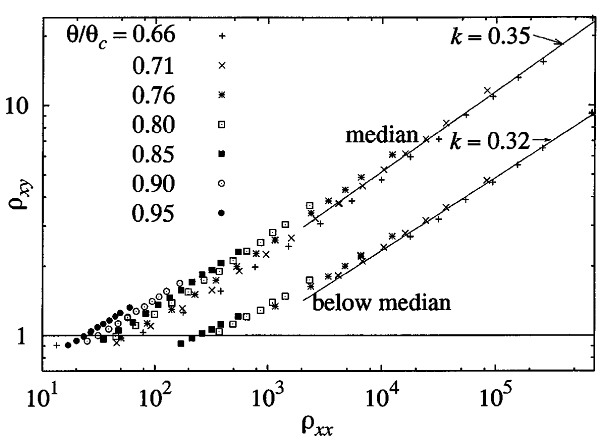

FIG. 3. Asymptotic correlation between ρ_{xx} and ρ_{xy} determined by two averaging procedures (see discussion in text). k is the slope on the log-log plot.

regime, the effective edge state transport theory for the fractional and integer cases is identical [2], up to factors of ν in the chemical potential relations (4). Thus, it is legitimate to use the CC model as a model of coherent quantum transport on edges of fractional QH puddles. However, the dephasing mechanism and determination of l_φ remains an interesting open problem, for both integer and fractional cases.

Quantum critical point and dephasing.—In a vicinity of the true quantum critical point, the effective dephasing length [17] should diverge at small temperatures as

$$l_\varphi(T) \sim T^{-1/z}. \tag{9}$$

Experimentally, resistivity and nonlinear resistivity data at transitions between plateaux have been collapsed onto universal curves using $z \approx 1.0$ (and independently determined $\nu_\xi \approx 2.4$) [13,24,25]. Based on our results, at a quantum critical point, and in the absence of additional phase-breaking mechanisms, one would also expect the Hall resistivity to diverge as

$$\rho_{xy} \sim (h/\nu e^2) \exp[kT^{-1/z}(B - B_c)^{7/3}]. \tag{10}$$

Experiments, however, have reported a constant or weakly B-dependent Hall resistivity on the insulating side of the transition. For similar samples, resistivity saturation at low temperatures has been reported [26]. Evidently, *both* effects are inconsistent with a true zero-temperature quantum Hall to insulator transition, characterized by a diverging dephasing length (9).

We conclude that a nearly quantized Hall resistivity indicates a strongly dephased regime where, i.e., $l_\varphi \leq l_V$. In contrast, the longitudinal resistivity in this regime is expected to diverge at large fields as [19]

$$\rho_{xx}(B) \sim (h/e^2) \exp[\nu(l_V/l)^2(B/B_c - 1)], \tag{11}$$

where l is the Landau length. This expression allows an independent estimation of l_V, and an upper bound on the dephasing length in the limit of small temperatures. Experimentally, the interplay between l_V, the scale of the long-range potential fluctuations, and the phase-breaking length l_φ was noticed in Ref. [27].

It is not yet clear what mechanism can explain zero-temperature finite resistivity in disordered QH systems, or why dephasing seems to be more pronounced in some particular experiments. One possible source might be a coupling of the edge excitations to nearby domains of the compressible $\nu = 1/2$ phase.

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