

Steady States of a Microwave-Irradiated Quantum-Hall Gas

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We consider effects of a long-wavelength disorder potential on the zero conductance state (ZCS) of the microwave-irradiated 2D electron gas. Assuming a uniform Hall conductivity, we construct a Lyapunov functional and derive stability conditions on the domain structure of the photogenerated fields. We solve the resulting equations for a general one-dimensional and certain two-dimensional disorder potentials, and find nonzero conductances, photovoltages, and circulating dissipative currents. In contrast, weak white-noise disorder does not destroy the ZCS, but induces mesoscopic current fluctuations.

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The observation of giant magnetoresistance oscillations in a microwave-irradiated two-dimensional electron gas (2DEG) [1], has spurred intensive theoretical activity. Two distinct microscopic mechanisms for conductivity corrections have been proposed: (i) The displacement photocurrent (DP) [2], which is caused by photoexcitation of electrons into displaced guiding centers and (ii) the distribution function (DF) mechanism, which involves redistribution of intra-Landau level population for large inelastic lifetimes [3].

Andreev, Aleiner, and Millis [4] have noted that irrespective of microscopic details, once the radiation is strong enough to render the *local* conductivity negative, the system as a whole will break into domains of photogenerated fields and spontaneous Hall currents. In their proposed domain phase, motion of domain walls can accommodate the external voltage, resulting in a zero conductance state (ZCS) in the Corbino geometry, or a zero resistance state for the Hall bar geometry, in apparent agreement with experimental reports [1]. However, one may ask, what should be the effects of long-wavelength (relative to the cyclotron radius) disorder, which is either naturally present or deliberately introduced? What is the nature of the coupling between a disorder potential and the photogenerated fields [5], and could the disorder pin domain walls? Such pinning would affect the macroscopic transport and could destroy the ZCS.

In this Letter we incorporate a long-wavelength disorder potential $\phi_d(\mathbf{x})$ into the nonlinear magnetotransport equations. We explore its effects on the domain structure and macroscopic transport coefficients. For the case of a constant Hall conductivity, we construct a *Lyapunov functional* [6] which greatly simplifies the determination of the stable steady states and their conductance. We use it to derive general stability conditions on domain walls in the strong radiation regime. We also show that weak “white-noise” disorder is an *irrelevant* perturbation, which does not destroy the ZCS. It does introduce, however, mesoscopic nonlinear current fluctuations. We find solu-

tions for the following disorder potentials: (i) The *general one-dimensional* potential, where domain walls are pinned to the potential extrema, which results in a nonzero conductance and photovoltage. (ii) The simple “egg-carton” potential solved variationally, and (iii) a generic nonseparable potential depicted in Fig. 1, which is solved numerically. (ii) and (iii) exhibit two-dimensional domain-wall pinning and *frustration* effects, which result in circulating dissipative currents.

Nonlinear magnetotransport.—In the presence of an external microwave field, we use a local relation between the dc current $\mathbf{j}(\mathbf{r})$ and the local electrostatic field $\mathbf{E}(\mathbf{r})$:

$$\mathbf{j} = \mathbf{j}^d(\mathbf{E}, \mathbf{r}) + \sigma^H \hat{\mathbf{z}} \times \mathbf{E}. \quad (1)$$

Here we assume at the outset that the Hall conductivity σ^H is a constant, independent of \mathbf{r} and \mathbf{E} , which leads to considerable simplifications. The dissipative conductivity,

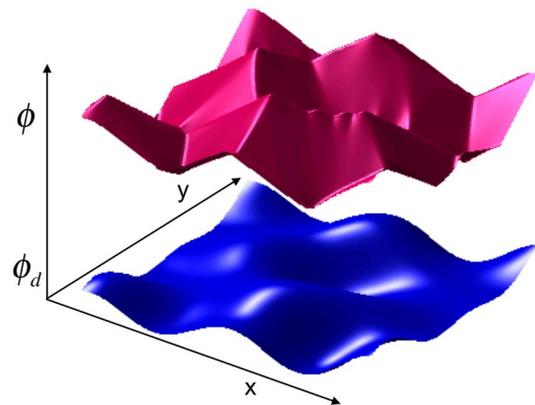


FIG. 1 (color online). Domain phase in the presence of disorder: the photogenerated potential $\phi(x, y)$ corresponding to a generic long-wavelength disorder potential ϕ_d . The steady state is determined by numerically minimizing the Lyapunov functional (see text). The domain walls (potential edges) are pinned by ϕ_d , yielding a finite conductance.

$\sigma_{\alpha\beta}^d(\mathbf{E}) \equiv \partial j_{\alpha}^d / \partial E_{\beta}$, satisfies

$$\sigma_{\alpha\beta}^d(\mathbf{E}, \mathbf{r}) = \sigma_{\beta\alpha}^d(\mathbf{E}, \mathbf{r}). \quad (2)$$

The vector function \mathbf{j}^d , in general, will depend explicitly on the position \mathbf{r} , due, e.g., to inhomogeneities in the 2DEG, and its direction may not be perfectly aligned with \mathbf{E} . Equation (1) is supplemented by the continuity equation, $\nabla \cdot \mathbf{j} = -\dot{\rho}$, where ρ is the charge density. We emphasize that Eq. (1) contains all microscopic interactions at length scales shorter than the cyclotron radius l_c , which serves as an ultraviolet cutoff, of order $1 \mu\text{m}$.

Writing $\mathbf{E} \equiv -\nabla\phi$, we may relate changes in the electrostatic potential ϕ to changes in ρ through the inverse capacitance matrix W :

$$\delta\phi(\mathbf{r}) = \int d^2r' W(\mathbf{r}, \mathbf{r}') \delta\rho(\mathbf{r}'). \quad (3)$$

If a time-independent steady state is reached, then we have simply $\nabla \cdot \mathbf{j} = 0$, and the precise form of W is unimportant.

In a Corbino geometry, one specifies the potential on the inner and outer boundaries of the sample, and one looks for a solution for $\phi(\mathbf{r})$ consistent with these boundary conditions. Since we assume σ^H to be a constant, the Hall current cannot contribute to $\nabla \cdot \mathbf{j}$ in the interior of the sample, so it does not appear in Kirchoff's equations. Consequently, the solution for $\phi(\mathbf{r})$ is independent of σ^H and we may, for simplicity set $\sigma^H = 0$. To recover the Hall current, one inserts the solution \mathbf{E} into the second term in (1).

Condition (2) on \mathbf{j}^d allows us to define a scalar *Lyapunov functional* as

$$G[\phi] = \int d^2r g(\mathbf{E}), \quad g = \int_0^{\mathbf{E}(\mathbf{r})} d\mathbf{E}' \cdot \mathbf{j}^d(\mathbf{E}'). \quad (4)$$

A variation of (4) is given by

$$\delta G = \int d^2r \nabla \cdot \mathbf{j}^d \delta\phi - \int_{\text{bound}} ds \hat{\mathbf{n}} \cdot \mathbf{j}^d \delta\phi. \quad (5)$$

The second integral vanishes on equipotential boundaries, or in the absence of external currents. The extrema of G are found to be steady states, with $\nabla \cdot \mathbf{j} = 0$. Using the positivity of the inverse capacitance matrix W , one may show that $G[\phi(t)]$ is indeed a Lyapunov functional, i.e., a nonincreasing function of time, so that its minima are stable steady states. In general, G may have multiple minima. Any initial choice of $\phi(\mathbf{r})$ will relax to some local minimum of G , but not necessarily the ‘‘ground state’’ with lowest G . Nevertheless, we expect that in the presence of noise, the system might tend to escape from high-lying minima and wind up in a state with G close to the absolute minimum.

Using the boundary term in (5), the current across a Corbino sample is equal to the first derivative of G with respect to the potential difference V between two edges, and the differential conductance is given by the stiffness, or

the second derivative:

$$\frac{dI}{dV} = \frac{d^2G}{dV^2}. \quad (6)$$

The domain phase.—We now consider a homogeneous system, in the regime of strong microwave radiation at frequencies slightly larger than the cyclotron harmonics $\omega > m\omega_c$, $m = 1, 2, \dots$, i.e., positive detuning. Both DP and DF mechanisms produce a regime of negative conductivity $\mathbf{j}(\mathbf{E}) \cdot \mathbf{E} < 0$, which implies a minimum of $g(E)$ at a finite field $|\mathbf{E}| = E_c > 0$, which was estimated [7] to be of order $\hbar\omega_c/(el_c)$. In order to satisfy equipotential boundary conditions, and the constraints $\oint dl \cdot \mathbf{E} = 0$, field discontinuities and charge density singularities must form.

A second order expansion of g about E_c reads as

$$g_0(\mathbf{E}) = g_0(E_c) + \frac{1}{2}(E - E_c)^2 \sigma_c + \lambda |\nabla \cdot \mathbf{E}|^2. \quad (7)$$

The clean system of Eq. (7), is governed by a ‘‘Mexican hat’’ Lyapunov density, with a flat valley along $|\mathbf{E}| = E_c$, i.e., the steady state local conductivity is ‘‘marginally’’ stable everywhere except inside the domain walls. The field-derivative coefficient $\lambda \approx \sigma_c l_{dw}^2$ implements the ultraviolet cutoff, introducing a domain-wall thickness scale l_{dw} assumed here to be of the order of l_c . Domain walls yield a positive contribution to G of order $\sigma_c E_c^2 l_{dw}$ per unit length. In the absence of disorder, the system will simply minimize total domain walls contribution, subject to aforementioned constraints, resulting in stable domain-wall patterns.

A change in the average field $\langle \mathbf{E} \rangle$, required if there is a change in the applied voltage V , can be accommodated by a motion of domain walls, or a reorientation of the local \mathbf{E} . The relative corrections to zero conductance vanish as l_{dw} over the sample length. This defines the clean ZCS phase described in Ref. [4].

Long-wavelength disorder.—In an inhomogeneous system, there will be a nonzero electrostatic field, $\mathbf{E}_d(\mathbf{r}) \equiv -\nabla\phi_d(\mathbf{r})$, present in the thermal equilibrium state, with no microwave radiation or bias voltage. We now see that disorder fields can modify domain-wall stability, and hence the zero conductance state itself.

At weak disorder field, $|\mathbf{E}_d| \ll E_c$, the Lyapunov density near $E \approx E_c$ is modified to

$$g(\mathbf{E}, \mathbf{E}_d) = g_0(\mathbf{E}) - \sigma_1(E) \mathbf{E} \cdot \mathbf{E}_d(\mathbf{r}) + \mathcal{O}(E_d^2), \quad (8)$$

which yields a current density

$$\mathbf{j}^d(\mathbf{r}) = -\sigma_1 \mathbf{E}_d + \frac{g'_0 - \sigma'_1 \mathbf{E} \cdot \mathbf{E}_d}{E} \mathbf{E}, \quad (9)$$

where $X' \equiv \partial X / \partial E$, and the coefficient σ_1 depends on microscopic mechanisms.

We wish to elaborate on a physical issue regarding Eq. (9): In the nonirradiated (dark) linear response theory, the current is driven by the *electrochemical* gradient

$\boldsymbol{\epsilon} = \mathbf{E} - \mathbf{E}_d$. Similarly, one may expect the photocurrent of the DF mechanism to also depend on $\boldsymbol{\epsilon}$. In contrast, the “upstream” photocurrent, pumped by the DP mechanism, involves transitions between single particle states which feel the local *electric* field $\mathbf{E}(\mathbf{r})$. Thus, due to both contributions, even if the DP mechanism is relatively weak, \mathbf{E}_d *cannot* be eliminated from Eq. (9) by a change of variables $\mathbf{E} \rightarrow \boldsymbol{\epsilon}$. By our microscopic estimate [8], for the pure DP mechanism, $\sigma_c, \sigma_1(E_c)$ are close to the dark conductivity, and $\sigma'_1 \ll \sigma_c/E_c$.

Local stability requires that $\sigma^d(\mathbf{r})$, of Eq. (2), has non-negative eigenvalues. The lower (transverse) eigenvalue is given by

$$\sigma_- = \frac{g'_0 - \sigma'_1 \mathbf{E} \cdot \mathbf{E}_d}{E} + \mathcal{O}(E_d)^2 \geq 0, \quad (10)$$

so marginal stability occurs at $E = E_c + \sigma'_1 \mathbf{E}_d \cdot \mathbf{E} / \sigma_c$.

In a steady state, the normal current density (in direction $\hat{\mathbf{n}}$) is continuous across a domain wall. If \mathbf{E}_1 and \mathbf{E}_2 are the fields on its two sides, we find by (8) and (10) that

$$\sigma_-(E_1) \mathbf{E}_1 \cdot \hat{\mathbf{n}} = \sigma_-(E_2) \mathbf{E}_2 \cdot \hat{\mathbf{n}} + \mathcal{O}(E_d^2). \quad (11)$$

When $\mathbf{E}_i \cdot \hat{\mathbf{n}}, i = 1, 2$ have opposite signs (as they do in the clean limit) (11) can only be satisfied for $\sigma_-(E_1) = \sigma_-(E_2) = \mathcal{O}(E_d)^2$. *This restricts the fields at the domain wall to be at their respective marginally stable values.* As a result, the current density (9) at the domain wall reduces to

$$\mathbf{j} = -\sigma_1(E_c) \mathbf{E}_d + \mathcal{O}(E_d)^2. \quad (12)$$

By Eq. (12), current conservation and Gauss’s theorem, and we obtain a global condition on any closed domain walls,

$$0 = \oint \mathbf{n} \cdot \mathbf{j} = -2\pi \sigma Q^{2D} + \mathcal{O}(E_d^2), \quad (13)$$

where Q^{2D} is the integral of the “2D disorder-charge density,” $\nabla \cdot \mathbf{E}_d / 2\pi$, over the area enclosed by the loop.

Finally, we note that *generically*, the differential conductance of a sample in the Corbino geometry can be obtained by solving for the conductance of a linear system with local conductivity given by $\sigma^d[\mathbf{E}(\mathbf{r})]$, in series with resistive elements along the domain walls, which arise from movement of the domain walls in response to a variation in the applied bias V . (There could also be discontinuities in the current at discrete values of V , if the system jumps discontinuously from one local minimum of G to another.) We shall see that for weak long-wavelength disorder, the scale of the macroscopic conductance is set by the domain-wall contribution.

White-noise disorder.—In the ZCS, we now show that a weak “white-noise” disorder potential, with a correlation length ξ_d (of the order $l_{dw} \sim l_c$), and root-mean-square value $\bar{\phi}_d \ll E_c \xi_d$ is an *irrelevant* perturbation which does not introduce new domain walls or destroy the ZCS. This is shown by using an Imry-Ma comparison [9] of surface to bulk contributions to the Lyapunov functional. By (8), for a

square domain of area L_d^2 , the negative contribution of aligning \mathbf{E} with the averaged disorder field \mathbf{E}_d , scales as $-\sigma_1 \bar{\phi}_d E_c \sqrt{L_d \xi_d}$. However, the linear cost of its domain walls grows as $+\sigma_c E_c L_d l_{dw}$. Therefore weak disorder cannot necessarily break the system up into smaller domains.

The disorder, however, will produce current fluctuations across preexisting domain walls, needed to satisfy boundary conditions on the sample. By Eq. (12), the current density integrates across the domain wall to yield a random number of order

$$\delta I = \pm \sigma_1 \frac{\delta V \bar{\phi}_d}{E_c \xi_d^{3/2}} L_d^{1/2}, \quad (14)$$

while the conductance will average out to zero at voltages $V \geq E_c \xi_d$ or if multiple domains are in series. The random currents and conductance fluctuations should be observable in small samples, or as harmonic noise generation for oscillatory bias voltage.

The arguments above also show that for a disorder potential ϕ_d which is correlated over a length scale $\xi_d \gg l_{dw}$, it will be favorable to introduce *additional* domain walls spaced at length scale ξ_d , when $E_d > E_c l_{dw} / \xi_d$. This occurs even at weak disorder if ξ_d is sufficiently large.

One-dimensional disorder.—In contrast to weak white-noise disorder, potentials with long-range correlations can be *relevant* perturbations. Consider the case of a general one-dimensional disorder [see Fig. 2(a)], where $\phi_d(y)$ is independent of the x coordinate. At wavelengths larger than l_{dw} , the Lyapunov functional is minimized if the system breaks up into parallel domains, so that \mathbf{E} is everywhere aligned with \mathbf{E}_d , and $\mathbf{j}^d = 0$. These conditions determine $\mathbf{E}(y)$ via (9), and the boundary conditions for a rectangular Corbino geometry (periodic boundary conditions at $x = 0$ and $x = L$). The voltage difference V between the leads at $y = 0$ and $y = W$, satisfies $V(I) = \int_0^W dy [E(y) - E_d(y)]$. At strong radiation intensity and zero current, (12) and transverse stability imposes that

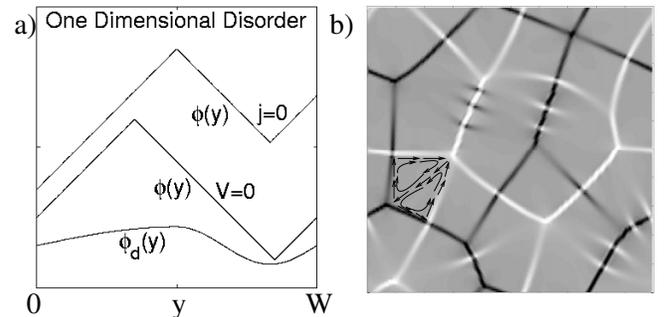


FIG. 2. (a) Photogenerated potential solutions ϕ , for one-dimensional disorder $\phi_d(y)$, with zero current and zero voltage boundary conditions. (b) Map of $\nabla \cdot \mathbf{E}$ (light: positive; dark: negative) for solution $\phi(x, y)$ of Fig. 1, showing the domain walls. Circulating dissipative currents are illustrated in one domain.

domain walls form precisely at maxima and minima of ϕ_d , given by the y_i , $i = 1, \dots, N$. To lowest order in ϕ_d , there will be a nonzero photovoltage which only depends on these positions:

$$V(0) = E_c \left((-1)^N W - 2 \sum_{i=1}^N (-1)^i y_i \right) + \mathcal{O}(E_d), \quad (15)$$

where $i = 1, 3 \dots$ are maxima. The differential conductivity σ_{yy} is given, to lowest order in ϕ_d , by

$$\frac{1}{\sigma_{yy}} = \frac{2E_c}{\sigma_1 W} \sum_i \frac{1}{|\phi_d''(y_i)|} \equiv \frac{2E_c}{\sigma_1 \tilde{E}_d}. \quad (16)$$

If $P(E_d)$ is the probability distribution for E_d at a random point, it can be shown that $\tilde{E}_d^{-1} = P(0)$. If E_d is taken from a Gaussian distribution, then $\tilde{E}_d = (2\pi)^{1/2} E_d^{\text{rms}}$, where $\tilde{E}_d^{-1} \equiv P(E_d = 0)$, with $P(E_d)$ the probability distribution for E_d at a chosen point. If ϕ_d has a Gaussian distribution, then $\tilde{E}_d = (2\pi)^{1/2} E_d^{\text{rms}}$. The differential conductivity σ_{xx} can also be calculated, using (10), and is given, to first order in ϕ_d by $\sigma_{xx} = \sigma_1 \langle |\mathbf{E}_d| \rangle / E_c$. For a Gaussian distribution, one has $\sigma_{xx} = (2/\pi) \sigma_{yy}$, while for a single sine wave, $\sigma_{xx} = (4/\pi^2) \sigma_{yy}$.

Two-dimensional potentials—The simplest 2D choice for ϕ_d is the separable “egg-carton” potential $\phi_d = (E_d^0/\sqrt{2})[\cos(x) + \cos(y)]$. We construct a zeroth order trial solution for zero bias current $\mathbf{j}_0^d = 0$ by placing domain walls on the lines $x = n\pi$ and $y = m\pi$, for integer n and m . This yields constant electric fields in each square domain $\mathbf{E}_0 = (E_c/\sqrt{2})(\pm\hat{x} \pm \hat{y})$. \mathbf{E}_0 is a gradient of a continuous potential, and satisfies charge neutrality (13). Upon application of an external voltage in an arbitrary direction, domain walls will move, as in the one-dimensional case, and also tilt with \mathbf{E} into a herringbone pattern. A variational calculation, assuming that within each domain \mathbf{E} is constant, finds, to first order in E_d , that $\sigma_{xx} = 0.83\sigma_1 E_d^0/E_c$. Numerical calculations confirm that the variational solution is at least close to the exact answer.

We have calculated analytically the first order (in E_d) corrections to \mathbf{E}_0 for zero external voltage, by integrating Kirchoff’s laws [8]. Away from the domain walls, we find $E > E_c$, which corresponds to circulating dissipative currents, which match onto the tangential currents at domain walls, given by Eq. (12). In Fig. 1, a generic two-dimensional example is displayed. ϕ_d contains 20 Fourier components chosen from Gaussian distributions with $\langle |\phi_d(\mathbf{k})|^2 \rangle$ independent of \mathbf{k} . The potential ϕ is found by numerically minimizing G . Figure 2(b) plots the 2D charge density where domain walls appear as line singularities. In both the egg carton and the potential of Fig. 1, G is frustrated from perfect alignment of \mathbf{E} and $\mathbf{E}_d(\mathbf{r})$ by the conditions $|\mathbf{E}| \geq E_c$ and $\nabla \times \mathbf{E} = 0$. This frustration underlies the circulating dissipative currents which are illustrated in one of the domains in Fig. 2(b).

In summary, we have introduced long-wavelength disorder into the transport theory of the microwave-irradiated quantum-Hall gas, using the Lyapunov functional as an organizing principle for the stability of steady states. We showed that weak white-noise disorder is irrelevant for the stability of the ZCS although it produces mesoscopic current fluctuations. For a strong and long-range potentials, the ZCS state breaks up into domains, which will generally result in a finite conductance and a photovoltaic effect. A microscopic theory necessary for the steady states dependence on microwave power and detuning frequency is deferred to a forthcoming publication [8]. We have not considered effects of conductivity anisotropy and variations in Hall coefficient. The latter will not affect the domain pattern or the longitudinal conductivity for one-dimensional disorder, but might have large effects and be experimentally relevant in other geometries.

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