THE INTERTIA TENSOR OF EQUILATERAL TRIANGLE

J.E. AVRON

Abstract. The solution to problem 4 in final, Fall 2002

1. Show that the frame is a principal frame

Since all three masses are on the $z = 0$ plane

$$I_{xz} = I_{yz} 0$$

For the remaining

$$I_{xy} = -m \sum x_i y_i = -m(x_1 y_1 + x_2 y_2 + x_3 y_3)$$

m_1 has $x_1 = 0$ and so does not contribute. For the remaining t20 $y_2 = y_3$ and $x_2 + x_3 = 0$ hence $I_{xy} = 0$.

2. The inertia tensor in frame I

From the figure, and elementary geometry

$$I_{xx} = ma^2 \left(\frac{1}{4} 2\right) = \frac{ma^2}{2}$$

the hight of the triangle is $h = \frac{a\sqrt{3}}{2}$ then

$$I_{yy} = mh^2 \left(2 \left(\frac{1}{3}\right)^2 + 1 \left(\frac{2}{3}\right)^2\right) = ma^2 = \frac{3}{4} \times \frac{6}{9} = \frac{ma^2}{2}$$

Since the triangle is thin

$$I_{zz} = I_{xx} + I_{yy} = ma^2$$

3. The Intertia tensor in a rotted frame

Since the top is symmetric, the inertia tensor is the same in all frames rotated in the x-y plane, i.e.

$$I = \frac{ma^2}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Department of physics, Technion, Haifa, Israel
E-mail address: avron@physics.technion.ac.il,

Date: February 23, 2002.
Thanks to Dan Gorbonos.