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Contents

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Cover picture: Schematic depiction of a chiral nematic liquid crystal phase.

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Open Problems in Mathematical Physics

by MICHAEL AIZENMAN (Princeton University, USA)

Around the the turn of the century a web page on Open Problems in Mathematical Physics (OPMP) was launched and originally cross-linked with the IAMP web site. It remains accessible at: http://web.math.princeton.edu/∼aizenman/OpenProblems_MathPhys/

In the ensuing 20 years two sets of problems from the list were solved. In each case the solutions were accomplished through non-trivial deepening of our understanding of the subject. The progress also opened the gates to other significant new insights and questions. Independently of the OPMP page, major awards and recognition were bestowed on the individuals involved.

Due to recent encouraging testimonials which we received about the role played by this initiative, the page will remain accessible. Also under consideration is the possibility of updating the OPMP list and reissuing it with about 20 interesting questions to be launched before the arrival of the year 2020.

In this issue of the IAMP News Bulletin we are including the most recent contribution to OPMP, an article by Yosi Avron on the solution of the problem of explaining why the Hall effect is quantized, taking into account electron-electron interactions. All IAMP members are encouraged to submit well formulated and documented proposals to: MA OPMP editor (aizenman@princeton.edu).

Constant Curvature: Solution of a 20-Year-Old Open Problem

by JOSEPH E. AVRON (Technion, Israel)

The adiabatic curvature associated with the bundle of ground states of interacting quantum particles approaches a constant as the system gets large. This fact, shown by Hastings and Michalakis [2, 10], together with complementary contributions by Giuliani, Mastropietro, and Porta, [8], Bachmann, De Roeck and Fraas [4], and Monaco and Teufel [7], solves an open problem advertised on the IAMP web site in 1999.

1 Introduction

Almost 20 years ago M. Aizenman initiated the page "Open problems in mathematical physics" on the IAMP web site[1]. In 1999, Ruedi Seiler and I advertised on this page the problem to put the theory of the Hall conductance[2] for large interacting systems on solid mathematical foundations.

The problem was dormant for about 10 years. In 2009 Matt Hastings and Spyridon Michalakis [1] put on the arXiv a paper which claimed to solve a central part of the problem, where they showed that the adiabatic curvature of the bundle of ground states of large, gapped,[3] two-dimensional systems, is almost constant. The paper introduced into the theory of the quantum Hall effect new techniques, (originally due to Hastings in different contexts). It took a long time, six years in fact, for the paper to get published [2], and even longer to be understood and gain the influence and impact that it deserved.

Alessandro Giuliani, Vieri Mastropietro, and Marcello Porta [8] derived partially overlapping and partially complementary results using different, field-theoretic, methods: multi-scale analysis and Ward identities.

Both Hastings *et al.* and Giuliani *et al.* identified the Hall conductance with the adiabatic curvature, a relation that follows from Kubo's formula. Putting Kubo's formula on firm mathematical foundation is one of B. Simon's 15 problems [3]. Sven Bachmann, Wojciech de Roeck, and Martin Fraas accomplished this for gapped systems in [4] making use of some of the machinery introduced in [2].

This note attempts to briefly review the status of the problem of bulk Hall conductance and offer some insights that I have gained reading and discussing with the authors of [2, 6].

2 Background

Before describing the recent progress in this field it is worthwhile to recall what was (rigorously) known about the theory of the quantum Hall effect when the problem was posed in 1999.

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[1]The open-problem page does not display anymore on the IAMP site, but it is stored at `http://web.math.princeton.edu/~aizenman/OpenProblems_MathPhys`.

[2]The quantum Hall effect had been studied from different theoretical perspectives including effective field theories and theories that focus on the edge. The problem formulated in 1999 focused on the Hall conductance of the bulk.

[3] The existence of a gap for certain weakly interacting systems is shown in [10].
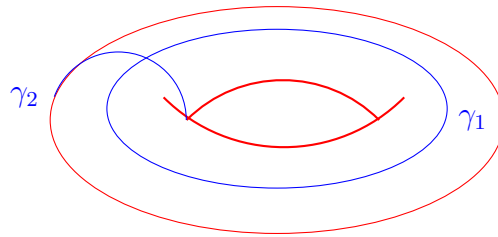
Figure 1: The bulk Hall conductance is concerned with the adiabatic curvature associated with a macroscopic collection of electrons confined to a two-dimensional torus. The electrons are acted on by an external gauge field A parametrized by two magnetic fluxes ϕ_j, given in Eq. (3). A gapped ground state then gives rise to a ground-state bundle with the flux space as its base. The main result of Hastings and Michalakis is that, for a macroscopic system, the curvature of the bundle is almost constant.

- In 1983 Thouless, Kohmoto, Nightingale, and den-Nijs showed that the (bulk) Hall conductance of non-interacting (two-dimensional) fermions in a periodic potential with the Fermi energy in a gap is quantized. This result was later interpreted, following Barry Simon, in terms of the Chern number of a line bundle over the Brillouin zone:

$$Chern(P) = \frac{1}{2\pi} \int \omega \in \mathbb{Z}, \tag{1}$$

where P is the bundle of spectral projections representing the occupied states and ω is the adiabatic (Berry's) curvature:

$$\omega = i\, Tr\,\Omega, \quad \Omega = P\,(dP)P_\perp(dP)\,P. \tag{2}$$

David Thouless received the 2016 Nobel Prize in Physics partly for this discovery.

In 1988 Jean Bellissard extended the results of Thouless *et al.* to non-interacting fermions in random and quasi-periodic potentials. There is no Brillouin zone in this setting, and Bellissard identified the Hall conductance with a suitable Fredholm index related to Chern numbers in non-commutative geometry.

- The geometric approach to the Hall conductance extends[4] to interacting Hamiltonians associated with multiply connected *finite* systems, as in Fig. 1, parametrized by two fluxes:

$$\phi_j = \oint_{\gamma_j} A. \tag{3}$$

Assuming that the ground state is separated by a gap, the associated projection P defines a bundle whose base space is flux space. Like the Brillouin zone, flux space is topologically a torus. Eqs. (1,2) then say that the Chern number is the *flux average* of the adiabatic curvature (=Hall conductance).

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[4]These results are due to Niu, Thouless, and Wu and independently to Avron and Seiler.

It was believed that for large interacting systems the adiabatic curvature becomes flux independent[5], and so quantized without flux averaging. Support for this belief comes from the works of Thouless *et al.* and Bellissard *et al.* for non-interacting fermions. If true, this is a remarkable fact: Normally, constant curvature reflects an underlying symmetry in the problem. Here, instead, it reflects an emergent symmetry in the thermodynamic limit.

The challenge was to show that the adiabatic curvature indeed approaches a constant for interacting electrons. This is what Hastings and Michalakis succeeded in doing.

3   Hastings and Michalakis: Quantized adiabatic curvature

Omitting technicalities, and stated somewhat cavalierly, Hastings and Michalakis proved [2]

Theorem 1. *Let $H(A, \ell)$ be a many-body Hamiltonian acting on the Fock space associated with the torus $\mathbb{Z}^2/(\ell\mathbb{Z})^2$ with a gapped ground state. Then*

$$\omega = \frac{n}{2\pi} \, d\phi_1 \wedge d\phi_2, \quad n \in \mathbb{Z},$$

up to an error that decays faster than any power of ℓ.

The proof rests on two new tools introduced into the theory of the quantum Hall effect: A generator of parallel transport acting on the ground-state bundle, with good localization properties, and Lieb-Robinson bounds that control the localization of observables evolving by local unitaries.

Bachmann *et al.* [6] gave a short and simpler proof of this result under the additional assumption of a uniform gap for *all* fluxes[6]. Below I shall attempt to give my interpretation of the insight behind the constancy of ω [2, 6].

The basic reason for the constancy of ω is an interplay between localization and gauge freedom: Gauge freedom allows one to pick a gauge so that the curvature Ω is localized on the torus near the region of the crossing of the blue stripes in Fig. 2. Gauge freedom also allows one to pick dK, a generator of parallel transport of the ground-state bundle, to be localized near the red stripes in the figure. dK allows the ground-state projection $P(\phi)$ to be deformed into the ground-state projection $P(\phi')$, and in particular to $\phi' = 0$. Since dK and Ω are localized far from each other, one can deform the flux ϕ to $\phi' = 0$ without changing Ω.

$|\psi\rangle \in \text{Range } P$ is said to undergo *parallel transport* if

$$|d\psi\rangle = (dP)|\psi\rangle$$

Let dK be

$$idP = [dK, P].$$

A dK with good localization properties is

$$dK = - \int dt \, W(t) e^{itH} (dH) e^{-itH}. \tag{4}$$

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[5]Under ϕ-dependent gauge transformation the adiabatic curvature changes by an exact 2-form. Strictly speaking, it is not simply a function of ϕ, but a function of the gauge field A.

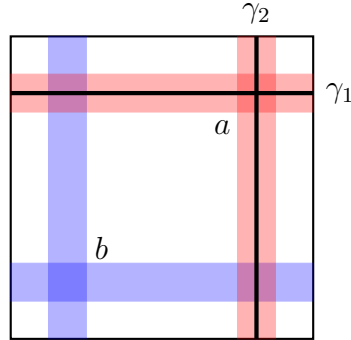[6]Hastings and Michalakis only assume a gap at zero flux.

Figure 2: The square represents a torus in Fig. 1. The fundamental periods γ_j are the black lines. The gauge fields a of Eq. 5, which determine the flux ϕ_0, live on the red strips. The gauge fields b live on the blue strips and determine $\delta\phi$.

$W(t)$, a real valued function, is chosen so that its Fourier transform $\widetilde{W}$ is smooth and

$$\widetilde{W}(\omega) = \begin{cases} i/\omega & \omega > \text{gap} \\ 0 & \omega = 0. \end{cases}$$

Since $\widetilde{W}(\omega)$ is smooth, $W(t)$ decays faster than any power for large times. Lieb-Robinson bounds then imply that dK is localized near dH.

We shall now introduce two choices for dK that correspond to splitting the flux ϕ into two pieces[7]:

$$A = A_0 + \phi_0\, a + \delta\phi\, b, \quad d_x a = d_x b = 0, \tag{5}$$

where $\phi = \phi_0 + \delta\phi$. The a part of the gauge field is chosen to be concentrated near the red strips[8] while the b part is chosen to be concentrated near the blue strips.

Taking $dH = d_{\phi_0}H$ in Eq. (4) gives a generator $d_{\phi_0}K$ which is localized near the red stripes and allows $P(\phi)$ to be deformed into $P(\phi')$ by deforming ϕ_0.

In the definition of the curvature Ω, we can trade dP for dK (up to a sign),

$$\Omega = -P\,(dK)P_\perp(dK)\,P.$$

Now choose dK as per Eq. (4) with $dH = d_{\delta\phi}H$. This choice localizes $d_{\delta\phi}K$ near the blue stripes and localizes Ω near the crossing of the blue stripes.

Since $d_{\phi_0}K$ and $d_{\delta\phi}K$ are far from each other, Lieb-Robinson types of estimates can be used to deform $\Omega(\phi)$ into any other $\Omega(\phi')$, while keeping ω almost constant.

3.1 Ward identities and cluster expansion

Using very different methods, Giuliani, Mastropietro, and Porta [8] proved the following theorem, glibly stated:

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[7]Hastings and Michalakis use a different argument.

[8]a is associated with stripes because one needs to respect the constraint that the magnetic field $d_x A$ remains the same.

Theorem 2. *By the Kubo formula, the Hall conductance for gapped non-interacting translation-invariant fermionic systems equals the conductance of the corresponding weakly interacting systems in the thermodynamic limit.*

This shows the universality of the quantum Hall conductivity in the sense that it is independent of the interaction strength (for weak interactions) and provides the value of the Hall coefficient.

The main ingredients of their proof are:

- Construction and proof of analyticity of the ground-state Euclidean correlations, uniformly in the system size, via *fermionic cluster-expansion* methods.

- Proof of the Wick rotation for the ground state Kubo conductivity. (I.e., the Kubo conductivity is equal to its Euclidean counterpart.)

- Proof that all the terms in the perturbation series in powers of the interaction series for the Euclidean Kubo conductivity vanish identically. This follows from a combination of Ward Identities with Schwinger-Dyson equations.

Giuliani et. al. [9] also proved the quantization and universality of the Hall coefficient in the gapless case of the the weakly interacting Haldane model.

4   Kubo's formula: Bachmann, de Roeck, and Fraas

In 1984 B. Simon formulated a list of 15 problems in mathematical physics. Problem 4 concerns transport theory. This is what he says:

There are also serious foundational questions in quantum transport. A basic formula in condensed matter physics is the Kubo formula for conduction . . . . Not only are the usual derivations suspect, but van Kampen [30], among others, has seriously questioned its validity on physical grounds.

Problem 4 B: Either justify Kubo's formula in a quantum model, or else find an alternate theory of conductivity.

Linear-response theory involves taking limits: The thermodynamic limit of a large system, the linear-response limit of weak perturbation and also the limit of adiabatic switching of the perturbation. Proving Kubo's formula requires controlling the limits and taking them in the correct order, where the thermodynamic limit is taken first.

Bachmann *et al.* [4, 5] proved the validity of Kubo's formula for gapped, translation-invariant, interacting spin systems with short-range interactions. In particular, they proved the commutativity of the thermodynamic and linear response limits. They rely on the tools introduced by Hastings: The Lieb-Robinson bounds and the generator of evolution dK of the previous section.

Bachmann *et al.* [4, 5] also adapted adiabatic theory to the setting of macroscopic systems. The usual Schrödinger picture of following the quantum state does not work for thermodynamic systems because tiny local errors accumulate as the system gets large and make the approximate state a poor approximant. The way out is to focus instead on the Heisenberg picture of adiabatic evolution of *local* observables. The adiabatic framework treats rigorously the

"adiabatic switching" which is central in linear response and proves Kubo's formula for static perturbations.

In subsequent works Teufel *et al.* [7] and [6] extended the results in [4, 5] to adiabatic transport in the quantum Hall effect. This involves, among other things, extending the theory from spins to fermions, and to currents generated by time-dependent gauge fields. The strategy of [4, 5] works in the quantum Hall effect for, as we have seen, the curvature is associated with a local observable, lying in the intersection of the blue stripes.

The results of [4, 5, 6] and [7] complement those of Hastings *et al.* and Giuliani *et al.*, who took Kubo formula for granted.

Acknowledgment

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Liquid Crystals and the Heilmann-Lieb Model

by IAN JAUSLIN (Institute of Advanced Study, Princeton, NJ, USA)

Liquid crystals are everywhere, and they do more for you than you might think. They tell you the weather, the time, and will kindly let you know that your connection to eduroam was rejected, and that you'll have to listen to the rest of the seminar. Indeed, most displays run on liquid crystals, from the small dials in digital wristwatches to large flat-screen televisions. In this paper, I will review some results pertaining to why (or, perhaps, how) liquid crystals exist in the first place. In particular, I will focus on a model introduced by Heilmann and Lieb in 1979, for which Lieb and I have recently proved the emergence of a nematic liquid crystal phase.

1   Gases, liquids and crystals

Let's start with a familiar paradigm: many materials (water, for example) occur in many different phases, each of which is stable in a given range of temperatures and pressures. At 101.3 kPa (the standard atmospheric pressure), water is gaseous above 100°C, liquid between 0°C and 100°C and solid below 0°C. These three phases are, rather dramatically, different, both from a microscopic and a macroscopic point of view.

Gases have a low density, so low, in fact, that it's constituent molecules barely notice each other's presence. Solids and liquids are much denser (by a factor of 10^3 in the case of water). Pushed together into close quarters, its constituent molecules interact strongly with each other. The absence (or, rather, weakness) of interactions between the molecules in a gas has observable macroscopic consequences: it implies the *ideal gas law*, $pV = nk_BT$, which, in particular, says that gases are rather amenable to being compressed or dilated, in contrast to liquids and solids.

In liquids, the molecules move around each other in a disorderly fashion, despite the smallness of the elbow-room allotted to them. Solids, on the other hand, cope with the density by spontaneously ordering: the molecules arrange themselves in a regular pattern. This is no small feat: an ice cube might contain a septillion (10^{24}) molecules, and almost all of these are aligned! As a consequence, liquids flow, whereas solids don't exactly do much of anything.
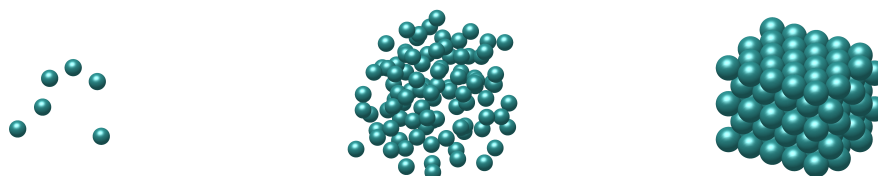

Figure 1: A caricatural depiction of a gas, liquid and crystalline phase. In the gaseous phase, the density is so low that the particles barely interact. The solid phase has a high density, and the molecules spontaneously align. The liquid phase has a large density, but displays no long range order.

The question of interest here, is whether one can prove that these phases actually occur in models of interacting molecules. To be definite, let us focus on the hard sphere model, in which each molecule is represented by a sphere, and particles interact solely through the condition that the spheres may not overlap. This system has a gas phase at low densities, and is expected to have a crystalline phase at high densities [WJ57, AW57, IK15], with a first order phase transition between the two.

The gas phase is rather well understood: at low densities, particles rarely ever run into each other, and one can compute many observables in terms of convergent power series in the density. These expansions are called *virial expansions*, and date back, at least, to the early years of the XX$^{\text{th}}$ century [Ka02]. Of particular note are two papers, one by Ursell [Ur27] and the other by Mayer [Ma37], in which a systematic recipe is introduced to compute virial expansions for any model with short-range (integrable) pair interactions. The convergence of this construction was later proved by Groeneveld [Gr62], Ruelle [Ru63] and Penrose [Pe63].

On the other side of the spectrum, at high densities, the situation is a bit more dicey, even for such a simple system as the hard sphere model. Even at close packing, that is, at the largest possible density, it is a challenge to prove that the system is crystalline. There is an old conjecture, often attributed to Kepler, which states that the maximal density configuration is either a face-centered cubic lattice, a hexagonal close packing, or a combination thereof. This is also called the *cannonball stacking* (or, for the more peaceful minded among us, the *orange stacking*) problem, since the question reduces to finding the optimal way of stacking cannonballs. Hales proved this conjecture [Ha05], using the assistance of a computer. There has been some debate over the validity of this proof, and it has recently been formalized and checked by a formal proof checker [HAe17]. However, at densities which are lower than close packing, there is, as of yet, no proof of crystallization in the hard sphere model.

In short, in the hard sphere model, one can get rigorous control over the low density gas phase and the maximal density crystalline phase, but there are no results for intermediate densities, and, in particular, no proof that the phase transition exists and is of first order.

The main difficulty in proving that the hard sphere model crystallizes at high density is that it is a *continuum* model: the positions of the spheres can take any value in $\mathbb{R}^3$. This spells out trouble for imposing translational order: if two neighboring spheres differ infinitesimally from the crystalline structure, and so do the next pair, and the next pair, and so on, then two spheres which are sufficiently far from each other could have completely decorrelated positions, which would break the crystalline structure.

In fact, if one puts the system on a lattice, that is, if one restricts the positions of the hard spheres (or, more generally, the particles) to a discrete set, then proving crystallization is feasible in certain cases. For example, Dobrushin [Do68] proved it for the nearest neighbor exclusion on the square lattice, Baxter [Ba82] on the hexagonal lattice, Heilmann and Præstgaard [HP74] for the third-nearest neighbor exclusion on the square lattice, a result which was generalized by Lebowitz and I [JL17] to a class of hard-core lattice particle models in $d \geqslant 2$ dimensions. In all of these cases, the basic idea is to map the dense particle system to a dilute model of holes, for which low-density methods can be used. The issue is that the interaction between holes can be difficult to treat, even on the lattice. The same could be done for the continuum model, but the interaction between holes is simply too complicated to control.

The hard sphere model does not have a liquid phase. A simple model which is expected to have a gas, a liquid and a crystalline phase is the *Widom-Rowlinson* model [WR70]. It consists of two species of particles, called A- and B-particles, which interact via a hard-core repulsion, like the hard sphere model. The difference with the hard spheres is that the radius of the interaction depends on whether it occurs between two A particles (R_{AA}), two B's (R_{BB}) or an A and a B (R_{AB}, with $R_{AB} > R_{AA}, R_{BB}$). In the gas phase, all particles intermingle, whereas in the 'liquid' phase, A's and B's are segregated and disordered. In the crystalline phase, they are ordered, like in the hard sphere model. Ruelle [Ru71] was the first to prove the existence of the 'liquid' phase in the case where the radii of the A-A and B-B interactions is 0. The result was generalized to a large class of Widom-Rowlinson-like models by Bricmont, Kuroda and Lebowitz [BKL84]. Proving crystallization, however, would be just as difficult as in the hard sphere model.

All the theorems I mentioned until now are about the existence of gas, liquid or crystalline phases, not about the nature of the phase transition between them. In the case of the gas-liquid transition Maxwell [Ma75] developed a heuristic construction to account for the corrections to the ideal gas law near the transition point. By introducing a family of forces acting between particles in a gas, Van der Waals derived an equation which refined the ideal gas law. The Van der Waals equation gives good agreement with experiments provided the density is small enough, but, at larger values, one finds an unphysical regime, in which the pressure increases with the volume. The Maxwell construction consists in flattening out that region, and interpreting it as the liquid-vapor phase transition. The construction was shown to be rigorous by Lebowitz and Penrose [LP66] in a system of particles with an infinitely weak and infinitely long range interaction called a *Kač* potential.

2 Liquid crystals

The take-home message which is buried somewhere in the previous section is that liquids are disordered and crystals are ordered, thus making the term 'liquid crystal' an oxymoron. Liquid crystals are phases which combine properties of ordered and disordered matter. They are typically found in systems of anisotropic molecules, which may be long and elongated like rods, or flat and wide like plates, or shaped like a boomerang, or a helix, et cætera... Whereas there are many types of liquid crystals, we will mostly focus on *nematic* liquid crystals, which occur in rod-shaped molecules (see figure 2). A rod has 5 degrees of freedom: 3 specify the position of the center of the rod and 2 its orientation. In a nematic phase, the orientational degrees of freedom exhibit long range order, that is, the rods are aligned, whereas the positional degrees of freedom are disordered (see figure 2). This phase is quite different from the ones mentioned until now, because it is *partially* ordered. From a macroscopic point of view, liquid crystals flow, like liquids, which is made possible by the positional disorder of the molecules. The orientational order manifests itself mostly through optical properties: a nematic liquid crystal is a polarizing filter, only allowing light polarized along the orientation of the molecules through.

Liquid crystals have come to play an important role in various display technologies, and have been used in digital watches, Game Boys, flat screen computer monitors, televisions, and smartphones. The basic mechanism underlying these technologies relies on liquid crystals that
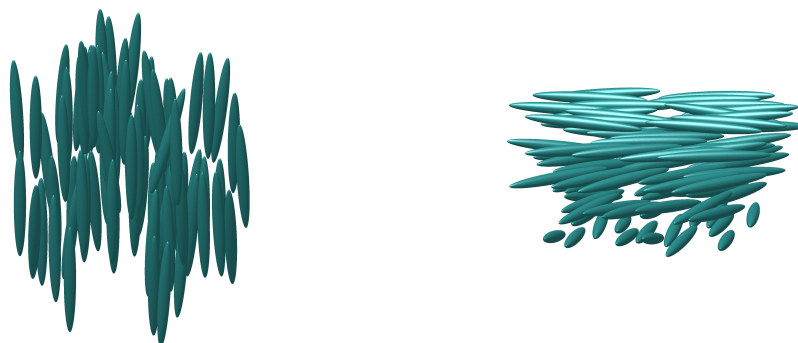
Figure 2: Caricatural depiction of a nematic liquid crystal phase and a chiral nematic liquid crystal phase. In the first the rod-shaped molecules are mostly aligned, but their positions are disordered. The second consists of a stack of horizontal planes, in each of which the rods are aligned but their positions are disordered. The orientation of the rods changes from one plane to the next.

can be switched between a nematic and a *chiral* nematic phase. In a chiral nematic phase, the molecules are arranged in a stack of planes, in each of which the molecules are aligned. The direction in each plane is rotated from one plane to the next (see figure 2). Because nematic liquid crystals are polarizing filters, by stacking the molecules in this way, chiral nematic liquid crystals alter the polarization of light. In particular, if horizontally polarized light is shined through a chiral nematic liquid crystal whose total change in the angle of the particles is $90°$, then it will come out with a vertical polarization.

Using this mechanism, one can design an electronic light filter. In its *off* (opaque) state, the filter consists of a chiral nematic liquid crystal sandwiched between two horizontal polarizing filters. In this state, the filter does not let any light through, since the first filter only allows horizontal light through, which is rotated by the liquid crystal and is vertically polarized by the time it reaches the second filter, which blocks the signal. To turn the filter *on* (transparent), an electric field is applied through the liquid crystal, which, by the *Twisted Nematic Field* effect, becomes nematic. At this point, the horizontally polarized light can make its way through the filter.

Modern LCD displays ('LCD' stands for Liquid Crystal Display) are made of an array of LEDs (Light Emitting Diodes), three for each pixel. Of these three, one is blue, another green, and the third red. In front of each of these LEDs, there is an electronic light filter, like the ones I just described. Pixels are turned on or off by applying an electric field through the filter or not. (By varying the strength of the field, one can adjust the proportion of the crystal that is chiral nematic, and thus adjust the brightness of the pixel, beyond a simple on/off switch.)

LCD technology is on its way out of smartphones, televisions and computer monitors, since, as it turns out, one gets better contrast by controlling the intensity of the LEDs directly. This is a bit of a technological challenge, which has been overcome only recently, through the introduction of OLEDs (Organic Light Emitting Diode). However, as of yet, OLED displays are still quite a bit more expensive than LCDs.

The question of interest here is why, or rather how, do liquid crystal phases exist? In other

words, can one construct a microscopic model in which one can prove the emergence of a liquid crystalline phase?

Onsager [On49] made a significant first step in this direction. He set out to understand a colloidal suspension of tobacco mosaic viruses, which consist of a single strand of RNA cozily nestled into a cylindrical shell of proteins. The virus is shaped like a rigid cylinder, about 18nm in diameter and 300nm long. Onsager modeled the suspension as a system of hard cylinders (in which the only interaction between cylinders is that they are not allowed to overlap), which are meant to represent the electrostatic repulsion between viruses, whose effective diameter far exceeds the size of the virus itself (Onsager noted that a nematic phase would appear even at densities as low as 2 percent, so the electrostatic repulsion must play a role in the formation of this phase). To study the system, Onsager computed a virial expansion, truncated at second order, and, by a (non-rigorous) self-consistency argument, conjectured that the nematic phase exists, and that the phase transition is of first order.

The earliest attempt (to my knowledge) at a rigorous proof of a nematic phase was undertaken by Heilmann and Lieb [HL79]. The model they considered is a dimer model on the square lattice $\mathbb{Z}^2$. A dimer is an object that occupies an edge of the lattice and its two endpoints (see figure 3), with the constraint that no two dimers can cover the same vertex. (This is sometimes called a *monomer-dimer* model, and the sites that are not covered by any dimers are called *monomers*.) Dimers represent molecules, which have a (discrete) position, and a (discrete) orientation, which can be either vertical or horizontal. One may, therefore, wonder whether the monomer-dimer model exhibits a nematic phase, but, in an earlier paper [HL72], Heilmann and Lieb had proved that there are no phase transitions in the monomer-dimer model, which, in particular, implies that there is no long range orientational order, and, therefore, no nematic phase. In order to favor orientational order, Heilmann and Lieb [HL79] introduced an interaction between dimers to induce alignment. The interaction favors configurations in which pairs of dimers are *adjacent* and *aligned*, that is, they are neighbors and are contained within the same row or column (see figure 3). They proved, using a reflection positivity argument, that dimers spontaneously align if the interaction is strong enough. In other words, there is long range orientational order. However, their argument does not show that there is no positional order, thus stopping short of proving the emergence of a nematic phase in this model.

Since then, other models have been introduced in which a nematic phase has been proved to form. Bricmont, Kuroda and Lebowitz [BKL84] noticed that a system of infinitely thin needles in two dimensions, in which the needles are allowed to be in one of two orientations (say, horizontal and vertical) is, essentially, a Widom-Rowlinson model. If more orientations are allowed (but a finite number of them), then the model closely resembles a *multi-component* Widom-Rowlinson model. They commented that the machinery developed in their paper [BKL84] for the Widom-Rowlinson model could easily be adapted to prove a nematic phase in this model.

Ioffe, Velenik and Zahradník [IVZ06] discussed a model of rods on $\mathbb{Z}^2$, in which rods consist of adjacent and aligned vertices, and are of varying length. The parameters of the model are tuned so as to make it *integrable*, and, by solving it exactly, they showed the emergence of a nematic phase. Disertori and Giuliani [DG13] considered a system of rods on $\mathbb{Z}^2$ of *fixed* length, which is *not* integrable, and showed the existence of a nematic phase.

There had also been some progress in proving the Heilmann-Lieb conjecture: Alberici-
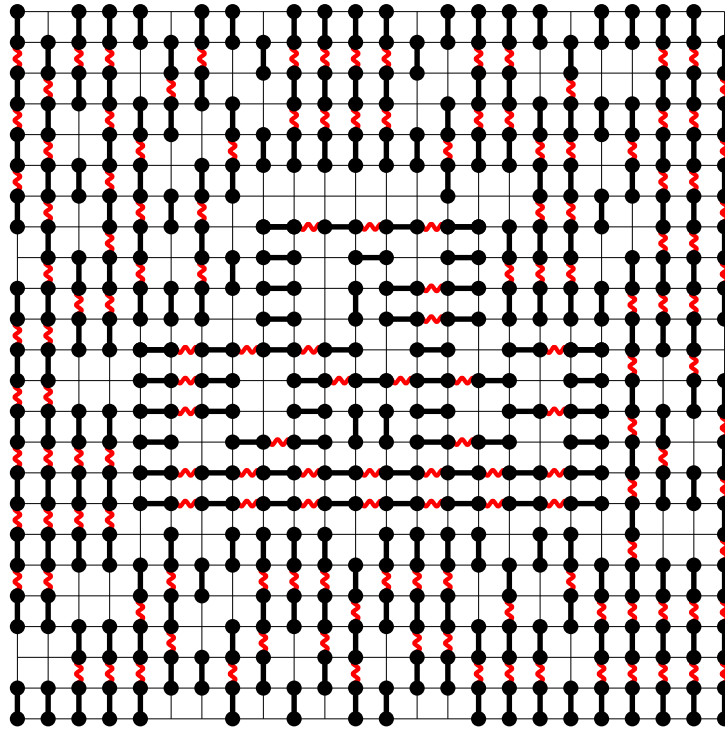
Figure 3: A dimer configuration in the Heilmann-Lieb model. Pairs of dimers which are adjacent and aligned interact, which is represented by a red wavy line.

[Al16] considered an interacting dimer model that is almost identical to the Heilmann-Lieb model, except that the chemical potentials of the horizontal and vertical dimers are different, which means that vertical dimers are favored over horizontal ones. This is a rather mild assumption, since Heilmann and Lieb proved [HL79] that dimers tend to align, so one of the orientations is spontaneously favored over the other, but in Alberici's model, this uniformity in the orientation is built into the model directly. Nevertheless, there was little doubt that the conjecture is true, and Papanikolaou, Charrier and Fradkin [PCF14] showed numerical evidence for the lack of positional order in the system.

Lieb and I [JL17b] have recently presented a proof of this conjecture, which I will discuss in the next section.

3   Nematic order in the Heilmann-Lieb model

I will now introduce the Heilmann-Lieb model on a more formal footing, state the theorem that implies the emergence of nematic order, and go through the main ideas of its proof.

3.1   The Heilmann-Lieb model

In [HL79], Heilmann Lieb actually introduced five models, in two and three dimensions, and proved long-range orientational order for each of them. The model which we consider here is called 'model I' in [HL79]. As was mentioned above, it is an interacting (monomer-)dimer

model on $\mathbb{Z}^2$. We consider the system in the *grand-canonical ensemble*, which means that we pick dimer configurations randomly, according to the Gibbs grand-canonical distribution. In order to define it, and, in the process, introduce some relevant notations, I will define the average of an observable A:

$$\langle A \rangle_{\mathrm{v}} = \lim_{L \to \infty} \frac{1}{\Xi_{\Lambda_L}(z, J)} \sum_{\underline{\delta} \in \Omega_{\mathrm{v}}(\Lambda_L)} A(\underline{\delta}) z^{|\underline{\delta}|} \prod_{\delta \neq \delta' \in \underline{\delta}} e^{\frac{1}{2} J \mathbb{1}_{\delta \sim \delta'}} \tag{6}$$

in which

- $\Lambda_L := \{1, \cdots, L\} \times \{1, \cdots, L\} \subset \mathbb{Z}^2$ is a *finite* subset of $\mathbb{Z}^2$ of volume L^2,

- $\Omega_{\mathrm{v}}(\Lambda_L)$ is the set of (monomer-)dimer configurations on Λ_L, with *vertical* boundary conditions (I will come back to these later),

- A is an observable, that is, a map from $\Omega_{\mathrm{v}}(\Lambda_L)$ to $\mathbb{R}$,

- $z \geqslant 0$ is a parameter called the *fugacity* (if β is the inverse temperature and μ is the chemical potential, then $z \equiv e^{\beta \mu}$),

- $|\underline{\delta}|$ is the number of dimers in $\underline{\delta}$

- $J \geqslant 0$ is the *interaction strength*

- $\mathbb{1}_{\delta \sim \delta'} \in \{0, 1\}$ is equal to 1 if and only if δ and δ' are adjacent and aligned

- $\Xi_{\Lambda_L}(z, J)$ is a normalization constant, called the *partition function*:

$$\Xi_{\Lambda_L}(z, J) := \sum_{\underline{\delta} \in \Omega_{\mathrm{v}}(\Lambda_L)} z^{|\underline{\delta}|} \prod_{\delta \neq \delta' \in \underline{\delta}} e^{\frac{1}{2} J \mathbb{1}_{\delta \sim \delta'}}. \tag{7}$$

Note that the fugacity z is related to the density of dimers: if z is large, then configurations with many dimers are favored. Similarly, when J is large, configurations with many interactions are more probable.

Heilmann and Lieb showed that, if $J \gg \max(1, -\log z)$, then, given a vertical edge e_{v} and a horizontal one e_{h}, the probability that they are both occupied is small. This implies that the system has at least two extremal Gibbs measures, one of which mostly contains vertical dimers, while the other mostly contains horizontal ones. In an effort to study each of these separately, I will impose a boundary condition that will select the vertical Gibbs measure (the same could, obviously, be done for the horizontal phase). This is done by isolating two horizontal strips of height $\ell_0 := e^{\frac{3}{2} J} z^{\frac{1}{2}}$ at the top and bottom of Λ_L (see figure 4). Horizontal dimers are forbidden from intersecting these strips. Since these regions have a large volume, they are likely to contain many vertical dimers, which will push the other dimers in the bulk to be vertical as well. The choice of the size $e^{\frac{3}{2} J} z^{\frac{1}{2}}$ of the strips will be clarified later.
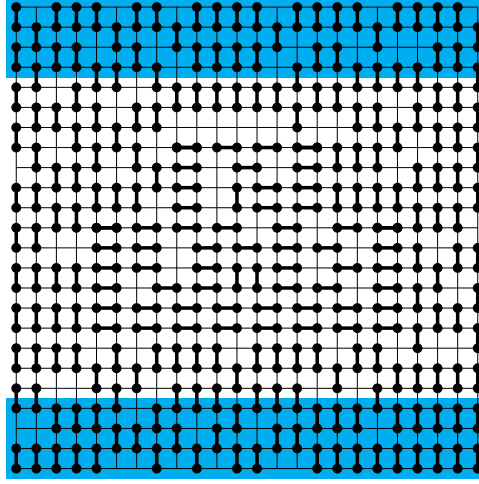
Figure 4: The boundary condition is chosen in such a way to favor vertical dimers. The blue strips, whose height is $\ell_0 := e^{\frac{3}{2}J}z^{\frac{1}{2}}$, are devoid of horizontal dimers.

3.2 Main result

Let me state the theorem that Lieb and I proved [JL17b], after which I will comment on why it implies the existence of a nematic phase.

Theorem 3.1. *We assume that*

$$J \gg z \gg 1. \tag{8}$$

Given an edge e, we define the observable $\mathbb{1}_e$, which returns 1 if e is occupied by a dimer, and 0 if not. Given two vertical edges e_{v} and e'_{v}, $\langle \mathbb{1}_{e_{\mathrm{v}}} \rangle_{\mathrm{v}}$ (that is, the probability that e_{v} is occupied) is independent of the choice of e_{v}, and

$$\langle \mathbb{1}_{e_{\mathrm{v}}} \rangle_{\mathrm{v}} = \frac{1}{2}\left(1 + O(z^{-\frac{1}{2}}e^{-\frac{1}{2}J})\right) \tag{9}$$

$$\left\langle \mathbb{1}_{e_{\mathrm{v}}}\mathbb{1}_{e'_{\mathrm{v}}} \right\rangle_{\mathrm{v}} - \left\langle \mathbb{1}_{e_{\mathrm{v}}} \right\rangle_{\mathrm{v}} \left\langle \mathbb{1}_{e'_{\mathrm{v}}} \right\rangle_{\mathrm{v}} = O\left(e^{-c\,\mathrm{dist}_{\mathrm{HL}}(e_{\mathrm{v}}, e'_{\mathrm{v}})}\right) \tag{10}$$

in which $\mathrm{dist}_{\mathrm{HL}}$ is the distance induced by the norm

$$\|(x, y)\|_{\mathrm{HL}} = J|x| + \ell_0^{-1}|y|. \tag{11}$$

Similarly, given two horizontal edges e_{h} and e'_{h}, $\langle \mathbb{1}_{e_{\mathrm{h}}} \rangle_{\mathrm{v}}$ is independent of the choice of e_{h}, and

$$\langle \mathbb{1}_{e_{\mathrm{h}}} \rangle_{\mathrm{v}} = O(e^{-3J}) \tag{12}$$

$$\left\langle \mathbb{1}_{e_{\mathrm{h}}}\mathbb{1}_{e'_{\mathrm{h}}} \right\rangle_{\mathrm{v}} - \left\langle \mathbb{1}_{e_{\mathrm{h}}} \right\rangle_{\mathrm{v}} \left\langle \mathbb{1}_{e'_{\mathrm{h}}} \right\rangle_{\mathrm{v}} = O\left(e^{-6J-c\,\mathrm{dist}_{\mathrm{HL}}(e_{\mathrm{h}}, e'_{\mathrm{h}})}\right). \tag{13}$$

Finally,

$$\left\langle \mathbb{1}_{e_{\mathrm{h}}}\mathbb{1}_{e_{\mathrm{v}}} \right\rangle_{\mathrm{v}} - \left\langle \mathbb{1}_{e_{\mathrm{h}}} \right\rangle_{\mathrm{v}} \left\langle \mathbb{1}_{e_{\mathrm{v}}} \right\rangle_{\mathrm{v}} = O\left(e^{-3J-c\,\mathrm{dist}_{\mathrm{HL}}(e_{\mathrm{h}}, e_{\mathrm{v}})}\right). \tag{14}$$

This theorem implies long range orientational order and the absence of positional order. Indeed, (12) and (9) imply that the probability of finding a horizontal dimer is much smaller than that of a vertical dimer. This is not a trivial fact, since the symmetry between vertical and horizontal dimers is only broken at the boundary, which is *infinitely* far away from e_h or e_v. In addition, (14) means that the joint probability of finding a vertical and a horizontal dimer is low (and, up to exponentially small terms, equal to the product of the probabilities of finding a dimer in each site). This implies orientational order

The probability that a vertical or horizontal edge is occupied is independent of the location of that edge. This is a good sign, but is not sufficient to prove that there is no positional order. Indeed, the Gibbs measure could be a mixture of two measures, in which half of the vertical edges are favored in one of the measures, while the other half is favored in the other. The decay of the correlation functions (10), (13) and (14) forbids this. Indeed, they show that the joint probability of finding a dimer at an edge e and another at e' is equal to a term that does not depend on e or e' plus an exponentially vanishing term.

Thus, there is long range orientational order, and no positional order: the symmetry of the system is partially broken. Therefore, in this range of parameters (8), the system is in a *nematic* phase.

The range of parameters (8) makes some sense, but is, presumably, not the optimal one. The orientational order is due to the interaction and to the large density of dimers. Therefore, it is natural to expect that J, z should be rather large. However, the range for which Heilmann and Lieb's proof holds is much wider:

$$J \gg \max(1, -\log z). \tag{15}$$

(This inequality does not even require $z \gg 1$, because, by making J big enough, the dimer density is already large.)

An interesting point is hidden in (11): the truncated correlation functions decay exponentially, but the rate of the decay is extremely anisotropic. In the horizontal direction, the rate is $J \gg 1$, which means that correlations are strongly dampened in the horizontal direction. In the vertical direction, it is $\ell_0^{-1} \equiv e^{-\frac{3}{2}J} z^{-\frac{1}{2}} \ll 1$. Therefore, the correlation length in the vertical direction is huge. This is the expected behavior, and is due to the large density of dimers. Indeed, in a column that contains only vertical dimers, that is, no horizontal dimers *and* no empty sites, the dimers can either only occupy even or only occupy odd edges. These dimers are strongly correlated. It is only when an empty site or a horizontal dimer is encountered that this restriction is relaxed. However, in this dense nematic phase, such events are *improbable*. On the other hand, it is *probable* to find an empty site in any interval of size $\approx \ell_0$, which is why correlations decay exponentially on that scale.

3.3 Ideas of the proof

It all starts with Heilmann and Lieb's result [HL79], which, I recall, ensures that most dimers are vertical. If *all* dimers were vertical, then the model would be solvable exactly: every column would be independent from the other columns (dimers are vertical and interact vertically), and each column is a one-dimensional system that can be solved by a transfer matrix technique.

The observables on the left sides of (9) and (10) can be computed exactly, so (9) and (10) can be checked explicitly. Incidentally, this is where $e^{\frac{3}{2}J}z^{\frac{1}{2}}$ appears, as the correlation length of the one-dimensional interacting dimer model. (As long as there are no horizontal dimers, the exponential decay in the horizontal direction could be replaced by a sharp step function, but this is not terribly important.) I will call the model with only vertical dimers the 'vertical dimer model', and the model with vertical and horizontal dimers the 'full model'. The proof is based on showing that the full model is *close*, in a sense to be made precise, to the vertical dimer model. Given Heilmann and Lieb's result, this is not a surprising claim.

Instead of focusing on dimer configurations, we will look at where *interfaces* between vertical and horizontal phases lie (see figure 5). The basic idea behind this approach is that interfaces should be unlikely, so there will be few of them and they will be far apart. Locating an interface is slightly ambiguous when there are monomers (empty sites) around (are monomers in the vertical or horizontal phase?), but this can be dealt with rather easily. With this in mind, we can rewrite the partition function (7), symbolically, as: (I will mostly be discussing the computation of the partition function, the correlation functions in theorem 3.1 can be computed in a similar way)

$$\frac{\Xi_{\Lambda_L}(z,J)}{Z_{\Lambda_L}(z,J)} = \sum_{\substack{\text{interface} \\ \text{configurations}}} e^{-W(\text{interfaces})} \left(\prod_{\substack{\text{interfaces} \\ \text{in configuration}}} \zeta(\text{interface}) \right) \tag{16}$$

where $Z_{\Lambda_L}(z,J)$ is the partition function of the vertical dimer model, ζ is the *effective activity* of each interface, and W is the *effective interaction* between interfaces. Once the interface configuration has been fixed, the partition function reduces to a product of partition functions in the areas between the interfaces. In these areas, dimers are either all horizontal or all vertical, so, using the exact solution of the vertical (horizontal) dimer model, one can compute these partition functions exactly. This yields an expression for the activity ζ and the interaction W.

The dominant factor in the activity comes from the interactions that are broken by the interface (since there is no force between a horizontal and a vertical dimer, there are no interactions along the interface, see figure 5). When the dimer density is large enough, each site contributes, roughly (ignoring empty sites), a half of an interaction (see figure 5). When an interface runs between two sites, it cuts one of the two half-interactions of the sites, so, roughly, the activity of an interface of length (that is, the number of edges the interface cuts) $|l|$ is

$$\zeta(\text{interface of length } |l|) \lesssim e^{-\frac{1}{2}J|l|}. \tag{17}$$

When J is large, this is a very small factor, which is consistent with the fact that interfaces are rare. In fact, if there were no interaction, then the system would reduce to a rarefied *gas* of interfaces, which, as was mentioned previously, can be solved by standard expansions (in this case, it would be called a *cluster expansion*, see, for instance, [Ru99, GBG04, KP86, BZ00] for details).

The interaction is much nastier. There are, essentially, two contributions, both of which cause trouble. The first is standard in problems which require Pirogov-Sinai theory [PS75, BKL84, KP84]. Interfaces have geometric constraints, which depend on whether they separate a vertical phase outside from a horizontal phase inside, or the other way around. It is, therefore,
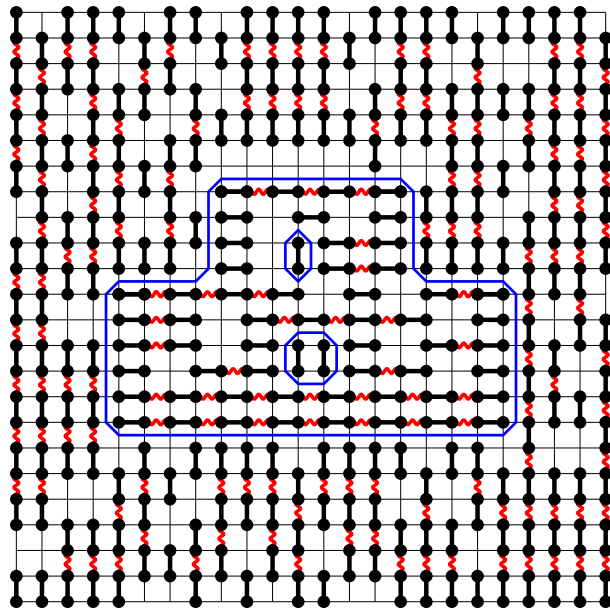
Figure 5: The interfaces in the configuration of figure 3. Each interface cuts interactions: whereas, in the uniform phases, if one were to neglect the presence of monomers (empty sites), there would be one half of an interaction for each site, whenever an interface runs between two sites, one of the half-interactions disappears.

important to keep track of the orientation of the phase outside an interface. However, if an interface has horizontal dimers outside it, it must be surrounded by an interface which has vertical dimers outside it. This induces an interaction between interfaces, which does not decay with the distance between them. A simple solution to this problem is to pretend that every interface has vertical dimers outside it. This can be done by rotating the dimers in each horizontal phase. However, the partition function of the horizontal dimer model and that of the vertical dimer model in an anisotropic region are different (there is no reason to assume that the horizontal regions are isotropic). It is expected that the ratio of these two partition functions is, at most, exponentially large in the size of the *boundary* of the region. If this is true, this rotating operation would yield a large factor $e^{|l|}$, which would be compensated by the gain $e^{-\frac{1}{2}J|l|}$ coming from the activity of the interface. To prove that the ratio is, indeed, exponentially large in the size of the boundary, we use the expression (16) (with Λ_L replaced by the horizontal region) to explicitly compute the ratio, and prove the appropriate bound.

The second contribution to the interaction comes from the fact that there are dimers between interfaces (see figure 6). Let us focus on the case in which these are vertical dimers. As was mentioned above, the correlation length in a column of vertical dimers is very large ($\ell_0 := e^{\frac{3}{2}J}z^{\frac{1}{2}} \gg 1$) which means that the partition function in a column of height $\ll \ell_0$ depends strongly on its height. This translates to an interaction between interfaces which decays exponentially with the distance between interfaces, but with a very small rate $\ell_0^{-1} \ll 1$. Therefore, the interaction is weak when the distance separating the interfaces is at least ℓ_0, but can be (and is) strong on shorter length scales. Overcoming this is the main difficulty (and novelty) of the proof.
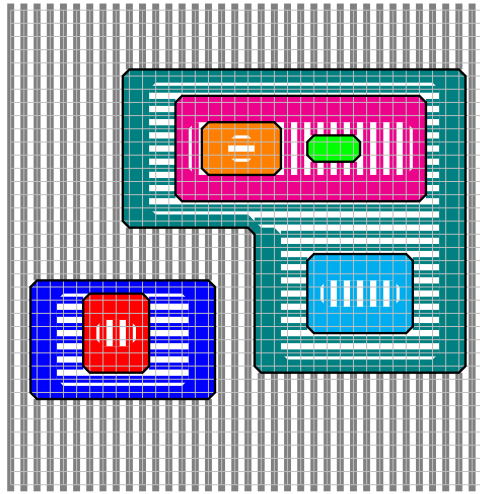
Figure 6: An interface configuration. The highlighted vertical and horizontal lines represent the interactions between the interfaces, which are mediated by the dimers between them.

In other words, there are two length scales in the problem: 1, which is the size of a dimer, and ℓ_0, the correlation length of the vertical dimer model. In addition, the system is very anisotropic (in a vertical phase, ℓ_0 appears in the vertical direction, and conversely in a horizontal phase). As a consequence, the two-scale nature of the problem cannot be dealt with by a coarse-graining procedure (as was done in [DG13]): the coarse-grained block would need to be a square of size ℓ_0 (because both columns of height ℓ_0 and rows of length ℓ_0 need to be taken into account). However, computing the partition function within this elementary block is already a hard problem. For the same reason, a reflection positivity argument similar to the one in [HL79] cannot be used.

At first glance, the situation is not terribly dire: the interaction is one-dimensional (vertical or horizontal) and, when summing over the positions of the interfaces, one only really needs to sum over the length of one interaction per interface. This yields a factor $\sum_l e^{-\ell_0^{-1}l} \approx \ell_0 \gg 1$, but each interface contributes, at least e^{-3J} (because its length is, at least, 6). Provided $z \ll e^{3J}$, $\ell_0 e^{-3J} \ll 1$, which is good. However, there also are columns of dimers that do not touch any interface, and go straight through Λ_L (see figure 6). These are called *trivial polymers*. If L is larger than ℓ_0, these cause no trouble, but, as was mentioned earlier, we need to compute the partition function for any horizontal region, which could contain columns of length $\ll \ell_0$. Therefore, we need to ensure that this situation never arises, which we accomplish by redefining the loop model to include interactions of length $< \ell_0$, thus removing them from the horizontal regions entirely.

After having dealt with the interactions as stated above, one can use a cluster expansion to compute the ratio in the left side of (16) as the exponential of an absolutely converging series. This actually tells us more than just the statement of theorem 3.1: the free energy and every correlation function can be computed, and estimated, in this way.

4 Concluding remarks

The Heilmann-Lieb model has now joined the few other models [BKL84, IVZ06, DG13] for which there is a proof of the existence of a nematic liquid crystal phase. There are a few open problems related to this result, some more important than others, which I would like to mention.

We have shown that the nematic phase appears in the range $J \gg z \gg 1$, but expect it to exist whenever $J \gg \max(1, -\log z)$. The reasons why we have restricted the range in such a way are extremely technical, so much so that I did not deem it appropriate to expound on them in this review, and I have little doubt that they can be relaxed.

As was mentioned above, Heilmann and Lieb [HL79] actually proved orientational ordering for *five* models, two of which are two-dimensional dimer models, two are three dimensional fourmer systems (a fourmer covers four vertices, and is shaped like a square), and the last is a three-dimensional dimer model. The proof that was discussed here only really works for the first of these five models, but it is not hard to imagine how to adapt it to the other two two-dimensional models. The situation could get more complicated in three dimensions. In fact, all four papers [BKL84, IVZ06, DG13, JL17b] cited here concern two-dimensional systems. Disertori, Giuliani and I are currently in the final stages of proving a result for a system of hard *plates* in three-dimensions. The plates we are considering are $k \times k^{\alpha} \times 1$ parallelepipeds with k large and $\alpha > \frac{3}{4}$, and they are allowed to be in any one of six orthogonal orientations. We show that a *uniaxial plate-nematic* phase emerges in a certain range of densities. In this phase, the short axis of the plates exhibits long range order, which means that the plates are, for the most part, horizontal. It would be rather interesting to prove the existence of a nematic phase in a three-dimensional *rod* model (note that Heilmann and Lieb did prove long range orientational order for a three-dimensional dimer model [HL79]).

The important open problem is to prove Onsager's heuristic result [On49], and show the existence of a nematic phase in a *continuous* system of rods. Since this would involve breaking a continuous symmetry, one would have to consider a three-dimensional continuum model, for which, as was just mentioned, there is no proof of a nematic phase even with discrete orientations.

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Editor-in-Chief, Journal of Mathematical Physics

AIP Publishing is seeking an Editor-in-Chief for *Journal of Mathematical Physics* (JMP).

Candidates are expected to be highly regarded scientists with research expertise in mathematical physics, broadly defined, such as partial differential equations, quantum mechanics, or representation theory. The successful candidate will bring vision and a passion for excellence to the role as well as strong leadership and interpersonal skills. Adopting a strategic view will be important, as will working collaboratively with a committed editorial team and the Publisher to translate that strategy into a vibrant editorial program.

The Editor-in-Chief will be eligible for an honorarium or compensatory payment to his or her institution. There are no geographic limitations to the Editor-in-Chief's location. The initial appointment is for five years with the possibility of renewal.

This is an exciting and challenging position leading the field's first journal dedicated to mathematical physics research. The search committee welcomes either applications or nominations of possible candidates. A nomination can be a short note suggesting a suitable candidate. An application should consist of a Curriculum Vitae and a statement of the applicant's vision and aspirations for the *Journal of Mathematical Physics.* This statement should be no more than 3 pages and should include a few paragraphs on your leadership experience and scientific accomplishments that make you uniquely qualified for the Editor-in-chief position. Applications or nominations received before **June 1, 2018** will receive full consideration. A Search Committee will advise AIP Publishing regarding suitable candidates for the position. Please send applications or nominations by e-mail with the subject line 'Editor Search' to the JMP search committee: jmp-journalmanager@aip.org.

About

Journal of Mathematical Physics is the first journal in the field of mathematical physics. It is a peer-reviewed journal with a focus on research areas that illustrate the application of mathematics to problems in physics, the development of mathematical methods for such applications, and for the formulation of physical theories. It published 463 articles in 2016 and was cited over 15,000 times. For more information on JMP visit jmp.aip.org. AIP Publishing is a wholly owned not-for-profit subsidiary of the American Institute of Physics (AIP). AIP Publishing's mission is to support the charitable, scientific and educational purposes of AIP through scholarly publishing activities in the fields of the physical and related sciences on its own behalf and on behalf of our publishing partners to help them proactively advance their missions. For more information visit publishing.aip.org.

Annales Henri Lebesgue

Translated from the French (*Gazette des Mathématiciens* **155**) by P. D. Hislop (University of Kentucky, Lexington, KY, USA), with the authorization of the *Gazette*.

1   Of fleeting and eternal mathematics

Mathematical theorems, their proofs and ideas that give them life, do not belong to anyone, not even their authors. Upon writing this, one can easily imagine the small smile appearing on the mouth of the reader, as if to betray the beginning of a small resistance to this statement. These somewhat exaggerated statements, however, are able to open the way to reflection. It is in the same way that from an indistinguishable block of stone, certain sculptors of Antiquity were able to carve beautiful and graceful figures rivaling nature. One has to mention the story of a certain Cypriot Pygmalion, who created such a lifelike sculpture and who loved it with such passion, that Venus gave her life[9]. There are also many legends in which men assume the character of demiurges[10] who give life to shapeless and inert objects. We remember, for example, the wise men Deucalion and Pyrrha, saved from the flood by Jupiter, who recreated humanity by throwing stones (probably clay stones) behind them[11]. Beyond the symbolism of these stories, it is in clay that the first mathematical calculations and the first recitation of these myths (eastern and western) were written, as if the authors were like the artisans and mythical creators. Of course, the Ancients not only wrote on tablets: the texts from Antiquity mention that the geometers drew their figures on sand in order to remember their reasoning and to transmit their ideas to future generations. In this manner, Socrates led a slave to publicly solve the problem of the duplication of a square[12]. But, dear reader, perhaps you did not choose to read this article in order that we snare you in Ovid's *Metamorphoses* or tell you Platonic reminiscences. What remains of the sand that anchored the geometers' arguments in the moment, or of the clay tablets of the scribes that was supposed to preserve their works?

2   A giant with feet of clay[13]

A little in the Platonic spirit that, worn down by time, joins desire and forgetfulness, let's leave antiquity and together leap over the centuries to the present. Chalk replaced sand, university amphitheaters and specialized schools welcome assemblies of students. Classes and recitations, at the core so fleeting, fight there regularly against the forgetfulness, and safeguard the fabulous sum of knowledge acquired since antiquity. This knowledge calls for our responsibility: the

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[9]Ovid: *Metamorphoses*, Book X, 243.

[10]From the Greek δημος (people) and εργον (work): originally meaning artisan, now creator

[11]Ovid: *Metamorphoses*, Book I, 325.

[12]Plato: *Meno*, 80d.

[13]Book of Daniel, verses 2.31–2.45, Bible.

Figure 7: Deucalion and Pyrrha, Rubens (1636), Prado Museum

question of scientific memory and its diffusion is urgent. But what has become of the clay tablets? Not so long ago, works of mathematics were exclusively published on paper. Perhaps, dear reader, you yourself have lingered in mathematics libraries and wandered from aisle to aisle in search of some elusive mathematical theorem? Perhaps you have sat in a comfortable chair, an article in one hand and a pen in the other, secretly charmed by this precious pleasure? Little by little, mathematical works have been digitalized. From now on, these works haunt many public and private servers; they are immediately accessible and are no longer weighed down by paper. Of course, they have not become pure spirits and printing them has not yet become a spiritual endeavor. They are still material and most of them are lodged in the servers of the commercial publishers that, by convention, we'll call Elsa and Sponz.

This commercialization exerts a continuous financial pressure on public institutions (laboratories, research centers, universities, etc.) serving science. Elsa and Sponz only care about the preservation of knowledge as an afterthought: they decide our needs to satisfy theirs. In this way, access to works of mathematics is not only for a fee, it is also submitted, for example, to the rule of the bouquet of journals: in order to access one journal, we must also access a collection of other journals that we might not desire. A research center might want a bouquet of roses and tulips, but the obscure florists require it to add some daisies, dandelions, and, sometimes, an entire haystack. Where is the scientific coherence in that strategy?[14] The more

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[14]One may consult the article by F. Hélein (*Gazette des Mathématiciens* **147**) in which the author considers this

that we consider these practices normal, the less we find them astounding, all the while we see considerable sums of money leave the budgets of research centers each year.

3   The birth of Annales Henri Lebesgue

Despite the fact that many colleagues regret this situation, many don't know how to change these publishing practices. They remark, however, that the authors of many of these articles are very often financed by public research agencies and that the editors and referees donate their work for free. How can one imagine that the fruits of this work are the source of private profits when the fruits depend on the public funds that finance the authors, editors, and referees? This question is all the more gripping when public means of distribution and conservation are readily available for mathematics articles. The recently-founded Centre Mersenne[15] is, in effect, able to furnish all the services necessary for the publication of mathematics articles: establishing a web site for the journal, standardization, distribution, and archiving of articles. To summarize: this can be done with public funds and for far less cost.

ANNALES
HENRI LEBESGUE
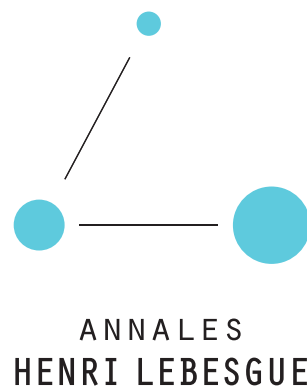
It is in this context that the *Annales Henri Lebesgue* came to life. For more than two years, researchers in the west of France have worked to create the journal. In the beginning, to be honest, the Annales was only a vague and elusive idea. However, the sentiment that these ideas incarnate: open access, free publication, and high standards, led them to return to these conversations with new vigor. The Centre Henri Lebesgue[16] aided in nourishing these ideas, molding them, and giving them structure.

We have contacted several colleagues in order to constitute a strong and motivated editorial board. These colleagues were enthusiastic to participate in this community movement of mathematicians supported by the Centre National de la Recherche Scientifique. The positive responses were overwhelming and we feared having too many editors! Zealous colleagues installed the Open Journals Systems[17] and adapted it to the needs of a mathematics journal. The

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question.

[15]The Centre Mersenne provides comprehensive scientific publishing infrastructure, and is a joint project of the CNRS and Université Grenoble Alpes: http://www.centre-mersenne.org/en/mersenne/

[16]https://www.lebesgue.fr/fr

[17]https://pkp.sfu.ca/ojs/

new journal was legally registered and a graphic artist designed a web site for article submission. The *Annales Henri Lebesgue*[18] became a real and independent journal.

4   Readership and Editorial Committee

The *Annales Henri Lebesgue* is a general mathematics journal, completely electronic, that strives to publish high-quality articles. It is freely accessible to all. Although the initiative was born in the west of France, the diverse editorial committee represents many fields of mathematics. Roughly half of the committee is composed of mathematicians from other regions, the majority of whom are foreigners. Of course, this new journal will not resolve all the problems of for-profit publishing by itself. It will join the collection of mathematics journals having these reasonable publishing practices[19]. The editorial committee will be renewed regularly in order to both involve other mathematicians as well as to cover, over time, a large spectrum of mathematical fields taking into account the broadness of mathematics.

5   Publish your papers in a free-access journal!

The *Annales Henri Lebesgue* is accessible and open to all, from advanced graduate students to experienced researchers. Many might hesitate to send a good paper to a newly-established journal whose reputation is not yet fully established. One might wonder if papers published in this journal will enjoy immediate recognition. One would be surprised, however, by the growing enthusiasm of mathematicians, especially young mathematicians, for these editorial initiatives and by their desire to be associated with the journal and these initiatives. In creating this journal, we are responding to this desire in offering them a journal worthy of their best papers. So it is without hesitation and with enthusiasm that we ask mathematicians to give life to the *Annales Henri Lebesgue.*

In fact, good reputations, for the most part, do not spring from the thigh of Jupiter: it is necessary to attract high-quality works, and that a serious editorial board be open to their evaluation. The research papers, in a certain sense, are more important than the journals themselves. Quality papers do not need journals to be well-written or to have an important scientific value. On the other hand, they need the care of the editorial board and quality referees. It is the work of these people who make, over time, the reputation of the journal. This idea has been key in the discussions around the creation of the *Annales Henri Lebesgue.*

Mathematicians have the means to supervise the totality of the publication process and to participate as well in a coherent editorial policy. The *Annales Henri Lebesgue* are among the clay stones that we wish to leave behind us. Contribute to giving them life!

> Xavier Caruso [xavier.caruso@normalesup.org] works at the Mathematic Institute of Rennes (IRMAR), France. His field of interest is number theory and especially p-adic numbers (p-adic Galois representations, explicit computations with p-adic objects).

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[18]https://annales.lebesgue.fr/index.php/AHL/
[19]Non-exhaustive lists may be found at http://cedram.org/ or http://www.ems-ph.org/journals/journals.php

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News from the IAMP Executive Committee

New individual members

IAMP welcomes the following new members

1. MS. COSTANZA BENASSI, University of Warwick, Coventry, UK

2. DR. FERENC BALOGH, John Abbott College, Sainte-Anne-de-Bellevue, Canada

3. DR. KARUPPAIYA SAKKARAVARTHI, Bharathidasan University, Tiruchirappalli, India

4. MR. CHRISTIAN BRENNECKE, University of Zurich, Switzerland

5. DR. JÉRÉMIE BOUTTIER, Commissariat á l' Énergie Atomique, Gif-sur-Yvette, France

6. PROF. CLAUDIO BONANNO, University of Pisa, Italy

7. PROF. EMIL PRODAN, Yeshiva University, New York, USA

8. DR. JACK SARKISSIAN, Algostox Trading, New York, USA

ICMP funding

Travel funding is still available for the XIX International Congress on Mathematical Physics to be held in Montreal on July 23-28, 2018, and for satellite events. Young researchers and others without their own grants are encouraged to apply for assistance to travel to the ICMP at

https://icmp2018.org/en/financial-aid-junior-participants.

IUPAP conference funding

The deadline for conference support through IUPAP is June 1, 2018. For further information and application instructions, see

http://iupap.org/sponsored-conferences/conference-policies/.

Dannie Heineman Prize for Mathematical Physics

Nominations for the Dannie Heineman Prize for Mathematical Physics are due by June 1, 2018. For further information, please consult

https://www.aps.org/programs/honors/prizes/heineman.cfm

Recent conference announcements

Quantum fields, scattering and spacetime horizons: mathematical challenges

May. 22-25, 2018. Les Houches School of Physics, France.
Organized by D. Häfner, M. Wrochna.

https://qft-horizons.sciencesconf.org

Third Great Lakes Mathematical Physics Meeting

June 22-24, 2018. Michigan State University, East Lansing, Michigan, USA.

https://sites.google.com/a/msu.edu/glamp2018/home

Quantum Roundabout

July 11-13, 2018. University of Nottingham, UK.
Organized by B. Morris, C. Napoli, G. Nocerino, B. Xu.

This conference is partially supported by IAMP.

https://quantumroundabout.weebly.com

Higher Algebra and Mathematical Physics

August 13-17, 2018.

This is a double conference, taking place at two locations:
MPIM Bonn (Germany) and Perimeter Institute, Waterloo (Canada)

Analysis of Differential Operators on Manifolds

Sept. 24-26, 2018. University of Freiburg (Germany).
Organized by K. Fedosova, N. Große, S. Murro.

http://home.mathematik.uni-freiburg.de/murro/workshop/index.html

Random Matrices, Integrability and Complex Systems

Oct. 3-8, 2018. Research Workshop of the Israel Science Foundation.
Yad Hashmona, Judean Hills (Israel).

This conference is partially supported by IAMP.

Progress and Visions in Quantum Theory in View of Gravity

Oct. 1-5, 2018. Max Planck Institut, Leipzig, Germany.
Organized by F. Finster, D. Giulini, J. Kleiner, J. Tolksdorf.

This conference is partially supported by IAMP.

https://www.mis.mpg.de/calendar/conferences/2018/qft2018.html

Random Matrices, Integrability and Complex Systems

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This conference is partially supported by IAMP.

School on Mathematics of Non-equilibrium Statistical Mechanics

Celebrating the 60-th birthday of Claude-Alain Pillet.

Oct. 24-26, 2018. CRM, Montreal, Canada.
Organized by J.-M. Barbaroux, H. Cornean, V. Jaksic, F. Koukiou, A. Shirikyan.

This conference is partially supported by IAMP.

Spectral Theory and Mathematical Physics 2018 (STMP 2018)

Dec. 3-7, 2018. Santiago de Chile.
Organized by C. Fernández, P. Miranda, N. Popoff, G. Raikov.

This conference is partially supported by IAMP.

https://sites.google.com/view/stmp2018/home

Results in Contemporary Mathematical Physics

A conference in honor of Rafael Benguria.

Dec. 17-21, 2018. Santiago de Chile.
Organized by E. Stockmeyer, H. Van Den Bosch

This conference is partially supported by IAMP.

Open positions

Postdoctoral Fellowship in Mathematical Physics, Czech Academy of Sciences

The Mathematical Physics Group in the Department of Theoretical Physics of the Nuclear Physics Institute of the Czech Academy of Sciences in Rez invites applications for a postdoctoral research position in Mathematical Physics within the scope of the project "Mathematical aspects of new challenges in physics", with a special emphasis put on phenomena encountered in the newly developing fields of quantum mechanics with non-Hermitian operators, metamaterials and spectral-geometric properties of nanostructures.

The position is for one year, starting before the end of 2018, with the extension option for a second year upon mutual agreement. The gross salary is approximately 30,000 CZK per month plus some extras for housing. There are no teaching duties associated with the position.

Applicants should have a PhD in Mathematics or Theoretical Physics (or equivalent) obtained preferably after January 1, 2014. They must show very strong research promise in at least one of the following research domains: geometric analysis, spectral theory, partial differential equations, semiclassical analysis. Previous experience in the area of the project is an advantage but not necessary.

The applications including cv (including list of publications), brief research statement (past, current and future interests) two letters of recommendation should be sent by e-mail to Pavel Exner, exner@ujf.cas.cz, and David Krejcirik, david@ujf.cas.cz. All the documents should be submitted as pdf files. The letters of recommendation should be sent directly by the persons providing the reference. Complete application packages should be received by May 31, 2018. For any further information about the fellowship please contact Pavel Exner and David Krejcirik at the above-mentioned e-mail addresses.

Professor Position in Mathematical Physics at Ecole Polytechnique

The Physics department of Ecole Polytechnique, Palaiseau, France, is advertising the opening of a Professor position in Mathematical Physics, with emphasis on statistical physics and quantum field theory. The deadline for applications is June 1, 2018.

The successful candidate will join the Physics Department and launch an independent research program at the Center for Theoretical Physics (CPHT). A strong commitment to excellence in teaching at all levels is expected. The position will be filled at the level of tenured Associate Professor or Full Professor, depending on seniority of the candidate. Generous startup funds, as well as administrative support and office space will be provided. Applicants are kindly asked to submit a CV, a list of publications - with brief comments on the five most significant ones - and a statement of research interests, including a sketch of projects to be developed on campus. They are asked to provide the names of three referees. Application packages in a single pdf file should be sent by email to secretariat-depphys@polytechnique.fr by June 1, 2018.

Ecole Polytechnique is a leading higher education and research institution in France. It is an equal-opportunity employer. It is located in Palaiseau, close to Paris, and a number of research institutions and facilities related to the field of this opening are located in its vicinity. For further information you can contact the President of the Physics Department, Christoph Kopper, Christoph.Kopper@polytechnique.edu, and the Director of CPHT, Jean-René Chazottes, Jean-Rene.Chazottes@cpht.polytechnique.fr.

For more information on these positions and for an updated list of academic job announcements in mathematical physics and related fields visit

http://www.iamp.org/page.php?page=page_positions

Benjamin Schlein (IAMP Secretary)

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