Instanton Operators and Enhanced Symmetries

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Challenges to quantum field theory in higher dimensions

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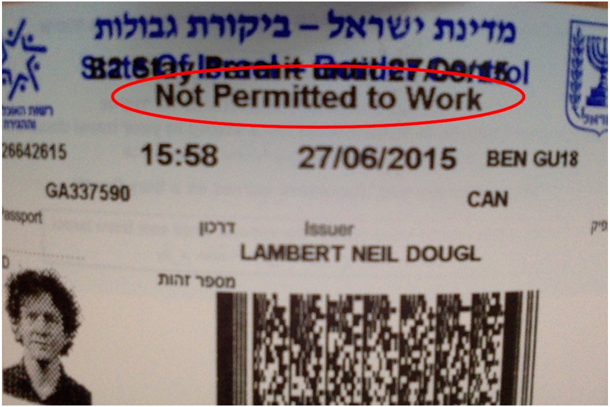
w/ C. Papageorgakis and M. Schmidt-Sommerfeld 1412.2789

KING'S College LONDON

I'd like to thank the organisers for my invitation, although I am not sure that I am allowed to give this seminar:



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On the other hand, there is always hope that the authorities do not view what we do as work.

Motivation

String/M-theory predicts the existence of interacting SCFTs in 5D and 6D [Witten,Seiberg,...]

Provide UV completions for a variety of 5D gauge theories

UV theories enjoy enhanced flavour or Lorentz symmetry, *e.g.*:

- ◇ $\mathcal{N} = 1$ SYM theories with $N_f \leq 7$
  $\Rightarrow$ $\mathcal{N} = 1$ 5D SCFT with E_{N_f+1} symmetry
- ◇ $\mathcal{N} = 2$ SYM theory $\Rightarrow$ $(2, 0)$ 6D SCFT

Can be seen in index calculations [Kim^2-Lee, Bashkirov, Hwang-Kim^2-Park,...]

Draw upon our knowledge of 3D theories where local monopole operators $\mathcal{M}_{Q_M}$ play important role:

Definition

$$\langle \mathcal{M}_{Q_M}(x)\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)\rangle = \int_{\frac{1}{2\pi}\oint_{S^2_x} F=Q_M} [DXDADψ]\ \mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)e^{-S_E}$$

i.e. inserts a Dirac monopole at x.

$\Rightarrow$ Global symmetry and susy enhancement in Chern-Simons CFT's [Borokhov-Kapustin-Wu, Gaiotto-Witten, ABJM, ...]

In particular for M2-branes one finds that the momentum around the M-theory circle is

$$P_{11} = \frac{k}{2\pi}\mathrm{Tr}\int F$$

So monopole operators ‘insert’ units of spacetime momentum

In 5D SYM the extra momentum appear as a central charge

$$P_5 = -\frac{1}{2g^2}\mathrm{Tr}\int F \wedge F$$

Indeed M2-branes can blow-up to M5-branes and monopoles are mapped to instantons [NL, Nastase, Papageorgakis]

This motivates an analogous notion of and instanton operator...

Outline

- ◇ Definition of instanton operators
- ◇ Supersymmetry
- ◇ Application: Lorentz symmetry enhancement
- ◇ Application: Flavour symmetry enhancement
- ◇ Application: Higgs Branch quantum corrections
- ◇ Chern-Simons terms
- ◇ Summary

Definition

5D gauge theories have a topologically conserved current

$$J = \frac{1}{8\pi^2} \mathrm{Tr} \star (F \wedge F)$$

Charged BPS-particle solutions: instanton solitons

Both global and Lorentz symmetry enhancement associated with instanton charge.

Intuatively: An instanton operator is a local operator which creates instanton-solitons out of vacuum

The OPE of this current with $\mathcal{I}_n(0)$ is given by

$$J^\mu(x)\mathcal{I}_n(0) \sim \frac{3n}{8\pi^2} \frac{x^\mu}{|x|^5} \mathcal{I}_n(0) + \cdots$$

More formally: Instanton operators, $\mathcal{I}_n(x)$, modify boundary conditions for gauge field at infinity in Eucliden path integral:

$$\langle \mathcal{I}_n(x)\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)\rangle = \int_{\frac{1}{8\pi^2}\mathrm{Tr}\oint_{S^4_x} F\wedge F=n}[DXDAD\psi]\;\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)e^{-S_E}$$

Fields need to satisfy classical eom near insertion point in $\mathbb{R}^5$

$$D^\mu F_{\mu\nu} = 0\;, \qquad D_{[\mu}F_{\nu\lambda]} = 0$$

but with non-vanishing

$$I = \frac{1}{8\pi^2}\mathrm{Tr}\oint_{S^4} F\wedge F$$

In spherical coordinates a simple solution is given by taking $A_r = F_{ri} = 0$ and the angular components satisfying

$$F = \pm \star_{S^4} F$$

This solution for $\mathrm{SU}(2)$ theory was found long ago by Yang, as static $\mathrm{SO}(5)$-symmetric particle in 6D ⇒ Yang monopole

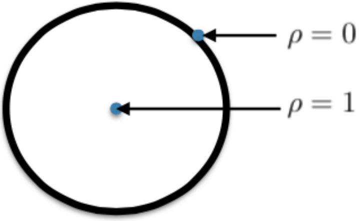
A DBI generalisation for $\mathrm{SU}(N)$ later given by Constable-Myers-Tafjord in context of D1$\perp$D5 intersections

Alternatively: Instanton operators defined by the condition that the gauge field has a Yang monopole singularity at the insertion point

Instantons on S^4 can be straightforwardly constructed by stereographic projection from $\mathbb{R}^4$

So the moduli space on S^4 is the same as on $\mathbb{R}^4$, except that there are no moduli from global gauge transformations. Fermion zero modes are also the same.

For example, for $n = 1$ there is a size modulus ρ as well as a centre of mass modulus $x_0 \in S^4$. Moduli space $\sim H^5$:



The solutions exhibit some amusing properties:

$$F \wedge F = \frac{8\rho^4 \ \sum_{i=1}^{3} T_i^2}{\left(1+\rho^2+(1-\rho^2)\cos\theta^1\right)^4} dvol(S^4)$$

with $[T_i, T_j] = 2i\epsilon_{ijk}T_k$ an $N \times N$ representation of $\mathfrak{su}(2)$

When $\rho = 1$ this reduces to the $\mathrm{SO}(5)$-symmetric

$$F \wedge F = \frac{1}{2}\sum_{i=1}^{3} T_i^2 dvol(S^4)$$

When $\rho \to 0$ this reduces to a delta-function at $\theta^1 = \pi$

When the T_i are irreps of $\mathfrak{su}(2)$ then

$$\sum_{i=1}^{3} T_i^2 = (N^2 - 1)\ \mathbb{1}_{N\times N}$$

and $F \wedge F$ is gauge invariant without the trace

By further considering (for generic ρ)

$$\begin{aligned} I &= \frac{1}{8\pi^2}\mathrm{Tr}\int F\wedge F \\ &= \frac{(N^2-1)}{6}\mathrm{Tr}\ \mathbb{1}_{N\times N} \\ &= \frac{N(N^2-1)}{6} \end{aligned}$$

Supersymmetry?

Instanton-solitons are supersymmetric but what about Instanton operators?

The supervariation of a fermion in the background of the Yang monopole is

$$\delta\psi = \frac{1}{2}\Gamma^{\mu\nu}F_{\mu\nu}\Gamma_5\varepsilon$$

The ε are 32-component spinors which also satisfy

$$\Gamma_{012345}\varepsilon = \varepsilon$$

Using that $F = \pm \star_{S^4} F$ one can satisfy $\delta\psi = 0$ if

$$\Big(\frac{x^\mu}{|x|}\Gamma_\mu\Gamma_5 \pm i\Big)\varepsilon = 0 \qquad (1)$$

This cannot hold for all x^μ and supersymmetry is broken.

But an exception to this is the case of $\rho = 0$. Here $F = 0$ except at one point on the S^4. Then we only need the projector (1) at one value of $x^\mu/|x|$.

- These are 1/2 BPS.
- Can also include an anti-instanton at the antipodal point.

They can be made supersymmetric by employing an R-symmetry twist [Rodríguez-Gómez, Schmude]

The 5D susy algebra contains a central charge

$$Z_5 = -\frac{1}{2g^2}\mathrm{Tr}\int F \wedge F$$

The instanton-solitons with charge n can be found after projecting out all other states

$$|n\rangle = \lim_{\tau\to\infty} e^{-(H-Z_5)\tau}\mathcal{I}_n(0)|0\rangle$$

Analogous to 3D where monopole operators and BPS vortices annihilated by different combinations of supercharges [Intriligator-Seiberg]

Application: Lorentz symmetry enhancement

The $(2,0)$ SCFT is completely described in terms of operator spectrum and OPE coefficients.

Consider

$$\langle \hat{\mathcal{O}}_1(\hat{x}_1)\hat{\mathcal{O}}_2(\hat{x}_2)\rangle = \frac{c_{12}}{|\hat{x}_1-\hat{x}_2|^{\Delta_1+\Delta_2}}$$

$\Rightarrow$ Use instanton operators to relate 6D to 5D $\mathcal{N}=2$ correlators by compactifying on S^1

Implement this by viewing S^1 as orbifold $\mathbb{R}/\Gamma$ and

$$\Gamma : (x, y) \mapsto (x, y + 2\pi R n) \text{ with } n \in \mathbb{Z}$$

For an operator on $\mathbb{R}^5 \times S^1$ write

$$\mathcal{O}(x, y) := \sum_{n\in\mathbb{Z}} \hat{\mathcal{O}}(x, y + 2\pi R n) = \sum_{m\in\mathbb{Z}} e^{imy/R} \mathcal{O}_m(x)$$

where $\mathcal{O}_m$ are Fourier modes

⇒ Not all operators can be related in this way as they have to satisfy linear equations - but it will give us something to work with.

Starting from the two-point function $\langle \mathcal{O}_1(x_1, y_1)\mathcal{O}_2(x_2, y_2)\rangle$, using the identification $R = g^2/4\pi^2$ one arrives at the following:

- For the zero modes (pertubative)

$$\langle \mathcal{O}_0(x_1)\mathcal{O}_0(x_2)\rangle = -\frac{c_{12}\pi^{\frac{3}{2}}}{g^2}\frac{\Gamma(\frac{\Delta_1+\Delta_2-1}{2})}{\Gamma(\frac{\Delta_1+\Delta_2}{2})}\frac{1}{|x_{12}|^{\Delta_1+\Delta_2-1}}$$

- For the non-zero modes (non-pertubative)

$$\langle \mathcal{O}_n(x_1)\mathcal{O}_m(x_2)\rangle = 0\ ,\ n \neq m$$

$$\langle \mathcal{O}_n(x_1)\mathcal{O}_{-n}(x_2)\rangle$$

$$= -\frac{c_{12}\pi^{\frac{\Delta_1+\Delta_2}{2}}}{2|n|\Gamma(\frac{\Delta_1+\Delta_2}{2})}\left(\frac{2\pi|n|}{g^2|x_{12}|}\right)^{\frac{\Delta_1+\Delta_2}{2}} e^{-\frac{4\pi^2}{g^2}|n||x_{12}|}\left(1+\mathcal{O}\Big(\frac{g^2}{|n||x_{12}|}\Big)\right)$$

The exponential terms looks like Yukawa interaction from exchanging particles of mass $|n|/R$.

We propose that the non-zero modes are related to instanton operators

$$\mathcal{O}_n(x) := \mathcal{I}_n(x)\mathcal{O}_0(x)$$

Compatible with:

- ◇ Momentum conservation on S^1
- ◇ The characteristic e^{-S_n} dependence of the non-zero mode correlators with

$$S_n = \frac{4\pi^2}{g^2}|n||x_1 - x_2|$$

Momentum Conservation

A single instanton operator has divergent action - coming from behaviour at infinity not from the singularity.

Introducing a long distance cut-off$R \to \infty$ we find

$$S_I = \frac{4\pi^2|n|}{g^2}R \to \infty$$

This implies that for correlators

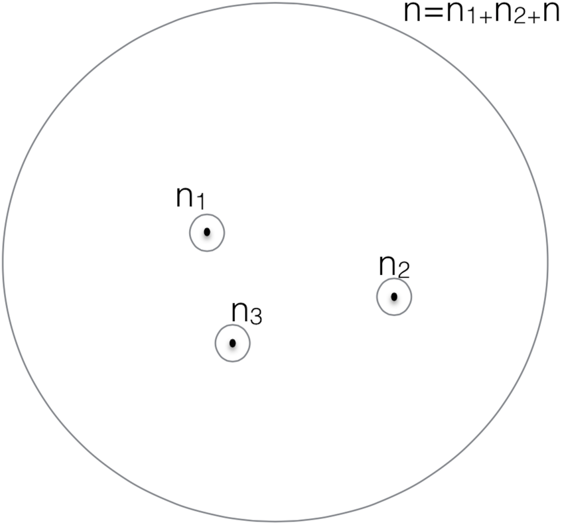
$$\langle \mathcal{O}_{n_1 1}(x_1)\mathcal{O}_{n_2 2}(x_2)...\mathcal{O}_{n_k k}(x_2)\rangle = 0$$

unless

$$\sum_{i=1}^{k} n_i = 0$$

$\Rightarrow$ Momentum conservation on S^1

$n=n_1+n_2+n_3$



Non-perturbative dependence

Finally, consider a correlator involving insertions of two instanton operators at x_1 and x_2 with charges n and $-n$.

By dimensional analysis the minimum action is:

$$S_{min} = \frac{1}{4g^2} \int d^5x F_{\mu\nu} F^{\mu\nu} = \frac{K}{g^2} |x_1 - x_2|$$

The coefficient K can be pinned down by taking the second point to ∞ and comparing with $S_I = \frac{4\pi^2 |n|}{g^2} R$ as $R \to \infty$

We get: $S_{min} = \frac{4\pi^2}{g^2} |n| |x_1 - x_2| = S_n$ as required

Application: flavour symmetry enhancement

Instanton Operators also play a role enhancing the flavour symmetries of $\mathcal{N} = 1$ 5D theories

Fermion zero-modes play a key role: there are 8 zero modes from gluinos plus additional matter zero-modes

Leads to a $2^4 = 16$-dim multiplet of operators with instanton charge in spinor reps. the of the $SO(2N_f)$ flavour group.

- ◇ Includes extra conserved currents [Tachikawa]
- ◇ Just right to enhance $SO(2N_f) \times U(1)$ to E_{N_f+1}. [Tachikawa] as well as in other gauge groups [Zafrir],[Yonekura]

Application: Higgs Branch

The Hilbert Series counts the number of operators on the Higgs branch and their relations. Can be computed for the UV theory from a knowledge of the moduli space

- Determined as the one-instanton moduli space of the flavour group, *i.e.* therefore different in the IR and UV

Can decompose into fugacities of the "manifest" flavour group plus instanton operator contributions [Cremonesi, Ferlito, Hanany, Mekareeya].

- leads to modification of the relations of chiral operators

Chern-Simons terms

In $\mathcal{N} = 1$ theories we can also add Chern-Simons terms

$$S_{CS} = \frac{k}{24\pi^2} \mathrm{Tr} \int (F \wedge F \wedge A + \frac{i}{2} F \wedge A \wedge A \wedge A - \frac{1}{10} A \wedge A \wedge A \wedge A \wedge A)$$

In the presence of such a term the instanton operators are not always gauge invariant:

$$\delta S_{CS} = \frac{k}{8\pi^2} \mathrm{Tr} \int F \wedge F \wedge \delta A$$

and by considering $\delta A = D\omega$ with $\omega = 0$ at ∞ one finds

$$\delta S_{CS} = -\frac{k}{8\pi^2} \mathrm{Tr} \left[\omega(x) \oint_{S^4_x} F \wedge F \right]$$

Inserting this into a correlator

$$\begin{aligned}
&\delta\langle \mathcal{I}_n(x)\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)\rangle = \\
&= -\int_{\frac{1}{8\pi^2}\mathrm{Tr}\oint_{S^4_x} F\wedge F = n} [DXDAD\psi]\,\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)\delta S_{CS} e^{-S_E} \\
&= \frac{k}{8\pi^2}\mathrm{Tr}\left[\omega(x)\oint_{S^4_x} F\wedge F\right]\langle \mathcal{I}_n(x)\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)\rangle
\end{aligned}$$

and one needs to understand the properties of

$$Q_I = \frac{1}{8\pi^2}\oint_{S^4_x} F\wedge F$$

Could this play a similar role to $Q_M = \frac{1}{2\pi}\oint_{S^2} F$ in GNO?

For the single instanton irreducible case

$$Q_I = \frac{1}{6}\sum_{i=1}^{3} T_i^2 = \frac{1}{6}(N^2-1)\ \mathbb{1}_{N\times N}$$

and Q_I is gauge invariant.

Introducing a basis t_a of full gauge group Lie algebra with metric $\kappa_{ab} = \mathrm{Tr}(t_a t_b)$ and symmetric tensor

$$d_{abc} = \mathrm{Tr}(t_{(a}t_{b)}t_c)$$

more generally we have

$$\delta\mathcal{I}_n = k d_{abc} Q_I^{ab} \omega^c \mathcal{I}_n$$

Vanishes in some cases (no 4D anomaly: $d_{abc} = 0$, *e.g.* pseudo real)

Summary

- Introduced instanton operators in 5D gauge theories
- Looked at some properties
- Argued that they are responsible for enhancement of Lorentz symmetry
- Used to relate correlators of $(2, 0)$ theory on $\mathbb{R}^5 \times S^1$ and $\mathcal{N} = 2$ SYM in 5D
- Flavour symmetry enhancement [tachikawa][Zafrir],[Yonekura]
- Instanton Operators correct the Higgs Branch [Cremonesi, Ferlito, Hanany, Mekareeya]