Gauge theories for 5d/6d solitons

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Challenges to QFT in higher dimensions, Technion
29 June 2015

The talk will overview some ideas discussed in :

Chiung Hwang, Joonho Kim, SK, Jaemo Park,
"General instanton counting and 5d SCFT" arXiv: 1406.6793

Joonho Kim, SK, Kimyeong Lee, Jaemo Park, Cumrun Vafa,
"Elliptic genus of E-strings" arXiv:1411.2324.

Abhijit Gadde, Babak Haghighat, Joonho Kim, SK, Guglielmo Lockhart, Cumrun Vafa,
"6d string chains," arXiv:1504.04614.

SK, Jaemo Park, "6d SU(3) minimal strings," work in progress.

Joonho Kim, SK, Kimyeong Lee, work in progress.

and also closely related to :

Haghighat, Klemm, Lockhart, Vafa, "Strings of minimal 6d SCFTs" arXiv:1412.3152.
Haghighat, Iqbal, Lockhart, Kozcaz, Vafa, "M-strings," arXiv:13nn.nnnn.
H.-C. Kim, SK, E. Koh, K. Lee, S. Lee,
"On instantons as Kaluza-Klein modes of M5-branes," arXiv:1110.nnnn.

Higher dimensional CFTs

- Constructed indirectly from string theory:
- brane systems in flat spacetime
- geometric engineering: e.g. M-/F-theory on singular CY_3

- No microscopic methods, apart from a small set of problems dictated by symmetries/dualities: SUSY, conformal symmetry, anomalies, AdS, …

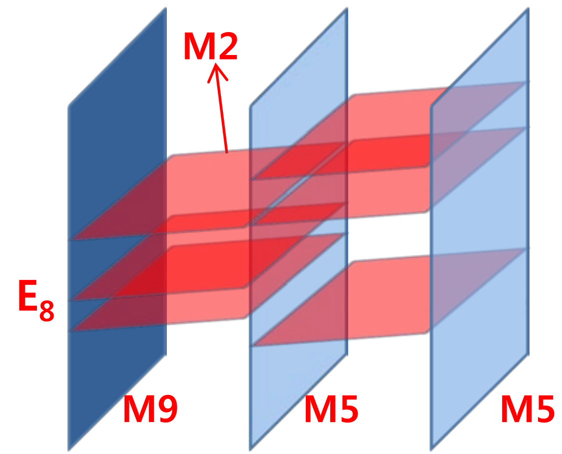
- Low E effective theories often provide good intuitions (e.g. 5d SYM)
- But hard to rigorously assess how reliable they are, beyond the cutoff scale.

- Goal: discover & study 1d/2d gauge theory descriptions…
- … **UV complete**
- … complete worldline/sheet descriptions of massive/tensionful objects in decoupling limits
- … combined with intuitions from 5d SYM, address more nontrivial CFT observables.

Setting: 6d CFT in Coulomb phase

- Coulomb branch: scalar VEV in tensor supermultiplet.

$$B_{\mu\nu} \text{ with } H = dB = \star dB \ , \quad \Psi^A \ , \quad \Phi$$



- **Self-dual strings**: similar to W-bosons in Yang-Mills
- (2,0) theory "M-strings": M2's suspended between M5's
- (1,0) theory for M5-M9: "E-strings"
- More challenging 6d (1,0) strings (later)

- These strings host 2d interacting SCFTs, at E << (tension)$^{1/2}$
- 2d gauge theories: weakly coupled in UV, flowing in IR to these SCFTs.
- Coulomb phase observables:
- E.g. elliptic genus: Z[T^2] ~ 6d QFT partition function on R^4 x T^2
- also related to the symmetric phase observables

$$Z[S^5\times S^1] = \int [d\phi] e^{-\frac{4\pi^2 \mathrm{tr}(\phi^2)}{\beta\omega_1\omega_2\omega_3}} Z^{\mathbb{R}^4\times T^2}\left(q = e^{-\frac{4\pi^2}{\beta\omega_1}}, \epsilon_1 = \frac{\omega_2-\omega_1}{\omega_1}, \epsilon_2 = \frac{\omega_3-\omega_1}{\omega_1}, \frac{m_i}{\omega_1}, \frac{\phi}{\omega_1}\right) Z^{\mathbb{R}^4\times T^2}(2) Z^{\mathbb{R}^4\times T^2}(3)$$

[H.-C. Kim SK] [Lockhart, Vafa] [H.–C. Kim, J. Kim, SK]
[H.-C. Kim, S.-S. Kim, SK, K. Lee] [Qiu, Zabzine]

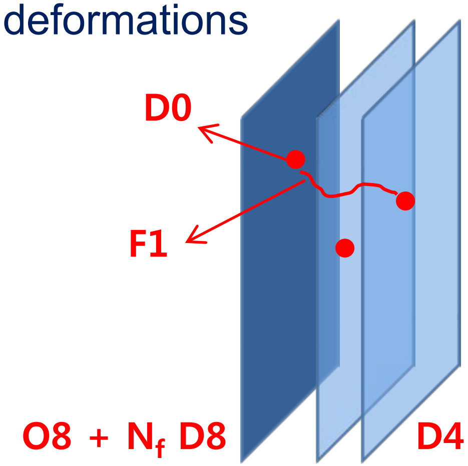
Setting: 5d CFT in Coulomb phase

- Coulomb branch: scalar VEV in vector supermultiplet.

$$A_\mu \ , \quad \lambda^A \ , \quad \phi$$

- Massive particles in deformed CFT: Coulomb branch, relevant deformations
- Heavy in IR, light in UV

- Classic example [Seiberg] '96: 5d CFT from D4-D8-O8
- Particles: D0 (~instantons) & F1 (~W-bosons)
- Many other examples from brane/geometric engineerings
- Infinitely many particles become light at UV fixed point



- These particles often host nontrivial superconformal QMs, at E << (mass)
- 1d gauge theories: weakly coupled in UV, flowing in IR to these SCQMs.
- Coulomb phase observables: Witten index, related to the symmetric phase observables

$$Z_{S^4\times S^1}[x=e^{-\epsilon_+}, y=e^{-\epsilon_-}, m_i, q] = \int [d\alpha] Z_{\rm Nek}^{\mathbb{R}^4\times S^1}(q,\epsilon_{1,2},m_i,\alpha) Z_{\rm Nek}^{\mathbb{R}^4\times S^1}(q^{-1},\epsilon_{1,2},m_i,\alpha)$$

[H.-C. Kim, S.-S. Kim, K. Lee]

Effective SYMs & solitons

- Relevant deformations of 5d SCFT: 5d N=1 SYMs

$$\mathcal{L} \leftarrow -\frac{1}{4g_{YM}^2}\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + (\text{other relevant deformations})$$

- Some particles are "**solitonic**": Yang-Mills instantons

$$F_{\mu\nu} = \star_4 F_{\mu\nu} \qquad\qquad k \equiv \frac{1}{8\pi^2}\int \mathrm{tr}\left(F\wedge F\right) \in \mathbb{Z}$$

- Coulomb branch of 6d CFTs w/ gauge symmetry: 6d N=1 SYMs
- anomaly-free: Green-Schwarz mechanism

$$\text{tree level anomaly}: \ \delta S \sim c\mathrm{tr}(\epsilon F)\wedge \mathrm{tr}(F\wedge F) \qquad\qquad S \leftarrow \int B\wedge \mathrm{tr}(F\wedge F)$$

- Self-dual strings are instanton string solitons

(Some 6d CFTs don't come with gauge symmetry, like M-/E-string theories. Although they don't admit a notion of "soliton," these strings can be treated similarly with 2d gauge theories.)

Instanton solitons & UV complete descriptions

- From the soliton viewpoint, one can use the moduli space approximation
- (0,4) SUSY non-linear sigma model w/ instanton moduli space as target.

$$\mathcal{L}_{1d/2d} = -g_{ij}(X)\partial^\mu X^I \partial_\mu X^j + \cdots$$

- The sigma model is incomplete: small instanton singularity. For SU(N) single instanton,

$$ds^2 = g_{MN}(X)dX^M dX^N = \underbrace{\boxed{ds^2(\mathbb{R}^4)}}_{\text{center-of-mass}} + \underbrace{d\lambda^2}_{\text{instanton “size”}} + \lambda^2\left[\underbrace{\boxed{ds^2(S^3/\mathbb{Z}_2)}}_{SU(2)\text{ orientation}} + \underbrace{\boxed{ds^2(\mathcal{M}_{4N-8})}}_{\frac{SU(N)}{SU(2)\times U(N-2)}}\right]$$

- Reflects UV incompleteness of 5d/6d SYM.

- Lagrangian UV completions using (0,4) gauge theories, in 1d/2d.

- Sometimes, UV completion unknown: e.g. exceptional instantons
- Sometimes, the UV completion is subtle, ambiguous or not unique. Different extra branch touching the singularity (definition of QFT unaffected), or extra d.o.f. stuck at singularity.

1d/2d gauge theories for 5d/6d solitons

- For instantons, this is the well-known ADHM descriptions (but subtleties later)
- Construction of instantons: E.g. for SU(N) k-instantons,

$$A_\mu = i v^\dagger \partial v \quad (v_{(N+2k)\times N},\ v^\dagger v = \mathbf{1}_{N\times N})$$

$$U^\dagger v = 0\ ,\quad U_{(N+2k)\times 2k} = \begin{pmatrix} \bar{q}_{N\times 2k} \\ (a_{\alpha\dot\beta})_{k\times k} - x_{\alpha\dot\beta}\otimes \mathbf{1}_{k\times k} \end{pmatrix} \qquad D^I \equiv q_{\dot\alpha}(\tau^I)^{\dot\alpha}{}_{\dot\beta}\bar{q}^{\dot\beta} + (\tau^I)^{\dot\alpha}{}_{\dot\beta}[a_{\alpha\dot\alpha}, a^{\alpha\dot\beta}] = 0$$

- (0,4) gauge theories for light open string modes on "Dp-D(p+4)-branes"

$$\mathcal{L} = \frac{1}{g^2_{\rm 1d/2d}}\mathrm{tr}\left[-\frac{1}{2}(D_\mu a_m)^2 - |D_\mu q_{\dot\alpha}|^2 - \frac{1}{2}(D^I)^2 - \frac{1}{4}(F_{\mu\nu})^2 + \text{fermions}\right]$$

- 2d gauge theory descriptions extend to 6d strings without SYM soliton picture
- Very powerful approach: Weakly coupled in UV. Ideal to study RG protected quantities (such as SUSY partition functions, indices, …)
- E.g. Nekrasov's instanton partition function…

Observables in (0,4) gauge theories

- In the remaining time, I'll explain aspects of the 2d (0,4) "ADHM gauge theories."
- Weakly-coupled in UV and describes strongly coupled IR SCFTs.

- Some RG invariant observables:

- 2d: elliptic genus partition function [Benini, Eager, Hori, Tachikawa] 1 & 2(2013)

$$Z_{2\mathrm{d}}(\tau,\epsilon_{1,2},\mu) = \mathrm{Tr}\left[(-1)^F e^{2\pi i\tau H_L + 2\pi i\bar{\tau}H_R} e^{2\pi i\epsilon_1(J_1+J_R)} e^{2\pi i\epsilon_2(J_2+J_R)} \cdot e^{2\pi i\mu_i(\mathrm{flavor}_i)}\right]$$

- 1d: Witten indices [Nekrasov] (2002) [Hwang, J. Kim, SK, Park] [Cordova, Shao] [Kim, Hori, Yi] (2014)

$$Z_{1\mathrm{d}}(\epsilon_{1,2},\mu) = \mathrm{Tr}\left[(-1)^F e^{-\beta\{Q,Q^\dagger\}} e^{-\epsilon_1(J_1+J_R)} e^{-\epsilon_2(J_2+J_R)} \cdot e^{-\mu_i(\mathrm{flavor}_i)}\right]$$

- All expressed in terms of contour integrals, given by "Jeffrey-Kirwan residue sum"

- Heavily used to cross-check the subtle gauge theory constructions
- Explores the 5d/6d CFT spectra (often combined with curved space partition function)

6d CFTs & strings from F-theory

- F-theory on elliptically fibered singular CY_3 (T^2 fibration over a base B_4)
- Blowing-up singularities ~ 6d tensor branch: classify 6d CFTs [Heckman, Morrison, Vafa]

- Basic "building blocks": **minimal SCFTs**
- Put F-theory on $B_4 = O(-n) \to P^1$ & maximally Higgs

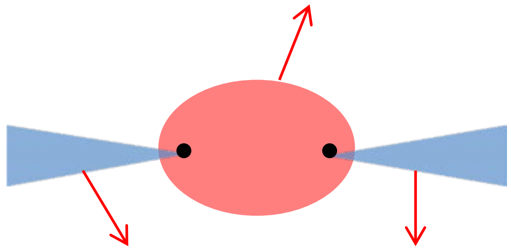
(~ E8 x E8 heterotic string on K3, w/ instanton # (12+n,12-n)…)

n	1	2	3	4	5	6	7	8	12
gauge symmetry	-	-	$SU(3)$	$SO(8)$	F_4	E_6	E_7	E_7	E_8
global symmetry	E_8	-	-	-	-	-	-	-	-
matters	-	-	-	-	-	-	$\frac{1}{2}\mathbf{56}$	-	-

E-string theory

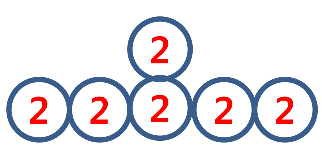
A_1 (2,0) SCFT

7-branes wrapping compact 2-cycle: 6d gauge symmetry

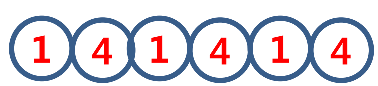


7-branes wraping noncompact 2-cycles: 6d global symmetry

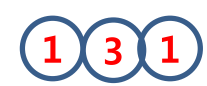
- Important "building blocks" of other CFTs: "glue minimal CFTs" For instance…



ADE (2,0) SCFTs



SCFT from D6-O6-NS5

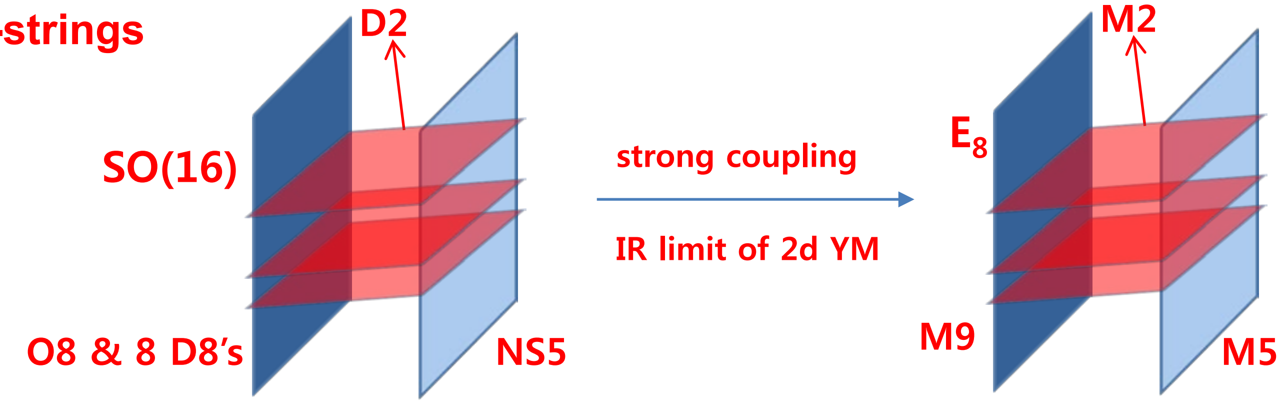


E_6 x E_6 conformal matter
[Del Zotto, Heckman, Tomasiello, Vafa]

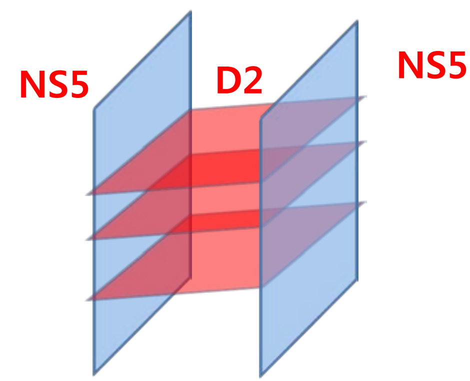
Strings of minimal SCFTs

- D3-branes wrapping compact cycle E: self-dual strings
- With alternative D-brane/open string descriptions, things get easier.

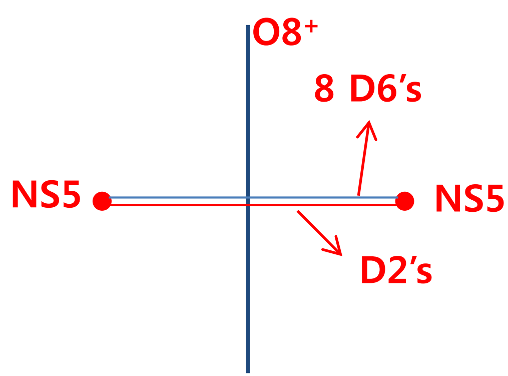
- n=1: **E-strings**



- n=2: **M-strings**. (4,4) gauge theory uplift ("2d Hanany-Witten"), or… a (0,4) uplift [Haghighat, Iqbal, Lockart, Kozcaz, Vafa]



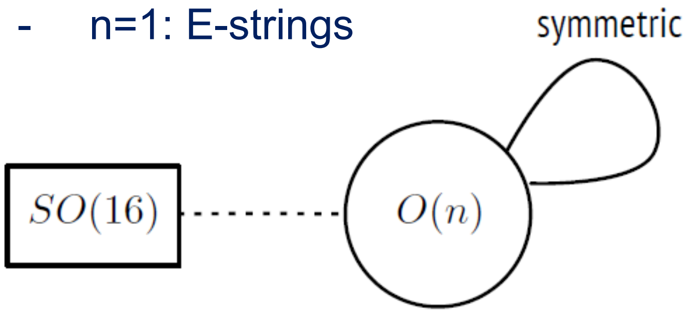
- n=4: **SO(8) instanton strings** from massive IIA



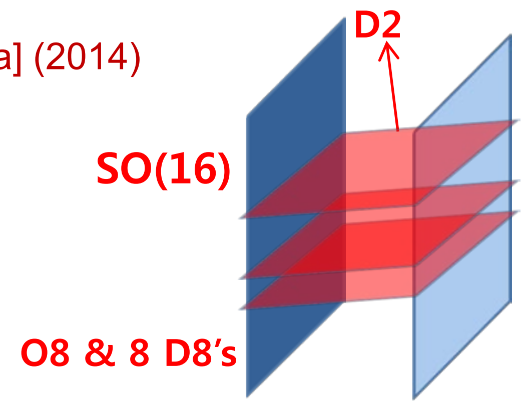
6d minimal strings from gauge theories

- With open string picture, found the worldsheet gauge theories: all anomaly-free

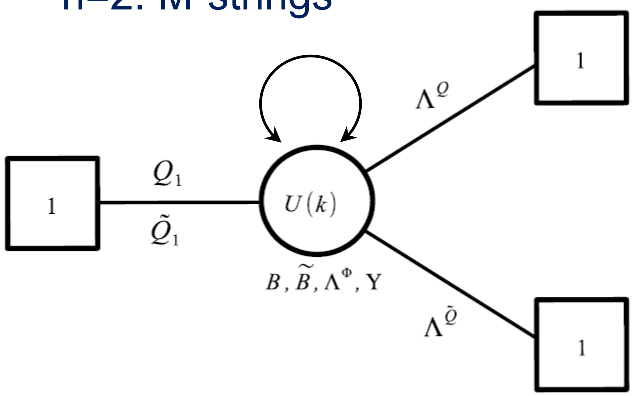
- n=1: E-strings [J. Kim, SK, K. Lee, J. Park, C. Vafa] (2014)



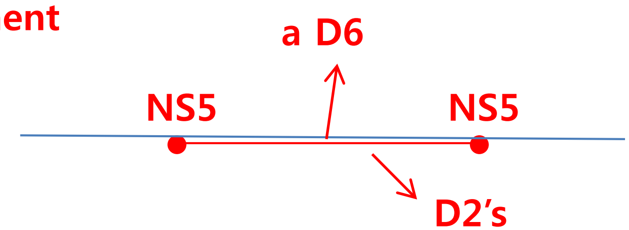
SO(16) → E_8 enhancement



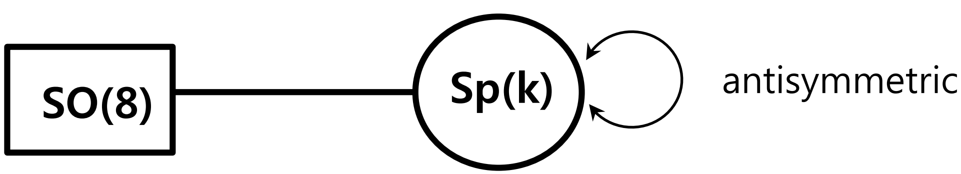
- n=2: M-strings [Haghighat, Iqbal, Kozcaz, Lockhart, Vafa] (2013)



(0,4) → (4,4) SUSY enhancement



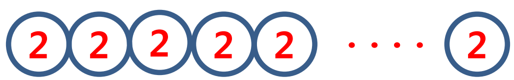
- n=4: SO(8) strings. simply the SO(8) ADHM quiver… [Haghighat, Klemm, Lockhart, Vafa] (2014)

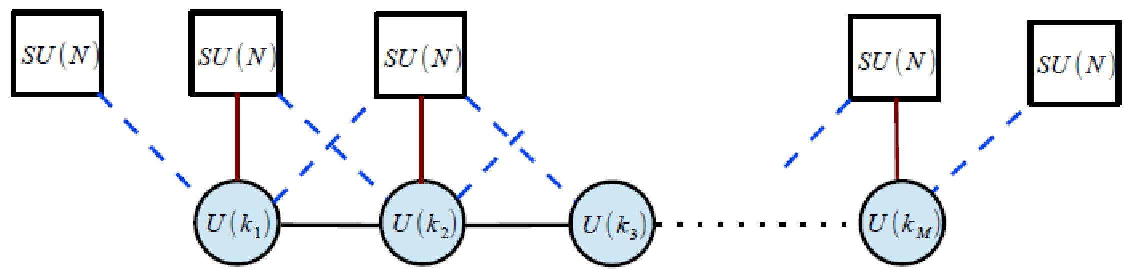


Chains of strings

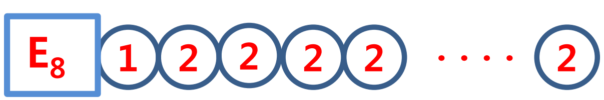
- One can “glue” the minimal CFTs or strings: [Haghighat, Gadde, Kim, SK, Lockhart, Vafa]
- These make 6d CFTs & strings with higher dimensional Coulomb branches

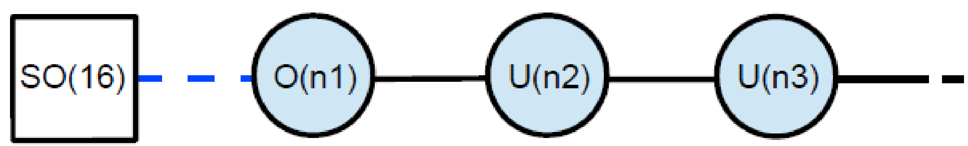
- A_M (2,0) CFT strings probing A_{N-1} orbifold [Haghighat, Iqbal, Kozcaz, Lockhart, Vafa] (2013)





- (1,0) CFT for M9-M5-M5- …. (higher rank E-strings):

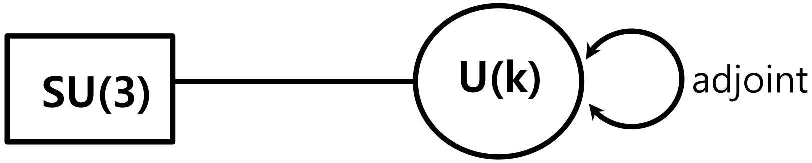




- And so on… : all uses n=1,2,4 mimimal strings. All engineered by D-branes/open strings

Challenges to minimal strings

- n > 4: exceptional gauge symmetry. The problem is "exceptional ADHM"
- n = 3: SU(3) instanton strings… try the "standard ADHM" quiver?



- The naïve ADHM quiver is anomalous.

fields	$U(k)$	$SU(3)$	$SU(2)_F$	$SU(2)_1$	$SU(2)_2$
$\lambda_{\dot{\alpha}A-}$	**adj**	**1**	**1**	**2**	**2**
$q_{\dot{\alpha}}(\rightarrow \psi_{A+})$	**k**	$\bar{\mathbf{3}}$	**1**	**2**	**1**
$a_{\alpha\dot{\beta}}(\rightarrow \chi_{\alpha A+})$	**adj**	**1**	**2**	**2**	**1**

- In certain sense, this failure is "expected"
- Here, SU(3) doesn't come from 3 D-branes: Rather, it comes from various light string junctions connecting 7-branes [Grassi, Halverson, Shaneson]

(For 5d particles, this quiver is completely fine. In 6d, much harder to get consistent quiver)

New ADHM for SU(3) minimal strings

- The anomaly-free quiver: [SK, Jaemo Park] work in progress

fields	$U(k)$	$SU(3)$	$SU(2)_F$	$SU(2)_1$	$SU(2)_2$
$\lambda_{\dot{\alpha}A-}$	**adj**	**1**	**1**	**2**	**2**
$\lambda^{\text{extra}}_{\dot{\alpha}A-}$	$\overline{\textbf{sym}}$	**1**	**1**	**2**	**2**
$q_{\dot{\alpha}}(\rightarrow \psi_{A+})$	**k**	$\bar{\mathbf{3}}$	**1**	**2**	**1**
$q^{\text{extra}}_{\dot{\alpha}}(\rightarrow \psi^{\text{extra}}_{A+})$	**k**	**3**	**1**	**2**	**1**
$a_{\alpha\dot{\beta}}(\rightarrow \chi_{\alpha A+})$	**adj**	**1**	**2**	**2**	**1**
$a^{\text{extra}}_{\alpha\dot{\beta}}(\rightarrow \chi^{\text{extra}}_{\alpha A+})$	$\textbf{anti} + \overline{\textbf{anti}}$	**1**	**2**	**2**	**1**
Ψ^{extra}_{-}	**k**	$\mathbf{3}+\mathbf{1}$	**1**	**1**	**1**
$\Lambda^{\text{extra}}_{\alpha-}$	**anti**	**1**	**2**	**1**	**1**

- Constructed via subtle procedures: start from SO(8) theory at $n_v = n_s = n_c = 1$ and then Higgs it to SU(3)… (as far as I can see, no hint from branes/open strings)

$$(E_7,\ n_{\frac{1}{2}\mathbf{56}} = 5) \rightarrow (E_6, n_{\mathbf{27}} = 3) \rightarrow (F_4, n_{\mathbf{26}} = 2)$$
$$\rightarrow \boxed{(SO(8), n_{\mathbf{8}_v,\mathbf{8}_s,\mathbf{8}_c} = 1) \rightarrow (G_2, n_{\mathbf{7}} = 1) \rightarrow (SU(3), n_{\mathbf{3}} = 0)}$$

Tests

- We can study their IR spectrum by computing the elliptic genus:
- E.g. single string: $$Z_1^{SU(3)}(v,\epsilon_{1,2}) = -\frac{\eta^2}{\theta_1(\epsilon_{1,2})}\sum_{i=1}^{3}\frac{\eta^4\theta_1(4\epsilon_+ - 2v_i)\theta_1(v_i)}{\prod_{j(\neq i)}\theta_1(v_{ij})\theta_1(2\epsilon_+ - v_{ij})\theta_1(2\epsilon_+ + v_j)}$$
- Partial data known from topological strings at k=1,2,3 [Haghighat, Klemm, Lockart, Vafa]

$$\log Z(\tau,\epsilon_+,\epsilon_+,\mu) = \sum_{g\geq 0,n\geq 0}(\epsilon_1\epsilon_2)^{g-1}(\epsilon_1+\epsilon_2)^n F_{g,n}(\tau,\mu)$$

$$\begin{aligned} F_{0,0} &= -\left[\frac{\theta_1(2v_1)\theta_1(v_1)}{\theta_1(v_{12})^2\theta_1(v_{13})^2\theta_1(v_2)\theta_1(v_3)} + (1,2,3\to 2,3,1) + (1,2,3\to 3,1,2)\right] \\ &= e^{-\pi i\tau + 2\pi i v_{12} + 2\pi i v_{23}}\sum_{d_0,d_1,d_2=0}^{\infty} N_{d_0,d_1,d_2}\left(\frac{e^{2\pi i\tau}}{e^{2\pi i v_{12}}e^{2\pi i v_{23}}}\right)^{d_0} e^{2\pi d_1 v_{12}} e^{2\pi d_2 v_{23}} \end{aligned}$$

$d_1 \setminus d_2$	0	1	2	3	4	5
0	1	3	5	7	9	11
1	3	4	8	12	16	20
2	5	8	9	15	21	27
3	7	12	15	16	24	32
4	9	16	21	24	25	35
5	11	20	27	32	35	36

Table 1: q^0

$d_1 \setminus d_2$	0	1	2	3	4	5
0	3	4	8	12	16	20
1	4	16	36	60	84	108
2	8	36	56	96	144	192
3	12	60	96	120	180	252
4	16	84	144	180	208	288
5	20	108	192	252	288	320

Table 2: q^1

complete agreement

Tests *(continued)*

$$F_{0,0} = -\left[\frac{\theta_1(2v_1)\theta_1(v_1)}{\theta_1(v_{12})^2\theta_1(v_{13})^2\theta_1(v_2)\theta_1(v_3)} + (1,2,3\to 2,3,1) + (1,2,3\to 3,1,2)\right]$$

$$= e^{-\pi i\tau+2\pi i v_{12}+2\pi i v_{23}} \sum_{d_0,d_1,d_2=0}^{\infty} N_{d_0,d_1,d_2}\left(\frac{e^{2\pi i\tau}}{e^{2\pi i v_{12}}e^{2\pi i v_{23}}}\right)^{d_0} e^{2\pi d_1 v_{12}} e^{2\pi d_2 v_{23}}$$

$d_1 \setminus d_2$	0	1	2	3	4	5
0	5	8	9	15	21	27
1	8	36	56	96	144	192
2	9	56	149	288	465	651
3	15	96	288	456	735	1080
4	21	144	465	735	954	1371
5	27	192	651	1080	1371	**1632**

Table 3: q^2

$d_1 \setminus d_2$	0	1	2	3	4	5
0	7	12	15	16	24	32
1	12	60	96	120*l*	180	252
2	15	96	288	456	735	1080
3	16	120	456	1012	1788	**2796**
4	24	180	735	1788	**2823**	**4356**
5	32	252	1080	**2796**	**4356**	**5760**

Table 4: q^3

complete agreement

- 1d limit $N_{0,d1,d2}$: agrees w/ Nekrasov partition function from standard ADHM quiver.

$$F_{0,0} \overset{q\ll 1}{\sim} -\frac{\sin(2\pi v_1)\sin(\pi v_1)}{4\sin^2(\pi v_{12})\sin^2(\pi v_{13})\sin(\pi v_2)\sin(\pi v_3)} + (2,3,1) + (3,1,2)$$

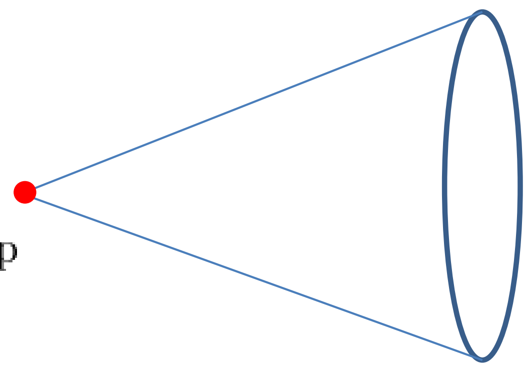
$$= \frac{1}{4\sin^2(\pi v_{12})\sin^2(\pi v_{13})} + (2,3,1) + (3,1,2) = Z^{SU(3),k=1}_{\text{Nekrasov}}(\mathbb{R}^4\times S^1)$$

- Alternative SU(3) ADHM in 1d/5d: different UV completion visible only in 2d/6d.

What's happening?

- In the Higgs branch, the non-linear sigma model description shouldn't change.
- All the other fields: extra degrees localized at the small instanton singularity

$$V(\phi_{\rm ADHM},\phi_{\rm extra}) \sim V(\phi_{\rm ADHM}) + V(\phi_{\rm extra}) + |\phi_{\rm extra}\phi_{\rm ADHM}|^2$$

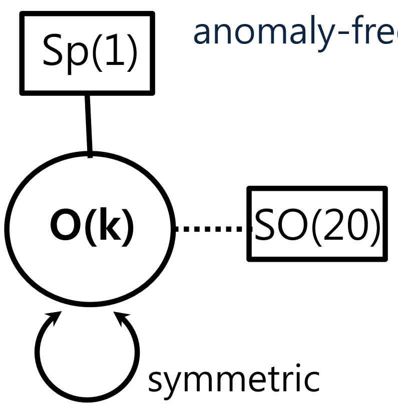


$\phi_{\rm extra}$ are massless only at the tip

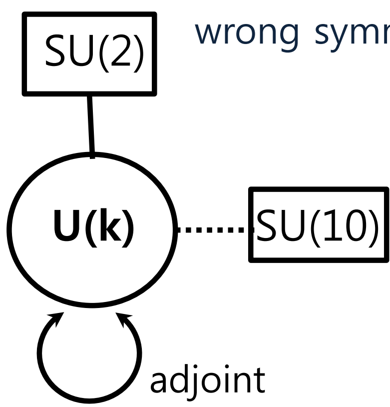
$$\mathcal{L}_{2d} = -g_{ij}(X)\partial^\mu X^I \partial_\mu X^j + \cdots$$

- It is so easy for the "standard ADHM" quivers to go bad in 2d.
- Another example: 6d SU(2) at N_f = 10 [Joonho Kim, SK, Kimyeong Lee] work in progress

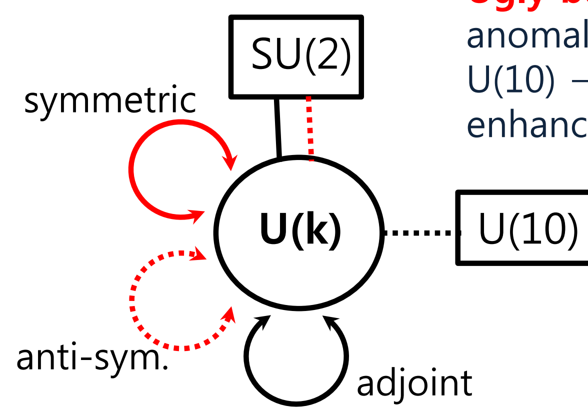
Good: anomaly-free



Bad: anomalous, & wrong symmetry



Ugly but OK: anomaly-free, U(10) →SO(20) enhancement



Concluding remarks

- 5d solitons' Witten indices (or $Z_{Nekrasov}$) are clearly understood only recently.
- New worldsheet descriptions of 6d self-dual strings are being discovered.

- 2d (0,4) gauge theories haven't been explored that much. But they are useful.
- Microscopic studies of 6d strings
- ADHM instantons, especially beyond open string constructions
- QFT dual of $AdS_3 \times S^3 \times S^3 \times S^1$ [Tong]

- General 2d (0,4) theories from F-theory? (wrapped 3-7-branes)
- How rich is this class? (Lagrangian or non-Lagrangian)
- Can we characterize and understand them better?

- Better understanding on the (1,0) SCFTs in the symmetric phase?
- E.g. various curved space partition functions? E.g. $S^5 \times S^1$?