6d SCFTs compactified on a circle/torus

Kazuya Yonekura, IAS

Mainly based on

- To appear and arXiv:1503.06217
  with K. Ohmori, H. Shimizu, Y. Tachikawa,

also

- arXiv:1410.6868 with H. Hayashi Y. Tachikawa,
- arXiv:1505.04743

Introduction

In six dimensions, we have many superconformal field theories!

- 6d N=(2,0) SCFTs: classified by ADE Lie algebra

- 6d N=(1,0) SCFTs: so many of them, classified recently in F-theory

[Heckman, Morrison,Vafa,2013]
[Del Zotto, Heckman,Tomasiello, Vafa,2014]
[Heckman,Morrison,Rudelius,Vafa,2015]

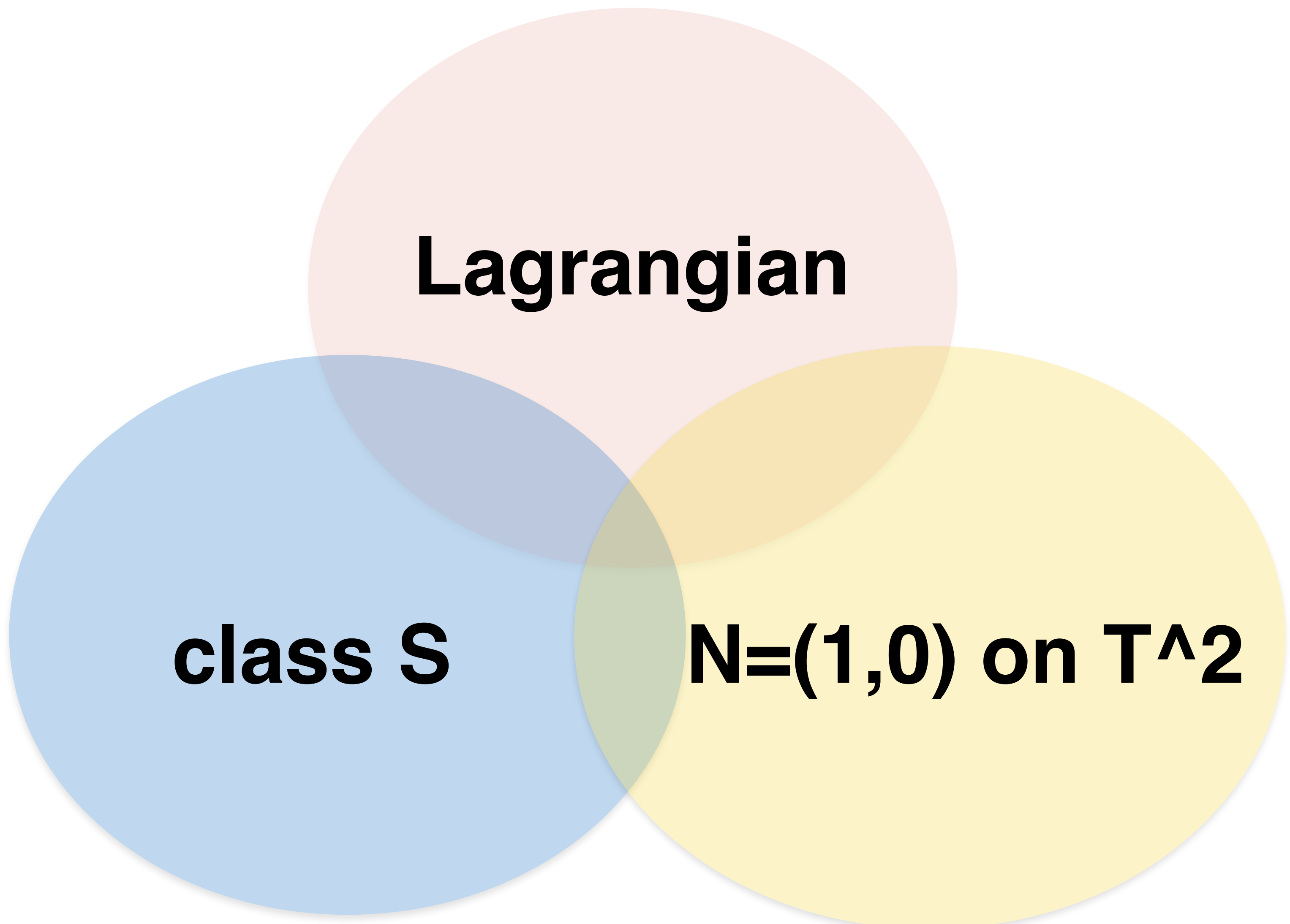
Introduction

There are at least three ways to get 4d N=2 theories.

- Lagrangian field theories.
- 6d N=(2,0) theories compactified on Riemann surfaces.
- 6d N=(1,0) theories compactified on a torus.

Compactification of N=(2,0) theories, called class S, have given us many new theories which might not have Lagrangian description.

Introduction



- What theories do we get from 6d N=(1,0) SCFTs on T^2?
- How much overlap do they have?

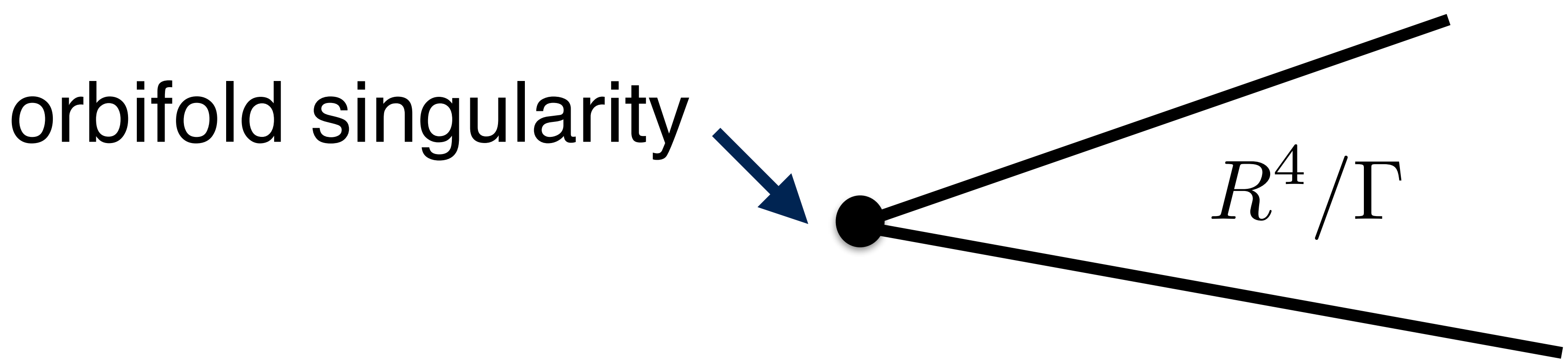
Contents

N=(2,0) in type IIB string

N=(2,0) theories can be constructed by type IIB string on

$$R^{1,5} \times (R^4/\Gamma)$$

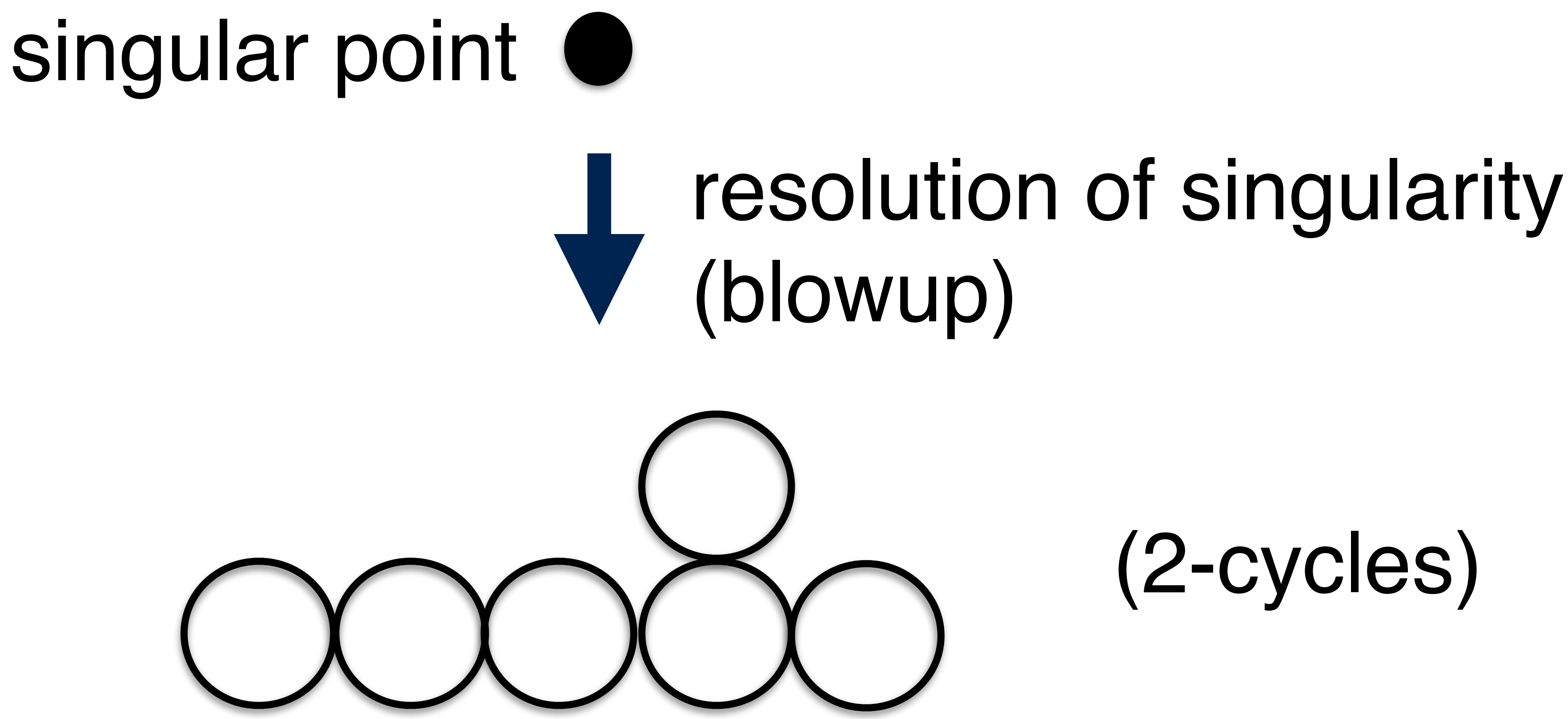
Γ is an orbifold group classified by ADE Dynkin diagram.
(E.g., for A-type, $\Gamma = Z_N$)



The N=(2,0) theory is realized on the singularity.

[Witten,1995]

N=(2,0) in type IIB string



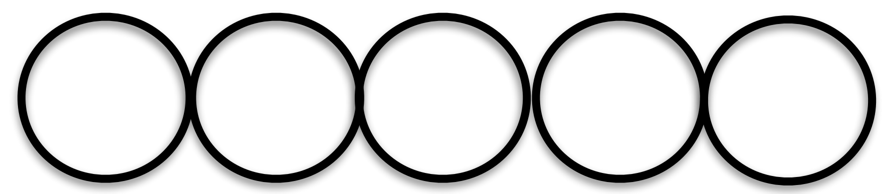
By the resolution of the singularity,
the singular point becomes several intersecting 2-cycles.

Field theoretically, this corresponds to giving vevs to some scalar fields of the N=(2,0) theory.

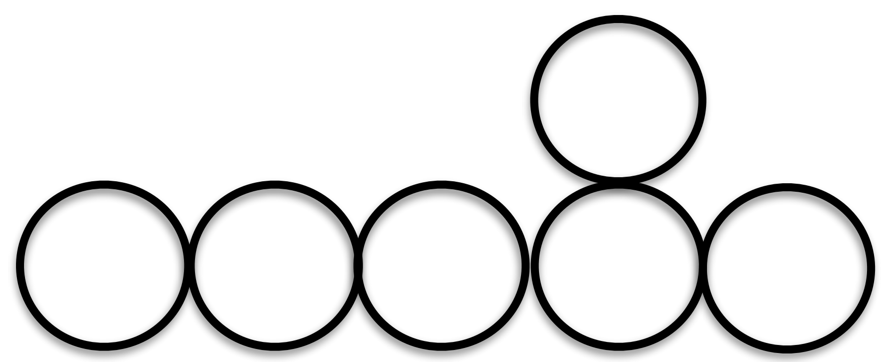
N=(2,0) in type IIB string

The intersection of the 2-cycles is determined by the Dynkin diagram corresponding to the orbifold group Γ.

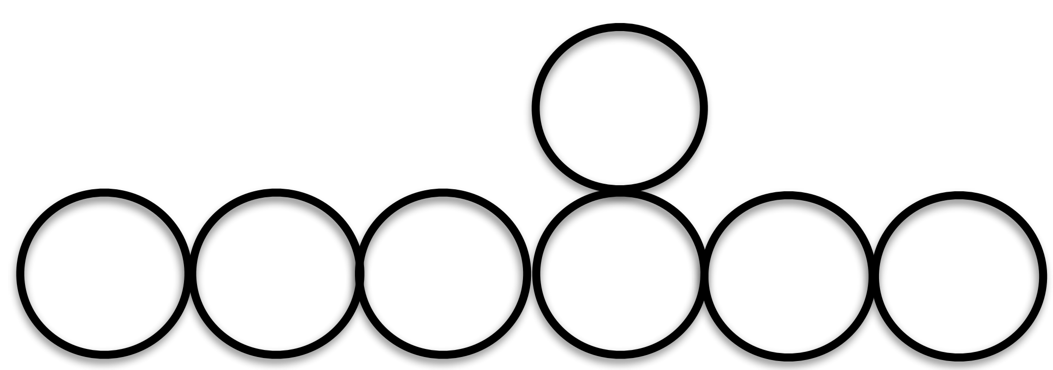
$\Gamma: \quad A_N$ type



$\Gamma: \quad D_N$ type

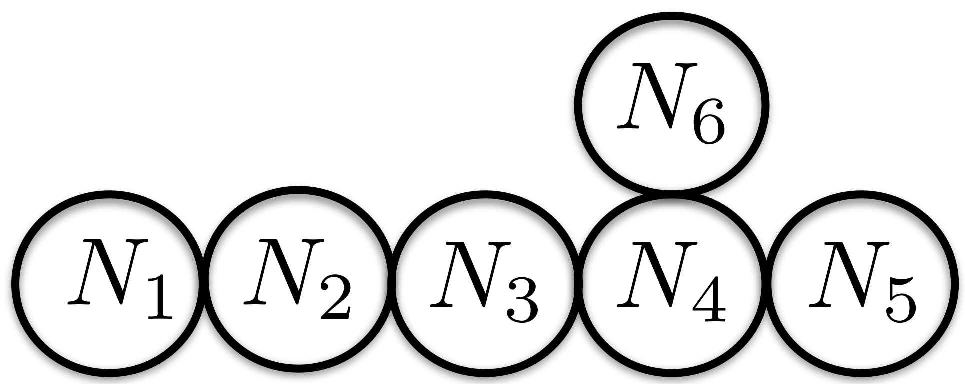


$\Gamma: \quad E_N$ type



Generalization to N=(1,0)

We can wrap D7-branes on each of the 2-cycles.

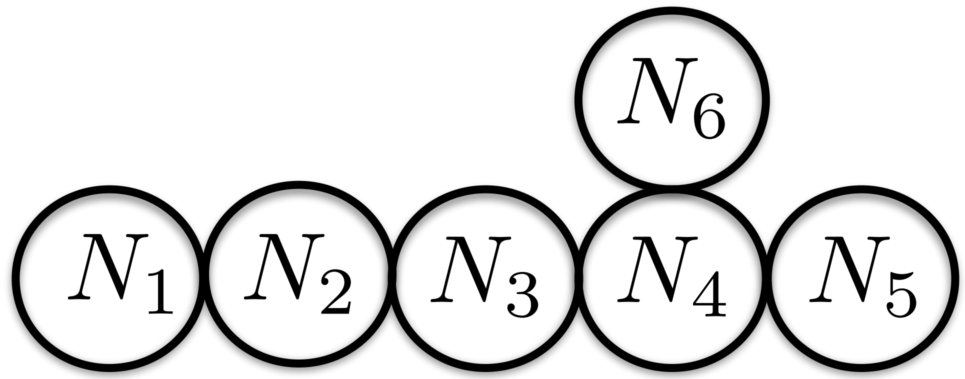


Each 2-cycle now supports $SU(N_i)$ gauge group.

By shrinking all the 2-cycles with D7-branes,
we get N=(1,0) SCFT at the singularity.

$\mathcal{L} \sim \phi \, \mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$ (ϕ : size of the 2-cycles.)

Generalization to N=(1,0)



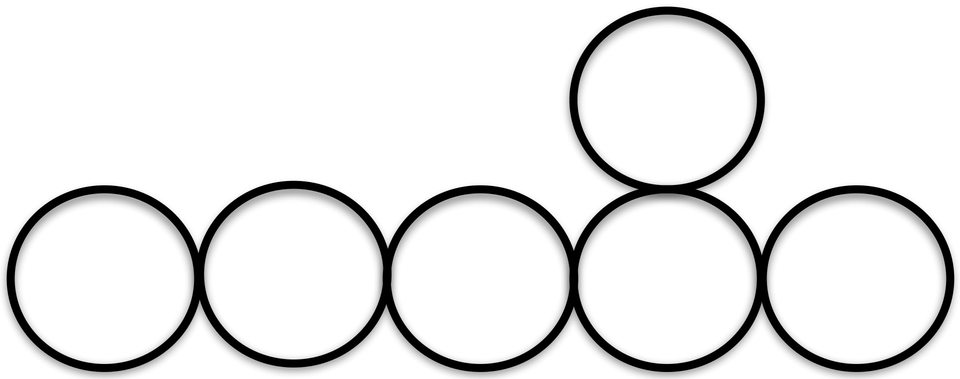
- Each 2-cycle has $SU(N_i)$ gauge group.
- There are bifundamental between adjacent 2-cycles
- Anomaly cancellation requires additional fundamentals such that each gauge group has

[Total flavor number]=2 [Color number]

We get a quiver gauge theory with the above properties.

Contents

N=(2,0) on a circle



The size of each 2-cycle is given by a vev of
a scalar field in the so-called tensor multiplet in 6d.

6d tensor multiplets

↓ on a circle

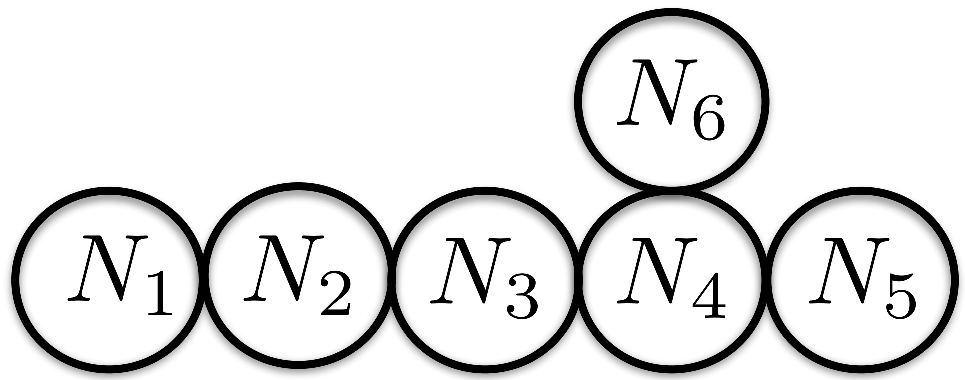
5d vector multiplets

↓ vevs → 0

5d N=2 Super-Yang-Mills of type G=ADE

N=(1,0) on a circle

Now we consider N=(1,0) theories on a circle.



- This gives a 5d SU(N) quiver after compactification.
- The gauge couplings are determined by the size of the 2-cycles, which are vevs of free vector multiplets.

$$\mathcal{L} \sim \phi \ \mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) \quad (\ \phi : \text{scalar in vector mult.})$$

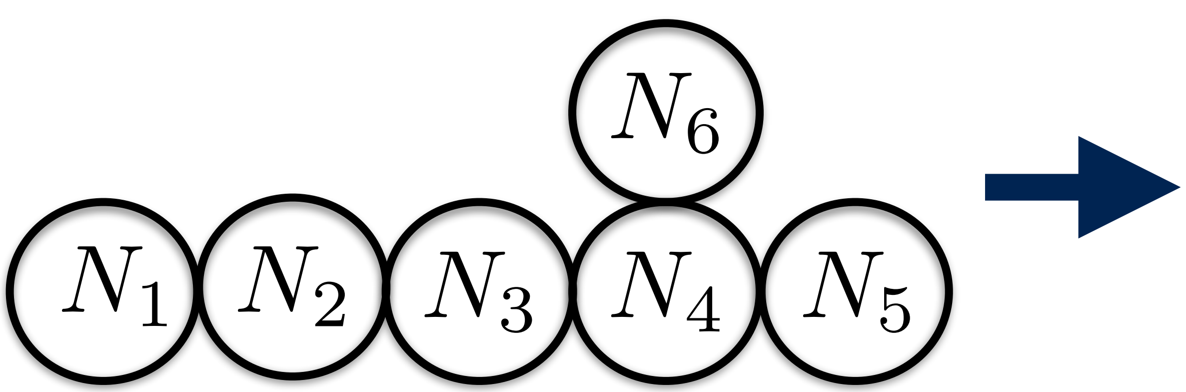
What happens when vevs →0 ?

N=(1,0) on a circle

The two effects of vevs→0;

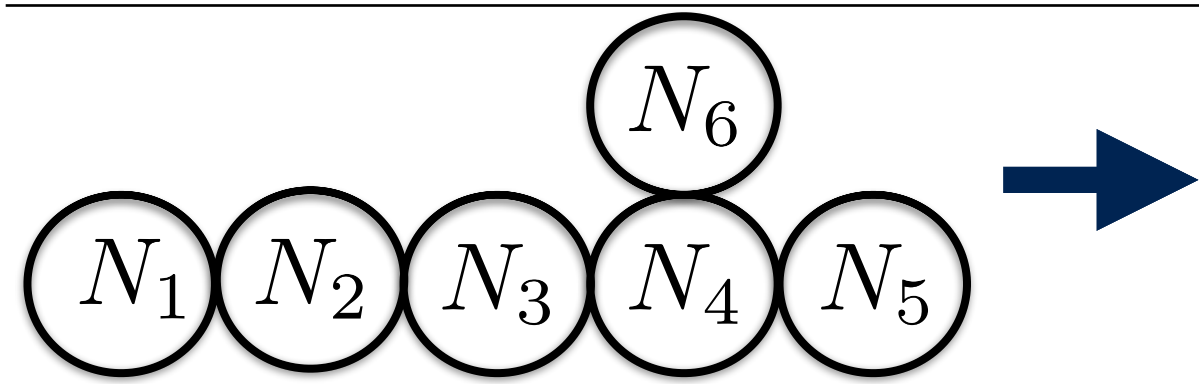
- 5d SU(N) gauge couplings become infinitely strong.
  → 5d SU(N) quiver becomes a 5d SCFT
- The gauge symmetry of the underlying N=(2,0) restored.
  →we get 5d SYM of type G

(G is determined by the ADE Dynkin diagram.)



5d SCFT + SYM of type G

N=(1,0) on a circle

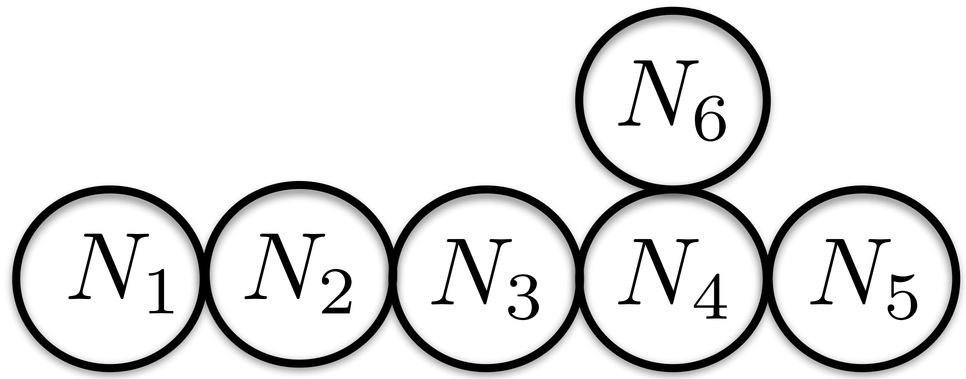


5d SCFT + SYM of type G

The 5d SCFT and SYM of type G must be coupled, so the 5d SCFT must have flavor G symmetry which is gauged by SYM of type G.

This is indeed the case by 5d symmetry enhancement!

N=(1,0) on a circle



In 5d, each gauge group has instanton symmetry and baryon symmetry $j_I \sim *(\mathrm{tr} F \wedge F) \quad j_B$

They give $U(1)^{2\mathrm{rank}G}$ flavor symmetries.

If each SU(N) gauge group has [flavor]=2[color] and zero Chern-Simons level, $G \times G$ flavor symmetry is enhanced. [Tachikawa,2015], [KY,2015]

Diagonal subgroup is gauged by SYM.

N=(1,0) on a circle

Summary of circle compactification

6d SCFT of the class we are considering

↓ on S^1

[5d SCFT with flavor G-symm.] + [5d SYM of type G]

The simplest example: in the case of N=(2,0),

[5d SCFT with G-symm.]=free adjoint hyper of G.

Contents

N=(1,0) on a torus

$T^2 \sim S^1 \times S^1$

6d SCFT

↓ on S^1

[5d SCFT with flavor G-symm.] + [5d SYM of type G]

↓ on S^1 (first bracket) ↓ on S^1 (second bracket)

???

[4d SYM of type G]

coupling $\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$

=complex structure of T^2

$SL(2,Z)$ S-duality

N=(1,0) on a torus

T^2 compactification of 6d SCFT is now reduced to S^1 compactification of 5d SCFT.

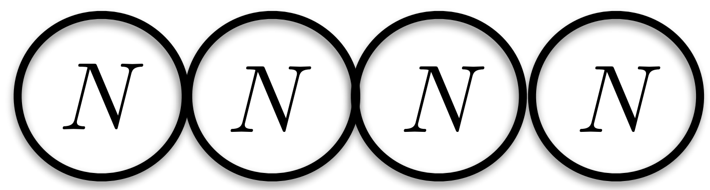
[5d SCFT with flavor G-symm.]

↓ on S^1

???

Example: Linear SU(N) quiver

As interesting example, consider the theory



$$G = A_{M-1}$$

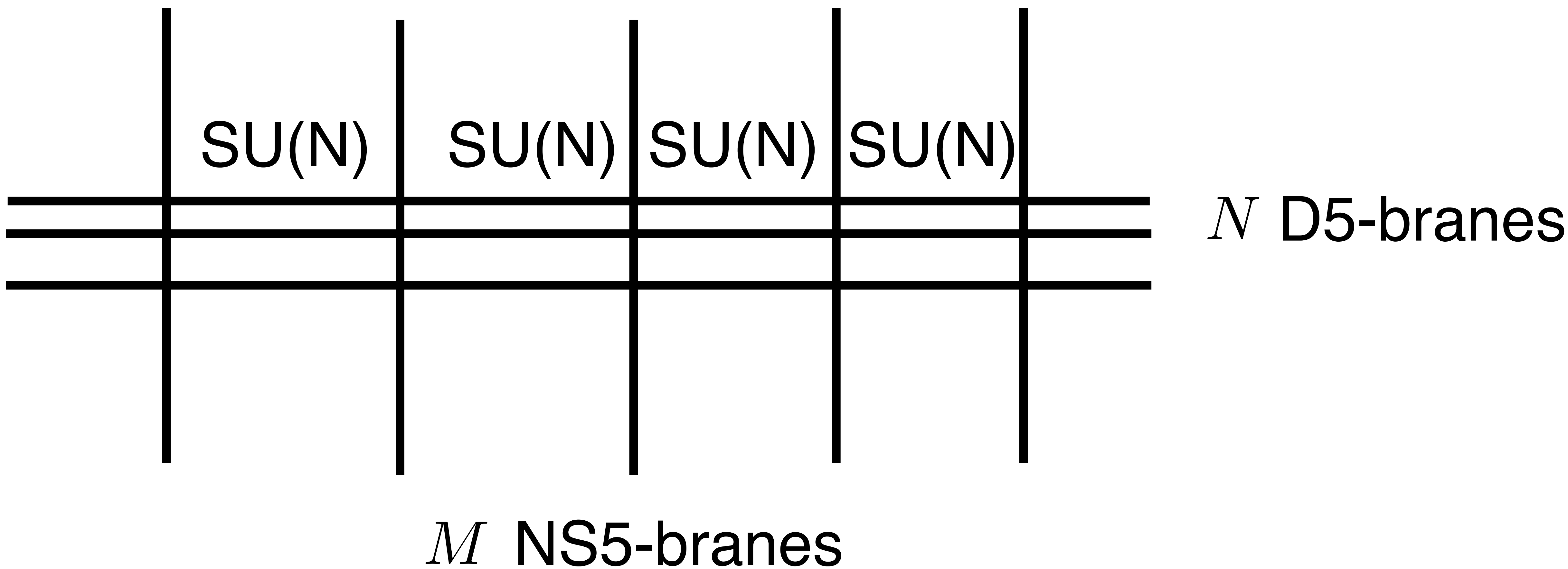
- All the gauge groups have the same color N.
- The underlying N=(2,0) theory is of type $G = A_{M-1}$
- The corresponding 5d quiver is

[N]—SU(N)—SU(N)—SU(N)—SU(N)—[N]

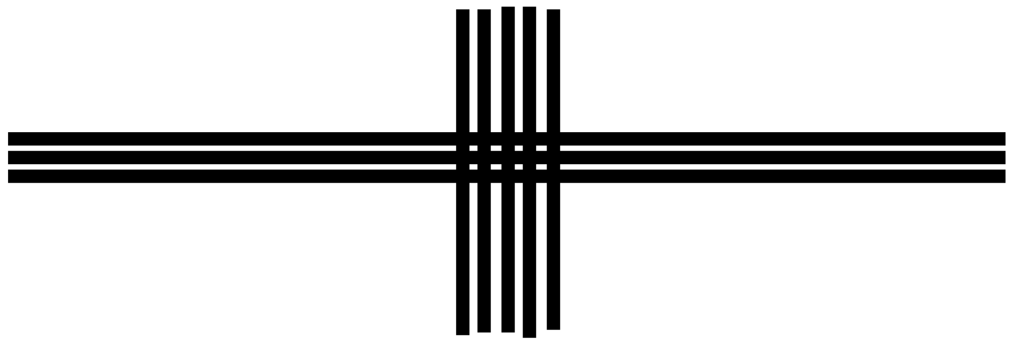
This is $SU(N)^{M-1}$ quiver.

Brane web realization

The 5d theory can be realized by a NS5-D5 system

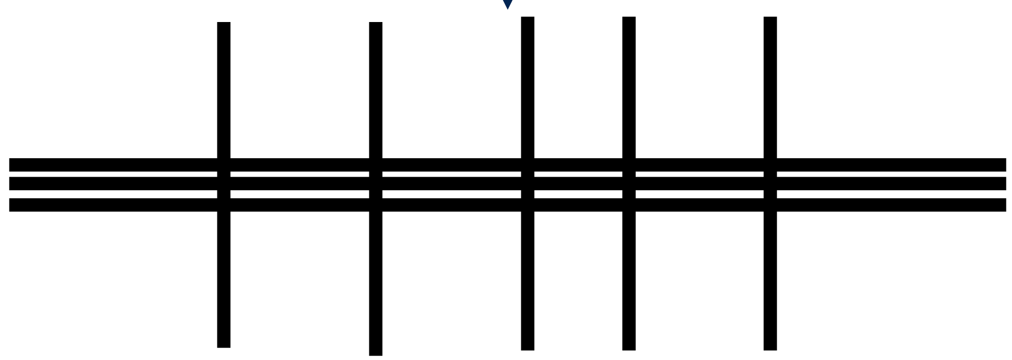


Brane web realization



5d SCFT limit
flavor symmetry
$SU(N)^2 \times SU(M)^2$

mass deformation of $SU(M)^2$



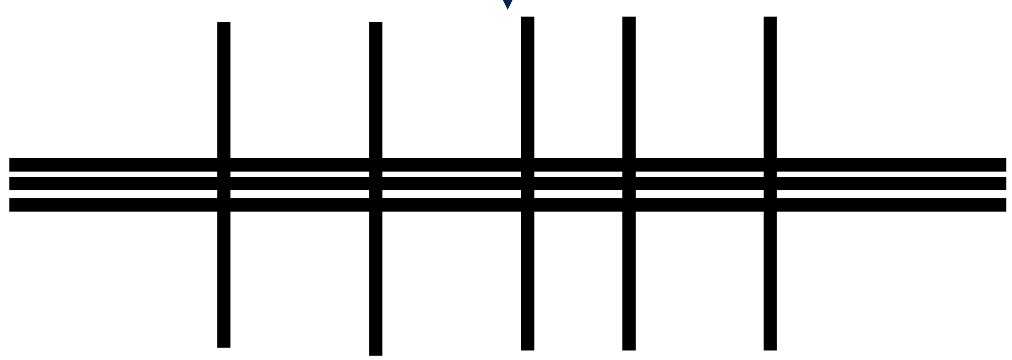
$SU(N)^{M-1}$ quiver

4d situation

???

(obtained from the 5d SCFT on circle compactification)

↓ mass deformation of $SU(M)^2$



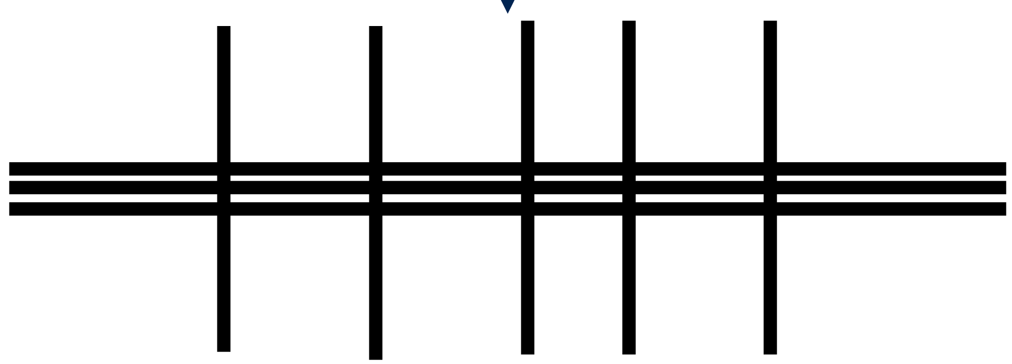
$SU(N)^{M-1}$ quiver in 4d

4d situation

???

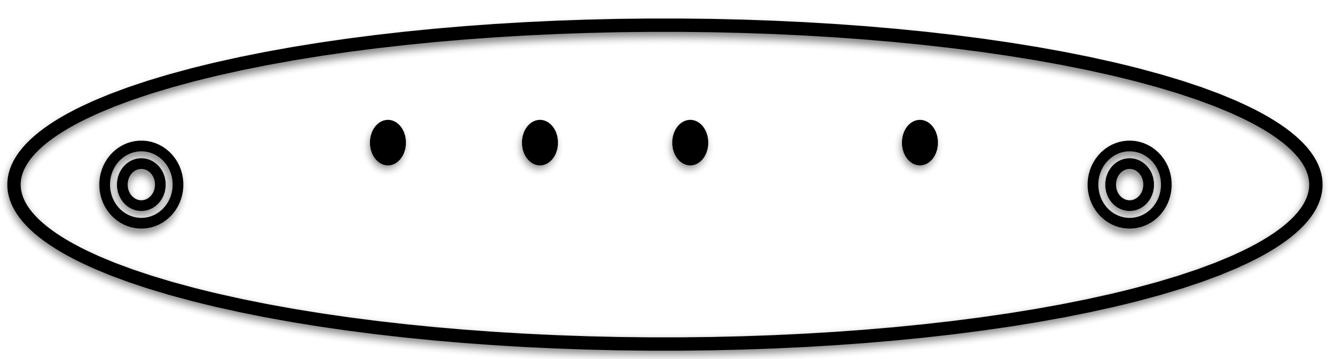
(obtained from the 5d SCFT on circle compactification)

mass deformation of $SU(M)^2$



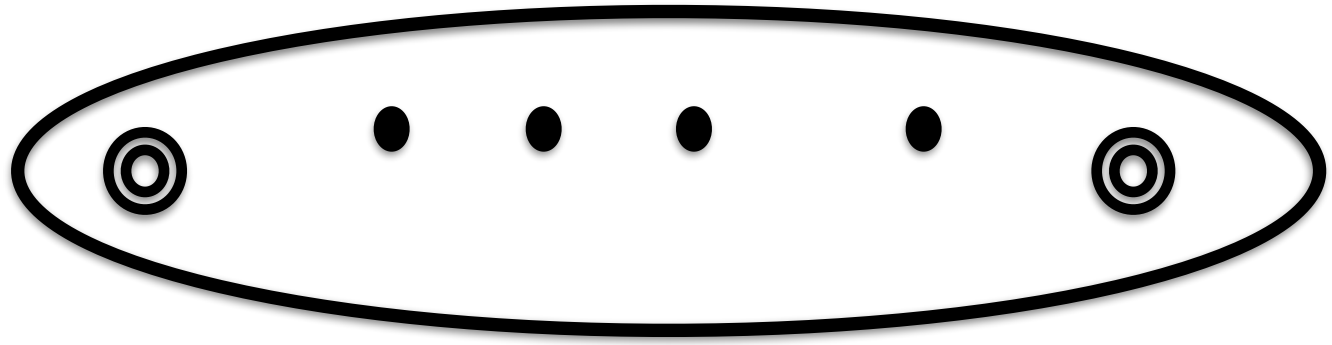
$SU(N)^{M-1}$ quiver in 4d

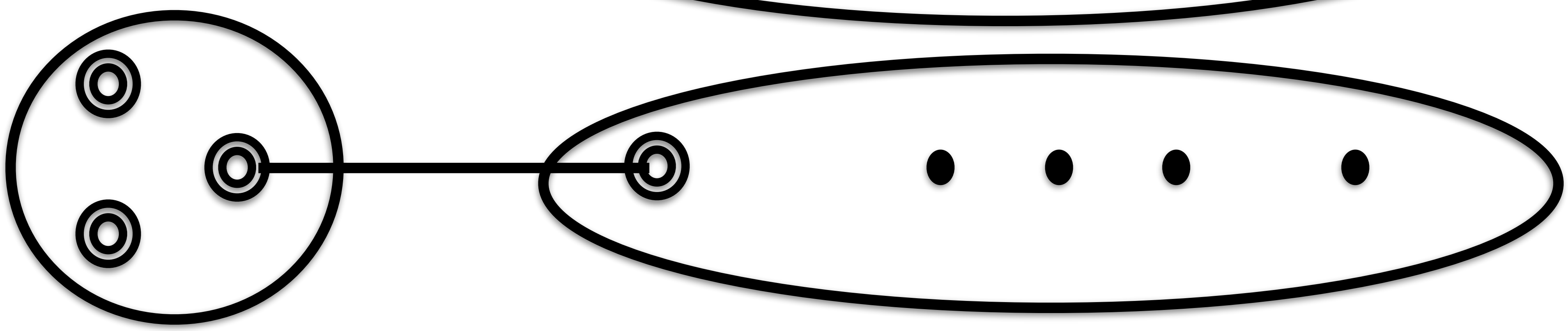
=



A sphere with two full punctures and M simple punctures in class S theory.

Gaiotto duality

$SU(N)^{M-1} =$ 

$=$ 

$$= T_N(\text{full}, \text{full}, Y) - SU(M) - SU(M-1) - \cdots - SU(1)$$

A relative of the so-called T_N theory appears.
(Just for simplicity I have taken $N > M$.)

$T_N(\text{full}, \text{full}, Y)$ is the theory with two full punctures and one puncture named Y . (Explicitly $Y = [N-M, 1^M]$)

Mass deformation

???

(obtained from the 5d SCFT on circle compactification)

↓ mass deformation of $SU(M)^2$

$$T_N(\text{full}, \text{full}, Y) - SU(M) - SU(M-1) - \cdots - SU(1)$$

<u>**A fact:**</u> $T_M(\text{full}, \text{full}, \text{full})$

↓ mass deformation of $SU(M)^2$

$$[M] - SU(M-1) - \cdots - SU(1)$$

[Bergman,Zafrir, 2014]
[Hayashi,Tachikawa,KY,2014]

The results

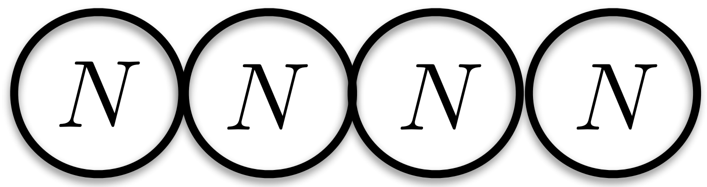
??? (obtained from the 5d SCFT on circle compactification)

= $T_N(\text{full}, \text{full}, Y) - SU(M) - T_M(\text{full}, \text{full}, \text{full})$

It turns out that the gauge group $SU(M)$ is infrared free by computing the beta function.

The two isolated SCFTs are connected by an IR free gauge group.

The results

6d SCFT 

$G = A_{M-1}$

↓ on S^1

[5d SCFT] + [5d SYM of type G=SU(M)]

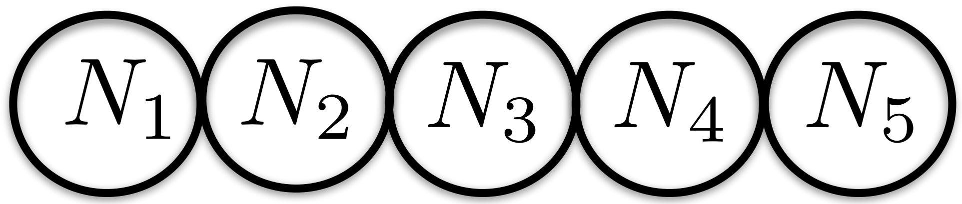
↓ on S^1

$$T_N(\text{full}, \text{full}, Y) - SU(M) - T_M(\text{full}, \text{full}, \text{full}) \quad SU(M)$$

The SU(M) is coupled to the diagonal subgroup of the SU(M)^2 of two full punctures.

The results

The result can also be generalized to



$G = A_{M-1}$

[Gaiotto,Tomasiello 2014]

on T^2

$$T_N(Y_L, Y_R, Y) - SU(M) - T_M(\text{full}, \text{full}, \text{full}) \quad SU(M)$$

- The punctures become more general.
- The case of N<M is also obtained in the same way.

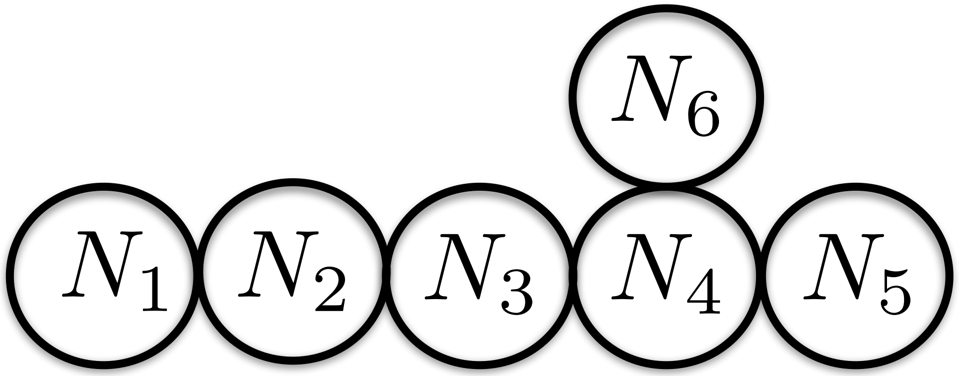
Conclusion:

The 4d theory is not exactly class S, but is described by using class S theories as building blocks.

Contents

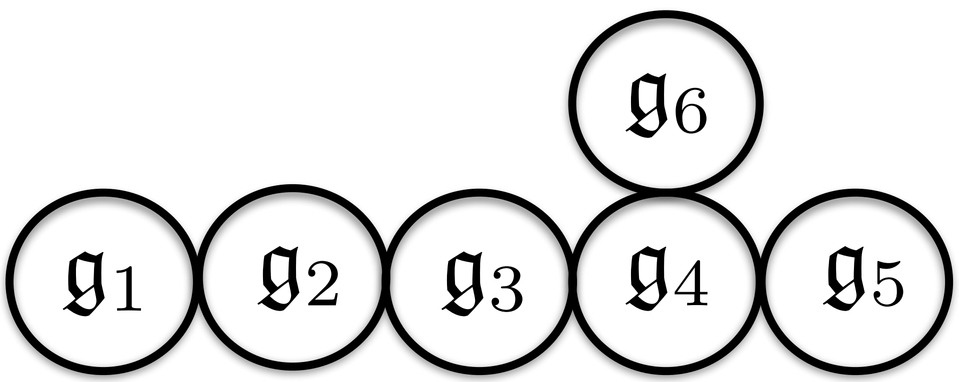
6d SCFTs in F-theory

We can wrap D7-branes on 2-cycles in Type IIB.



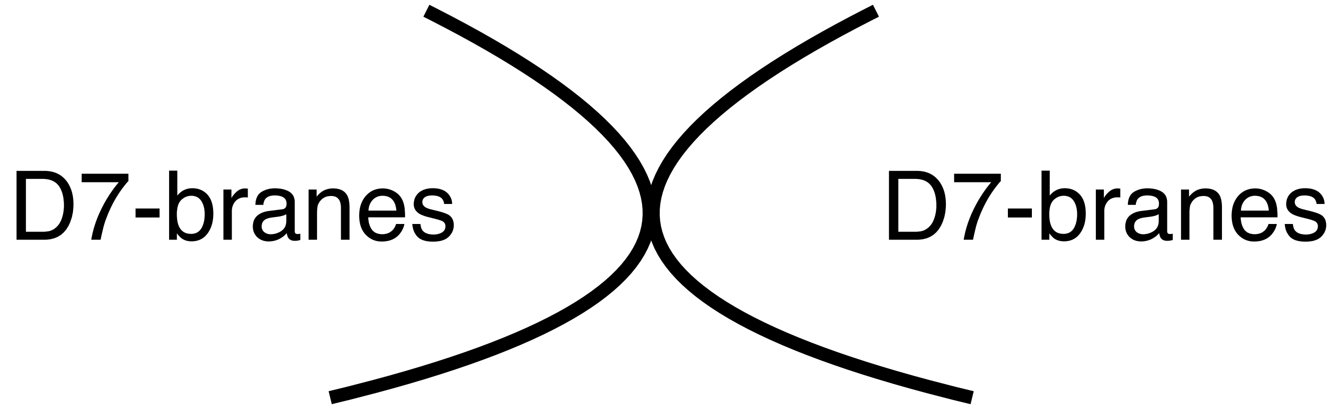
Type IIB → F-theory

D7-branes → singular Kodaira fibers with gauge algebra $\mathfrak{g}$

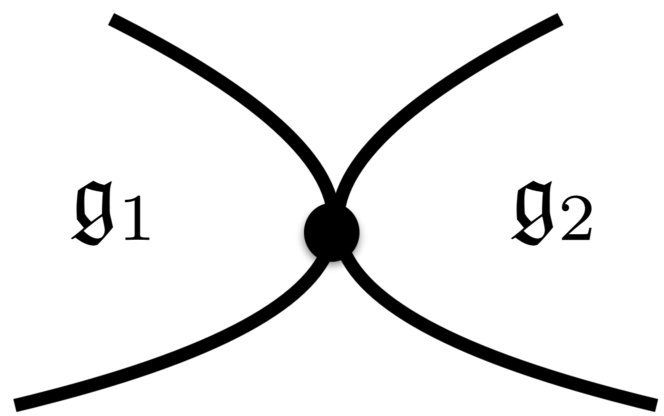


We focus on the case in which the intersection matrix of the 2-cycles is the same as N=(2,0) of type G=ADE.

6d SCFTs in F-theory



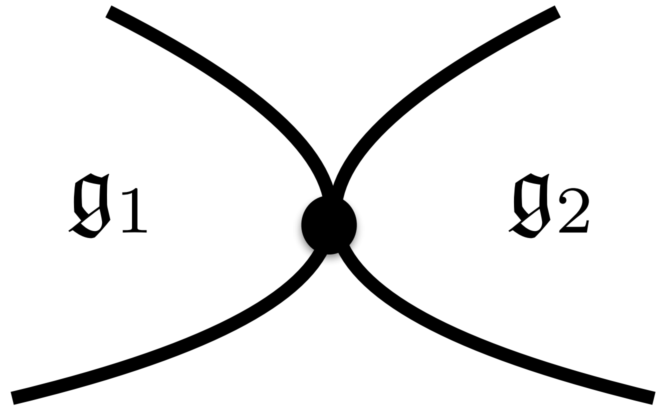
Bifundamental matters appear at the intersection of the D7-branes.



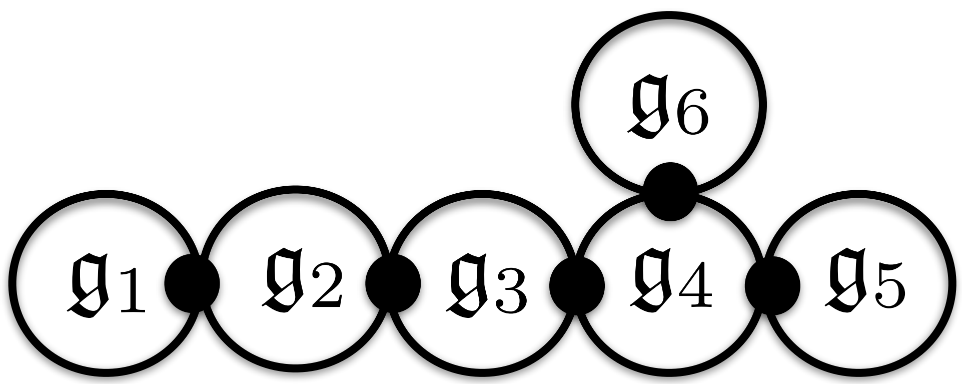
In F-theory, when two singular fibers intersect, we get strongly interacting 6d SCFTs, called conformal matter.

[Del Zotto, Heckman,Tomasiello, Vafa,2014]

Conformal matter



Conformal matter has at least $\mathfrak{g}_1 \times \mathfrak{g}_2$ flavor symmetries and it is regarded as a generalized bifundamental in quivers



More generally, they are very-Higgsable theories, but I will not talk about them. See our paper.

[Ohmori,Shimizu,Tachikawa,KY]

Conformal matter on circle

D_N conformal matter may be tractable for 5d theorists.

(D_N, D_N) conformal matter

↓ on S^1 with flavor holonomies

SU(N-2) gauge theory with 2N flavors

[Hayashi,Kim,Lee,Taki,Yagi,2015]
[KY, 2015]

Remark:

Without holonomies, the 5d version of the conformal matter is a strongly coupled theory.

Conformal matter on torus

$(\mathfrak{g}, \mathfrak{g})$ conformal matter

↓ on T^2

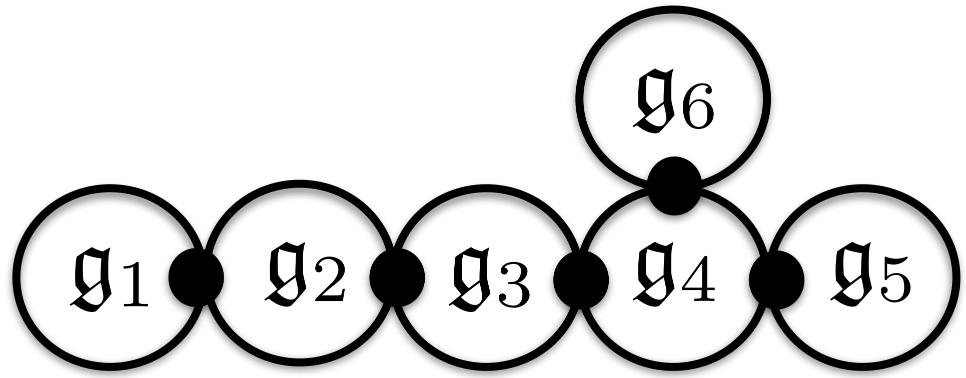
Class S theory of type $\mathfrak{g}$ with two full punctures and one simple puncture.

- This is a strongly interacting isolated theory.
- When $\mathfrak{g} = A$, it is just an ordinary bifundamental.

[Ohmori,Shimizu,Tachikawa,KY,2015]
[Del Zotto, Vafa, Xie, 2015]

General case

6d SCFT

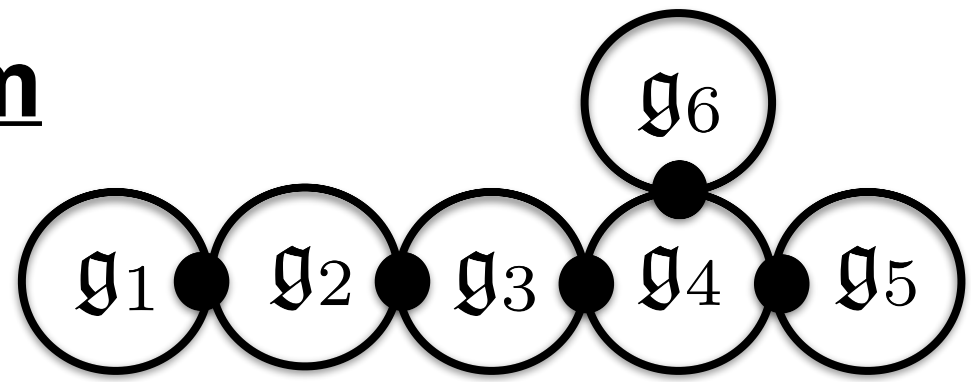


We want to find the compactification of the 6d SCFTs of the class whose base is the N=(2,0) theory of type G.

General proposal

<u>Our general claim</u>

6d SCFT



on S^1

[5d SCFT with flavor G-symm.] + [5d SYM of type G]

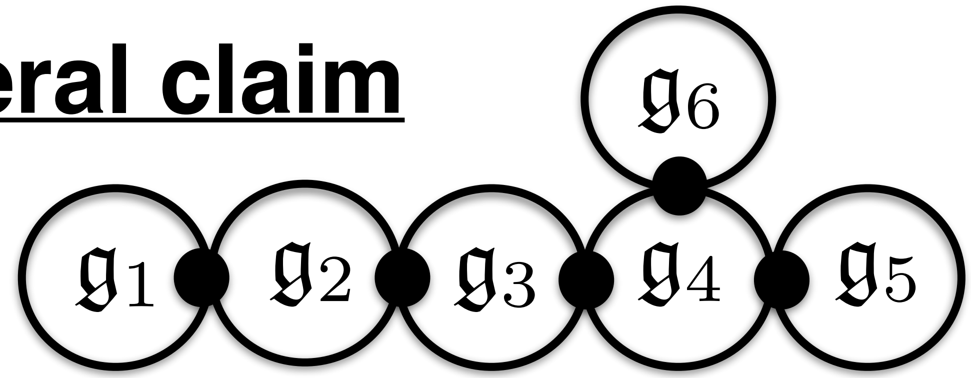
on S^1

[4d SCFT(1)] — [IR free gauge group]—[4d SCFT(2)]
|
[4d SYM of type G]

Justification of general claim (1)

First part of general claim

6d SCFT

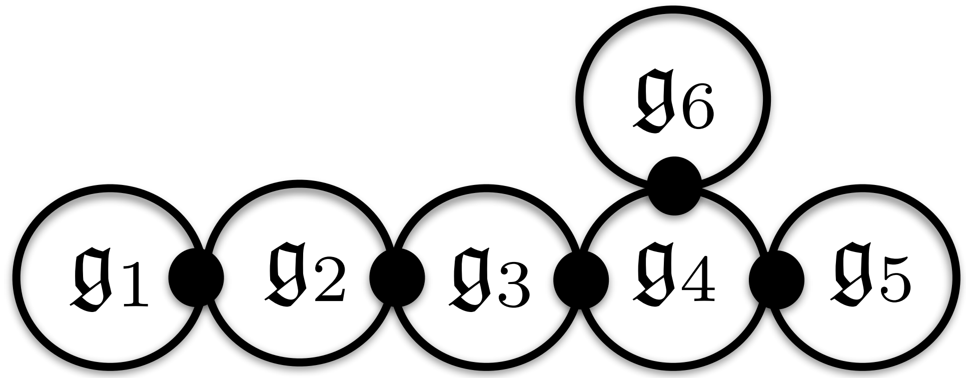


on S^1

[5d SCFT with flavor G-symm.] + [5d SYM of type G]

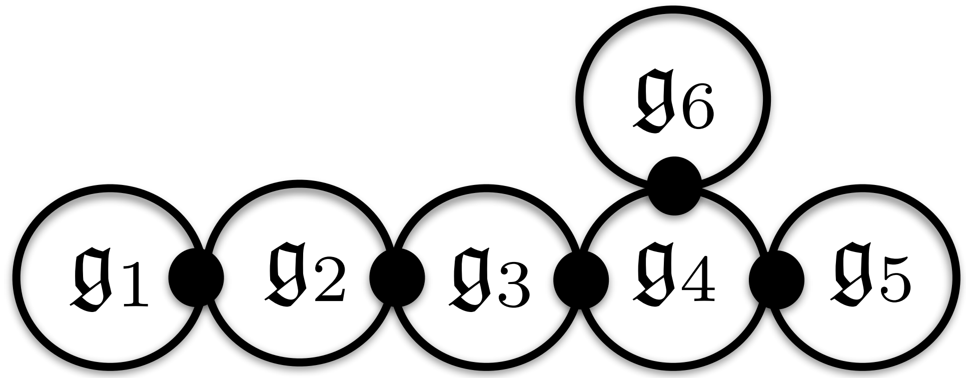
Justification of general claim (1)

6d SCFT

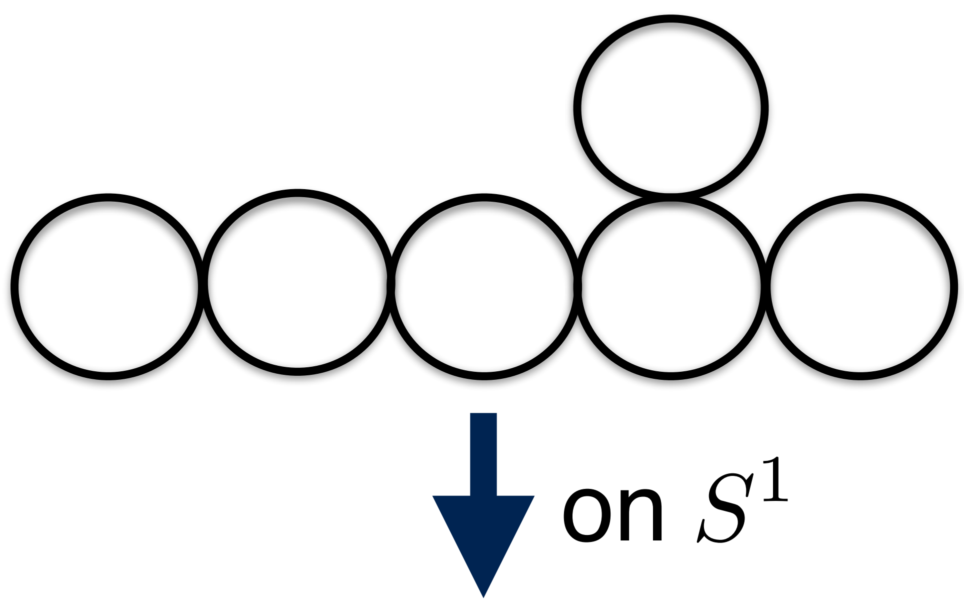


Justification of general claim (1)

6d SCFT

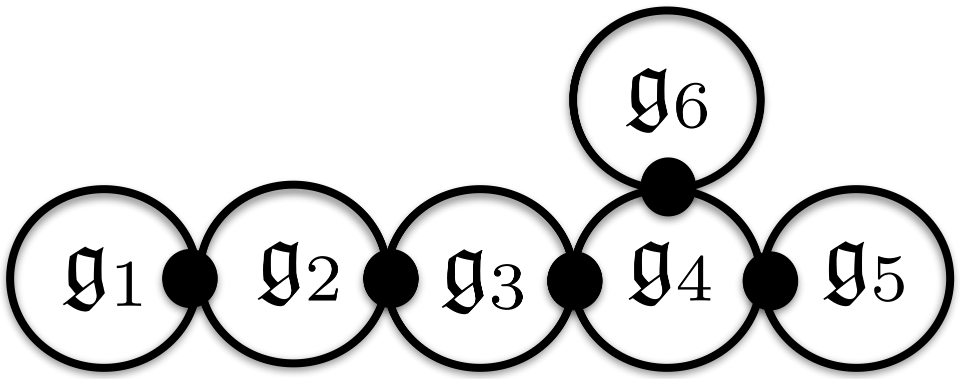


We can always Higgs this theory to the N=(2,0) theory

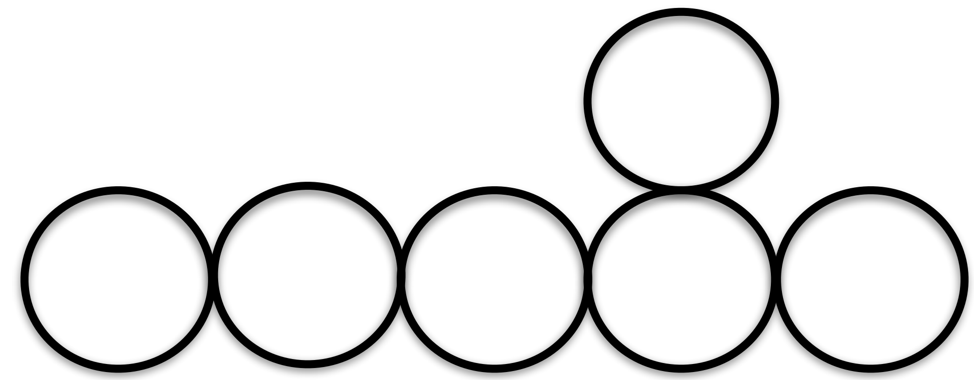


[5d N=2 SYM of type G]

Justification of general claim (1)



Before Higgsing



After Higgsing

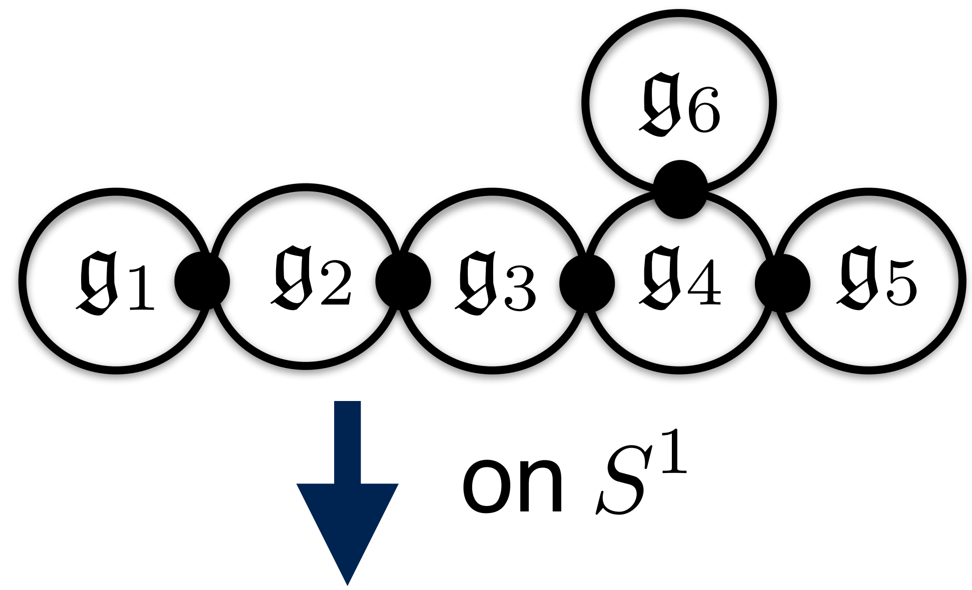
General fact about theories with 8 supercharges:

Higgs branch fields and tensor/vector fields do not mix.

If the theory contains 5d SYM after Higgsing, it contains 5d SYM before Higgsing.

Justification of general claim (1)

6d SCFT



[Something] + [5d SYM of type G]

We conjecture that something is always 5d SCFT.

Gauge couplings of $\mathfrak{g}$'s → infinity when vevs → zero

Justification of general claim (2)

Second part of general claim

[5d SCFT with flavor G-symm.] + [5d SYM of type G]

↓ on S^1

[4d SCFT(1)] — [IR free gauge group]—[4d SCFT(2)]
|
[4d SYM of type G]

Justification of this claim is more difficult…
See our paper.

Implication of General proposal

For example, 5d version of a generalized quiver like

$$CM_{\mathfrak{g}} - \mathfrak{g} - CM_{\mathfrak{g}} - \text{........} - \mathfrak{g} - CM_{\mathfrak{g}}$$

where $CM_{\mathfrak{g}}$ is the conformal matter of type $(\mathfrak{g}, \mathfrak{g})$

This quiver must have a UV fixed point as a 5d SCFT!
The UV fixed point has enhanced G=SU(N) symmetry.

More studies need to be done…..

Contents

Summary

- There is a class of 6d SCFTs which is a certain generalization of the N=(2,0) theories of type G=ADE.

- The circle compactification always gives

  [5d SCFT with flavor G-symm.] + [5d SYM of type G]

- The problem of the torus compactification of 6d SCFTs is reduced to the circle compactification of the corresponding 5d SCFTs.

Thank you very much!