Introduction to the mesoscopic physics of electrons and photons

From mesoscopic metals to cold atomic gases

Eric Akkermans ESPCI January 2007

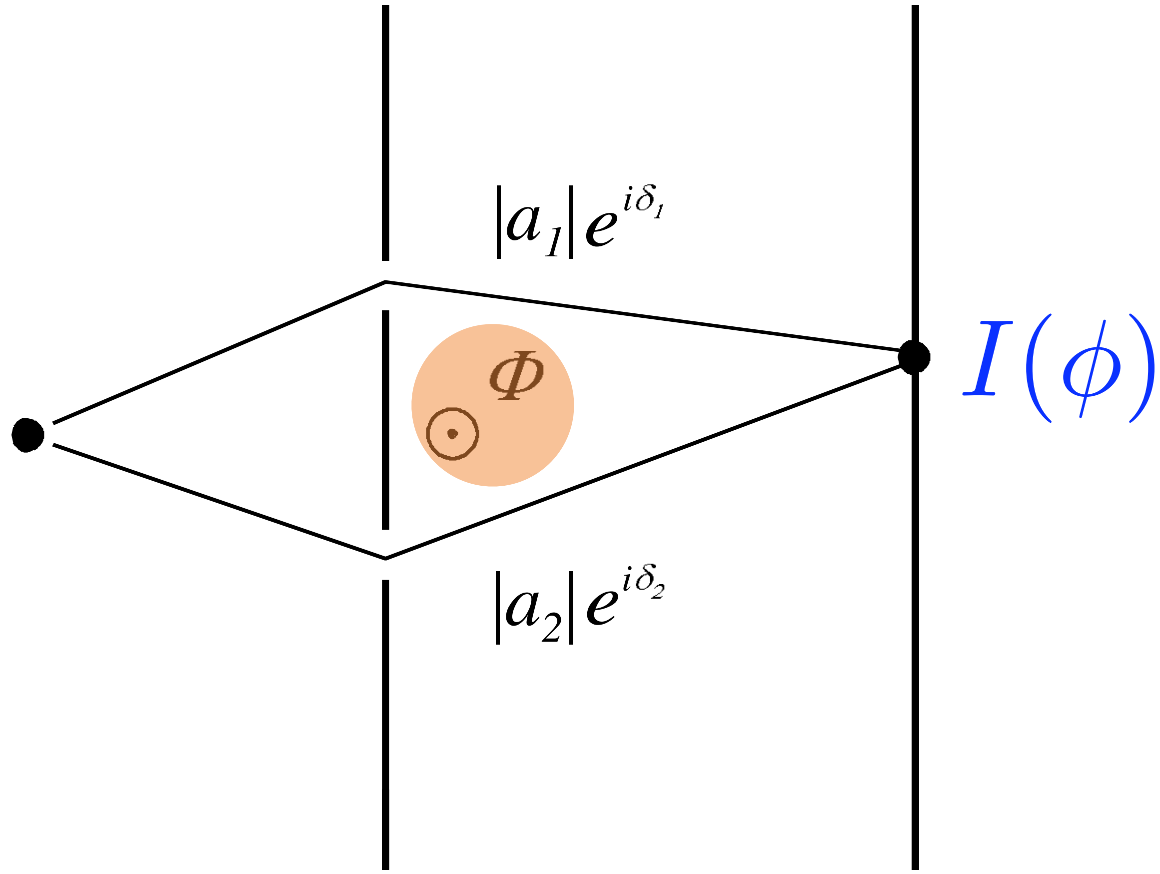
Based on *Mesoscopic physics of electrons and photons*, by Eric Akkermans and Gilles Montambaux, Cambridge University Press, 2007

Lecture 1

Introduction to mesoscopic physics

- The Aharonov-Bohm effect in disordered conductors.
- Phase coherence and effect of disorder.
- Average coherence: Sharvin^2 effect and coherent backscattering.
- Phase coherence and self-averaging: universal fluctuations.
- Classical probability and quantum crossings.

- # Aharonov-Bohm effect in disordered metals



The quantum amplitudes $a_{1,2} = |a_{1,2}|e^{i\delta_{1,2}}$ have phases:

$$\delta_1 = \delta_1^{(0)} - \frac{e}{\hbar}\int_1 \mathbf{A}.d\mathbf{l} \quad \text{and} \quad \delta_2 = \delta_2^{(0)} - \frac{e}{\hbar}\int_2 \mathbf{A}.d\mathbf{l}$$

The intensity $I(\phi)$ is given by

$$I(\phi) = |a_1 + a_2|^2 = |a_1|^2 + |a_2|^2 + 2|a_1 a_2| \cos(\delta_1 - \delta_2)$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\delta_1 - \delta_2)$$

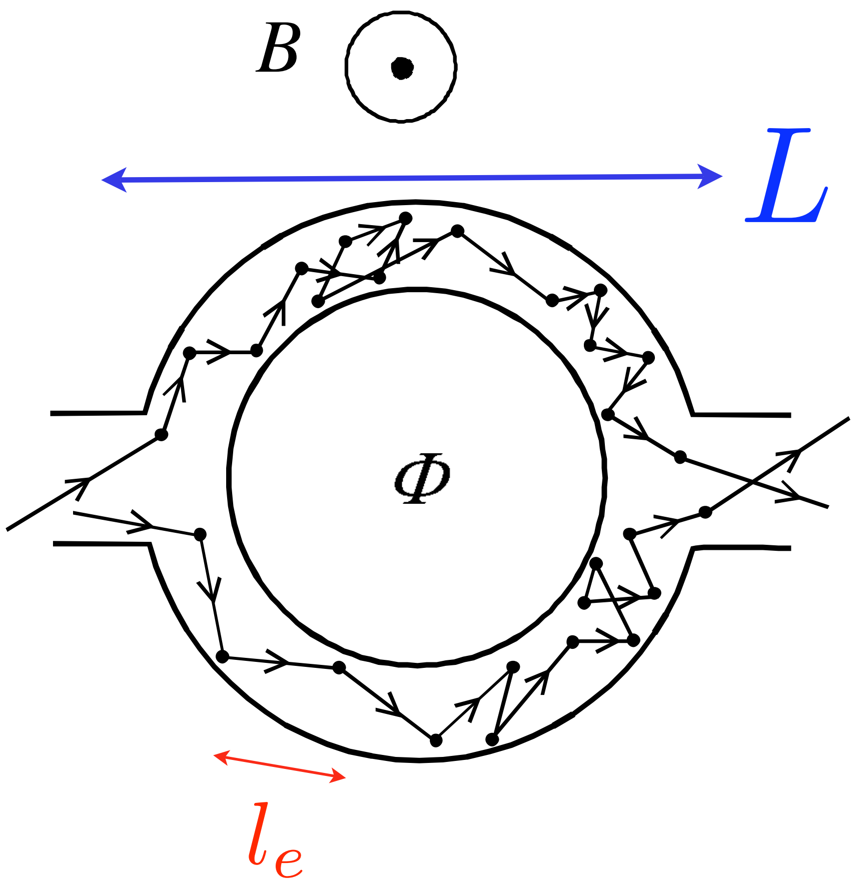
The phase difference $\Delta\delta(\phi) = \delta_1 - \delta_2$ is modulated by the magnetic flux ϕ :

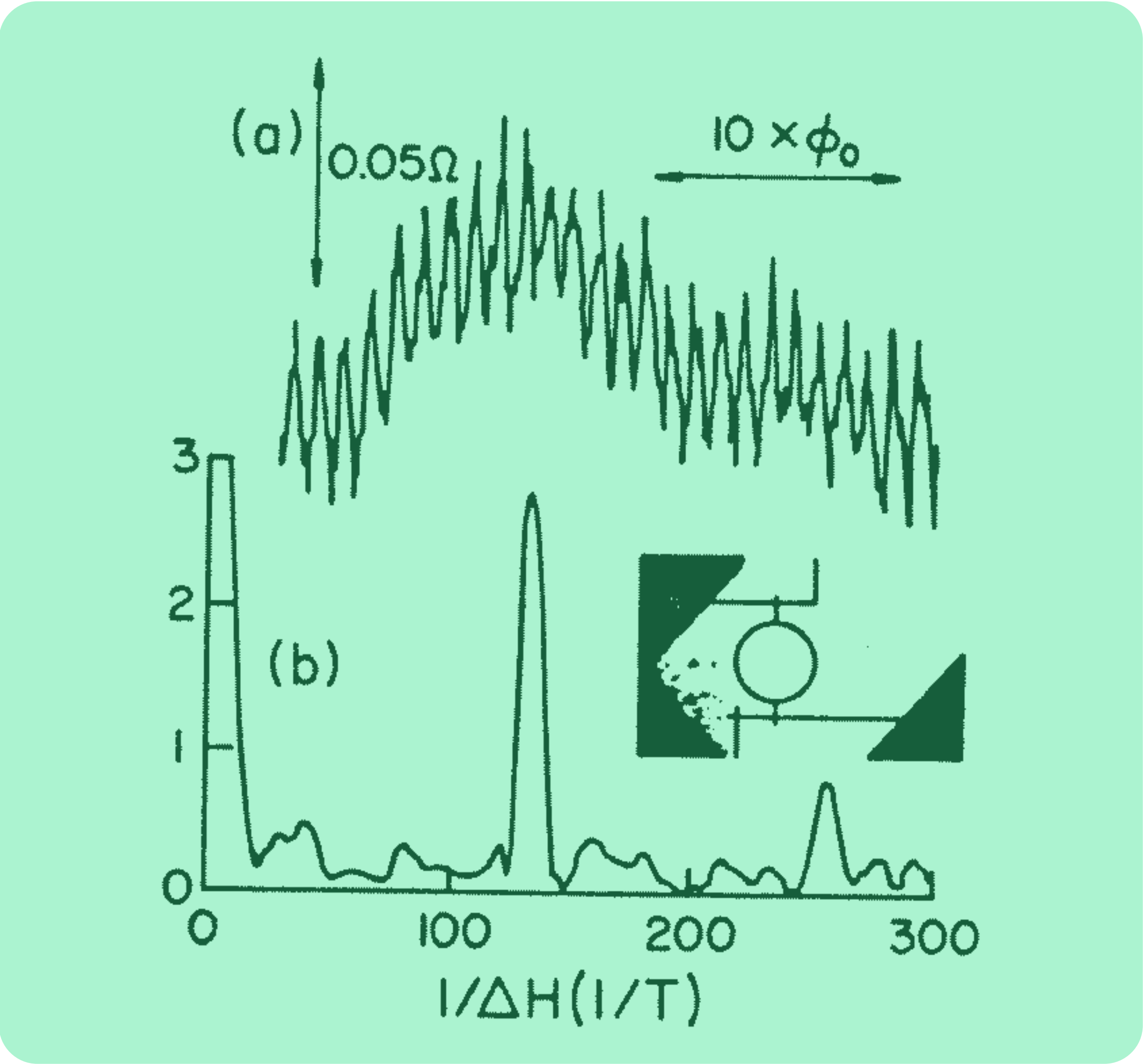
$$\Delta\delta(\phi) = \delta_1^{(0)} - \delta_2^{(0)} + \frac{e}{\hbar} \oint \mathbf{A}.d\mathbf{l} = \Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0}$$

where $\phi_0 = h/e$ is the quantum of magnetic flux.
Continuous change of the state of interference at each point:

Aharonov-Bohm effect.

Implementation in metals : the conductance $G(\phi)$ is the analog of the intensity.





$$G(\phi) = G_0 + \delta G \cos(\Delta\delta^{(0)} + 2\pi\frac{\phi}{\phi_0})$$

Webb et al.

Phase coherent effects subsist in metals in the multiple scattering regime.

Reconsider the Drude theory of metals.

Phase coherence and effect of disorder

The Webb experiment has been realized on a ring of size $L \simeq 1\mu$
For a macroscopic system coherent effects are washed out in normal metals.

⇨ It must exist a characteristic length L_ϕ called phase coherence length beyond which all coherent effects disappear.

Quantum coherence: gas of quantum particles in a finite volume

Quantum states of the gas are coherent superposition of single particle states and they extend over the total volume (ex. superconductivity, superfluidity, free electron gas, coherent states of the photon field).

For the electron gas, coherence disappears at non zero temperature so that we can use a classical description of transport and thermodynamics

Vanishing of quantum coherence results from the existence of incoherent and irreversible processes associated to the coupling of electrons to their surrounding (additional degrees of freedom) :

Coupling to a bath of excitations: thermal excitations of the lattice (phonons)
Chaotic dynamical systems (large recurrence times, Feynman chain)
Bath of virtual photons (Lamb shift,...)
Impurities with internal degrees of freedom (magnetic impurities)
Electron-electron interactions,....

The understanding of decoherence is difficult. It is one of the main challenges in quantum mesoscopic physics. The phase coherence length L_ϕ accounts in a generic way for decoherence processes.

The observation of coherent effects require that

$$l_e \ll L \ll L_\phi$$

Average coherence and multiple scattering

What is the role of disorder ? Does it erase coherent effects ?

Phase coherence leads to interference effects for a *given realization of disorder.*

The Webb experiment corresponds to a given configuration of disorder.

Averaging over disorder ⇒ vanishing of the Aharonov-Bohm effect

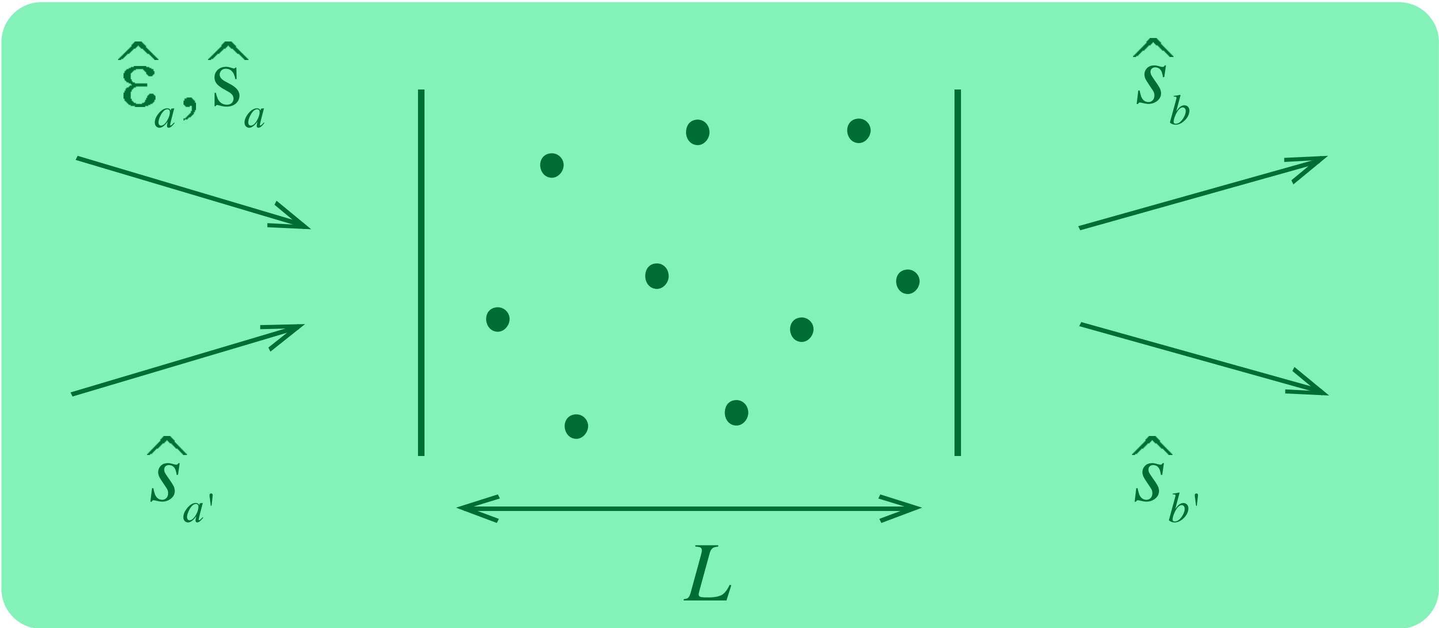
$$G(\phi) = G_0 + \delta G \cos(\Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0})$$

$$\Rightarrow \quad \langle G(\phi) \rangle = G_0$$

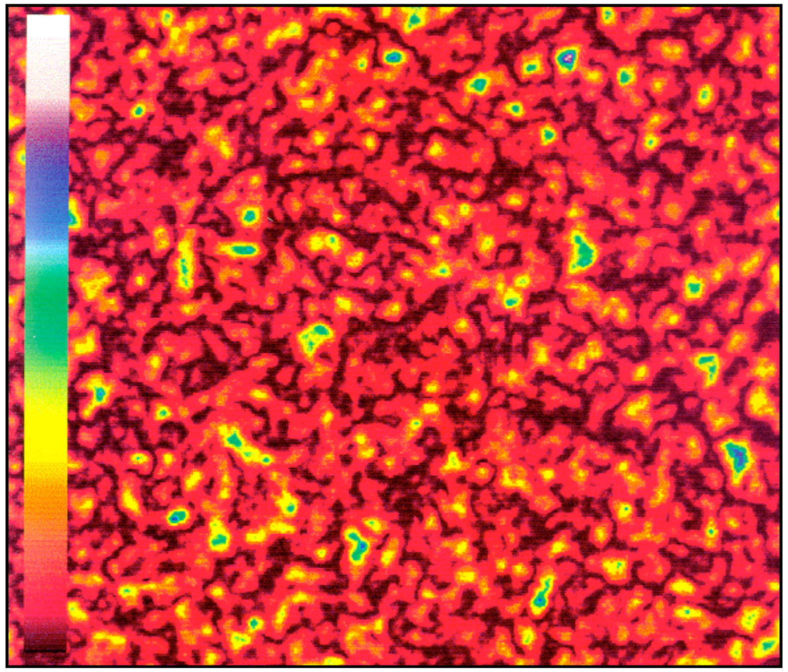
Disorder seems to erase coherent effects....

An analogous problem: Speckle patterns in optics

Consider the elastic multiple scattering of light transmitted through a disordered suspension.

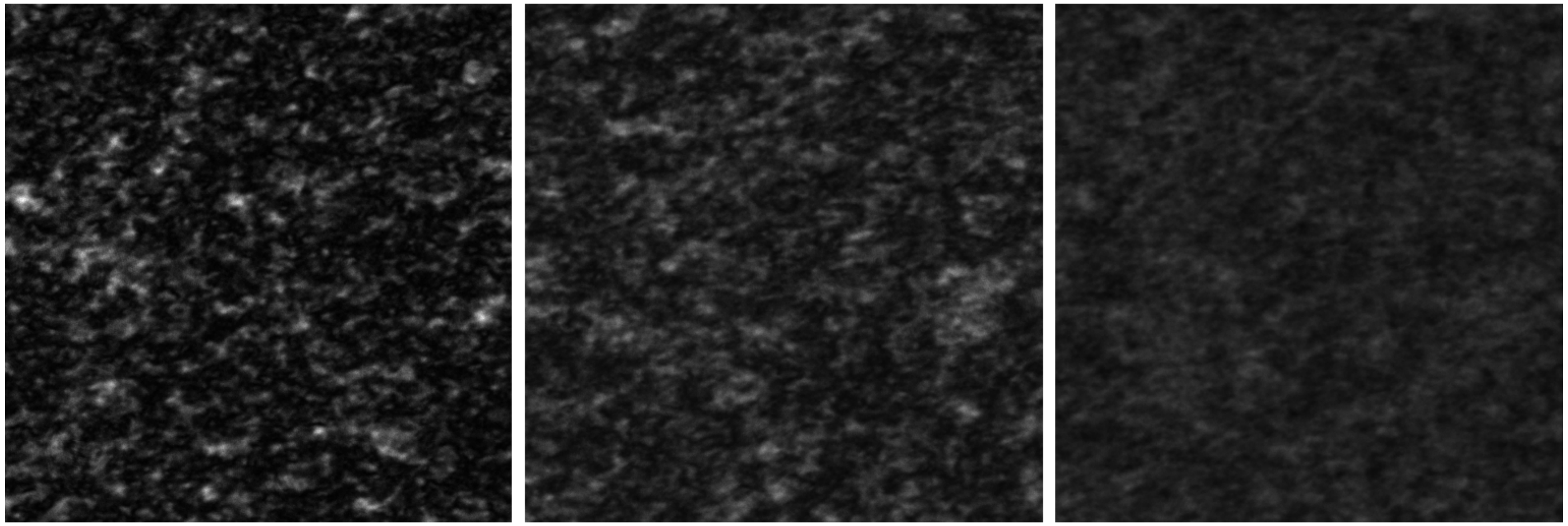


Outgoing light build a speckle pattern i.e., an interference picture:



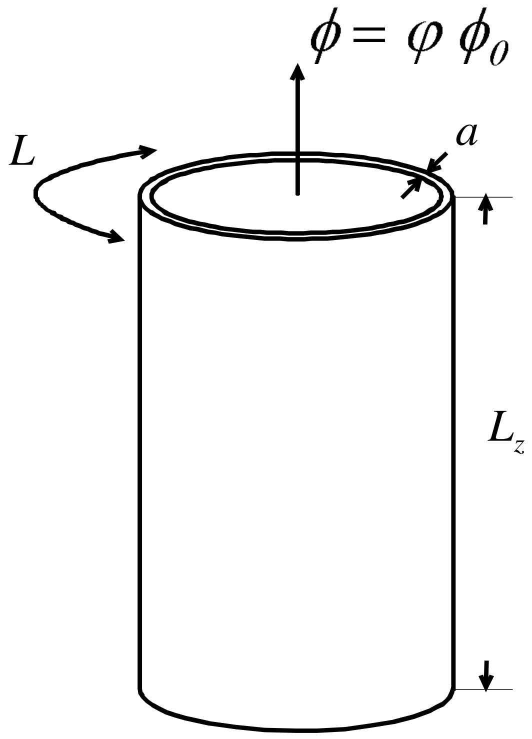
Averaging over disorder erases the speckle pattern:

Integration over the motion of the scatterers leads to averaging

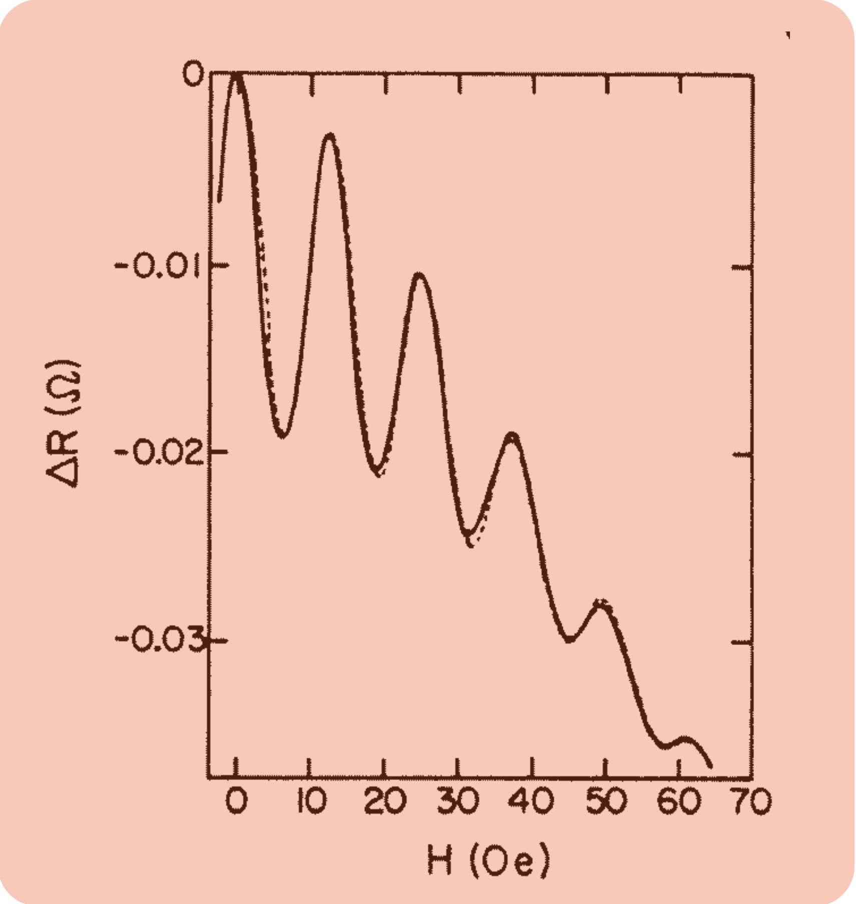


(a) (b) (c)

The Sharvin^2 experiment



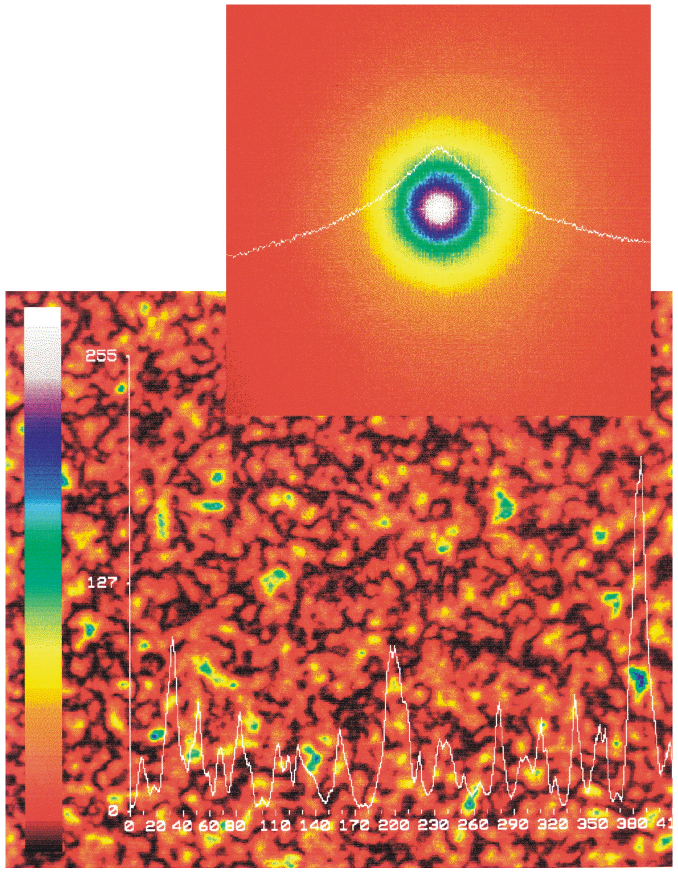
Experiment analogous to that of Webb but performed on a hollow cylinder of height larger than L_ϕ pierced by a Aharonov-Bohm flux.
Ensemble of rings identical to those of Webb but incoherent between them.

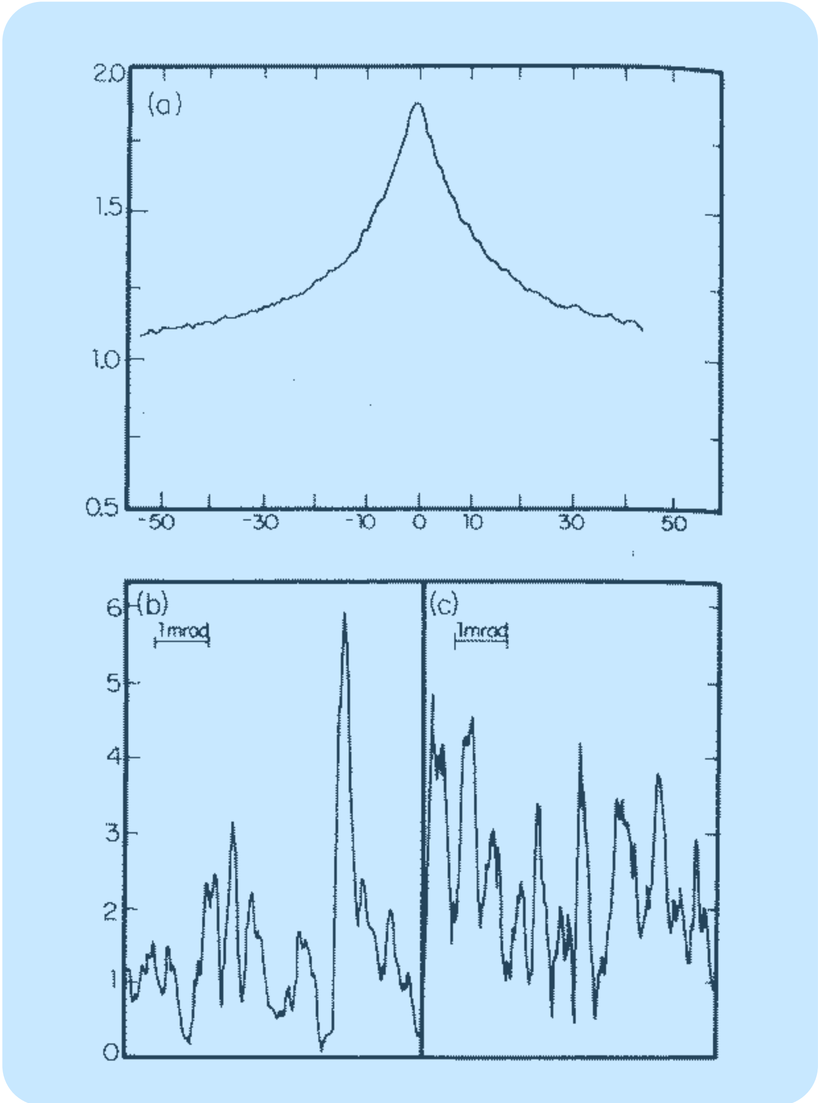


The signal modulated at ϕ_0 disappears but, instead, it appears a new contribution modulated at $\phi_0/2$

After all, disorder does not seem to erase coherent effects, but to modify them....

What about speckle patterns ?

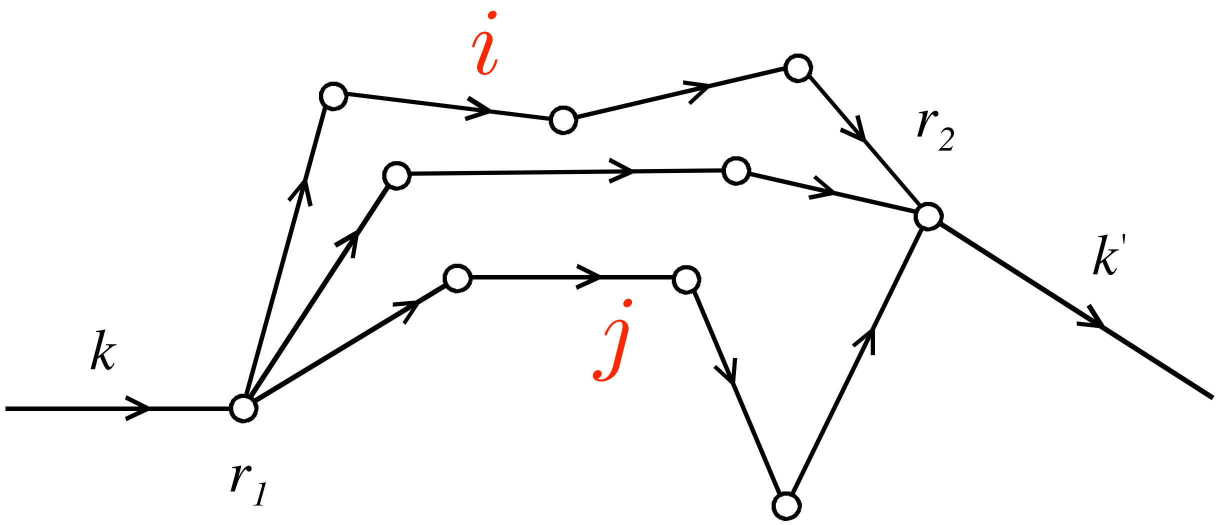




Averaging over disorder does not produce incoherent intensity only, but also an angular dependent part, the coherent backscattering, which is a coherence effect. We may conclude:

Elastic disorder is not related to decoherence : disorder does not destroy phase coherence and does not introduce irreversibility.

How to understand average coherent effects ?



Complex amplitude $A(\mathbf{k},\mathbf{k}')$ associated to the multiple scattering of a wave (electron or photon) incident with a wave vector $\mathbf{k}$ and outgoing with $\mathbf{k}'$

$$A(\mathbf{k},\mathbf{k}') = \sum_{\mathbf{r_1},\mathbf{r_2}} f(\mathbf{r_1},\mathbf{r_2}) e^{i(\mathbf{k}.\mathbf{r_1} - \mathbf{k}'.\mathbf{r_2})}$$

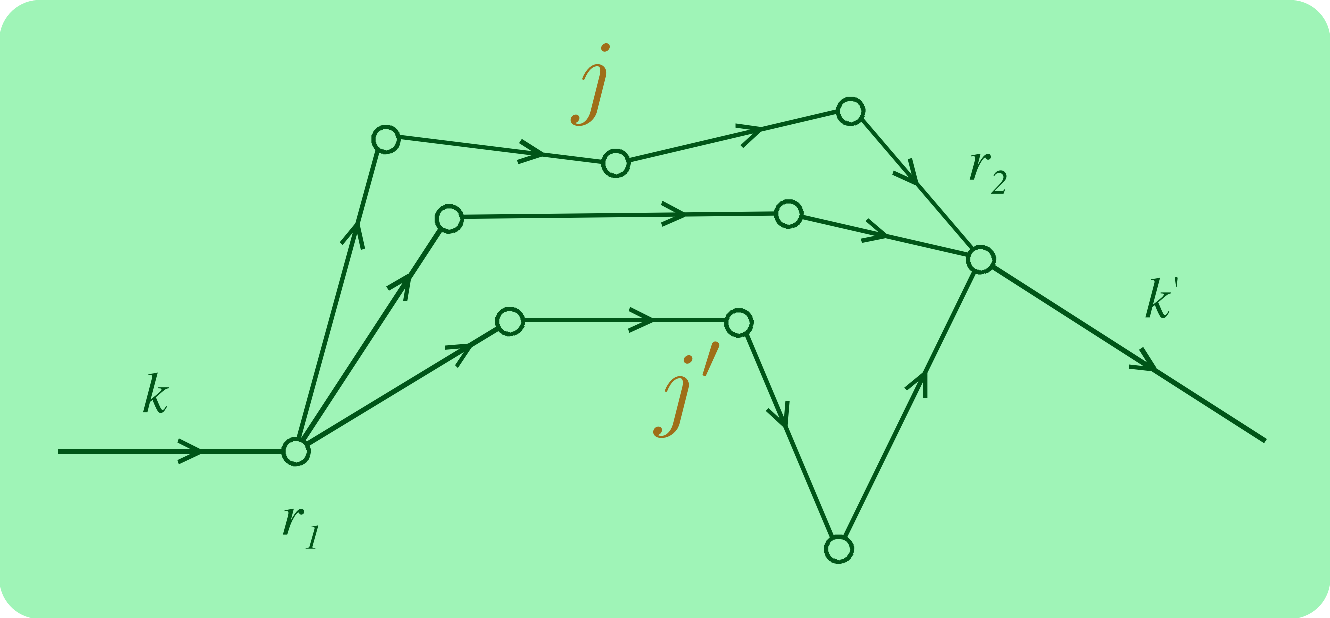
where the complex amplitude $f(\mathbf{r_1},\mathbf{r_2}) = \sum_j |a_j| e^{i\delta_j}$ describes the propagation of the wave between $\mathbf{r_1}$ and $\mathbf{r_2}$.

The related intensity is

$$|A(\mathbf{k},\mathbf{k}')|^2 = \sum_{\mathbf{r_1},\mathbf{r_2}} \sum_{\mathbf{r_3},\mathbf{r_4}} f(\mathbf{r_1},\mathbf{r_2}) f^*(\mathbf{r_3},\mathbf{r_4}) e^{i(\mathbf{k}.\mathbf{r_1}-\mathbf{k}'.\mathbf{r_2})} e^{-i(\mathbf{k}.\mathbf{r_3}-\mathbf{k}'.\mathbf{r_4})}$$

with

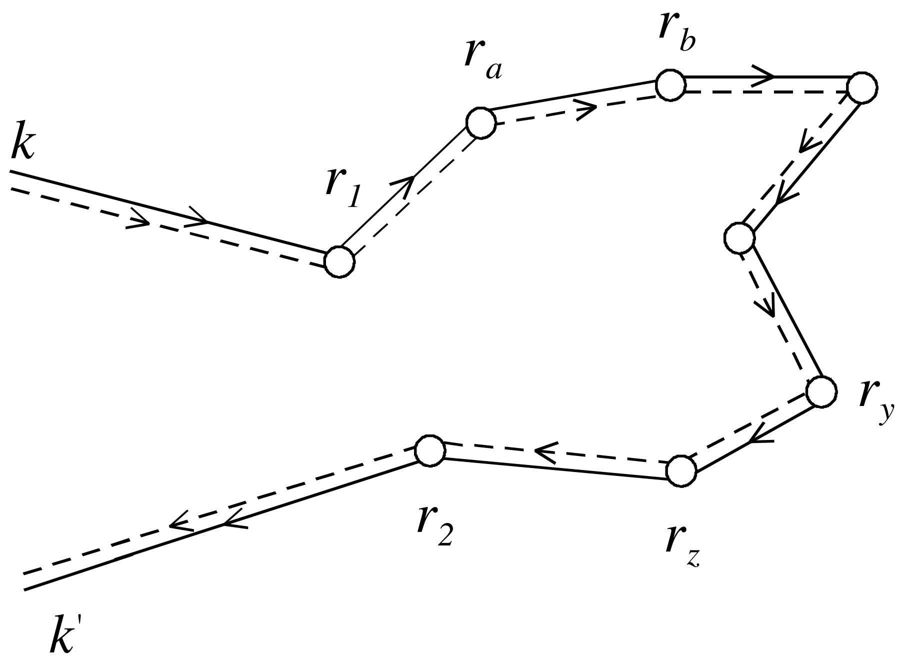
$$f(\mathbf{r_1},\mathbf{r_2}) f^*(\mathbf{r_3},\mathbf{r_4}) = \sum_{j,j'} a_j(\mathbf{r_1},\mathbf{r_2}) a^*_{j'}(\mathbf{r_3},\mathbf{r_4}) = \sum_{j,j'} |a_j||a_{j'}| e^{i(\delta_j - \delta_{j'})}$$



On average over disorder, most contributions to $f f^*$ disappear since the dephasing $\delta_j - \delta_{j'} \gg 1$

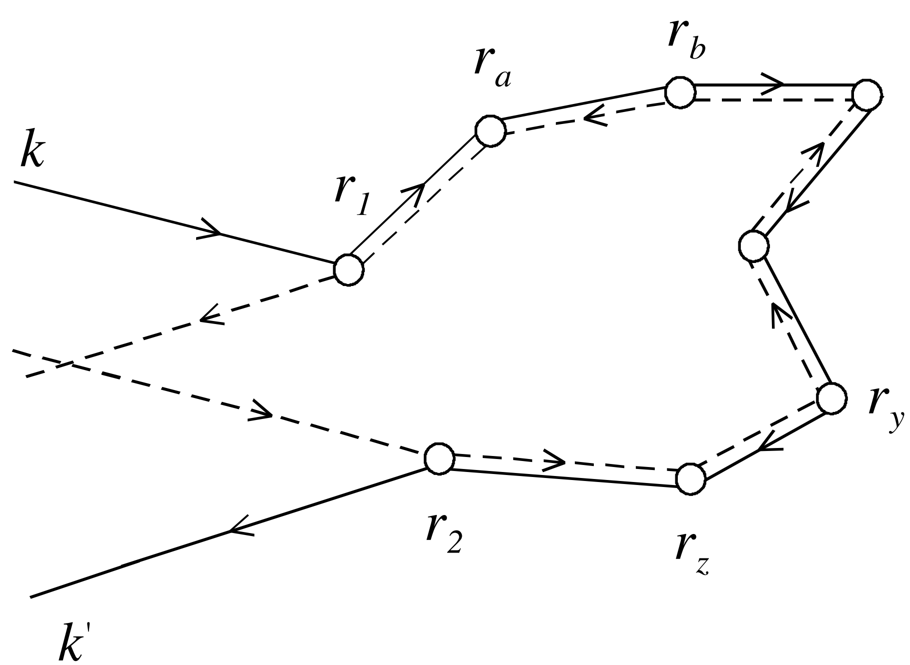
The only remaining contributions to the intensity correspond to terms with zero dephasing, i.e., to identical trajectories.

(a)



$$\mathbf{r_1} \longrightarrow \mathbf{r_a} \longrightarrow \mathbf{r_b} \cdots \longrightarrow \mathbf{r_y} \longrightarrow \mathbf{r_z} \longrightarrow \mathbf{r_2}$$

(b)



$$\mathbf{r_2} \longrightarrow \mathbf{r_z} \longrightarrow \mathbf{r_y} \cdots \longrightarrow \mathbf{r_b} \longrightarrow \mathbf{r_a} \longrightarrow \mathbf{r_1}$$

incoherent classical term

interference term

The total average intensity is:

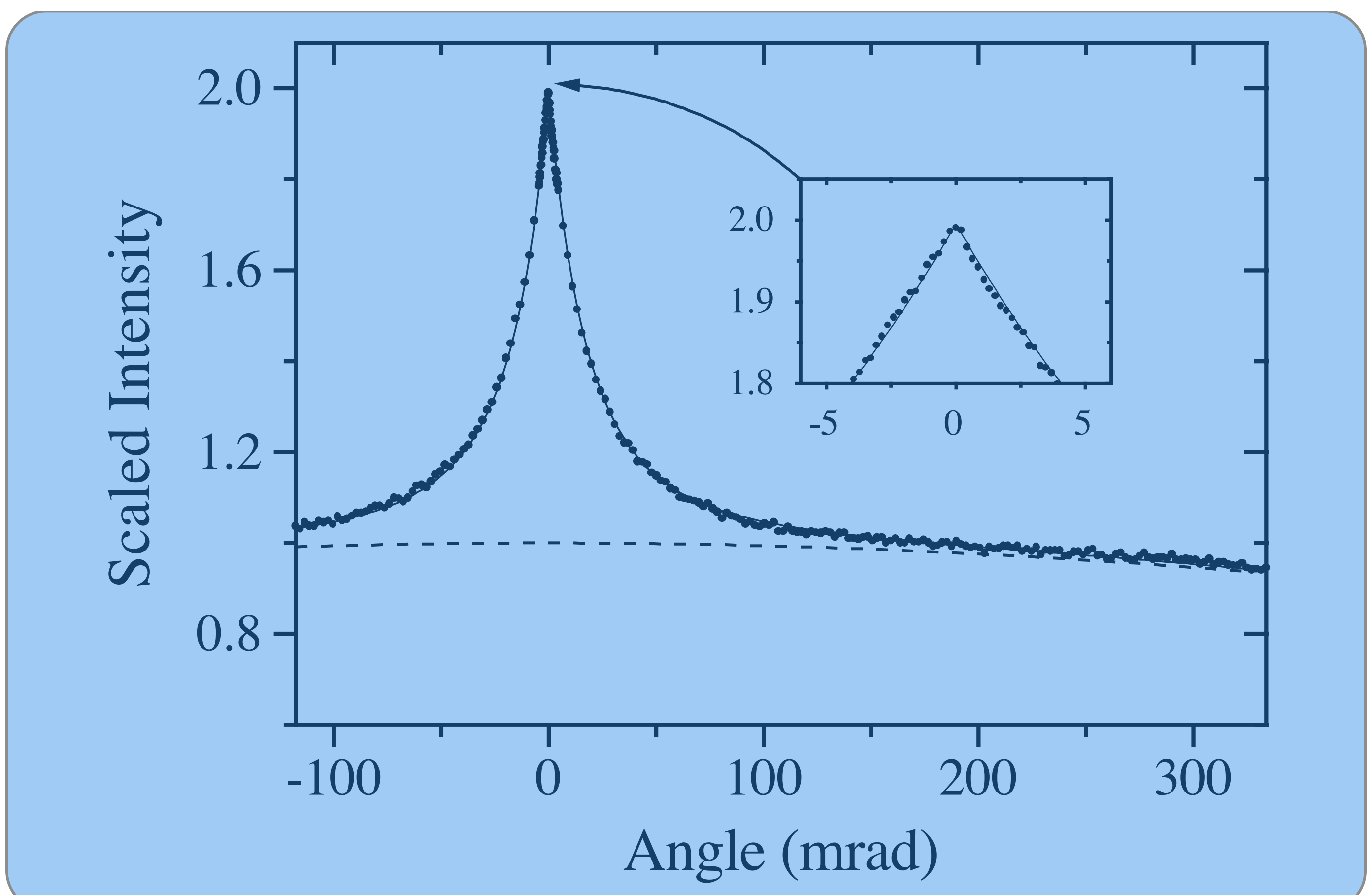
$$\overline{|A(\mathbf{k},\mathbf{k}')|^2} = \overline{\sum_{\mathbf{r_1},\mathbf{r_2}} |f(\mathbf{r_1},\mathbf{r_2})|^2 \left[1 + e^{i(\mathbf{k}+\mathbf{k}').(\mathbf{r_1}-\mathbf{r_2})}\right]}$$

$$\overline{|A(\mathbf{k},\mathbf{k}')|^2} = \overline{\sum_{\mathbf{r_1},\mathbf{r_2}} |f(\mathbf{r_1},\mathbf{r_2})|^2 \left[1 + e^{i(\mathbf{k}+\mathbf{k}').(\mathbf{r_1}-\mathbf{r_2})}\right]}$$

Generally, the interference term vanishes due to the sum over $\mathbf{r_1}$ and $\mathbf{r_2}$, except for two notable cases:

$\mathbf{k}+\mathbf{k}' \simeq 0$: Coherent backscattering

$\mathbf{r_1}-\mathbf{r_2} \simeq 0$: closed loops, weak localization and $\phi_0/2$ periodicity of the Sharvin effect.



Coherent backscattering

Random quantum systems (quantum complexity)

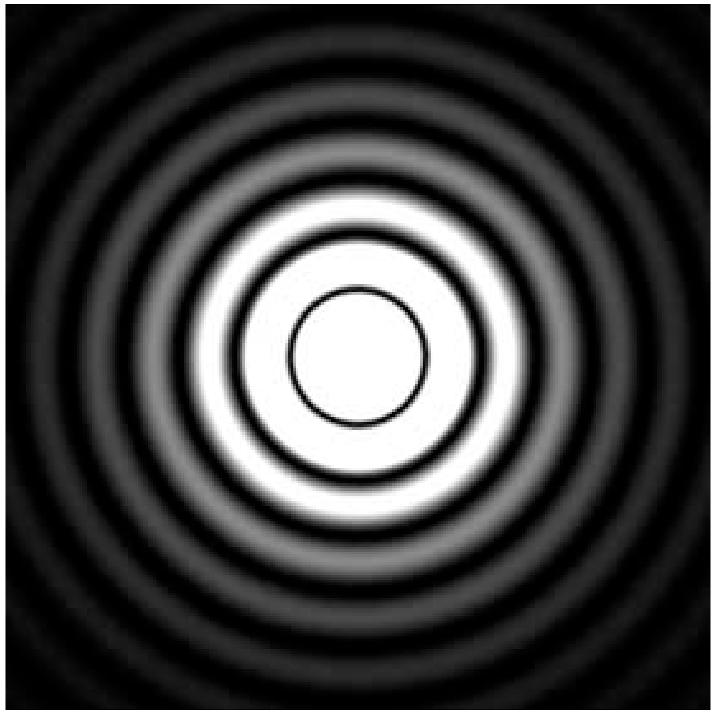
Disorder does not break phase coherence and it does not introduce irreversibility

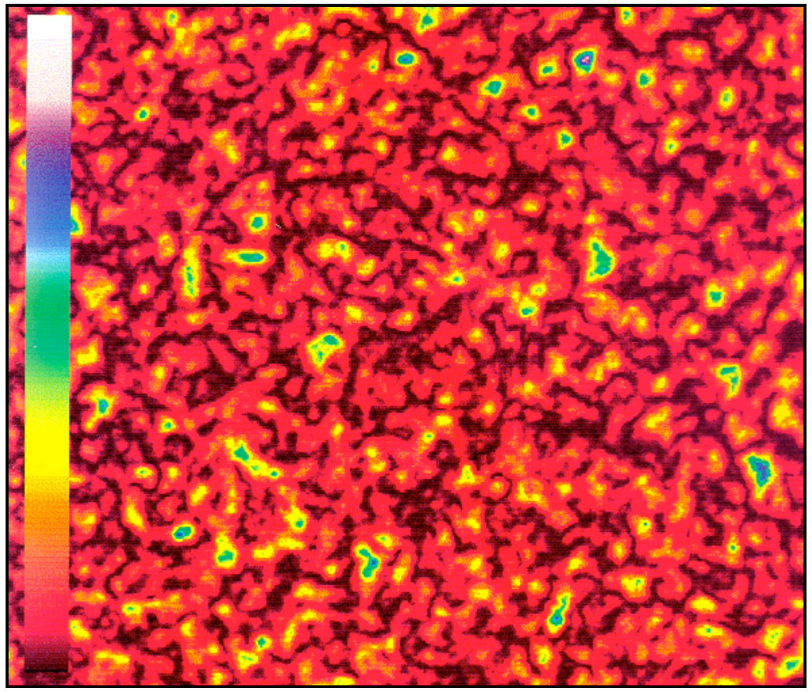
It introduces randomness and complexity: all symmetries are lost, there are no good quantum numbers.

Each quantum observable of a quantum complex system depends on the specific realization of disorder.

Exemple: speckle patterns in optics

Diffraction through a circular aperture: order in interference





Transmission of light through a disordered suspension: complex system

Mesoscopic quantum systems

- Most (all ?) quantum systems are complex
- Complexity (randomness) and decoherence are separate and independent notions.
- Complexity: loss of symmetries (good quantum numbers)
- Decoherence: irreversible loss of quantum coherence $L \gg L_\varphi$

Mesoscopic quantum system is a coherent complex quantum system with $L \leq L_\varphi$

Phase coherence and self-averaging: universal fluctuations.

Classical limit : $L \gg L_\varphi$

The system is a collection of $N = (L/L_\varphi)^d \gg 1$ statistically independent subsystems.

A macroscopic observable defined in each subsystem takes independent random values in each of the N pieces.

Law of large numbers: any macroscopic observable is equal with probability one to its average value.

The system performs an average over realizations of the disorder.

For $L \ll L_\varphi$, we expect deviations from self-averaging which reflect the underlying quantum coherence.

Need:

- a good understanding of the phase coherence length L_φ
- a description of fluctuations and coherence in a quantum complex system.
- If disorder (complexity) is strong enough, the system may undergo a *quantum phase transition*

Exemple: electrical conductance of a metal

A metal can be modeled as a quantum gas of electrons scattered by an elastic disorder.

At T=0 and in the absence of decoherence, it is a complex quantum system.

Due to disorder there is a finite conductance which is a quantum observable.

Classically, the conductance of a cubic sample of size L^d is given by Ohm's law: $G = \sigma L^{d-2}$ where σ is the conductivity.

- The units of electrical conductance in quantum mechanics is $$e^2/h \simeq 1/25.8k\Omega$$

- Units of classical conductance is given by the electromagnetic impedance of the vacuum:

$$\sqrt{\epsilon_0/\mu_0} \simeq 1/377\Omega$$

- The ratio of quantum to classical conductance is the fine structure constant:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$$

quantum effect in the electrical conductance are small and can be treated using perturbation theory like in quantum electrodynamics.

Quantum conductance fluctuations

Classical self-averaging limit : $\frac{\delta G}{\overline{G}} = \frac{1}{N} = \left(\frac{L_\varphi}{L}\right)^{d/2}$

where $\delta G = \sqrt{\overline{G^2} - \overline{G}^2}$ and $\overline{G} = \sigma L^{d-2}$

$\overline{\cdots}$ is the average over disorder.

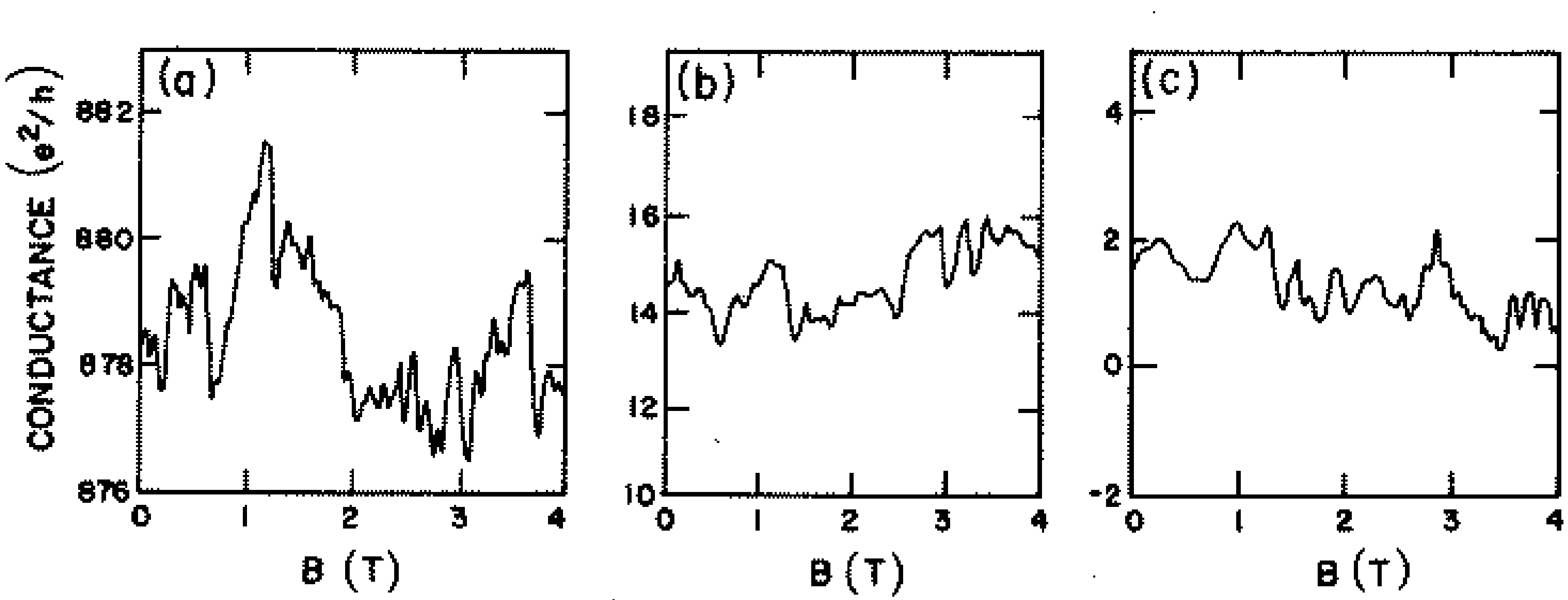
$$\delta G^2 \propto L^{d-4}$$

In contrast, a mesocopic quantum system is such that :

$$\delta G \simeq \frac{e^2}{h}$$

Fluctuations are quantum, large and independent of the source of disorder : they are called universal.

In the mesoscopic limit, the electrical conductance is not self-averaging.



Gold ring

Si-MOSFET

NUMERICS ON
THE ANDERSON MODEL