

## Multiple scattering of photons by a cold atomic gas

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We show that photons propagating in a mesoscopic gas of atoms acquire a correlation stronger than the value predicted by Rayleigh law. This enhanced correlation results from cross-terms based on the effects of spatial disorder and atomic quantum internal degrees of freedom. The great sensitivity of these cross-terms upon dephasing, such as a Zeeman splitting induced by a magnetic field, opens the possibility of a new kind of high precision spectroscopy.

### 1. Introduction

A wave propagating in a random medium undergoes scattering and the intensity pattern resulting from interferences of the scattered waves with each other is known as a speckle pattern [1]. The angular and time-dependent properties of these patterns have been extensively studied [2–6]. For a large enough scattering system, speckle patterns present universal features such as the Rayleigh law, which states that the variance  $\overline{\delta T^2} = \overline{T^2} - \overline{T}^2$  of the transmission coefficient  $T$  of the wave through the scattering sample, is simply related to its average by  $\overline{\delta T^2} = \overline{T}^2$  [1], where  $\overline{\quad}$  means a configuration average. The Rayleigh law is independent of the exact nature of the wave and of the scatterers as long as they are classical. It simply expresses that a speckle pattern results from the coherent superposition of a large number of uncorrelated random and complex valued amplitudes of the scattered wave. Our aim is to show that speckle correlations are significantly enhanced above the Rayleigh value for the case of scatterers having quantum degrees of freedom. Here we consider atoms with degenerate Zeeman sublevels. This enhancement results from cross-terms of transition amplitudes connecting two different sets of quantum states. For atoms with a degenerate ground state, the cross-terms involve the corresponding ground state sublevels. This suggests that the enhanced correlations are very sensitive to the Zeeman splitting induced by an applied magnetic field. We show that it is indeed the case and that the amplified correlation displays a resonant-like behaviour whose linewidth is, in principle, not limited. This might prove useful in level-crossing spectroscopy measurements.

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We refer to the setup shown in figure 1. An ensemble of atoms placed at random positions  $\mathbf{R}_i$ , which scatter a monochromatic beam of wave vector  $\mathbf{k}_a$ , with  $\hat{\mathbf{s}}_a = \mathbf{k}_a/|\mathbf{k}_a|$ , and polarization  $\hat{\mathbf{e}}_a$ . At some time, which we choose to be zero ( $t=0$ ), scattered photons of wave vector  $\mathbf{k}_b$  ( $\hat{\mathbf{s}}_b = \mathbf{k}_b/|\mathbf{k}_b|$ ) and polarization  $\hat{\mathbf{e}}_b$ , are detected far away by the detector  $D$  after being multiply scattered. This process will be referred to as  $ab$  channel. A time  $\tau$  later a second measurement is performed, which corresponds to an incident beam incoming along  $\hat{\mathbf{s}}_{a'}$ , while the scattered photons are measured along  $\hat{\mathbf{s}}_{b'}$  ( $a'b'$  channel). We assume that the atoms are cold enough, and the time  $\tau$  is short enough, so that the scatterers stay at rest between the two measurements. Considering that atoms have a diffusive motion characterized by a diffusion coefficient  $D_B$  due to fluctuations of the atomic momentum, they stay at rest so long as  $\tau \ll \tau_B$ , where  $\tau_B \simeq \lambda^2/D_B$ . In optical molasses, the diffusion coefficient  $D_B$  is about a few  $\text{mm}^2\text{s}^{-1}$ . For visible light with  $\lambda \sim 5 \times 10^{-4} \text{ mm}$  we find the order of magnitude  $\tau_B \sim 10^{-7} \text{ s}$ . We also assume  $\tau > 1/\Gamma$  in order not to probe atomic relaxation processes. For example, referring to the above orders of magnitude,  $\Gamma$  should be larger than  $10^7 \text{ s}^{-1}$ , otherwise there is no  $\tau$  that fits all requirements. A similar couple of measurements is then repeated after a relatively long time  $T \gg \tau$ , during which the scatterers move. This motion requires the averaging over atom position, namely, a *disorder average*.

Atoms are modelled as degenerate two-level systems. We denote by  $|m_i\rangle$  the ground state Zeeman sublevels with a total angular momentum quantum number  $j_g$ ,  $|m_e\rangle$  are the excited states sublevels of linewidth  $\Gamma$  and total angular momentum  $j_e$ , and  $m$  is the projection on a quantization axis. We consider resonant elastic scattering so that the length of the wave vectors is fixed,  $|\mathbf{k}_{a,b}| = |\mathbf{k}_{a',b'}| = k = 2\pi/\lambda$ . For the sake of simplicity we shall assume that all ground state sublevels

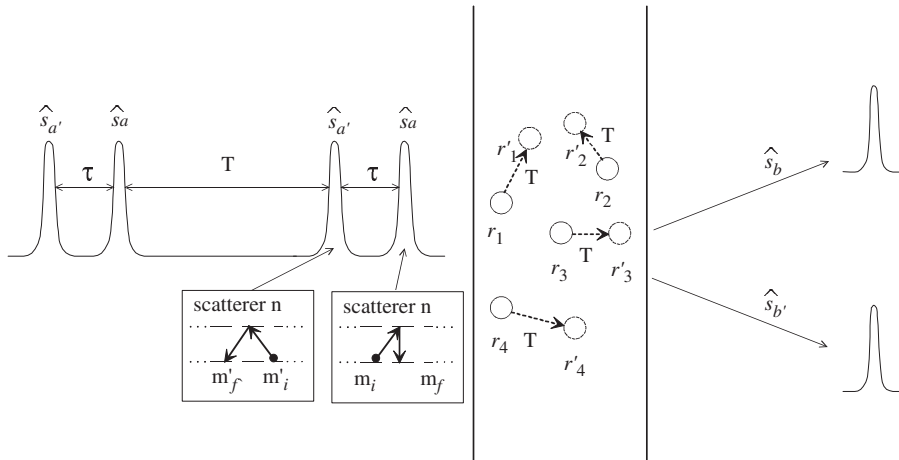


Figure 1. The suggested experimental setup. Each pair of pulses, incident along  $\hat{\mathbf{s}}_a$  and  $\hat{\mathbf{s}}_{a'}$ , and detected along  $\hat{\mathbf{s}}_b$  and  $\hat{\mathbf{s}}_{b'}$ , respectively, corresponds to a measurement of the non-averaged quantity  $\mathcal{T}_{ab}\mathcal{T}_{a'b'}$ . During the time  $\tau$  the atoms stay at rest. Repeating this measurement many times yields  $\overline{\mathcal{T}_{ab}\mathcal{T}_{a'b'}}$ , provided  $T$  is sufficiently long so as to allow the atoms to move appreciably.

are *a priori* equiprobable. We use here a lowest order perturbation theory, meaning that the incoming intensity is supposed to be sufficiently weak so that each atom undergoes, at most, one lowest order scattering event during a period of time that corresponds to a single measurement. On the other hand, notice that photons stay within the atomic gas for a relatively long time during a multiple scattering sequence, so that other photons are also passing through the medium. This eliminates any possible ‘which path’ information, since there is no way by which one is able to assign a certain atomic transition to a certain photon. Moreover, during the time interval  $\tau$  many photons are scattered by the gas, so that there is no correlation between the atomic states probed by photons that correspond to the first measurement, and that of the second measurement.

The purpose of this work is to study the *static normalized angular correlation function* of photons performing coherent multiple scattering in a cold atomic gas. This function is defined by [1, 8]

$$C_{aba'b'} = \frac{\overline{\delta\mathcal{T}_{ab}\delta\mathcal{T}_{a'b'}}}{\overline{\mathcal{T}_{ab}}\overline{\mathcal{T}_{a'b'}}}. \quad (1)$$

For classical scatterers, intensity fluctuations obey the Rayleigh law which rewrites  $C_{abab} = 1$ . In the presence of a Zeeman degeneracy, angular correlations of speckle patterns and intensity fluctuations become larger than one. This is a new and genuine mesoscopic effect specific to multiple scattering of photons by atoms and directly related to properties of atomic quantum states.

## 2. The single scattering case

It is instructive to begin with the simple example of only two atoms in the gas cloud, and a ‘beam’ consisting of only two incident photons having a wave vector  $\mathbf{k}_a$ . At  $t=0$ , these two photons are scattered by the system of two atoms. The interaction of the two photons with the atoms is assumed to be simultaneous, meaning that there is no way by which one can obtain the intermediate state of the system between individual scattering events. This point is crucial, since it distinguishes our case from others [7], in which the low intensity limit is assumed, thus excluding the situation of two photons scattered simultaneously off the atoms. Regarding the scattering system, thus, one can only obtain its initial and final state, i.e. the internal quantum states of both atoms (in addition to their positions) before and after the ‘beam’ has interacted with them. Suppose that at  $t=0$  the atoms were in the Zeeman sublevels  $|m_1\rangle$  and  $|m_3\rangle$ , and after scattering they were in the  $|m_2\rangle$  and  $|m_4\rangle$  sublevels, respectively. Since the scattering of the two photons happens simultaneously, there is no way by which we could assign a given transition (say  $|m_1\rangle \rightarrow |m_2\rangle$  of atom 1) to a certain photon. In other words, the fact that the two scattering events are indistinguishable makes any ‘which path’ information to disappear. Of course, photon polarization can also be used to obtain ‘which path’ information in certain cases, but we will not consider them here because they are not applicable in the case of multiple scattering. Assuming that each photon is scattered, at most, only once, our aim is to find the transmission coefficient  $\mathcal{T}_{ab}$  along a direction  $\mathbf{k}_b$ .

For a very large number of incoming photons,  $\mathcal{T}_{ab}$  is approximated by the probability of one photon to emerge along  $\mathbf{k}_b$ . Under the above assumptions  $\mathcal{T}_{ab}$  equals, to lowest order

$$\mathcal{T}_{ab} = \mathcal{F} |\langle m_2 | U_1 | m_1 \rangle \exp(-i\mathbf{Q} \cdot \mathbf{R}_1) + \langle m_4 | U_2 | m_3 \rangle \exp(-i\mathbf{Q} \cdot \mathbf{R}_2)|^2, \quad (2)$$

where we have defined the operators  $U_i = \sum_{m_e} (\mathbf{d}_i \cdot \hat{\boldsymbol{\epsilon}}_a) |m_e\rangle \langle m_e| (\mathbf{d}_i \cdot \hat{\boldsymbol{\epsilon}}_b)$ ,  $m_e$  being the Zeeman quantum numbers of the atomic excited state,  $\mathbf{R}_i$  is the position and  $\mathbf{d}_i$  is the electric dipole moment of atom  $i$ ,  $\mathbf{Q} = \mathbf{k}_b - \mathbf{k}_a$ , and  $\mathcal{F}$  is a proportionality factor† containing the photon frequency  $\omega$ , the quantization volume  $L^3$ , the natural width of the excited atomic state  $\Gamma$ , etc. We also assume a purely resonant scattering. The average transmission coefficient  $\overline{\mathcal{T}}_{ab}$  is found by averaging  $\mathcal{T}_{ab}$  both over the positions  $\mathbf{R}_{1,2}$  and the initial quantum states  $|m_1\rangle$ ,  $|m_3\rangle$ . Also, summation over undetected final quantum states must be performed. Assuming  $k|\mathbf{R}_2 - \mathbf{R}_1| \gg 1$ , the cross-terms involving products like  $\exp(-i\mathbf{Q} \cdot \mathbf{R}_1) \exp(i\mathbf{Q} \cdot \mathbf{R}_2)$  vanish upon averaging over the positions of the atoms (disorder average) because of the rapidly fluctuating phase difference. Thus, denoting  $J = 2j_g + 1$ , we obtain

$$\overline{\mathcal{T}}_{ab} = \frac{\mathcal{F}}{J} \sum_{m_{1,2}} |\langle m_2 | U_1 | m_1 \rangle|^2 + \frac{\mathcal{F}}{J} \sum_{m_{3,4}} |\langle m_4 | U_2 | m_3 \rangle|^2. \quad (3)$$

We now wish to find the (not normalized) correlation function between  $\mathcal{T}_{ab}$  and  $\mathcal{T}_{a'b'}$ , namely

$$\text{Corr}(a, b, a', b') = \overline{\mathcal{T}_{ab} \mathcal{T}_{a'b'}} - \overline{\mathcal{T}_{ab}} \overline{\mathcal{T}_{a'b'}}. \quad (4)$$

Denoting  $A_i^{\{mm'\}} = \mathcal{F}^{1/2} \langle m' | U_i | m \rangle \exp(-i\mathbf{Q} \cdot \mathbf{R}_i)$ , we have

$$\overline{\mathcal{T}}_{ab} = \frac{1}{J} \sum_{m_{1,2}} |A_1^{\{m_1 m_2\}}|^2 + \frac{1}{J} \sum_{m_{3,4}} |A_2^{\{m_3 m_4\}}|^2 \quad (5)$$

with a similar expression for  $\overline{\mathcal{T}}_{a'b'}$ , and

$$\mathcal{T}_{ab} \mathcal{T}_{a'b'} = |A_1^{\{m_1 m_2\}} + A_2^{\{m_3 m_4\}}|^2 |A_1^{\{m'_1 m'_2\}} + A_2^{\{m'_3 m'_4\}}|^2. \quad (6)$$

Note that the prime designates only the internal quantum numbers  $m_i$ . This is because the other degrees of freedom, namely the positions  $\mathbf{R}_i$ , are assumed to be the same for  $\mathcal{T}_{ab}$  and  $\mathcal{T}_{a'b'}$ . It should be kept in mind, however, that the incoming and outgoing polarizations corresponding to the two transmission coefficients are generally different. To find  $\text{Corr}(a, b, a', b')$  we now have to average (6). We first notice that (6) contains products of four amplitudes corresponding to all the possible combinations of internal and external degrees of freedom. Performing first the disorder average, only products involving zero phase difference survive, the others

†This factor is of no importance since it cancels in the normalized correlation function (to be defined later). Therefore, it will not be given explicitly.

contain rapidly fluctuating phases and vanish on average, leaving only

$$A_1^{\{m_1 m_2\}} A_2^{*\{m_3 m_4\}} A_1^{*\{m'_1 m'_2\}} A_2^{\{m'_3 m'_4\}} + \text{c.c.} \quad (7)$$

Averaging over internal quantum numbers we find, using the definition (4)

$$\text{Corr}(a, b, a', b') = \frac{1}{J^4} \sum_{m_i, m'_i} A_1^{\{m_1 m_2\}} A_1^{*\{m'_1 m'_2\}} \sum_{m_i, m'_i} A_2^{*\{m_3 m_4\}} A_2^{\{m'_3 m'_4\}} + \text{c.c.}, \quad (8)$$

where the pre-factor  $1/J^4$  corresponds to averaging over initial internal states of the atoms.

Looking at the rhs of (8), we see that the correlation consists of a product of two cross-sections, one for each of the two atoms. But as opposed to the usual, intensity cross-section, each of the cross-sections appearing in (8) involves the coupling of two scattering amplitudes that correspond to different Zeeman quantum numbers. To be more explicit we write, up to a factor, the two kinds of cross-sections, which we also term *vertices*

$$\mathcal{V}^{(i)} \propto \frac{1}{J} \sum_{m_{1,2}} |\langle m_2 | U | m_1 \rangle|^2 \quad (9)$$

for the usual, intensity cross-section, and

$$\mathcal{V}^{(c)} \propto \frac{1}{J^2} \sum_{m_{1,2,3,4}} \langle m_2 | U | m_1 \rangle \langle m_4 | U | m_3 \rangle^* \quad (10)$$

for the correlation cross-section. For non-degenerate atomic states we have  $J=1$ , and the two vertices coincide. Therefore, focusing on the case  $a = a'$ ,  $b = b'$

$$\text{Corr}(a, b, a, b) = 2|A_1|^2 |A_2|^2 \quad (\text{non-degenerate}). \quad (11)$$

Since the two atoms are identical we have  $|A_1|^2 = |A_2|^2$  so that

$$C_{abab} = \frac{\text{Corr}(a, b, a, b)}{\overline{\mathcal{T}}_{ab}^2} = \frac{1}{2} \quad (\text{non-degenerate}). \quad (12)$$

This is the Rayleigh law for only two scattering atoms (the result  $C_{abab} = 1$  is the Rayleigh law for many atoms). Comparing (12) to (8) and (5) we notice that the correlation in the degenerate ( $J > 1$ ) case generally differs significantly from that of the non-degenerate case. An upper bound of the correlation in the degenerate case is obtained by taking in (8) all the amplitudes  $A_i^{\{mm'\}}$  to be equal. This leads to  $C_{abab} = J^2/2$ , a much larger value than  $1/2$  obtained for non-degenerate atomic levels.

### 3. Multiple scattering

We now turn to the multiple scattering regime, the limit of a large number of atoms and photons. The average transmission coefficient  $\overline{\mathcal{T}}_{ab}$  is obtained by summing all

the possible scattering amplitudes  $A_i^{\{R,m\}}$ , corresponding to a given configuration  $\{R,m\}$ . Here  $\{R\}$  accounts for the spatial positions of all scatterers, and  $\{m\}$  is a notation for their internal Zeeman states both before and after the scattering. The index  $i$  denotes one possible multiple scattering path. Squaring the sum of amplitudes we have

$$\overline{\mathcal{T}}_{ab} = \overline{\left| \sum_i A_i^{\{R,m\}} \right|^2} = \sum_{i \neq i'} \overline{A_i^{\{R,m\}} A_{i'}^{\{R,m\}*}}. \tag{13}$$

When averaging over  $\{R\}$ , all cross-terms  $i \neq i'$  vanish because of large fluctuating phase shifts, so that  $\overline{\mathcal{T}}_{ab} = \sum_i \overline{|A_i^{\{m\}}|^2}$ , where now the averaging sign corresponds only to the  $\{m\}$  degrees of freedom. This expression, termed *Diffuson* [5], is the leading approximation in the weak disorder limit  $kl \gg 1$ , where  $l$  is the elastic mean free path of the photons inside the gas. Similarly to the simple case considered above, the correlation of the transmission coefficients is

$$\overline{\mathcal{T}}_{ab} \overline{\mathcal{T}}_{a'b'} = \overline{\mathcal{T}_{ab}^{\{R,m\}} \mathcal{T}_{a'b'}^{\{R,m'\}}} = \overline{\sum_{ijkl} A_i^{\{R,m\}} A_j^{\{R,m\}*} A_k^{\{R,m'\}} A_l^{\{R,m'\}*}}. \tag{14}$$

Here again, the averaging over  $\{R\}$  leaves only pairs of amplitudes having exactly opposite phase shifts. For weak disorder, the only non-vanishing contributions involve two possible pairings of amplitudes, either  $i = j, k = l$ , which gives  $\overline{\mathcal{T}}_{ab} \overline{\mathcal{T}}_{a'b'}$ , or  $i = l, j = k$  so that

$$\overline{\delta \mathcal{T}_{ab} \delta \mathcal{T}_{a'b'}} = \sum_{ij} \overline{A_i^{\{m\}} A_i^{\{m'\}*} A_j^{\{m'\}} A_j^{\{m\}*}}. \tag{15}$$

The correlation function thus, appears as products of two amplitudes, that correspond to different internal configurations  $\{m\}$  and  $\{m'\}$ , but to identical scattering paths  $i$  (or  $j$ ). Most of multiple scattering paths  $i$  and  $j$  do not share common scatterers so that we can average  $A_i^{\{m\}} A_i^{\{m'\}*}$  and  $A_j^{\{m'\}} A_j^{\{m\}*}$  separately, since these averages are taken upon different atoms, and finally,

$$\overline{\delta \mathcal{T}_{ab} \delta \mathcal{T}_{a'b'}} = \left| \sum_i \overline{A_i^{\{m\}} A_i^{\{m'\}*}} \right|^2. \tag{16}$$

We now consider the special case  $a = a'$  and  $b = b'$ . Performing both the disorder and internal averages, the generalization of (12) is

$$C_{abab}^{1/2} = \frac{\sum_i \sum_{\{m_i\}, \{m_i'\}} A_i^{\{m_i\}} A_i^{\{m_i'\}*} / J^{2n_i}}{\sum_i \sum_{\{m_i\}} |A_i^{\{m_i\}}|^2 / J^{n_i}}. \tag{17}$$

Here  $n_i$  is the number of atoms involved in the multiple scattering path  $i$ . The non-degenerate limit is obtained by taking  $J = 1$ , which leads immediately to the Rayleigh law  $C_{abab} = 1$ . To illustrate the fact that the ‘quantum’ correlation (i.e. for  $J > 1$ ) can exceed the Rayleigh law, we assume hypothetically, as in section 2, that all amplitudes are equal. This gives  $C_{abab}^{1/2} = \sum_i J^{2n_i} / \sum_i J^{n_i}$  which, for  $n_i \gg 1$  and  $J > 1$ , is much larger than 1.

To proceed further, it is useful to consider a continuous description [8]. We define two *Diffuson functions*  $\mathcal{D}^{(i,c)}$  by†

$$\overline{T}_{ab} = \int d\mathbf{r}d\mathbf{r}' \mathcal{D}^{(i)}(\mathbf{r}, \mathbf{r}') \quad (18)$$

and

$$\overline{\delta T_{ab} \delta T_{a'b'}} = \left| \int d\mathbf{r}d\mathbf{r}' \exp(ik_0[\Delta \hat{\mathbf{s}}_a \cdot \mathbf{r} - \Delta \hat{\mathbf{s}}_b \cdot \mathbf{r}']) \mathcal{D}^{(c)}(\mathbf{r}, \mathbf{r}') \right|^2, \quad (19)$$

where  $\Delta \hat{\mathbf{s}}_{a,b} = \hat{\mathbf{s}}_{a,b} - \hat{\mathbf{s}}_{a',b'}$ . The intensity Diffuson  $\mathcal{D}^{(i)}(\mathbf{r}, \mathbf{r}')$  is the sum of the terms  $|A_h^{(m)}(\mathbf{r}, \mathbf{r}')|^2$  between endpoints  $\mathbf{r}$  and  $\mathbf{r}'$ , whereas the correlation Diffuson  $\mathcal{D}^{(c)}(\mathbf{r}, \mathbf{r}')$  is the sum of the terms  $A_i^{(m)}(\mathbf{r}, \mathbf{r}') A_i^{(m')*}(\mathbf{r}, \mathbf{r}')$ , i.e. that involve uncorrelated configurations  $\{m\}$  and  $\{m'\}$ . The two functions  $\mathcal{D}^{(i,c)}$  are obtained from the iteration of vertices  $\mathcal{V}^{(i,c)}$ , that describe the microscopic details of the scattering process. The iteration is written symbolically (either for  $\mathcal{D}^{(i,c)}$ ,  $\mathcal{V}^{(i,c)}$  we shall denote by  $\mathcal{D}, \mathcal{V}$ ) as

$$\mathcal{D} = \mathcal{V} + \mathcal{V} \mathcal{W} \mathcal{V} + \dots = \mathcal{V} + \mathcal{D} \mathcal{W} \mathcal{V}. \quad (20)$$

The term  $\mathcal{V}$  accounts for a single scattering and  $\mathcal{D} \mathcal{W} \mathcal{V}$  represents its iteration. The quantity  $\mathcal{W}$  describes the propagation of the photon intensity between successive scattering events. Because the vertex depends on the incoming and outgoing polarizations, all quantities in (20) are four-rank tensors: they depend on two incoming and two outgoing polarization *components*. Finding the correlation function  $C_{aba'b'}$  amounts to calculating the two Diffusons  $\mathcal{D}^{(i)}$  and  $\mathcal{D}^{(c)}$ . This calculation is, however, quite complicated [9]. We have calculated numerically the dependence of  $C_{aba'b'}$  on  $\Delta \hat{\mathbf{s}}_{a,b}$  (see figure 2). Analytically we have

$$C_{abab} = Y \left( \frac{\sin^2(X/b)}{X \sin X} - 2 \sin^2\left(\frac{\pi}{b}\right) \frac{\exp(X^2 - \pi^2)}{\pi^2 - X^2} \right)^2. \quad (21)$$

Here  $b = L/l$  is the optical depth of the atomic cloud ( $l$  is the elastic mean free path), and  $X = L/L_0$ , where  $L_0 = (cl/3|\gamma_0|)^{1/2}$ . The quantity  $|\gamma_0|$  is proportional to  $\mathcal{V}^{(c)} - \mathcal{V}^{(i)}$ , namely, to the difference between the correlation and intensity cross-sections [2]. For  $\mathcal{V}^{(c)} = \mathcal{V}^{(i)}$  we have  $|\gamma_0| = 0$ , so that  $X=0$  and the exponential term on the rhs of (21) is negligible. This term represents the correlation enhancement above the classical value, which occurs thus only for  $\mathcal{V}^{(c)} > \mathcal{V}^{(i)}$ . The factor  $Y$  results from projecting the transmitted intensity onto the specific polarization channels being detected. For the transition  $J = 3 \rightarrow J_e \equiv 2j_e + 1 = 5$ , and a polarization channel  $\sigma^+$  to  $\sigma^+$ , we have  $\gamma_0 l/c \sim 0.1$  and  $Y \sim 0.2$ .

†All prefactors that depend on the strength and shape of the photon source and on the quantization volume have been absorbed in the definition of the functions  $\mathcal{D}^{(i,c)}$ .

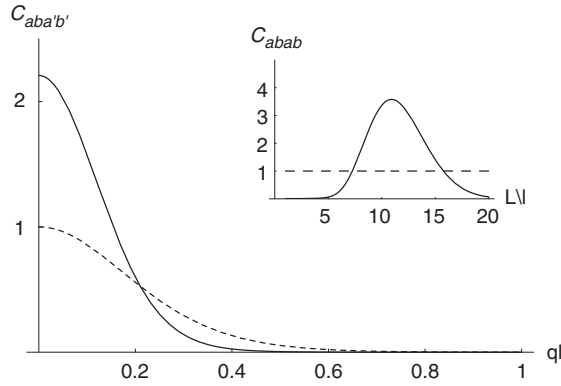


Figure 2. The correlation  $C_{aba'b'}$  as a function of  $q = k\Delta\hat{\mathbf{s}}_q$ . The dashed line accounts for the non-degenerate transition  $J=1$  to  $J_c=3$  and thus displays the classical Rayleigh law. The solid line corresponds to the Zeeman degenerate atomic transition  $J=3$  to  $J_c=5$ , where the above-Rayleigh correlations are seen. The inset gives the dependence of  $C_{abab}$  upon  $L$ , where the dashed line indicates the Rayleigh value, and the peak results from a phenomenological cutoff limiting the correlation enhancement. These results are obtained for  $L=7l$ .

**4. Effect of a magnetic field**

From equation (17) it is clear that cross-terms, involving the coupling of amplitudes corresponding to different Zeeman configurations, play a central role in the quantum correlation. If an external magnetic field  $H$ , is now applied, the Zeeman splitting suppresses all the contributions to  $\mathcal{V}^{(i,c)}$  that correspond to transitions between different magnetic quantum numbers. For a large enough field  $H$  these terms vanish, and the vertices become independent of  $H$  and proportional to

$$\mathcal{V}^{(i)} \propto \frac{1}{J} \sum_m |\langle m|U|m\rangle|^2 \quad (\text{large } H) \tag{22}$$

and

$$\mathcal{V}^{(c)} \propto \left| \frac{1}{J} \sum_m \langle m|U|m\rangle \right|^2 \quad (\text{large } H). \tag{23}$$

Using again the illustrative assumption that all amplitudes are equal, we have  $\mathcal{V}^{(i)} = \mathcal{V}^{(c)}$ . This means that the correlation cross-section is equal to the intensity cross-section, which characterizes the non-degenerate ( $J=1$ ) case, thus leading immediately [2] to the Rayleigh law  $C_{abab} = 1$ . A detailed calculation [10] shows that this is indeed the case, and the dependence of the normalized correlation  $C_{abab}$  on the dimensionless field  $s = g\mu_B H/\hbar\Gamma$  is shown in figure 3. To modify equation (21) so as to include the magnetic field, one has to make the replacement

$$X^2 \rightarrow X^2(1 - s^2). \tag{24}$$



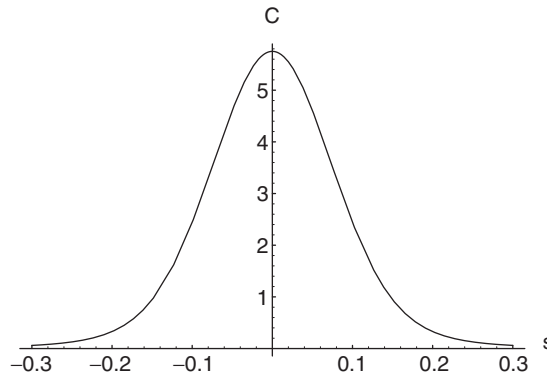


Figure 3. The dependence of  $C \equiv C_{abab}$  on  $s = g\mu_B H / \hbar\Gamma$ . This plot corresponds to the transition  $J=3$  to  $J_c=5$  and  $L=5l$ . The FWHM is  $g\mu_B H \simeq 0.2\hbar\Gamma$ .

The full width at half maximum (FWHM) of  $C_{abab}$ , as a function of the field strength, behaves linearly with  $1/b$  for large values of  $b$ . Its slope is calculated from (21) and (24) to give

$$\Delta H = a \frac{\hbar\Gamma}{g\mu_B} \frac{l}{L}. \quad (25)$$

Here  $a$  is a constant of the order unity. Result (25) is very interesting. The dependence of the FWHM on  $l/L$ , a parameter which can be controlled and is usually of the order  $1/10$  or less, predicts the possibility of performing level-crossing spectroscopy [11] with an, *in principle*, unlimited resolution. However, it must be remembered that we have neglected various mechanisms that might reduce the coherence in the system. For example, if  $L/l$  is very large, phenomena like Doppler shift, recoil, and motion of the atoms, which destroy the coherence among the possible amplitudes, become important.

The enhanced correlation is thus, much more sensitive to the application of a magnetic field, than other phenomena such as Hanle or Franken effects [11, 12]. In level-crossing spectroscopy one uses the Franken or Hanle effects to measure the energy width of atomic states. The FWHM of these phenomena is given by  $g\mu_B H \simeq \hbar\Gamma$ , and is directly related to the optimal resolution that can be achieved in the experiment. It is clear, therefore, that a phenomena as described here, which is orders of magnitude more sensitive to a magnetic field, might be of a great usefulness in improving these experiments precision.

## 5. Conclusions

We have found that when a light beam is multiply scattered in a gas of Zeeman degenerate atoms, the transmitted intensity exhibits large correlations, which exceed Rayleigh law. They result from the existence of internal Zeeman degrees of freedom

and their interplay with the random position of scatterers, which leads to the standard Rayleigh correlation. We have shown that the enhanced correlations decrease dramatically for relatively very small values of an applied magnetic field, showing a sharp resonance-like curve. The FWHM of this curve is inversely proportional to the linear size of the atomic cloud, and thus is *in principle* unlimited. In level-crossing spectroscopy, one uses magnetic field-dependent phenomena (Hanle and Franken effects) to measure atomic parameters. The narrowness of the resonance peak of these phenomena is directly related to the experimental resolution. The enhanced correlation and its very sharply peaked field dependence are, therefore, a candidate for a new kind of high resolution spectroscopy.

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