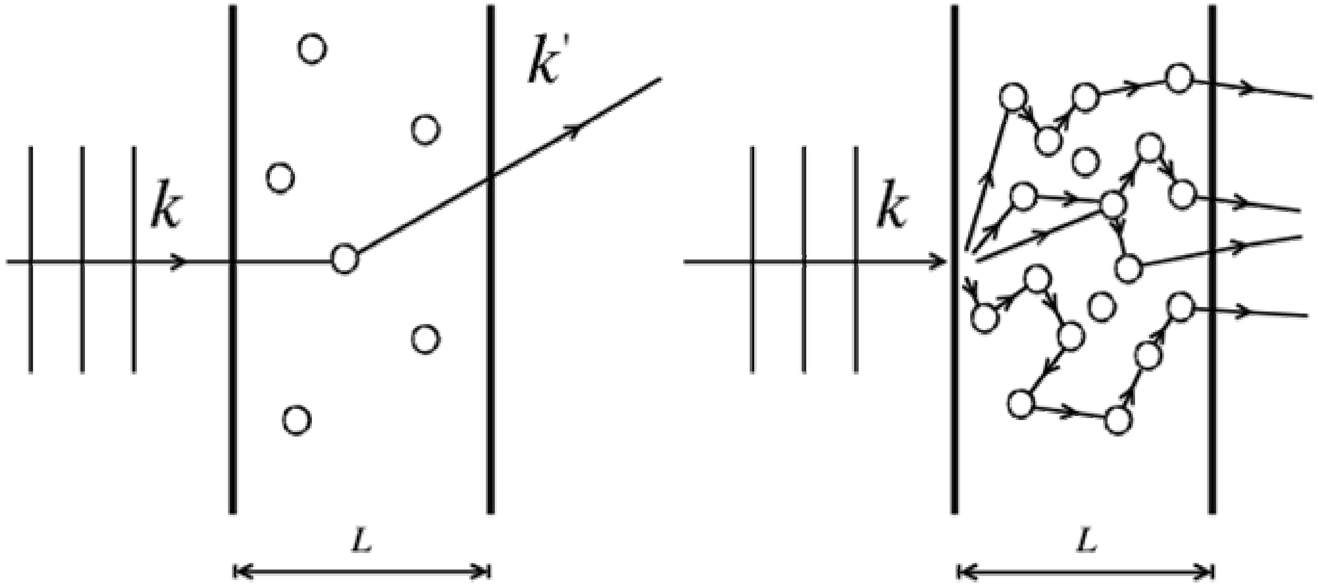
Superradiance-
mesoscopic fluctuations
and
localization in cold atomic gases

Aim of the talk

- To present recent results obtained in quantum mesoscopic physics of photons in cold atomic gases and of matter waves (Bose-Einstein condensates) :
- Intensity fluctuations and deviations from the Rayleigh law resulting from quantum interferences between Zeeman states.
- Cooperative effects (superradiance): strong decrease of the group velocity and of the diffusion coefficient.
- Interplay between disorder and non-linearities: localization of one dimensional matter waves.

Photon multiple scattering



Characteristic lengths:

Wavelength: λ_0

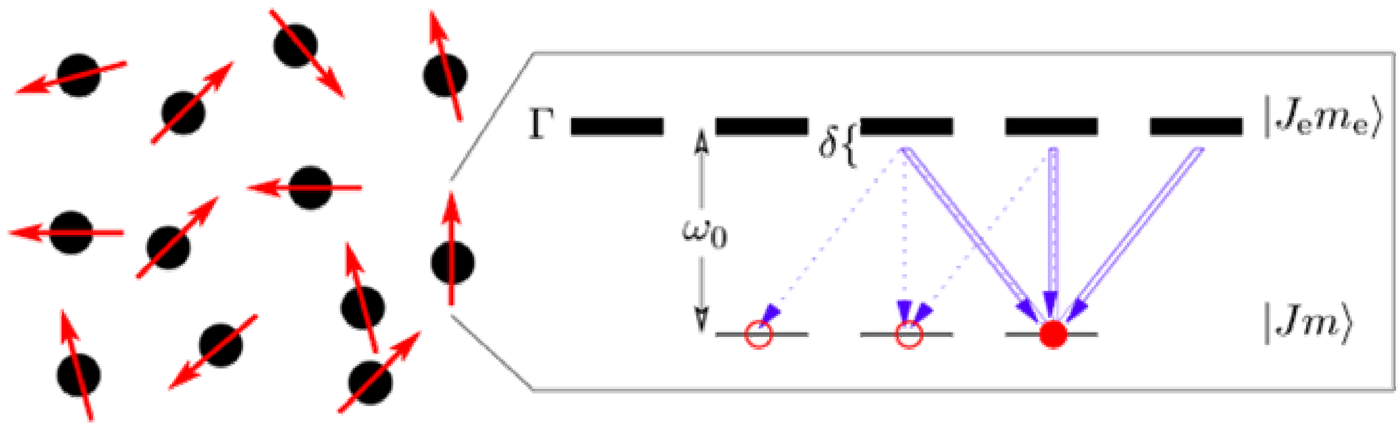
Elastic mean free path: $l = \frac{1}{n_i \sigma} \gg n_i^{-1/d}$

density of scatterers

scattering cross section

Weak disorder $\lambda_0 = l \Leftrightarrow$ independent scattering events

Light scattering in a dilute cloud of cold atoms



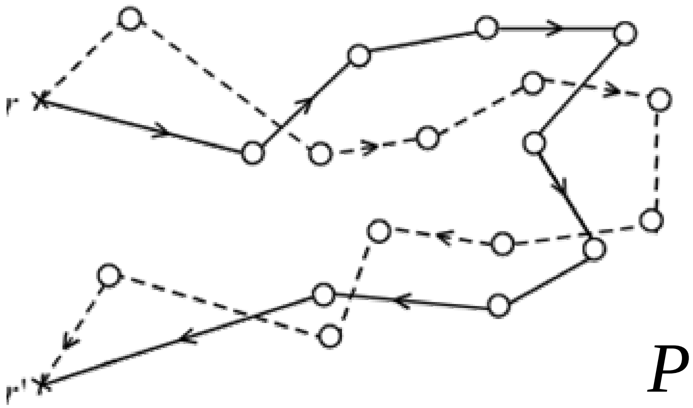
- Photon-atom interaction: dipolar interaction $\hat{V} = -\hat{d} \cdot \hat{E}(r)$

A degenerate atomic dipole transition (J, J_e) allows

Rayleigh scattering $(m = m')$ and Raman scattering $(m \neq m')$

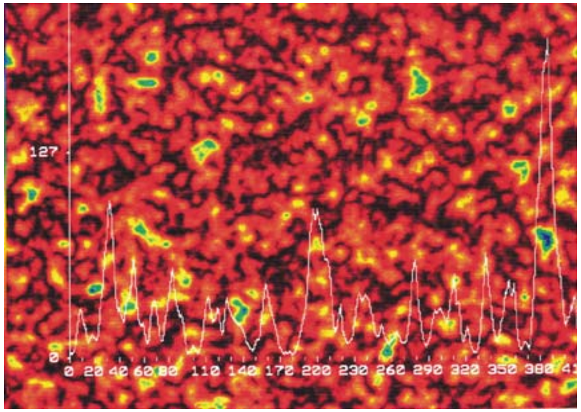
- **Average light propagation in a cold atomic gas:**
- Trace over the positions of the atoms
- Trace over the quantum numbers m with a scalar atomic density matrix.
- Dilute medium $n_i \lambda_0^3 \ll 1$ of weak and resonant scatterers $l \gg \lambda_0$

Motion of a wavepacket and probability of quantum diffusion



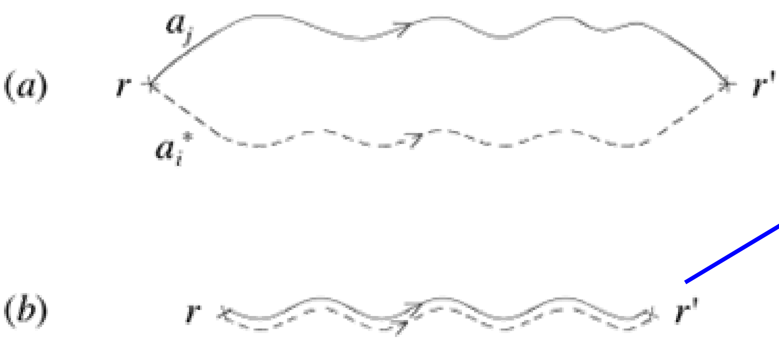
$$P(r,r') = \sum_{i,j} \overline{a_i^*(r,r')\, a_j(r,r')}$$

$$P(r,r') = \sum_{j} \overline{\left|a_j(r,r')\right|^2} + \sum_{i \neq j} \overline{a_i^*(r,r')a_j(r,r')}$$



Before averaging : coherent speckle pattern

Configuration average: most contributions vanish because of large and random phases.



Diffuson:

$$D^{(i)}(r,r') = \sum_{j} \overline{\left|a_j(r,r')\right|^2}$$

Part I

- Speckle correlations and intensity fluctuations of diffusing photons in cold atoms

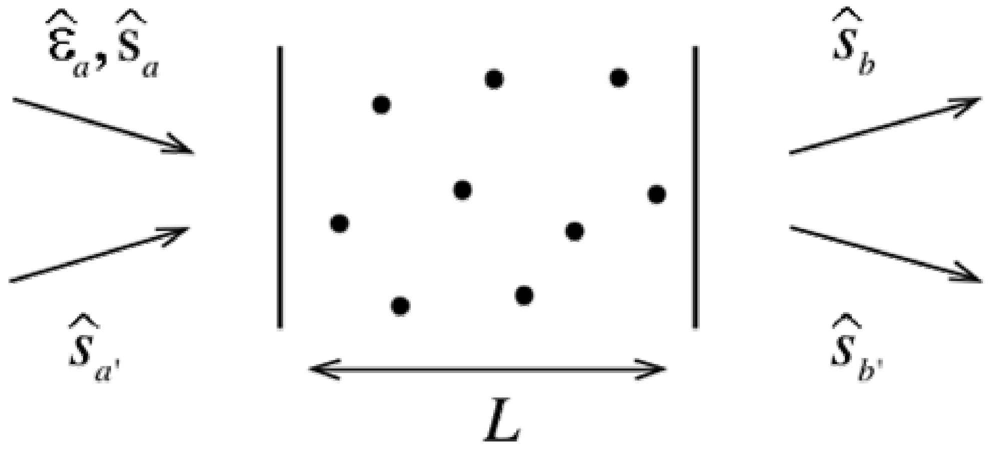
Ohad Assaf and E.A. Technion

Speckle correlations and intensity fluctuations of diffusing photons in cold atoms

We are interested in obtaining the *angular correlation function of atomic speckle patterns* at the $C^{(1)}$ approximation i.e. without quantum crossings.

$$C^{(1)}_{aba'b'} = \frac{\overline{\delta T_{ab}\delta T_{a'b'}}}{\bar{T}_{ab}\bar{T}_{a'b'}}$$

Slab geometry:



$$\psi_{ab} = \int dr\, dr'\, e^{ik_0(\hat{s}_a \cdot r - \hat{s}_b \cdot r')} \sum_C E_C(r, r')$$

$$\overline{T}_{ab} = \overline{|\psi_{ab}|^2}$$

$$\overline{T}_{ab}\overline{T}_{a'b'}$$

Intensity Diffuson

$$D^{(i)}$$

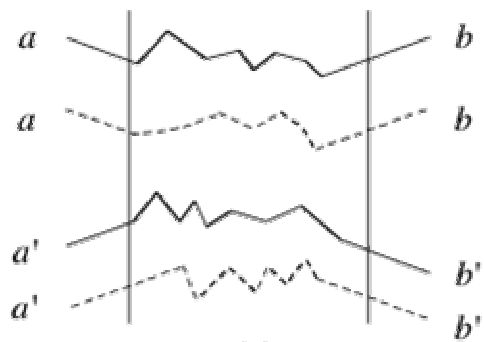
Paired amplitudes are from the same realization.

$$\overline{\delta T_{ab} \delta T_{a'b'}} = \left| \overline{\psi_{ab}\psi_{a'b'}} \right|^2$$

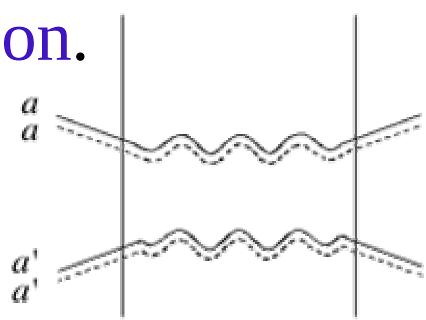
Correlation Diffuson

$$D^{(c)}$$

Paired amplitudes are from distinct realizations.

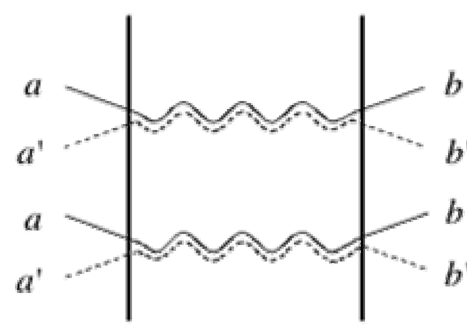


(a)



(b)

+



(c)

The average transmission coefficient $\bar{T}_{ab}$ involves the Diffuson $D^{(i)}(r,r')$

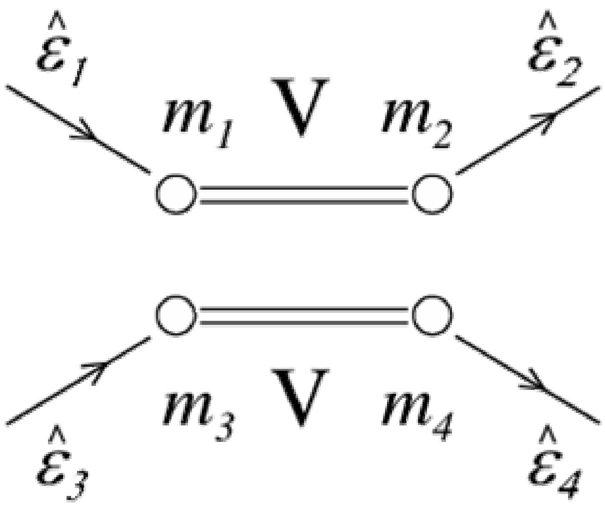
$$\bar{T}_{ab} = \int dr\, dr'\, D^{(i)}(r,r')$$

The angular correlation function involves the Diffuson $D^{(c)}(r,r')$

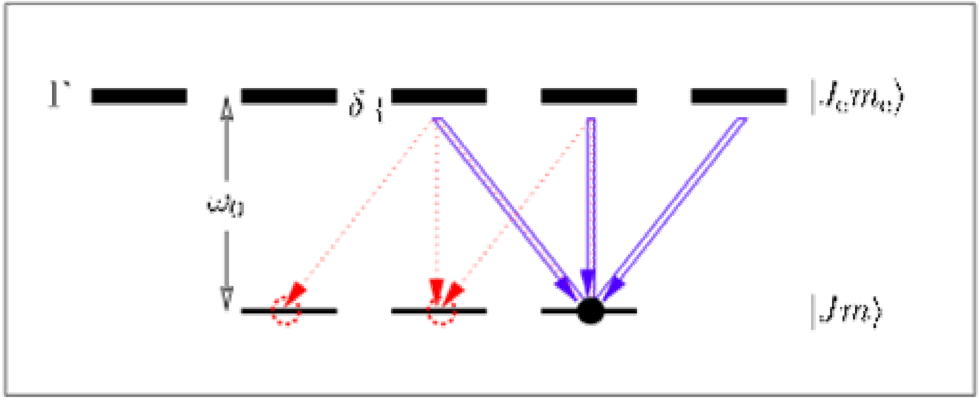
$$\left|\overline{\psi_{ab}\psi_{a'b'}}\right|^2 = \int dr\, dr'\, e^{i k_0 \left[\Delta\hat{s}_a \cdot r - \Delta\hat{s}_b \cdot r'\right]} D^{(c)}(r,r')$$

with $\Delta\hat{s}_{a,b} = \hat{s}_{a,b} - \hat{s}_{a',b'}$

$D^{(i)}$ and $D^{(c)}$ are obtained from the iteration of single scattering.
For the atom-photon system, *single scattering is obtained from the dipolar interaction energy* $\mathbf{d} \cdot \mathbf{E}$ where $\mathbf{d}$ is the atomic dipole operator and $\mathbf{E}$ the electric field.



$$V(\hat{\varepsilon}, \hat{\varepsilon}') = \sum_{m_e} \hat{\varepsilon} \cdot \mathbf{d} | j_e m_e \rangle \langle j_e m_e | \mathbf{d} \cdot \hat{\varepsilon}'^*$$



$$\mathcal{V} = \gamma_e \sum_{m_i} \langle J m_2 | V(\hat{\varepsilon}_1, \hat{\varepsilon}_2) | J m_1 \rangle \langle J m_4 | V(\hat{\varepsilon}_3, \hat{\varepsilon}_4) | J m_3 \rangle^*$$

Intensity Diffuson $D^{(i)}$: $m_1 = m_3$ *and* $m_2 = m_4$ and $\hat{\varepsilon}_1 = \hat{\varepsilon}_3$ *and* $\hat{\varepsilon}_2 = \hat{\varepsilon}_4$

$$\gamma_e = 4\pi n_i \sigma$$

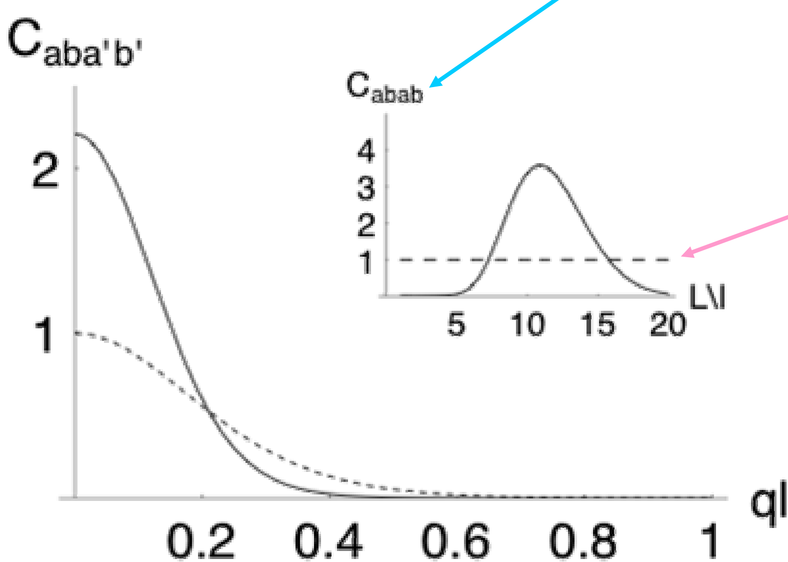
$$C_{aba'b'} = \frac{\left|\overline{\psi_{ab}\psi^{*}_{a'b'}}\right|^2}{\bar{T}_{ab}\bar{T}_{a'b'}}$$

Intensity fluctuations:

$$C_{abab} = \frac{\overline{\delta T_{ab}^{2}}}{\bar{T}_{ab}^{2}}$$

Rayleigh law:

$$C_{abab} = 1 \quad \Leftrightarrow \quad \overline{\delta T_{ab}^{2}} = \bar{T}_{ab}^{2}$$



$$C_{abab} \simeq \pi^4 \left(\frac{e^{X\Lambda D/L^2} - 1}{X} \right)^2$$

Inelastic mean free time, Doppler shift, finite size absorption…

$$X \equiv \left(\frac{L}{L_0^{(c)}} \right)^2 - \pi^2 \quad and \quad L_0^{(c)} = \sqrt{D\left|\tau_0^{(c)}\right|}$$

Part II

- Cooperative effects between atoms: Superradiance-Diffusion coefficient and group velocity.

Aharon Gero and E.A. Technion

Scalar waves in random media

Monochromatic electromagnetic wave (λ_0) at the scalar approximation : $\psi(r)$ is the electric field solution of the Helmholtz wave equation:

$$-\Delta\psi(r) - k_0^2\,\mu(r)\,\psi(r) = k_0^2\;\psi(r)$$

Disorder potential is continuous : fluctuations of dielectric constant

$$V(r) = -k_0^2\,\mu(r) = \delta\varepsilon / \varepsilon$$

Gaussian white noise model:

easy to do calculations

$$\langle V(r)\rangle = 0$$

$$\langle V(r)V(r')\rangle = B\,\delta(r-r')$$

Relate to scattering properties of individual scatterers

Edwards model for disorder: N_i identical localized, randomly distributed scatterers,

$$V(r) = \sum_{j=1}^{N_i} v(r - r_j)$$

The potential $v(r)$ is short range compared to λ_0 so that

$$v(r - r_j) = v_0 \delta(r - r_j)$$

In weak potential limit (Born approximation) the scattering cross section of a single scatterer is

$$\sigma = \frac{1}{16\pi^2} v^2(k - k') \approx \frac{v_0^2}{4\pi}$$

and $B = n_i v_0^2$ where $n_i = \frac{N_i}{\Omega} = density\ of\ scatterers$

Average amplitude of the field

Solution of the wave eq. with a source $j(r)$ is given in terms of the Green's function $G(r,r')$:

$$\psi(r) = \int dr' \, j(r') G(r,r')$$

Solution of $\left[\Delta_r + k_0^2 - V(r)\right] G(r,r_i) = \delta(r - r_i)$

$G(r,r')$ may be expressed in terms of the free Green's function $G_0(r,r')$ without scattering potential:

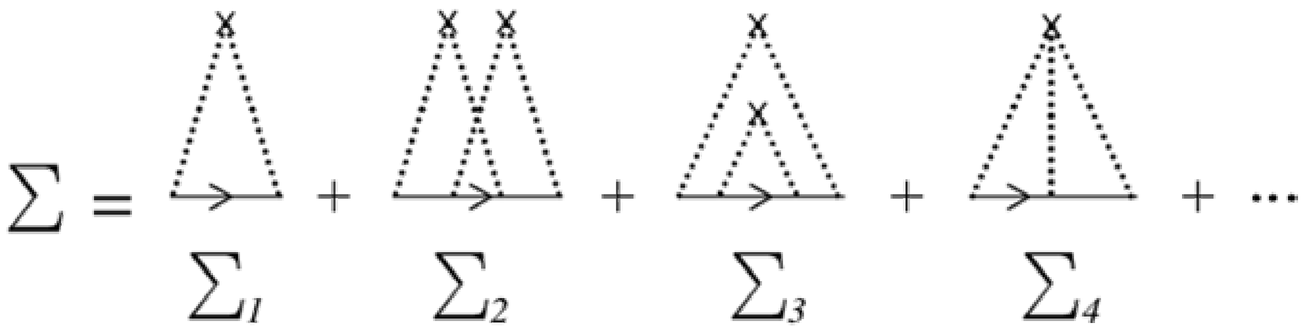
$$G(r,r') = G_0(r,r') - \int dr_1 \, G(r,r_1) V(r_1) G_0(r_1,r')$$

Disorder average restores translational invariance and the Fourier transform of the Green's function is $\bar{G}(k)$

$\bar{G}(k)$ can be expressed in terms of the self-energy $\Sigma(k)$ as

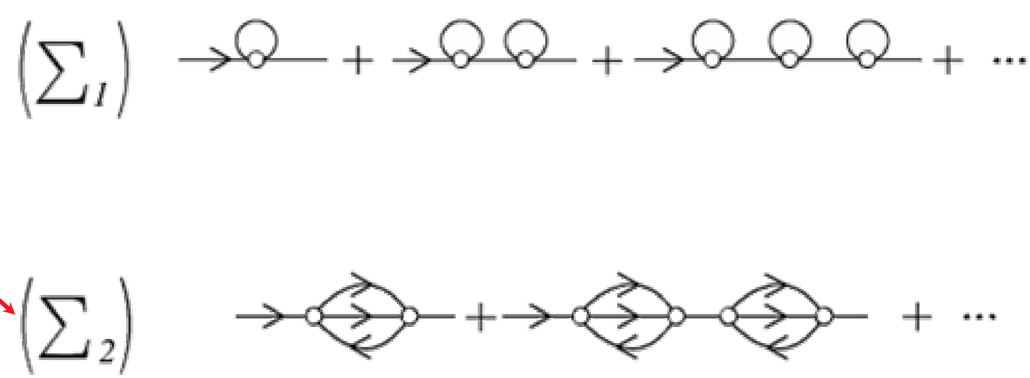
$$\bar{G}(k) = G_0(k)\left[1 + \Sigma(k)\bar{G}(k)\right]$$

and the self-energy is given by the sum of irreducible scattering events



The main contribution to the self-energy $\Sigma(k)$ neglects interference effects between scatterers,

In real space:



The self-energy is proportional to the average polarizability of the scattering medium, so that its real part gives the average index of refraction $\eta = {ck}/{\omega}$

$$\eta = \left(1 - \left(\frac{c}{\omega} \right)^2 \mathrm{Re}\,\Sigma_1 \right)^{1/2}$$

and the group velocity $v_g \equiv \dfrac{d\omega}{dk}$ of the wave inside the medium,

$$\frac{c}{v_g} = \eta + \omega \frac{d\eta}{d\omega} = \frac{1}{\eta} \left(1 - \left(\frac{c^2}{2\omega} \right) \frac{d}{d\omega} \mathrm{Re}\,\Sigma_1 \right)$$

We have used a model of disorder where scatterers are independent :
Edwards model or white noise

T
e
x
t

In atomic gases, there are cooperative effects *(superradiance, subradiance)* that lead to an interacting potential between pairs of atoms.

Dicke states and pairs of degenerate two-level atoms:

$$|g\rangle = |J_g = 0, m_g = 0\rangle$$

$$|e\rangle = |J_e = 1, m_e\rangle \; natural\ width\ \Gamma$$

Pair of two-level atoms in their ground state + absorption of a photon.
Unperturbed and degenerate 0-photon states

$$Singlet\ \ Dicke\ state: |00\rangle = \frac{1}{\sqrt{2}}\left[|e_1 g_2\rangle - |g_1 e_2\rangle\right]$$

$$Triplet\ \ Dicke\ states:$$

$$|11\rangle = |e_1 e_2\rangle, |10\rangle = \frac{1}{\sqrt{2}}\left[|e_1 g_2\rangle + |g_1 e_2\rangle\right], |1-1\rangle = |g_1 g_2\rangle$$

Second order in perturbation theory in the coupling to photons

$$\varepsilon V_e(r) = -\varepsilon \frac{\hbar\Gamma}{2} \frac{\cos k_0 r}{k_0 r}$$

$$\Gamma^{(\varepsilon)} = \Gamma\left(1 + \varepsilon \frac{\sin k_0 r}{k_0 r}\right)$$

Superradiance

$$V_e(r) \propto -\frac{1}{r}$$

(attractive at short distance)

Superradiant state $\varepsilon = +1$

$$\frac{1}{\sqrt{2}}\left[\left|e_1 g_2\right\rangle + \left|g_1 e_2\right\rangle\right]$$

$$\Gamma^{(+1)} = 2\Gamma \quad (\text{for } r = 0)$$

Characteristics of superradiance

Subradiant state $\varepsilon = -1$

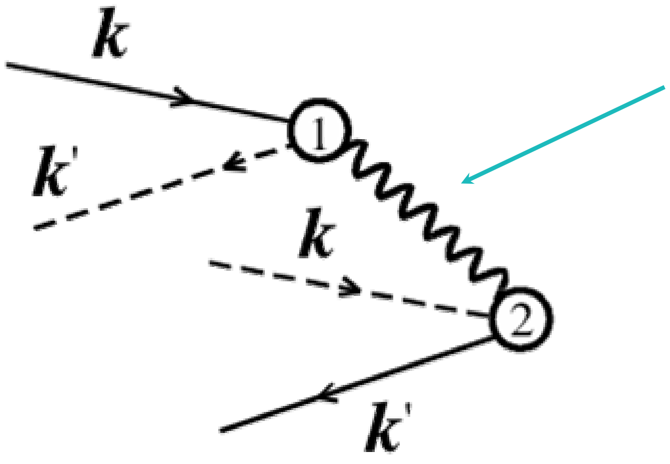
$$\frac{1}{\sqrt{2}}\left[\left|e_1 g_2\right\rangle - \left|g_1 e_2\right\rangle\right]$$

$$\Gamma^{(-1)} = 0$$

Photon is trapped by the two atoms

Superradiance and Cooperon

- Scattering diagram of a photon on a superradiant state is analogous to a Cooperon



infinite number of exchanges of a virtual photon

Multiple scattering and superradiance

Consider multiple scattering of a photon by atoms in superradiant states, i.e. coupled by the attractive potential $V_e(r) \propto -1/r$

Use Edwards model to calculate the self-energy $\Sigma_e^{(1)}$ in the weak disorder limit $k_0 l ?\ 1$

$$\Sigma_e^{(1)} = \frac{6\pi n_i}{k_0 r_m} \int_0^{r_m} \frac{dr}{\frac{\delta}{\Gamma} + \frac{1}{2k_0 r} + i}$$

n_i atomic density

r_m maximum separation between the two atoms.

$$\frac{k_0}{l} = -\mathrm{Im}\Sigma_e^{(1)}$$

Elastic mean free path

$$\frac{c}{v_g} = \frac{1}{\eta}\left(1 - \frac{c^2}{2\omega}\frac{d}{d\omega}\mathrm{Re}\Sigma_e^{(1)}\right)$$

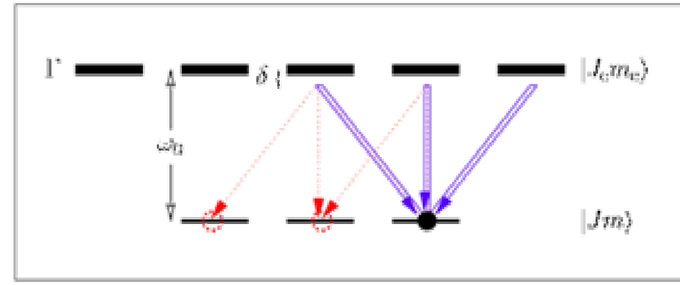
Group velocity

Index of refraction: $\eta = \left(1 - \left(\frac{c}{\omega}\right)^2 \mathrm{Re}\Sigma_e^{(1)}\right)^{1/2}$

Absence of divergence of the group velocity

- The group velocity at resonance is

$$\frac{c}{v_g(0)} = 1 + \frac{4\pi n_i}{k_0^3}\frac{\omega_0}{\Gamma}(k_0 r_m)^2$$

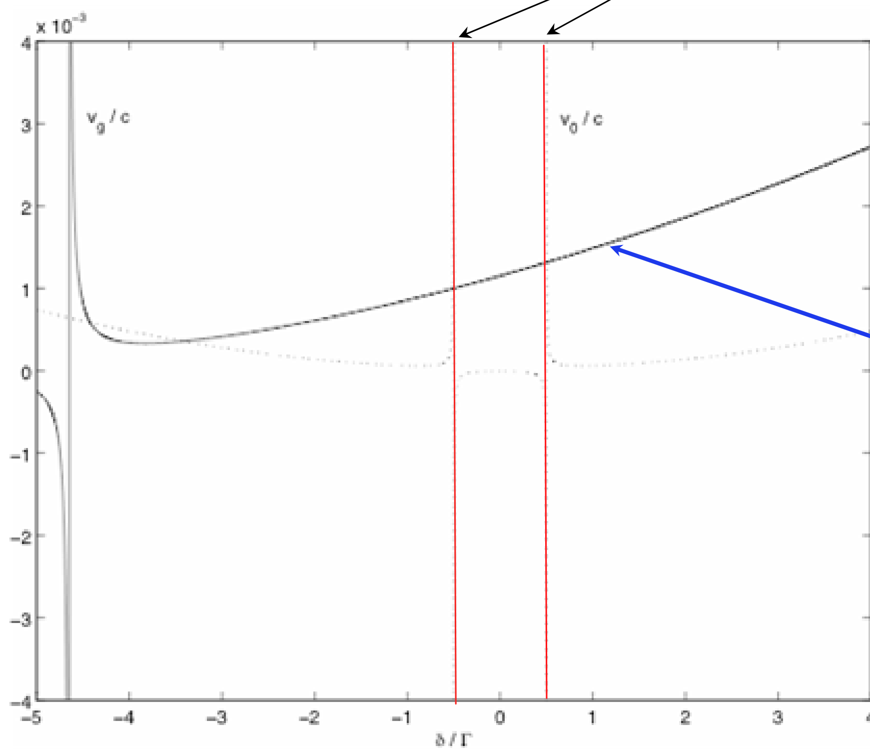


$\simeq 0.26$

$\simeq 10^5$ for ^{85}Rb

$$\frac{v_g(0)}{c} \simeq 3.84 \times 10^{-5}$$

Divergence of the group velocity for scattering by independent atoms



Group velocity for superradiant states

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Slow Diffusion of Light in a Cold Atomic Cloud

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We study the diffusive propagation of multiply scattered light in an optically thick cloud of cold rubidium atoms illuminated by a quasiresonant laser beam. In the vicinity of a sharp atomic resonance, the energy transport velocity of the scattered light is almost 5 orders of magnitude smaller than the vacuum speed of light, reducing strongly the diffusion constant. We verify the theoretical prediction of a frequency-independent transport time around the resonance. We also observe the effect of the residual velocity of the atoms at long times.

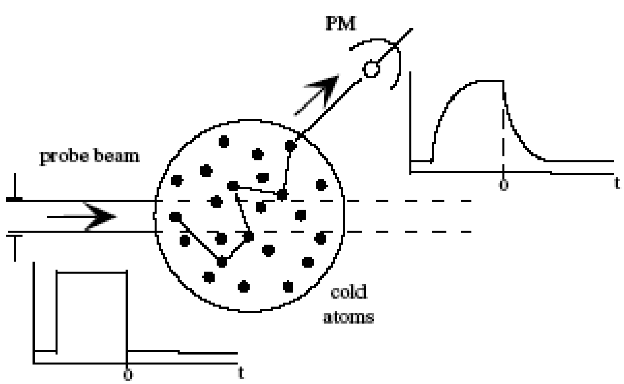


FIG. 1. Radiation trapping (RT) experimental scheme. A pulsed probe beam is sent through the center of a laser-cooled atomic cloud. The transmitted diffuse light is collected as a function of time in a solid angle close to the forward direction.

Weak disorder: $k_0 l \simeq 2 \times 10^3 \gg 1$

$$\frac{v_g(0)}{c} \simeq 3.1 \times 10^{-5}$$

$$D \simeq 0.66 m^2/s$$

$\longrightarrow$ $k_0 r_m \simeq 0.51$

Part III

- Interplay between disorder and non-linearities : localization of matter waves.

Sankalpa Gosh, IIT New Delhi
Ziad Muslimani, U. Florida
E.A. Technion

One-dimensional and interacting BEC's in a random potential

Dimensionless Gross-Pitaevskii equation (mean field approx.)

$$i\partial_t \Psi + \partial_z^2 \Psi - z^2 \Psi - 2\alpha_{1d} |\Psi|^2 \Psi = 0$$

with the dimensionless interaction parameter

confining potential: $a_z, a_\perp$

$$\alpha_{1d} = \frac{2aa_z}{a_\perp^2} = \frac{1}{2\xi^2}$$

a: *scattering length*

Stationary limit:

$$\mu\phi + \partial_z^2 \phi - z^2 \phi - 2\alpha_{1d} |\phi|^2 \phi = 0$$

Interesting cases:

1. Thomas-Fermi limit: $\mu \gg$ kinetic energy and $\mu \gg \hbar\omega_z$

2. Bright soliton limit: in the absence of confinement and for $\alpha_{1d} = -1$

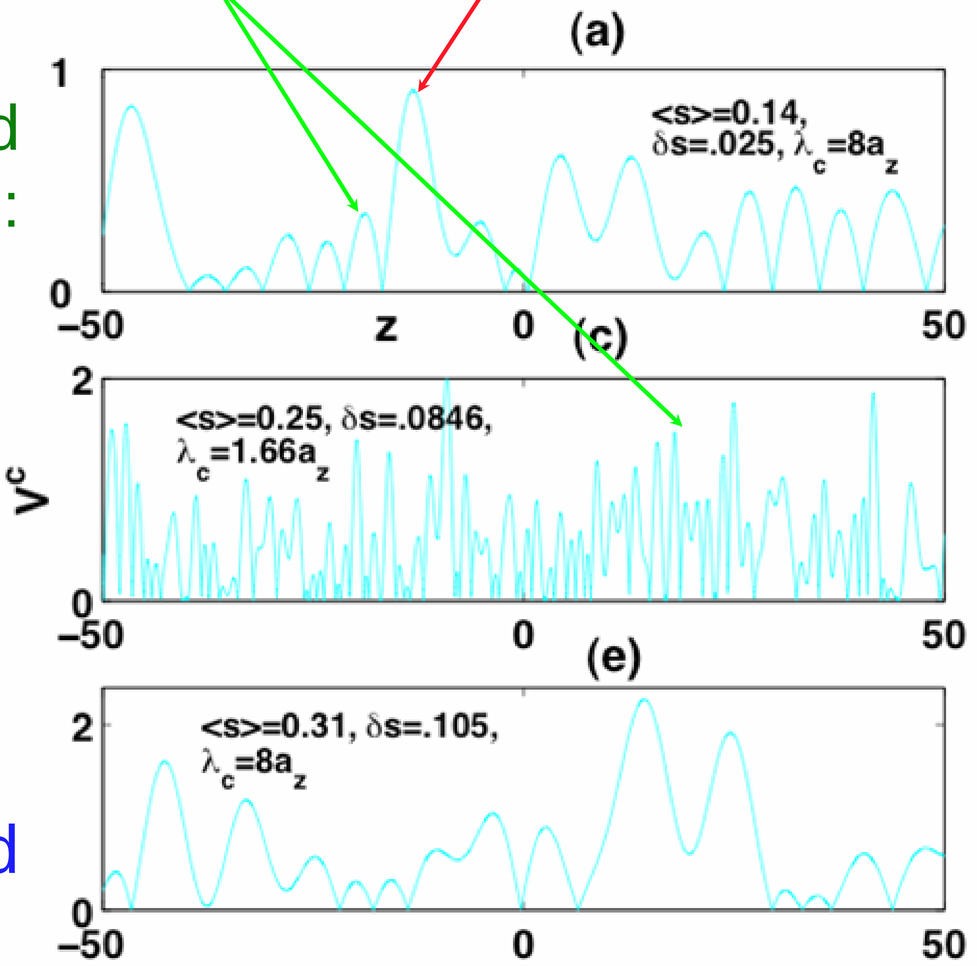
$$\mu\phi + \partial_z^2\phi - z^2\phi(z) - V_d(z)\phi - 2\alpha_{1d}|\phi|^2\phi = 0$$

In the presence of disorder characterized by its amplitude and Fourier spectrum

Relative weight between disorder and interaction characterized by the ratio:

$$\mathcal{R} = 2\xi^2 \frac{\int dz \left(\frac{\partial\phi_\omega}{\partial z}\right)^2}{\int dz|\phi_\omega|^4}$$

between average kinetic energy and interaction energy of the cloud

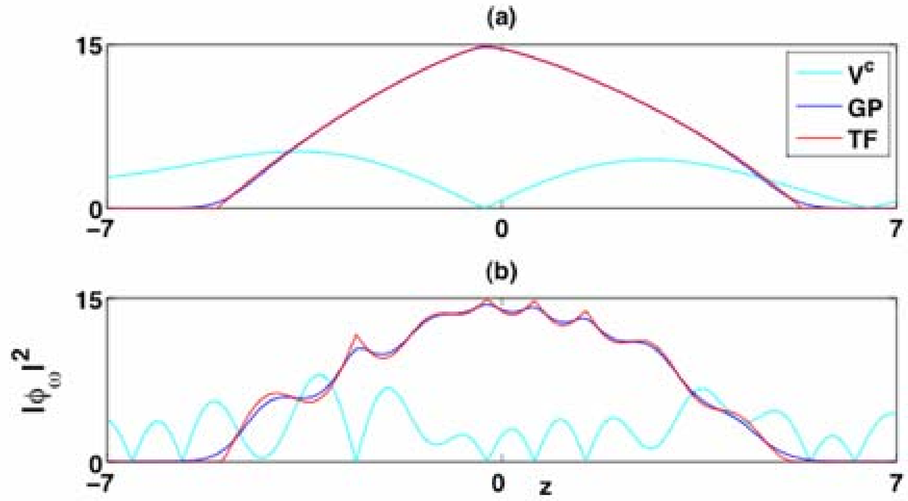


Thomas-Fermi limit

Stationary solutions:

$$\rho_{TF}(z) = \frac{\mu - z^2 - V^c}{2\alpha_{1d}}, \quad \mu \geq z^2 + V^c$$

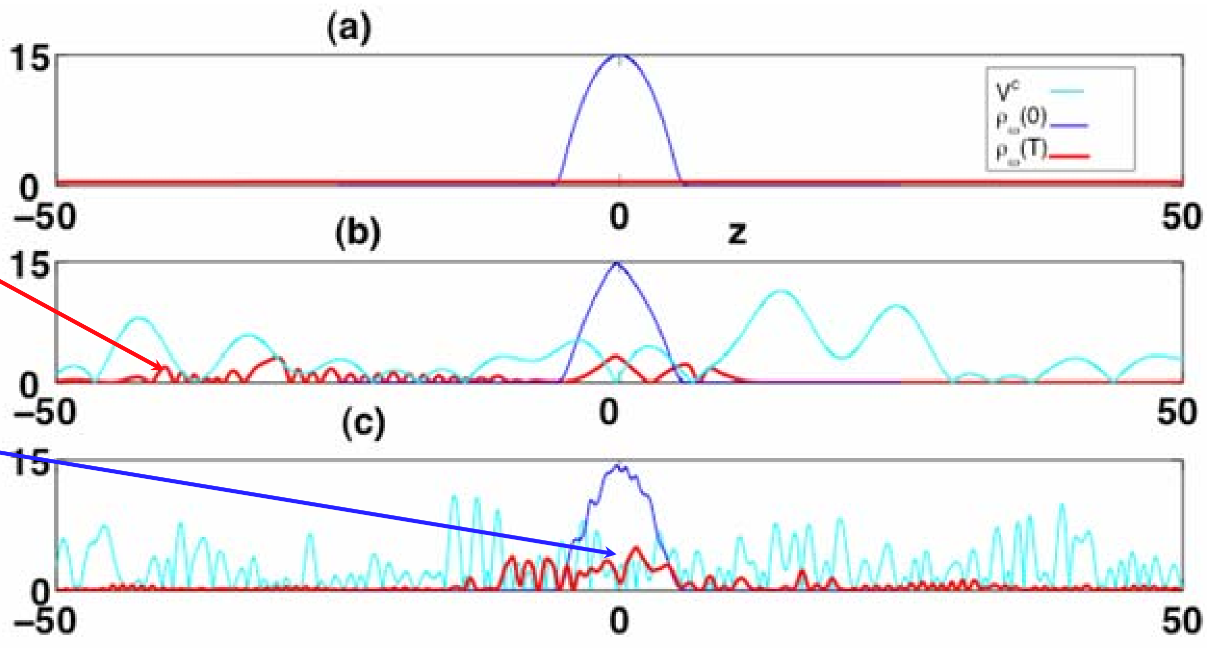
$$\rho_{TF}(z) = 0, \quad \mu < z^2 + V^c$$



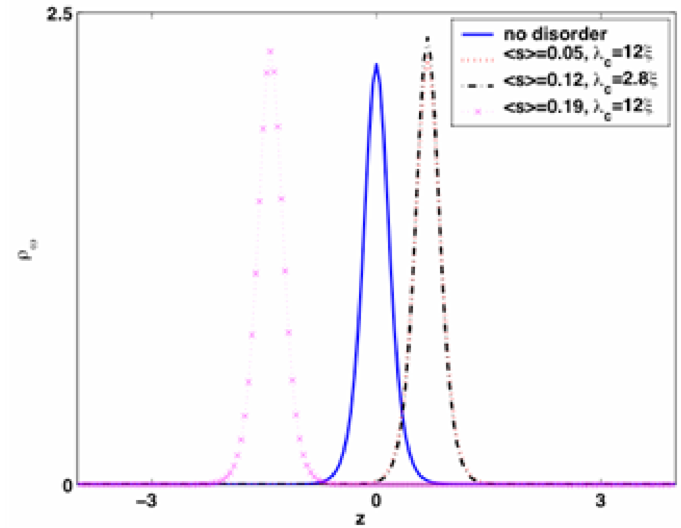
Time-dependent solution:

Larger kinetic energy:
localization effects

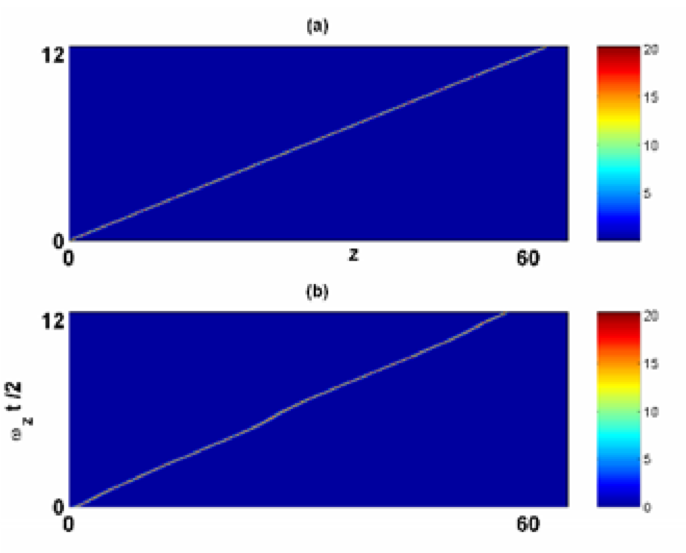
Larger density:
larger interaction energy:
no Anderson localization

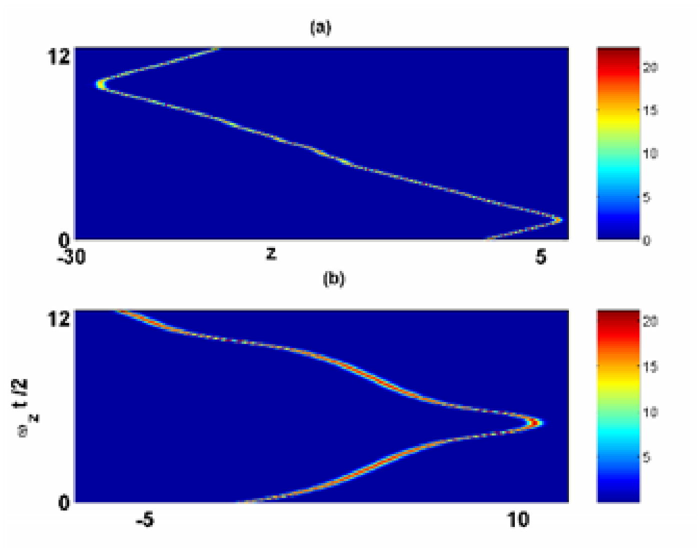


Stationary solution:
keep the same shape as in the absence of disorder but with a broader width.



Time-dependent solution





Weak disorder

Strong disorder

Conclusions

1. Intensity fluctuations in multiple scattering constitute a genuine mesoscopic effect which allows to obtain accurate spectroscopic measurements of atomic structures.

2. Cooperative effects such as Superradiance play in important role and cannot be neglected especially if we are interested in reaching the photon localization threshold. They give rise to long range forces that modify substantially the nature and description (e.g. critical exponents or critical dimensionality) of the Anderson localization transition.

3. Non linearities obscure localization effects due to disorder : Photon localization seems more appropriate than localization of matter waves (except perhaps for well tuned Feshbach resonances).

Some relevant bibliography

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