Hong-Ou-Mandel Experiment Review

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Measurement of Subpicosecond Time Intervals between Two Photons by Interference

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HOM Experiment Review

Outline



- 2 Two photon interference
- 3 Joint probability of photo detection
- 4 Coincidence measurement
- 5 Experimental setup
- 6 Results
- Classical case

8 Conclusions

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• This technique is useful for single photon and distinguishably measure

Two photons in a beamsplitter



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Quantum description of beamsplitter

• Beamsplitter described by
$$U_{\rm BS} = \begin{pmatrix} \sqrt{T} & i\sqrt{R} \\ i\sqrt{R} & \sqrt{T} \end{pmatrix}$$
, where $T+R=1$

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 $= \underbrace{(T - R)}_{11} |1_1, 1_2\rangle + i\sqrt{2RT} |2_10_2\rangle + i\sqrt{2RT} |0_12_2\rangle$

• For $T = R = \frac{1}{2}$, destructive interference results no coincidences detection

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Non-monochromatic photons

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$$\left|1\right\rangle = \int \mathrm{d}\omega f\left(\omega\right) a^{\dagger}\left(\omega\right) \left|0\right\rangle$$

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 $\phi\left(\omega_{1},\omega_{2}
ight)$ is some weight function peaked at $\omega_{1}=\omega_{2}=rac{1}{2}\omega_{0}$

• The joint probability of detection for both D1 and D2 at times $t, t + \tau$ $P_{12}(\tau) = \left\langle \hat{E}_{1}^{(-)}(t) \, \hat{E}_{2}^{(-)}(t+\tau) \, \hat{E}_{2}^{(+)}(t+\tau) \, \hat{E}_{1}^{(+)}(t) \right\rangle$

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- For the described photons state we get

$$P_{12}(\tau) = \left\{ \begin{array}{c} T^2 |G(\tau)|^2 + R^2 |G(\tau - 2\delta\tau)|^2 \\ -RT \left[\frac{G^*(\tau)G(\tau - 2\delta\tau)}{G^2(0)} + \text{c.c.} \right] \end{array} \right\}$$

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- For Gaussian $G\left(\tau\right)=e^{-\left(\Delta\omega\tau\right)^{2}/2}$, observed number of coincidences

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The vanishing of the coincidence rate is purely quantum





























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For $\tau \to \infty$

quantum and classical $P_{12}\left(au ightarrow\infty ight) ightarrow 1$

For $\tau \to \infty$



For $\tau \to 0$

quantum case $P_{12}\left(0
ight)
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For $\tau \to \infty$



For $\tau \to 0$



For $\tau \to \infty$



For $\tau \to 0$



Classical case

$$P_{12}^{\rm C}(\tau) = 1 - \frac{2\left\langle |e_1|^2 |e_2|^2 \right\rangle}{\left\langle \left(|e_1|^2 + |e_2|^2 \right)^2 \right\rangle} \cos\left(\Delta \omega \tau\right)$$

Classical case

٥

$$\begin{split} P_{12}^{\rm C}\left(\tau\right) &= 1 - \frac{2\left\langle |e_1|^2 |e_2|^2 \right\rangle}{\left\langle \left(|e_1|^2 + |e_2|^2\right)^2 \right\rangle} \cos\left(\Delta \omega \tau\right) \\ \text{Notice that } \frac{2\left\langle |e_1|^2 |e_2|^2 \right\rangle}{\left\langle \left(|e_1|^2 + |e_2|^2\right)^2 \right\rangle} &\leq \frac{1}{2}, \end{split}$$

Classical case

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Classical case

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Classical case

$$\begin{split} P_{12}^{\rm C}(\tau) &= 1 - \frac{2\left\langle |e_1|^2 |e_2|^2 \right\rangle}{\left\langle \left(|e_1|^2 + |e_2|^2 \right)^2 \right\rangle} \cos\left(\Delta\omega\tau\right) \geq \frac{1}{2} \\ \bullet \text{ Notice that } \frac{2\left\langle |e_1|^2 |e_2|^2 \right\rangle}{\left\langle \left(|e_1|^2 + |e_2|^2 \right)^2 \right\rangle} \leq \frac{1}{2}, \\ 4\left\langle |e_1|^2 |e_2|^2 \right\rangle \leq \left\langle |e_1|^4 \right\rangle + 2\left\langle |e_1|^2 |e_2|^2 \right\rangle + \left\langle |e_2|^4 \right\rangle \\ &\quad 0 \leq \left\langle |e_1|^4 - 2\left| e_1 \right|^2 |e_2|^2 + \left| e_2 \right|^4 \right\rangle \\ &\quad 0 \leq \left\langle \left(|e_1|^2 - |e_2|^2 \right)^2 \right\rangle \end{split}$$

For $\tau \to \infty$



For $\tau \to 0$



• The observed fourth-order destructive interference is a quantum effect

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Bonus - a citation from *The Principles of Quantum Mechanics* by P.A.M. Dirac (1939) p.9

"Each *Photon* then interferes only with itself. Interference between two different photons never occurs."

Hope you slept comfortably!