HOT NEW SPIN-1/2 PERFECT KAGOMÉ COMPOUND

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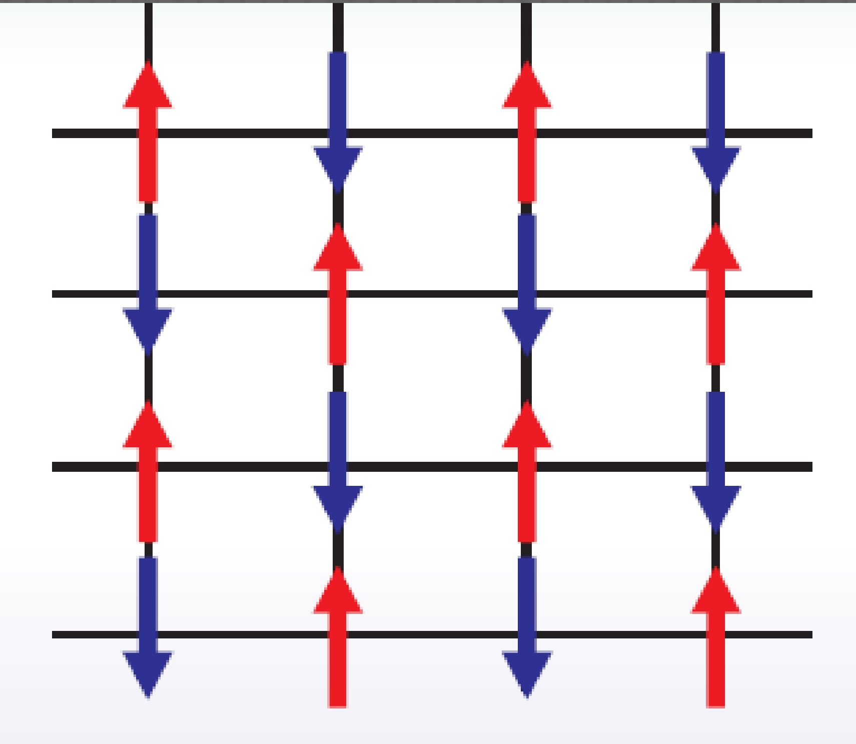
Outline

- Kagomé, what is it ? Why is it interesting ?
- The search for kagomé realization.
- Cu(1,3-bdc) compound.
- Research questions.
- Experimental methods and their results.
- Summary.

Frustration !?

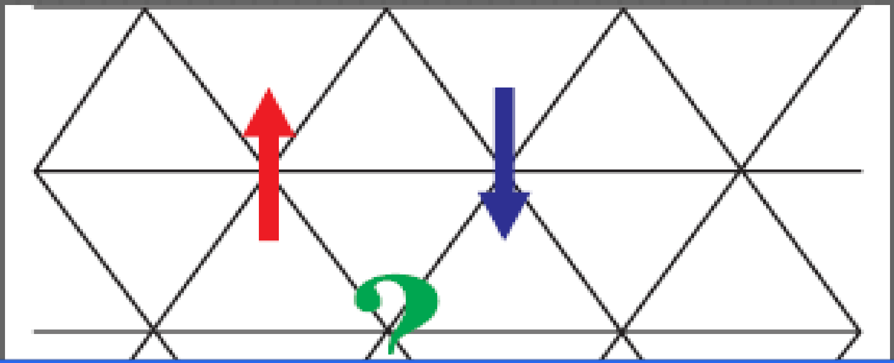
Antiferromagentic (AFM) arrangement of spins can be easily provided in a square lattice.

$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} S_i \cdot S_j$$



Frustration?

AFM arrangement of spins in a triangle cannot be satisfied. The frustration is caused by the geometry of the lattice.



Geometrical frustration is a phenomenon in condensed matter in which the geometrical properties of the crystal lattice forbid simultaneous minimization of all interactions. This may lead to highly degenerate ground state with a nonzero entropy at zero temperature.
Kagmé lattice is one form of frustrated lattices.

Kagomé, what is it? Why is it interesting?

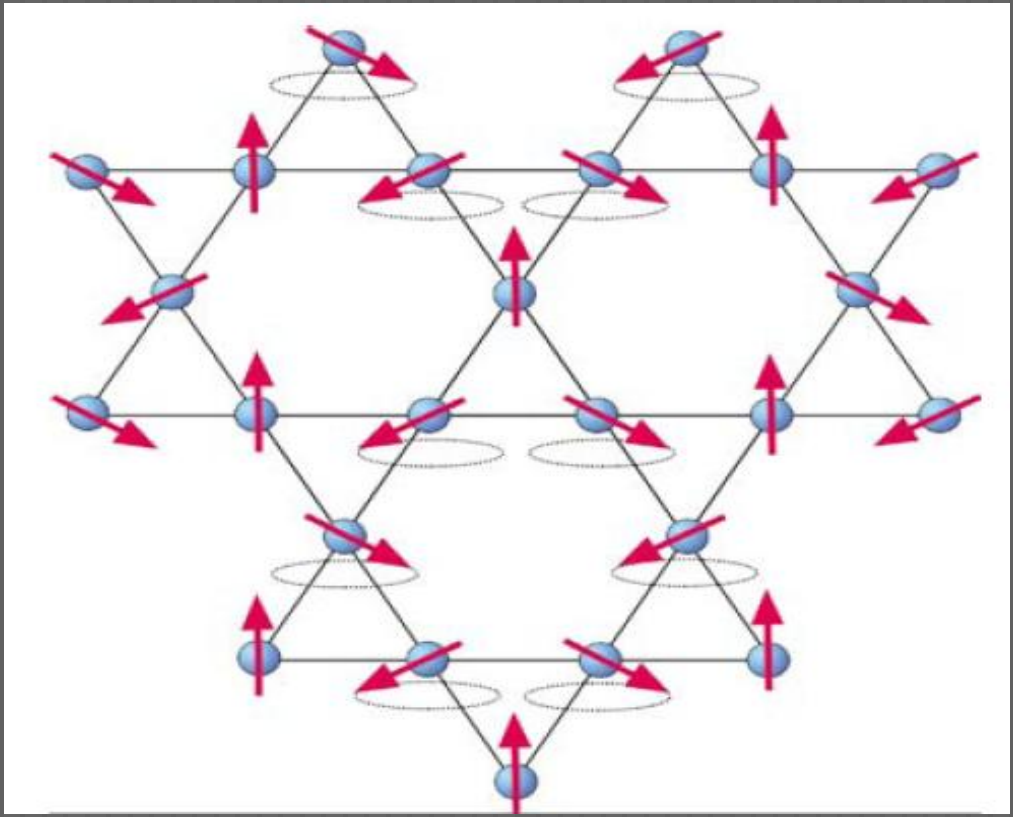
The 2D antiferromagnetic (AFM) kagomé lattice is a highly frustrated lattice, being comprised of corner sharing triangles.

The classical ground state :

The energy can be minimized by providing the 120^0 condition.

<u>Kagomé has a unique property:</u> The spins can oscillate but still maintain the 120^0 condition even at a lowest temperature of T→0.

The ground state of a kagomé AFM is infinitely degenerate.



T → 0: dynamics + no long-range order.

The search for kagomé realization

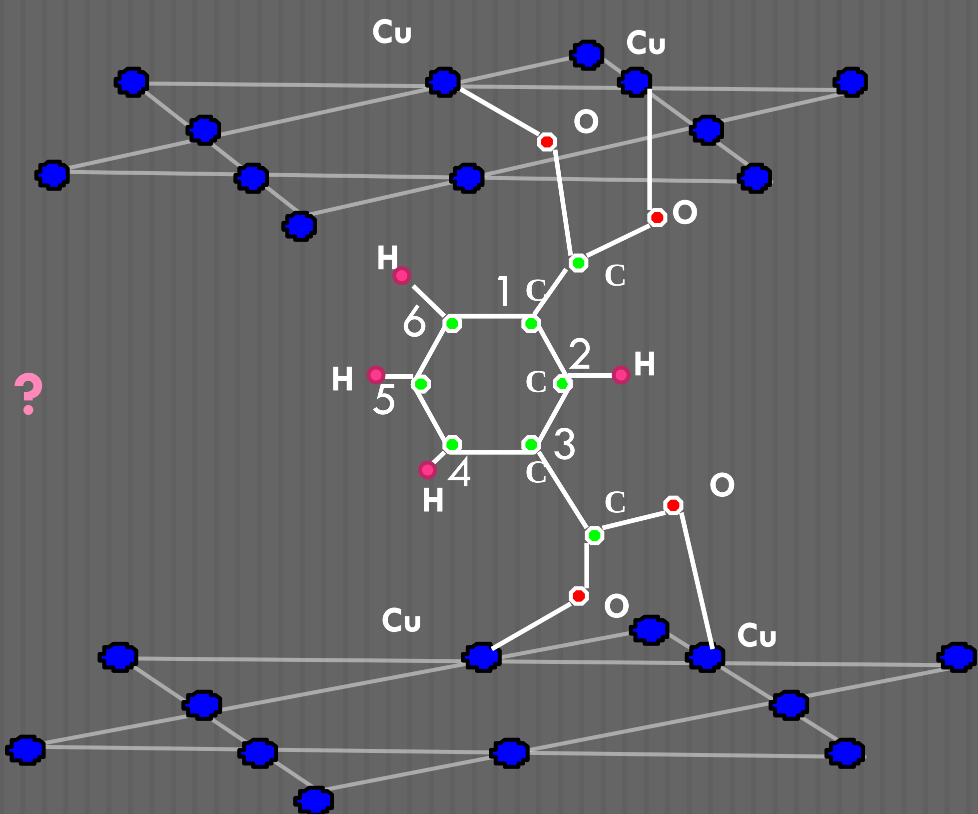
- SCGO – Spin 3/2, alternating kagome and triangle planes.
- Volborthite – distortion.
- Jarosite – large spin number, long-range order.
- Vesignieite –impurities.
- Herbertsmithite – impurities.

Cu(1,3-bdc) –$C_8H_4CuO_4$

Cu(1,3-bdc) is shorthand for Cu(1,3-benzendicarboxylate). Synthesized by Nytko *et al*. at MIT.

The organometallic hybrid compound Cu(1,3-bdc), which has structurally perfect spin ½ copper kagomé planes separated by pure organic linkers.

Each linker contains a benzene molecule.

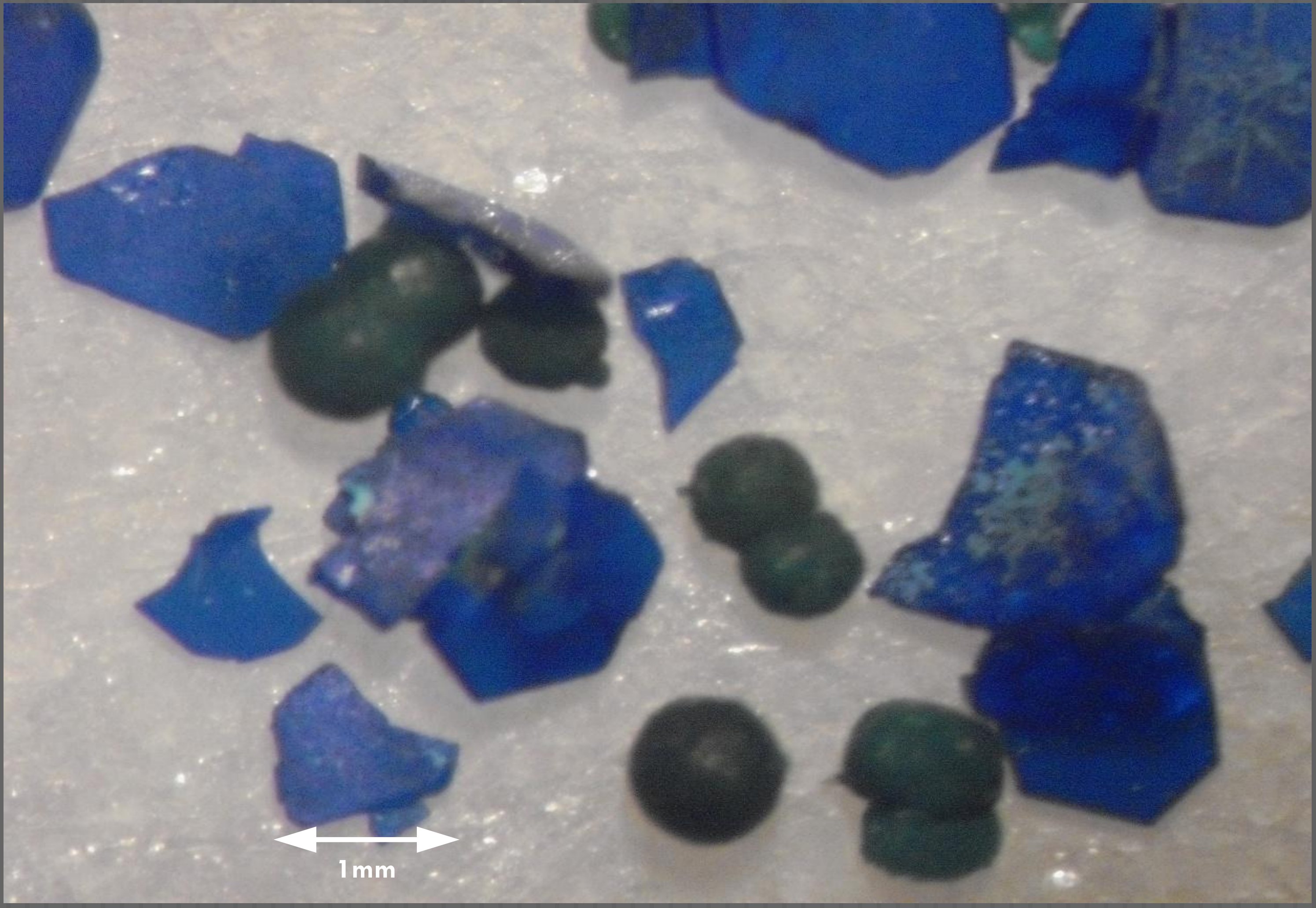


What does Cu(1,3-bdc) mean ?

- Θ_{cw} = -33 K.
- C (heat capacity) has a peak at 1.8K.
- χ saturates at 1.8K

E. A. Nytko at al. J. Am. Chem. Soc. 130, 2922 (2008)

Cu(1,3-bdc)

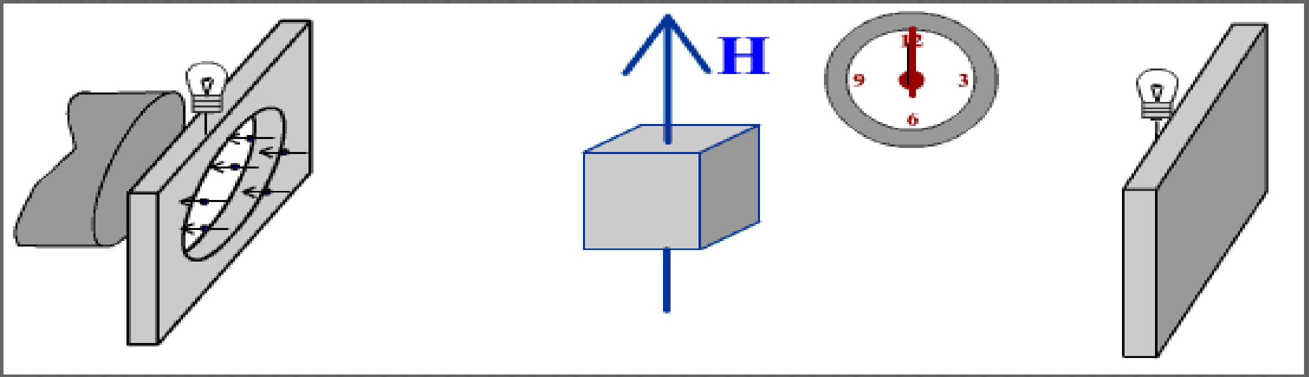


Research Questions

- Does this compound behave as expected from a kagome compound:
  - Does it have a long-range order?
  - Is the limit of T→ 0 dynamic?

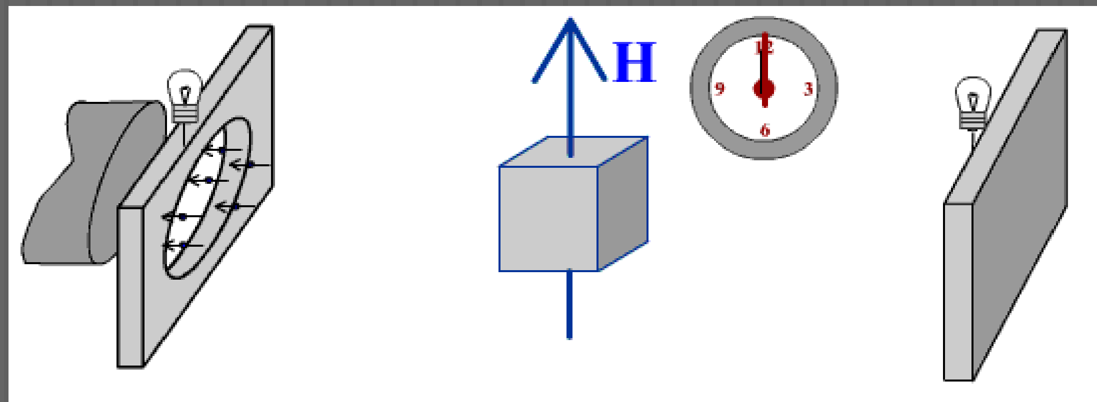
- Does it have anisotropic behavior ?
- What is the Hamiltonian of this compound ? Can we characterize it?

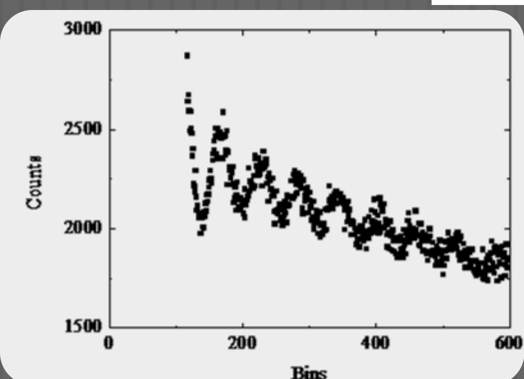
Muon Spin Resonance (μSR)



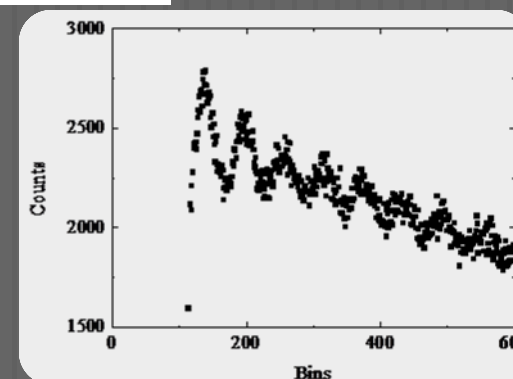
- Perfectly spin polarized muons.
- Muons are implanted into the sample.
- Muons rotate at a larmor frequency, $\omega = \gamma_\mu H$.
- The Muons decay after mean-life-time of 2.2μsec.
- Positrons are emitted preferentially in the muon spin direction.
- We can reconstruct the muon polarization.

Principals of μSR



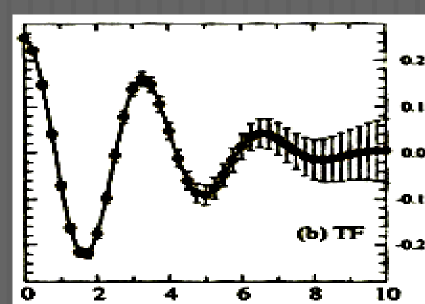


$$\text{Asymmetry} = \frac{(F-B)}{(F+B)} \propto Pz(t).$$



Transverse Field

Asymmetry



Time

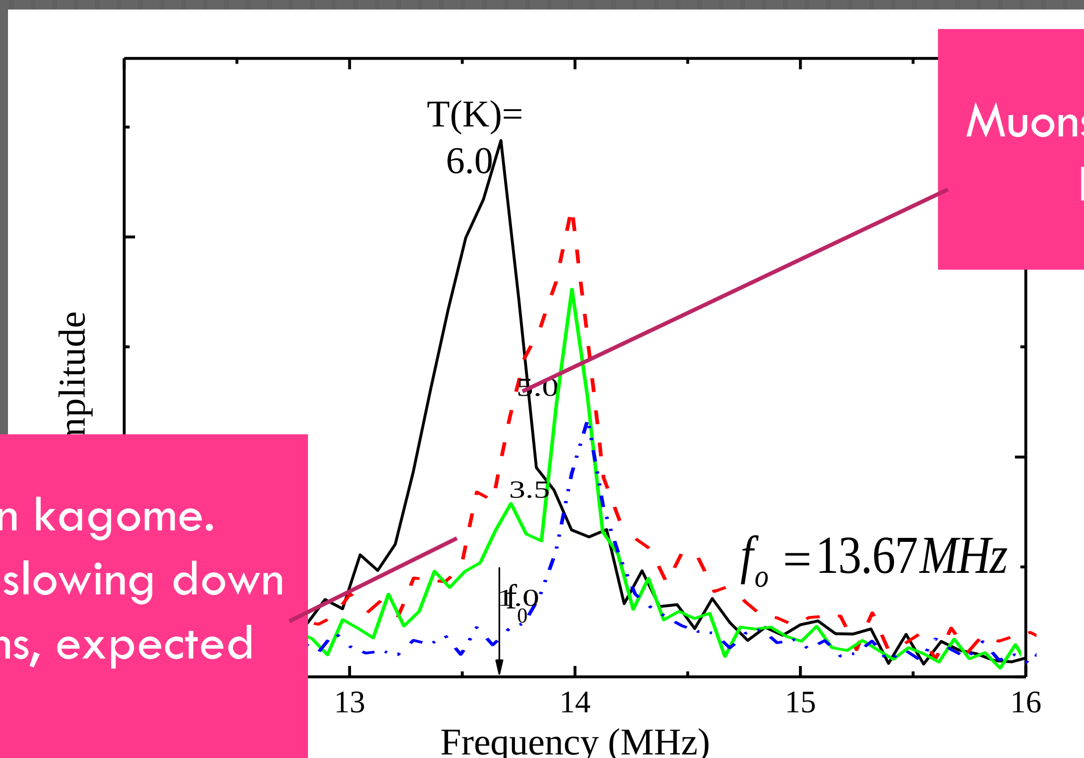
μSR TF Results

- ❑ Does this compound behave as expected from a kagome compound:
- ❑ Does it have a long-range order?
- ❑ Is the limit of T→ 0 dynamics?

Fast Fourier Transform (FFT) of the TF data at **H=1kOe.** : T = 6K, there is a wide asymmetric peak.

This peak separates into two different peaks as we lower the temperature down to 3.5K.

At even lower temperature the low frequency peak vanishes.



Muons that stop at the by-product.

Muons that stop in kagome. Typical signal of slowing down of spin fluctuations, expected near 2K.

L. Marcipar *et al.* PRB **80** 132402 (2009)

μSR TF Results

- ❑ Does this compound behave as expected from a kagome compound:
- ❑ Does it have a long-range order?
- ❑ Is the limit of T→ 0 dynamics?

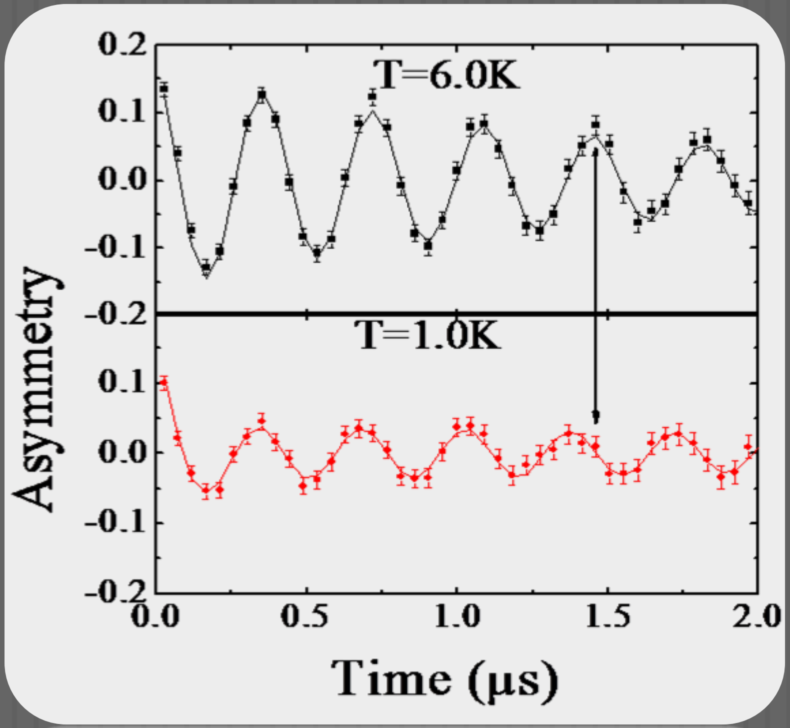
Transverse Field (TF) measurements: 0.9K < T < 6K , H=1kOe.

TF data displayed in rotating-reference-frame, H=800Oe.

$$A_{TF}(t) = A_1 e^{-(R_1 t)^2/2} \cos(\omega_1 t + \varphi) + A_2 e^{-R_2 t} \cos(\omega_2 t + \varphi) + Bg$$

$$\omega_1, R_1 - By\ product$$

$$\omega_2, R_2 - kagome\ part$$



L. Marcipar *et al.* PRB **80** 132402 (2009)

μSR TF Results

- ❑ Does this compound behave as expected from a kagome compound:
- ❑ Does it have a long-range order?
- ❑ Is the limit of T→ 0 dynamics?

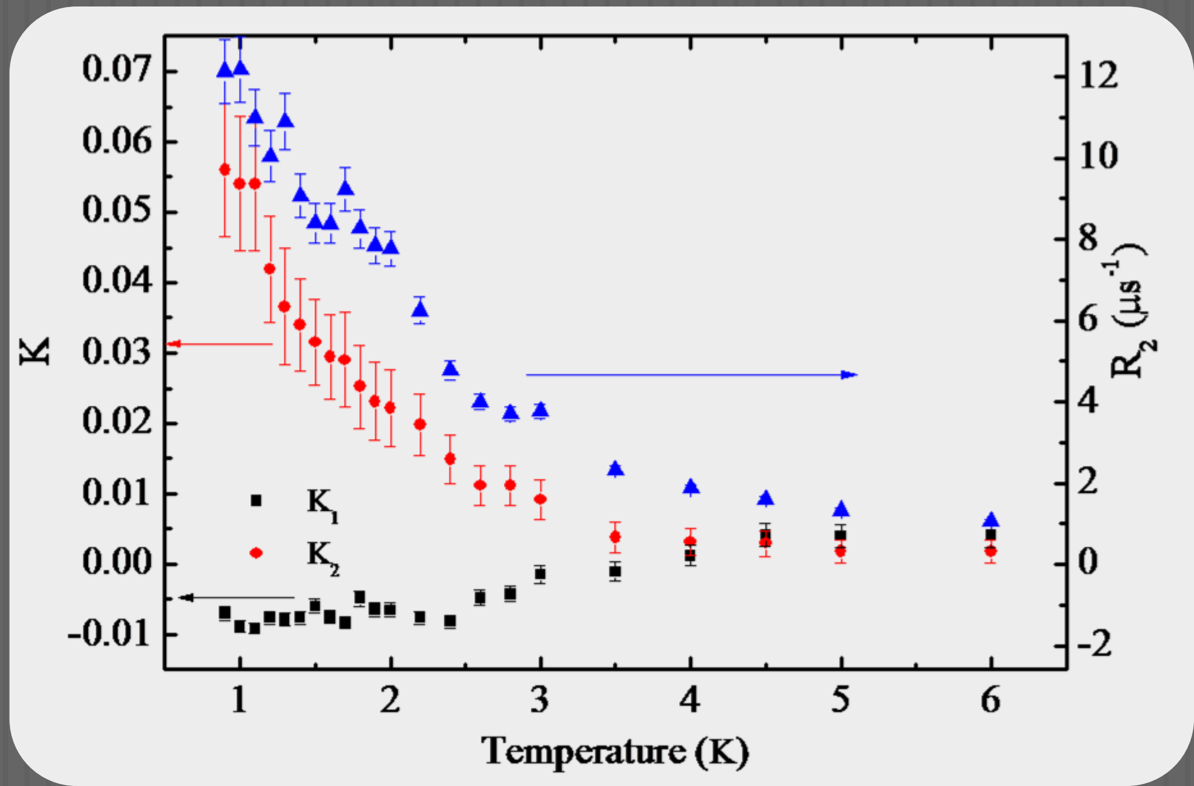
❑ The muon shift of the kagomé part , K_2, increases with decreasing the temperature

→ as expected.

❑ The muon transverse relaxation, R_2 , has the same temperature behavior as the shift,K_2.

$$K_{1,2} = \frac{\omega_s - \omega_{1,2}}{\omega_s}$$

$$\omega_s = 2\pi f_s$$



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μSR TF Results

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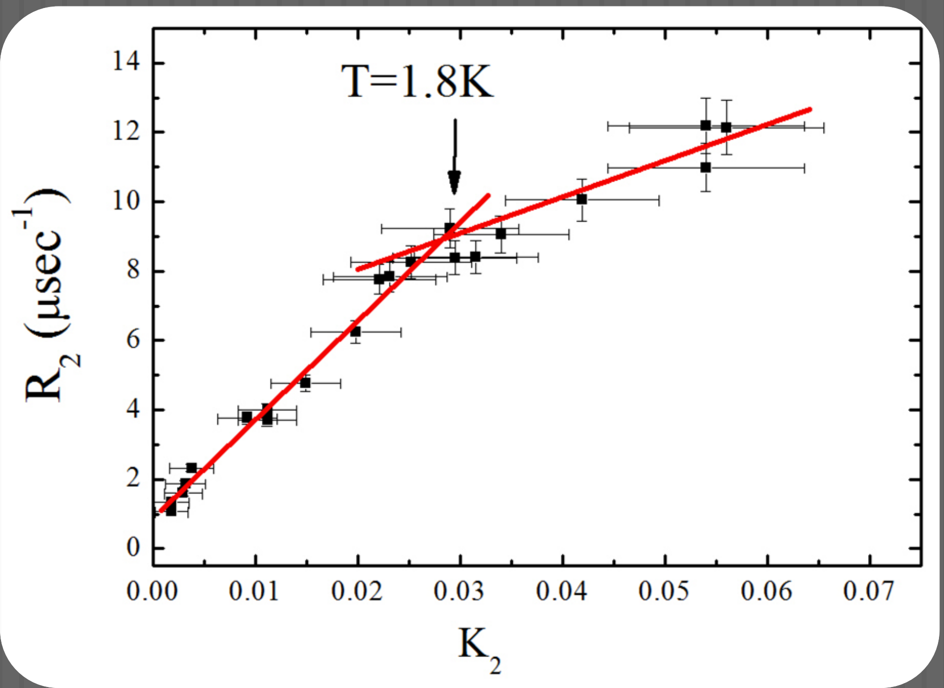
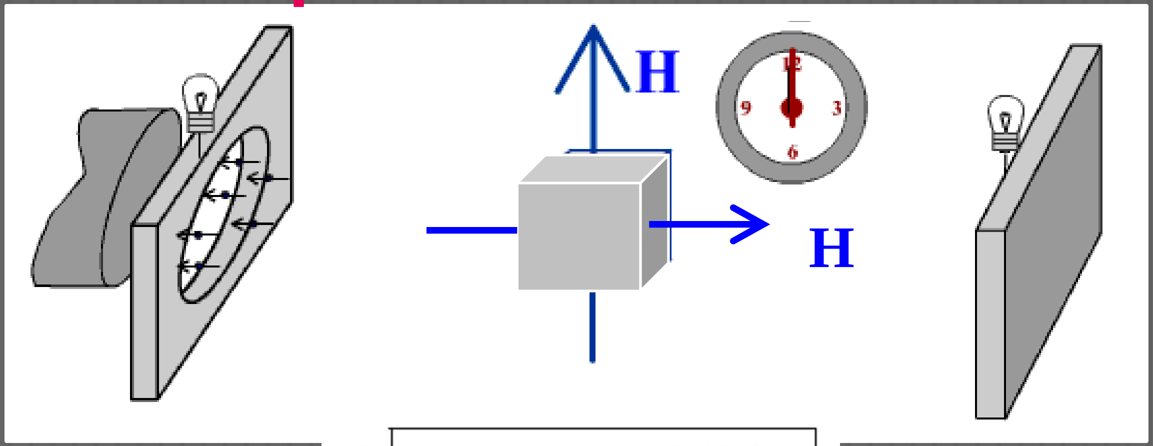
$$\left.\begin{aligned}&\langle S\rangle = M = \chi \mathrm{H}\\ &R_2 = \gamma_\mu \langle S\rangle \sum_k \sigma_k\ ,\quad K_2 = \frac{\langle S\rangle}{H}\sum_k \bar{A}_k(r)\end{aligned}\right\} R_2 \propto K_2$$

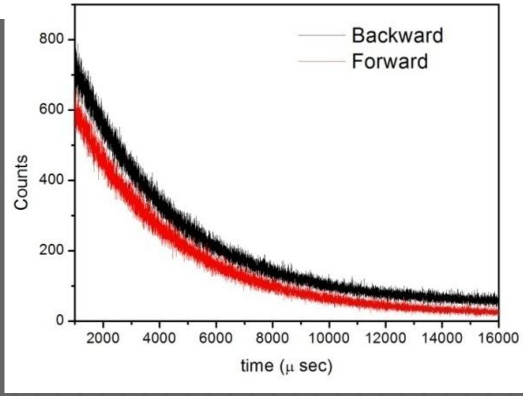


$$R_2 \not\propto K_2$$

- ❑ This result suggests a change in the hyperfine field distribution at T_0. There is a change in the hyperfine coupling $\sum_k \bar{A}_k(r)$ due to change in the distance between the muon spin and the electron.

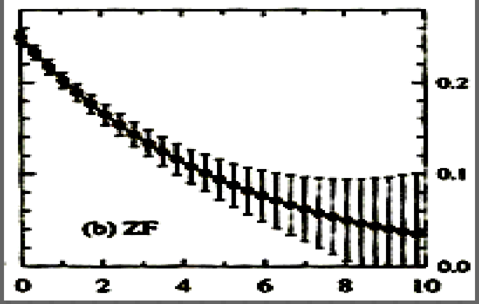
Principals of μSR





$$\text{Asymmetry} = \frac{(F-B)}{(F+B)} \propto Pz(t).$$

Longitudinal Field



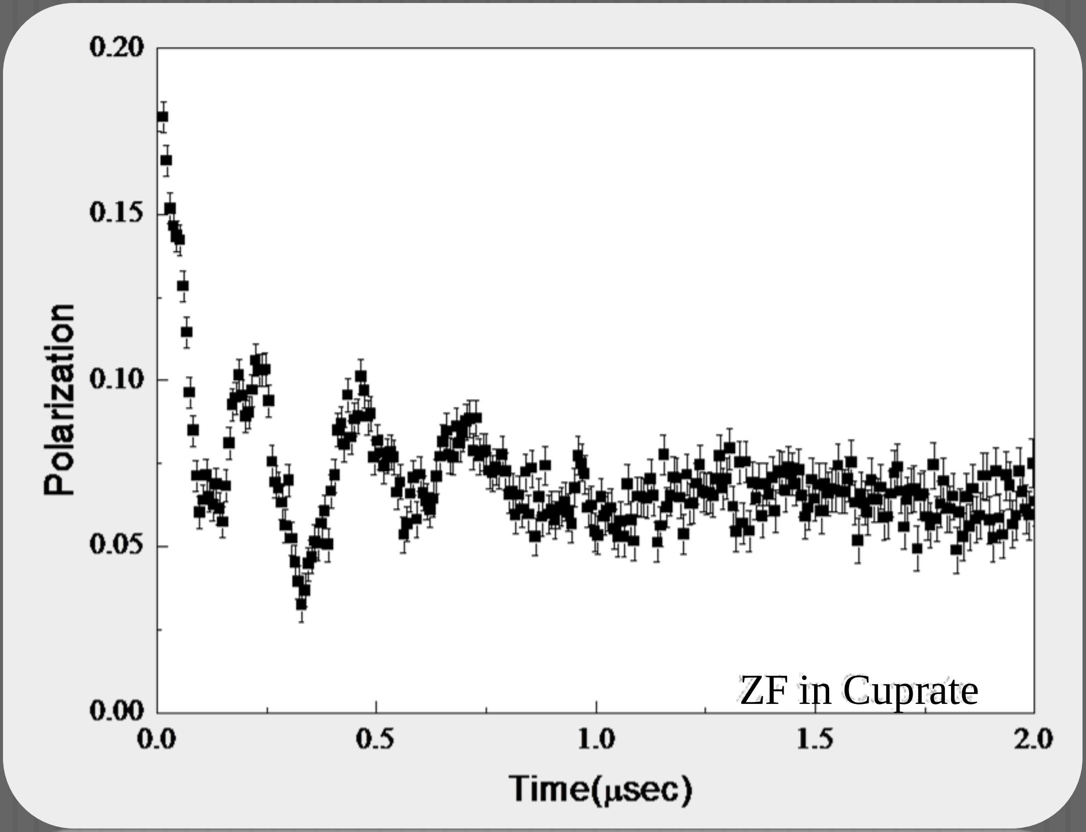
Asymmetry

Time

Theoretical expectations – ZF μSR

Long-range order in a system → the muons oscillate several times.

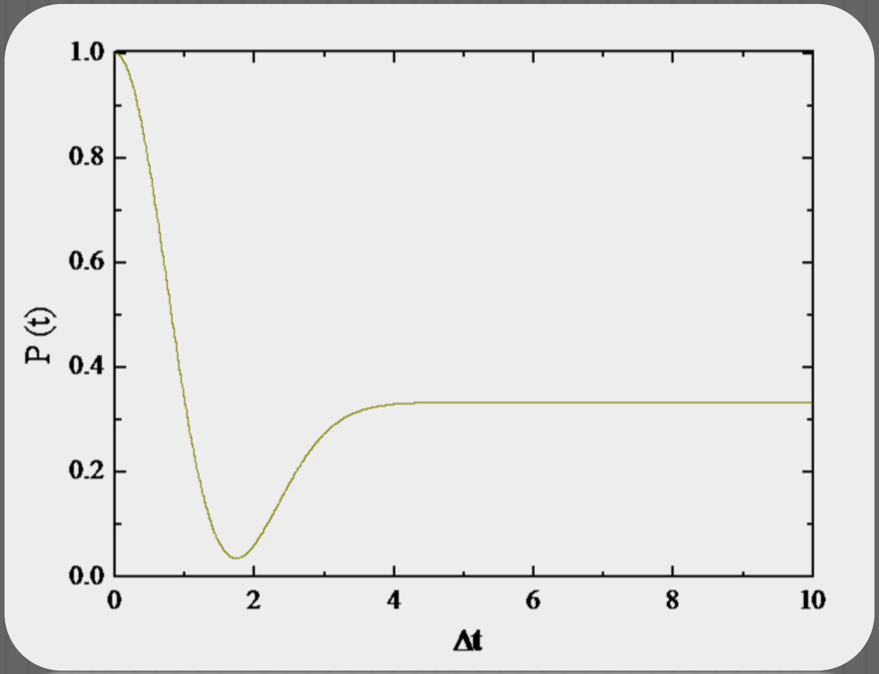
Typical data in a system with long range order:



Static random internal field (H=0).

The muons hardly complete an oscillation but saturate at 1/3 of the initial asymmetry due to parallel internal fields.

Expected behavior (Δ field distribution width):

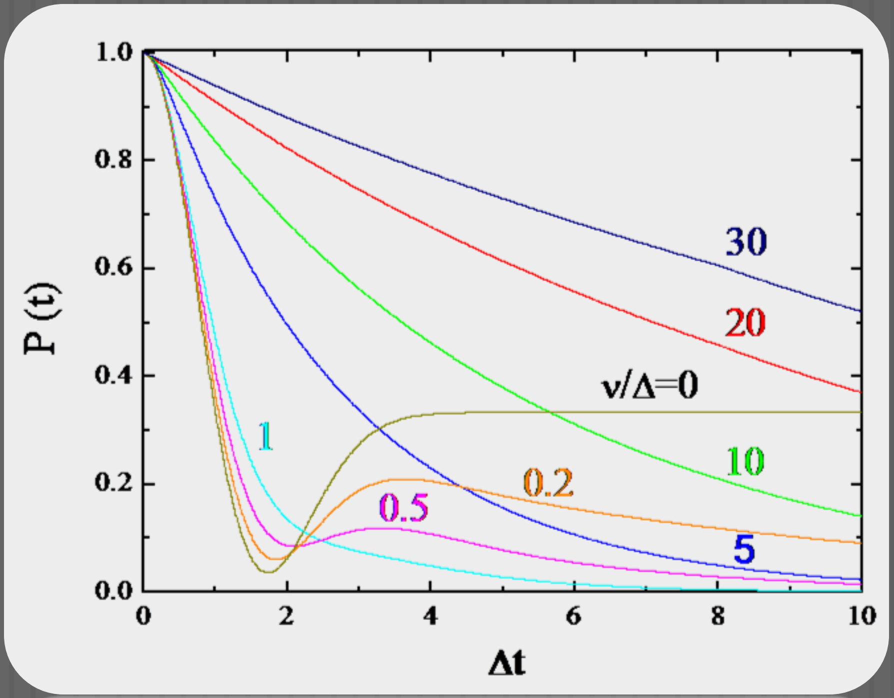


Dynamic random internal fields (H=0)

The muons hardly complete an oscillation and can't even saturate.

Expected behavior (ν is fluctuation rate) :

- ❑ The 1/3 recovery is lost.
- ❑ $\nu \gg \Delta$ the relaxation becomes exponential.



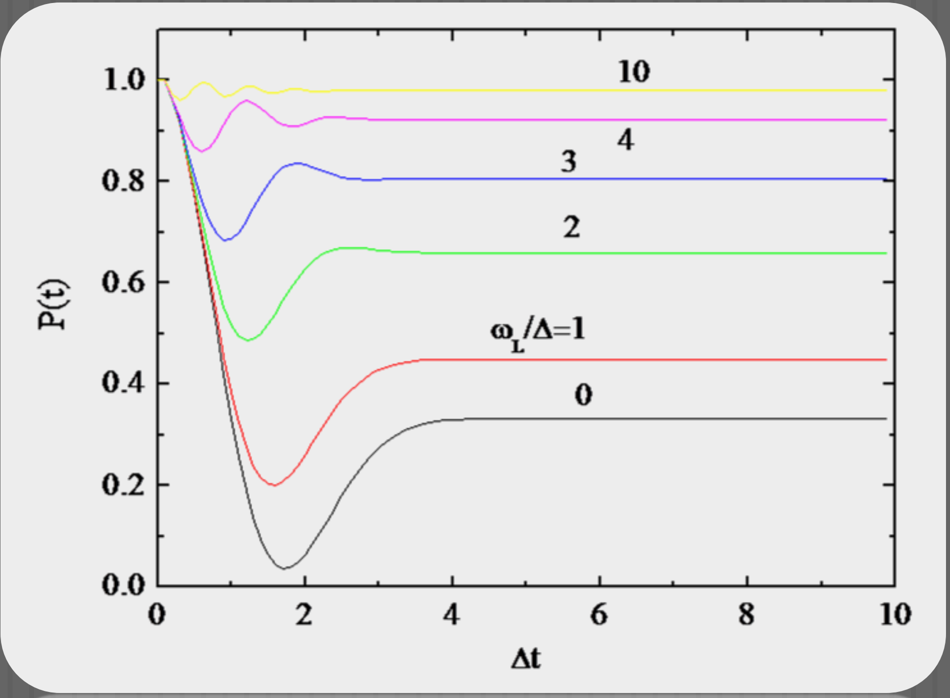
With longitudinal field (H>0)

Expected behavior $\left(\omega_L = \gamma_\mu H\right)$:

❑**The dip** is due to the presence of a typical field scale around which the muon spin nearly completes an oscillation . However, wide field distribution causes quick damping of the oscillations.

❑**The recovery** is due to the fact that some muons experience nearly static field during the measurement.

❑ $\omega_L >> \Delta$, the field at the muon site is nearly parallel to the initial muon spin direction → the muon does not relax.



μSR LF Results

- ❑ Does this compound behave as expected from a kagome compound:
- ❑ Does it have a long-range order?
- ❑ Is the limit of T→ 0 dynamics?

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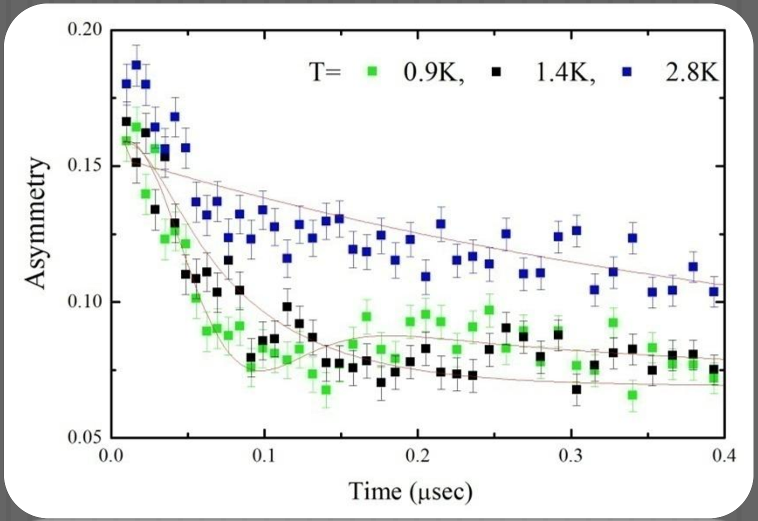
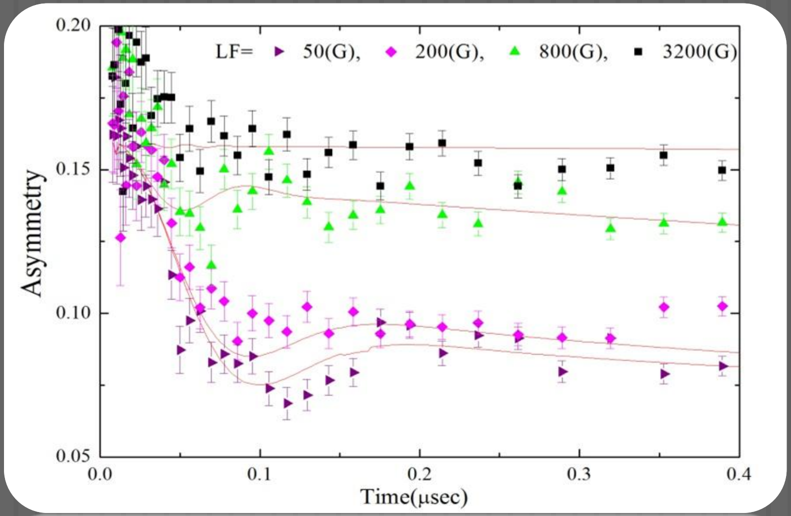
Zero-Field(ZF) measurements:
0.9K< T < 2.8K .



- No long range order at 0.9K (and below).
- Relaxation increases as T decrease.
- There is a dip at around 0.1μsec.
- The ZF data fits the expected theory.

Longitudinal-Field(LF) measurements:
50Oe < H < 3.2kOe , T=0.9K.



- The asymmetry recovers at more than 1/3 of the height.
- The LF data fits the same expected theory.

μSR LF Results

All those fits are done with one function ,
The Dynamical Gaussian Kubo Toyabe with one free parameter, $\nu(t)$.

These are unusual μSR data in a kagomé magnet.

Other kagomé magnets show the same general behavior but without this dip.

The data indicates the absence of long-range order and the presence of quasi-static field fluctuation.

We can extract the fluctuation rate ν.

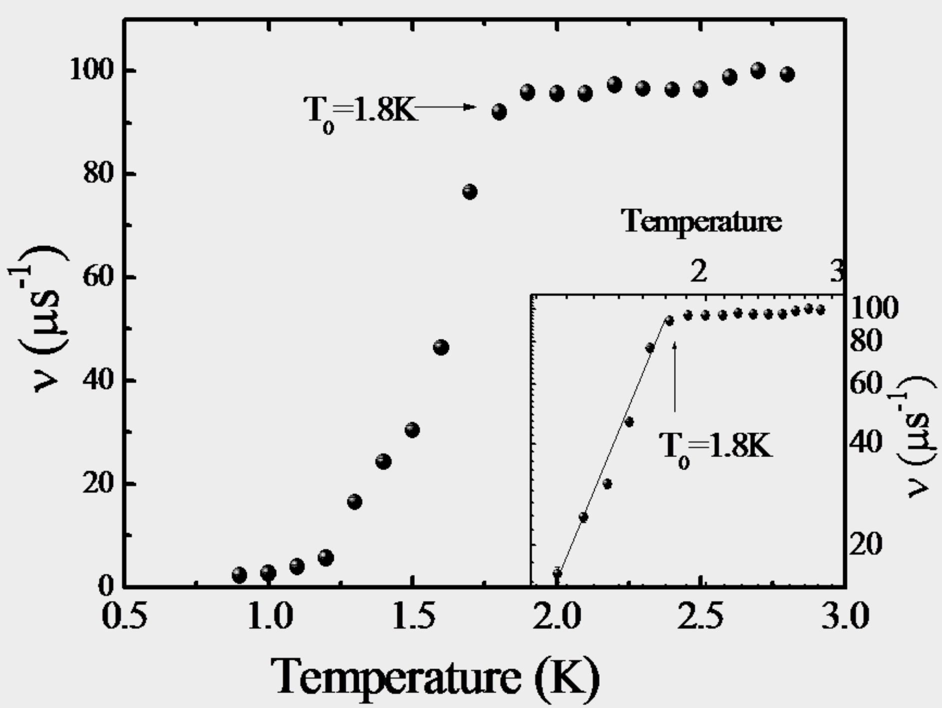
Spin fluctuation rate

- Does this compound behave as expected from a kagome compound:
- Does it have a long-range order?
- Is the limit of T→ 0 dynamics?

J. Robert *et al.* PRL **101** 117207 (2009), L. Marcipar *et al.* PRB **80** 132402 (2009)

- 2.8K < $T < T_0$ =1.8K
  ν hardly changes.
- $T < T_0$ =1.8K, ν decreases but saturates below 1K.
- ν remains small → the spins remain dynamic with <u>no long-range order</u>.

$$\nu = 3.6(2)\mu s^{-1}$$

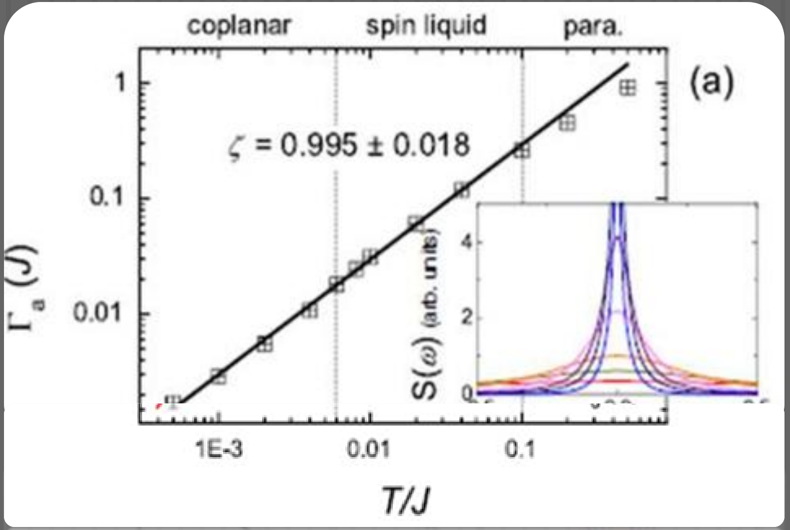


Spin fluctuation rate

Linear relation near T_0 , $\nu - \nu_\infty = \nu_0 (T - T_0)$

The high temperature fluctuation rate

Classical numerical simulations:



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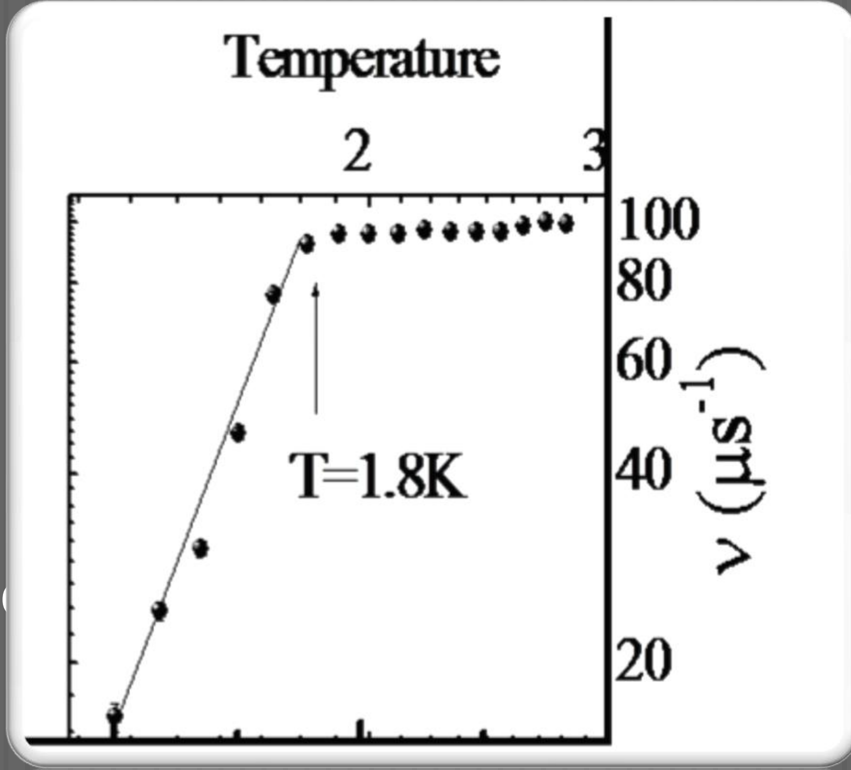


Research Questions

- Does this compound behave as expected from a kagome compound:
    - Does it have a long-order? **NO**
    - Is the limit of T->0 dynamics? **YES**

- Does it have anisotropic behavior ?
- What is the Hamiltonian of this compound ? Can we characterize it?

Susceptibility

- **Does it have anisotropy behavior ?**
- What is the Hamiltonian of this compound ?
  Can we characterize it?

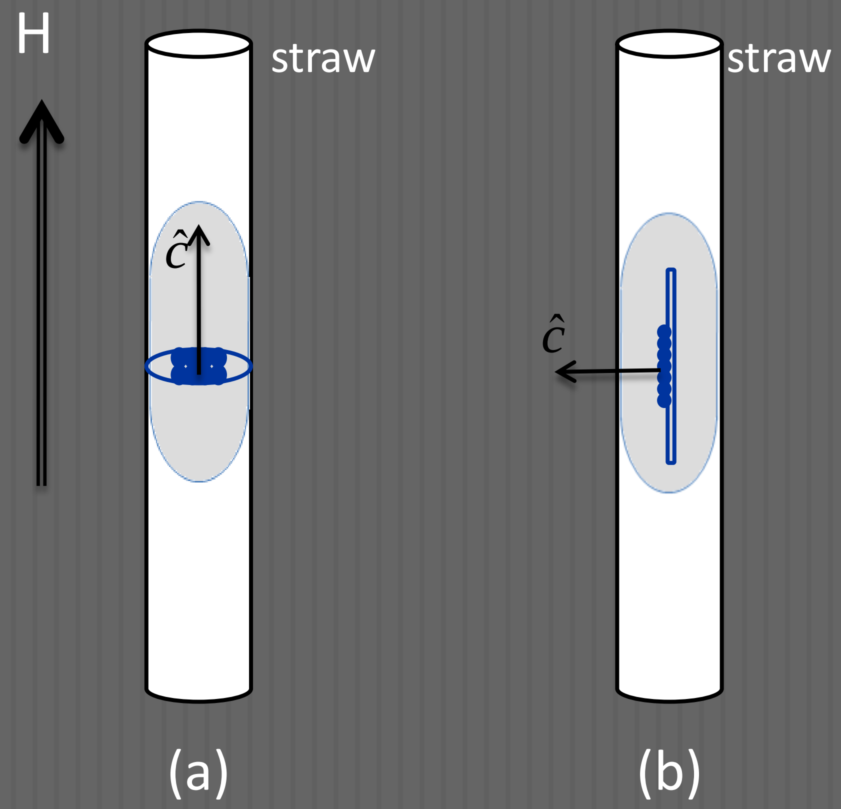
- Measure sample magnetization with SQUID (Superconducting Quantum Interference Device)
- Susceptibility measurements are the first step towards the Hamiltonian characterization.

Susceptibility

- **Does it have anisotropy behavior ?**
- What is the Hamiltonian of this compound ?

Can we characterize it?

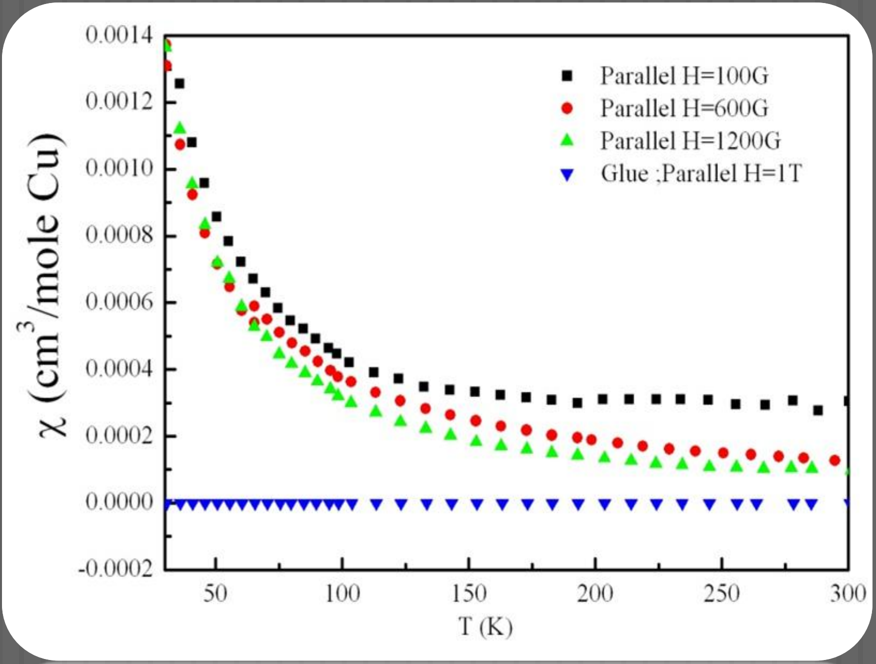
Gluing the single-crystal plates allow us to perform magnetization measurements at different directions .



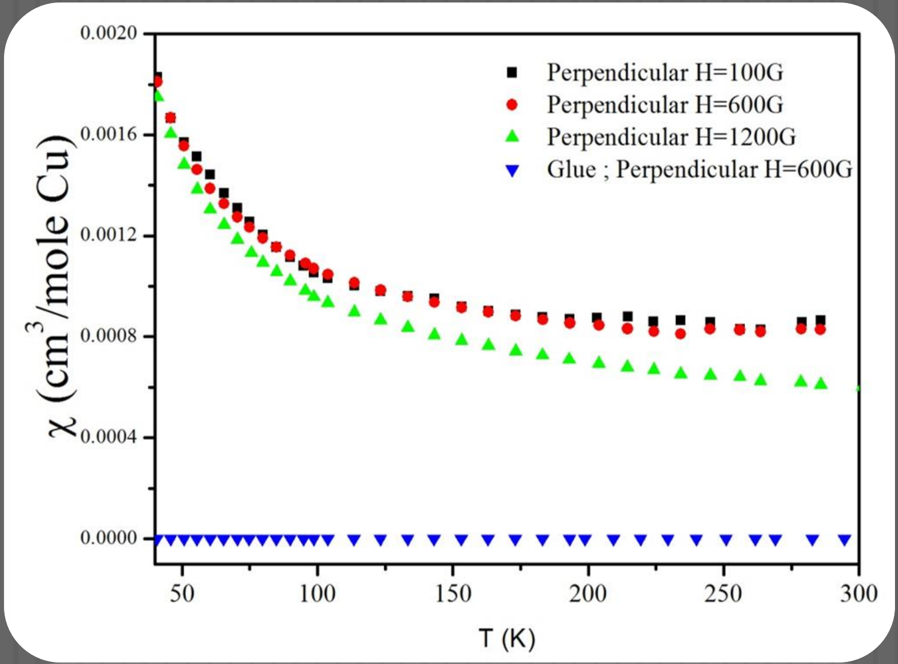
Susceptibility

- **Does it have anisotropy behavior ?**
- What is the Hamiltonian of this compound ?
  Can we characterize it?

Parallel Fields



Perpendicular Fields



Single-crystal magnetic susceptibility measurements display a pronounced anisotropy, $\chi_{\perp} > \chi_{\parallel}$.

Susceptibility

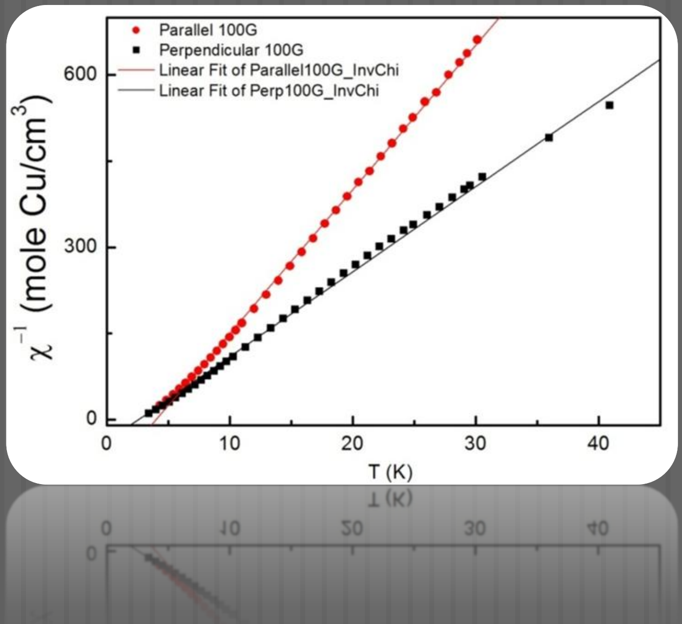
- **Does it have anisotropy behavior ?**
- What is the Hamiltonian of this compound ?

  Can we characterize it?

$$\chi_{\perp,\|}^{-1}=\frac{3k_B}{(g\mu_B)^2 S(S+1)}\left(T-\theta_{\perp,\|}^{cw}\right)$$

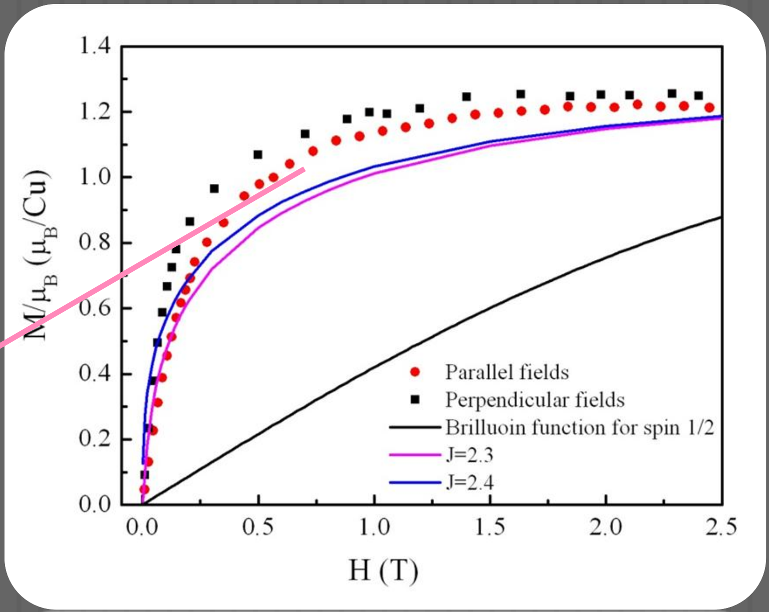
Using the curie-wiess law we can determine $\theta_{CW}^{\|}=4.03(1)K$.
The perpendicular direction could not be fitted to a linear function.
$\theta_{CW}^{\perp}$ is not reliable.

Indications of ferromagnetic interactions



Susceptibility

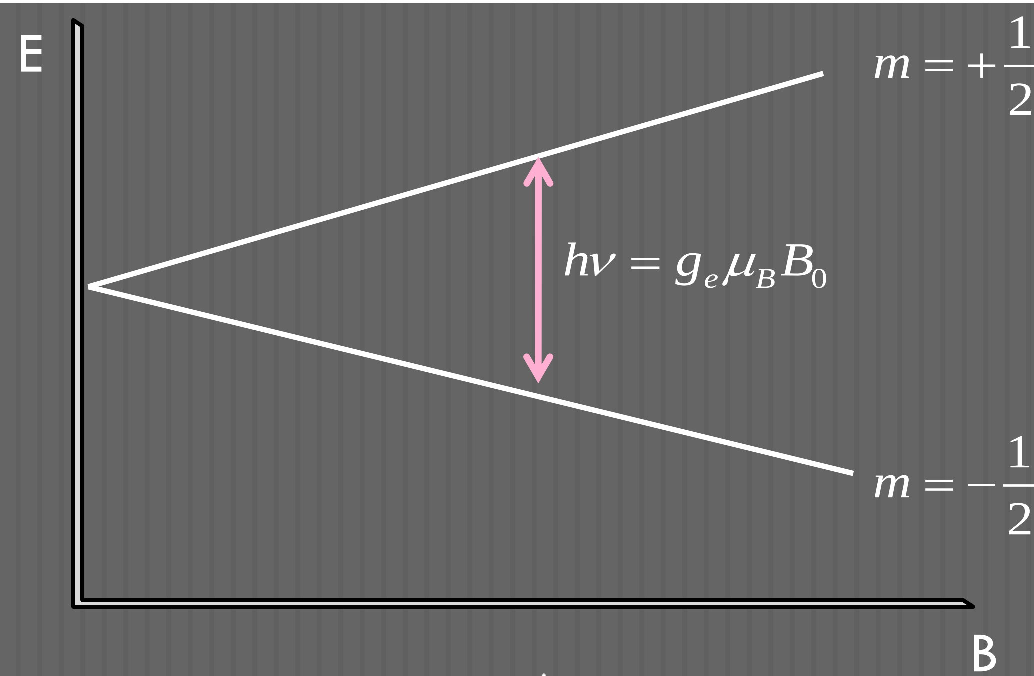
- Magnetization versus magnetic field at T=2.4 K
- The magnetization saturates around $1.2\,\mu_B / Cu$.
- At low-T, $g = 2.42(2)$



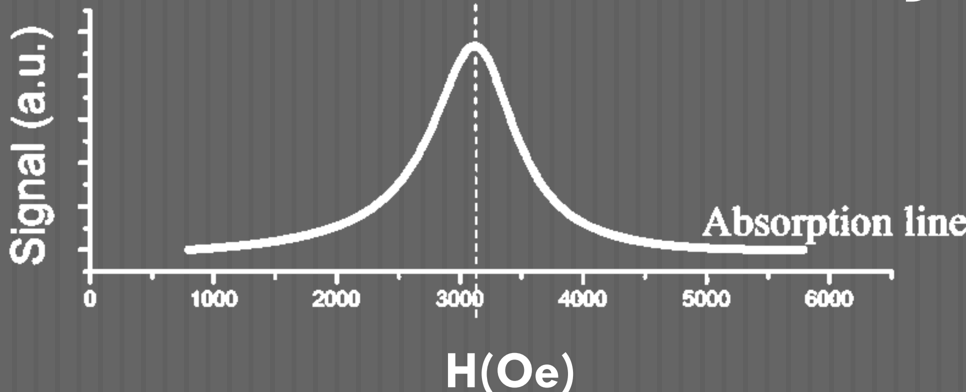
Indications of ferromagnetic interactions

Electron Spin Resonance (ESR)

- ESR is a technique for studying species that have one or more unpaired electrons.
- No magnetic field → the 2 energy states of the electron are degenerate.
- Applying an external magnetic field, those electron energy levels split into 2 energy levels.



We use fixed microwave radiation and sweep the magnetic field. Once the resonance condition $\Delta E = g_e\mu_B B_0$ is satisfied,
energy is absorbed by the sample and we get the absorption line.



Continuous Wave (CW) ESR

- CW ESR consists a source of microwave radiation, a resonator cavity and a detector.
- The frequency, ω, is fixed (~9.5GHz) and the magnetic field H, is swept. The absorption power:

$$\frac{\Delta P_c}{P_w} \simeq \chi'' \eta Q_u$$

ΔP_c – the change in the reflected power

P_w – the power reaching the sample

Q_u – unloaded quality factor

η - filling factor

χ'' - the imaginary part of the susceptibility

CW ESR

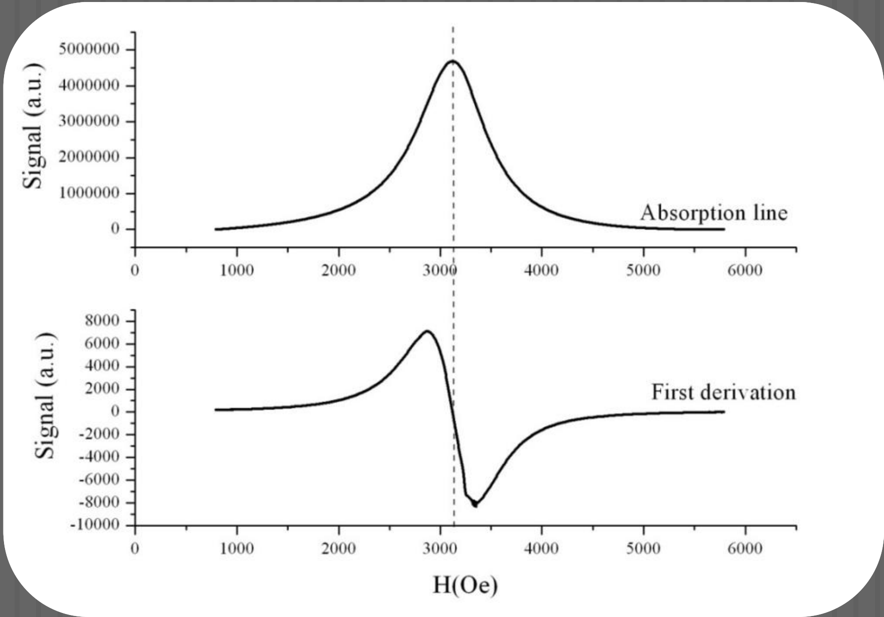
Modulation field is used to improve sensitivity:

$$H = St + H_m \cos(\omega_m t)$$

$$\chi''(H,\omega) = \chi''(St,\omega) + \frac{\partial \chi''(H,\omega)}{\partial H} H_m \cos(\omega_m t)$$

A lock-in detects the amplitude of the fast modulations in the absorption power, therefore the outcome of the measurement is:

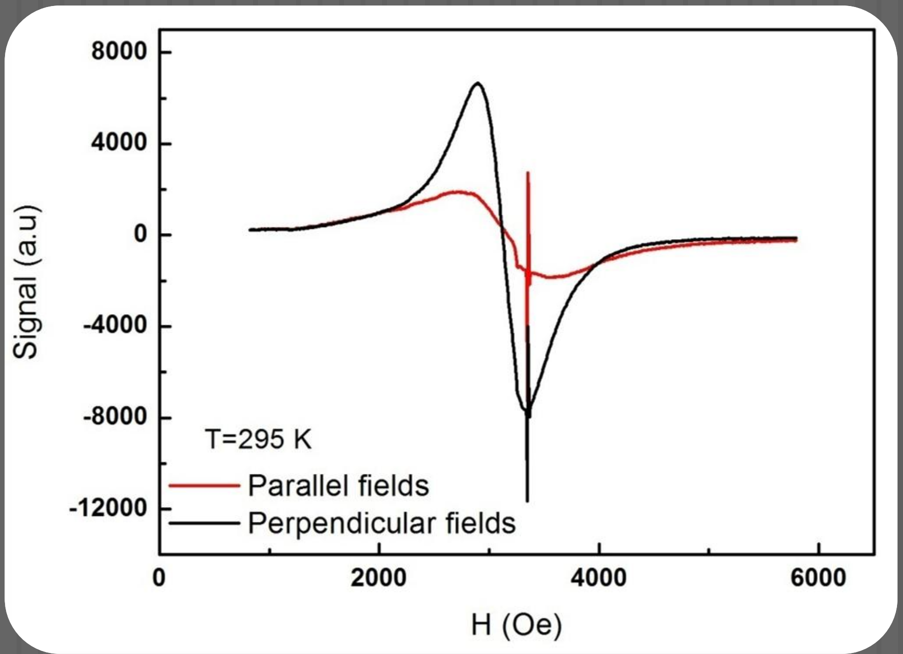
$$\frac{\partial \chi''(H,\omega)}{\partial H} \xrightarrow[INTEGRATION]{} \chi''(H,\omega)$$

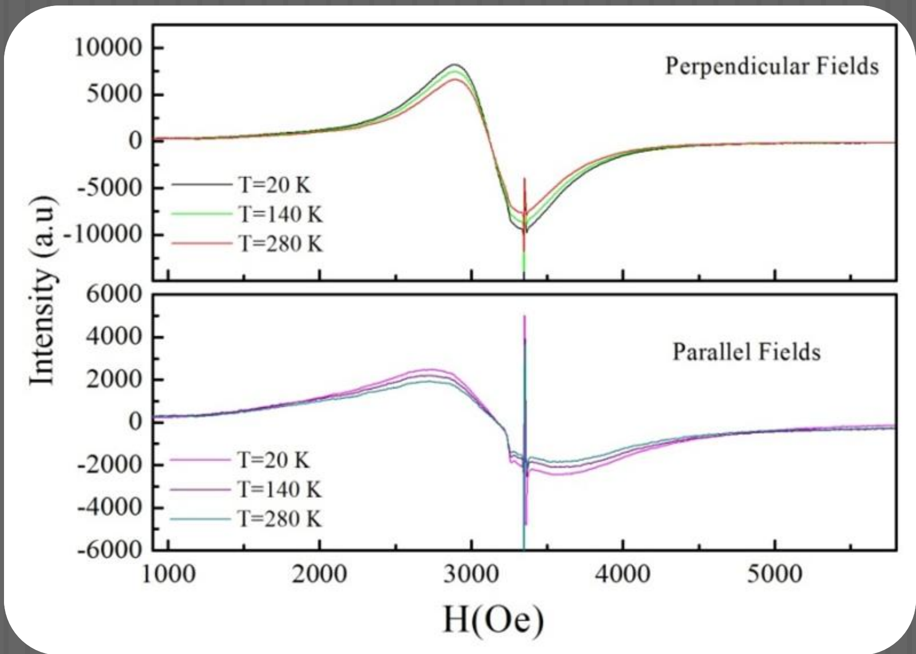


ESR Results

- **Does it have anisotropy behavior ?**
- What is the Hamiltonian of this compound ?
  Can we characterize it?

- 15K < T< 300K.
- Distinct anisotropy seen in the ESR results. consistent with susceptibility measurements.
- Intensity increase as T decrease. Expected behavior.





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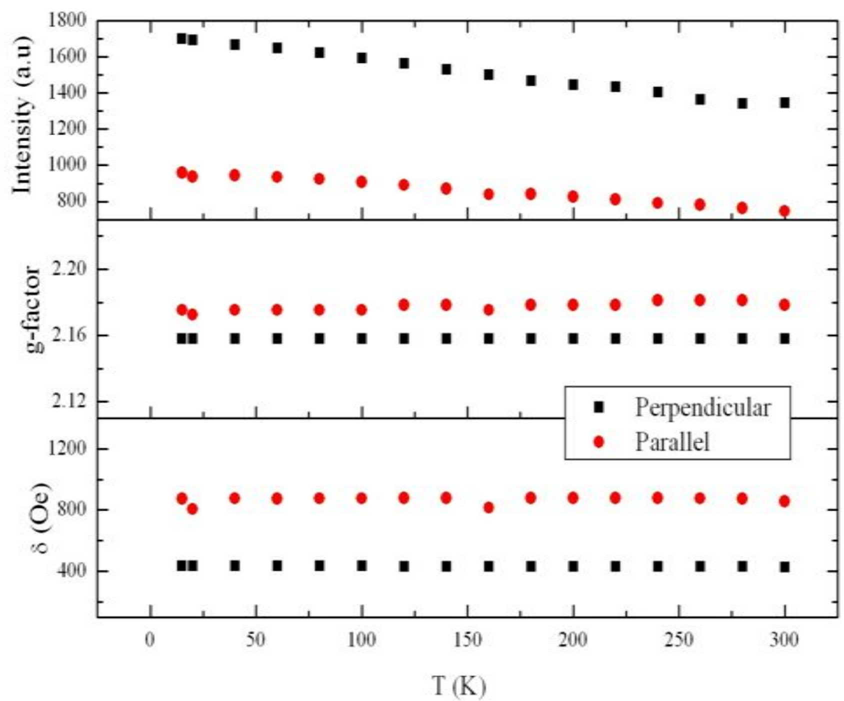

- The g-factor at each temperature can be calculated by

$$g_{\perp,\parallel} = \frac{\Delta H}{H_r} \cdot 2.0023 + 2.0023$$

H_r -resonance field of a reference sample

ΔH- difference between resonance fields of the sample and the reference

$$g_{\perp} = 2.158(5), \quad g_{\parallel} = 2.175(5)$$

- Two different g-factors and two absorption line-widths.
- Temperature independence.

Method of Moments

- Does it have anisotropy behavior ?
- What is the Hamiltonian of this compound?
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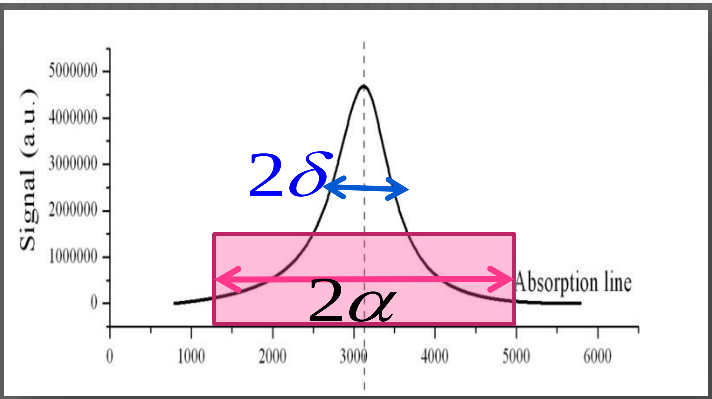
Method of Moments



- The data can be fitted to Lorentzian:

$$f(x)=\frac{1}{\pi}\frac{\delta}{\delta^2+(x-x_0)^2}$$

- The *n*th moment of a resonance curve (Lorentzian):

$$M_n \equiv \int (x-x_0)^n f(x)dx$$

- A cut-off must be used to calculate M_2 and M_4 .

$$|x-x_0| \le \alpha \ , \quad \alpha \gg \delta$$

$$M_2=\frac{2\alpha\delta}{\pi},\ M_4=\frac{2\alpha^3\delta}{3\pi} \Rightarrow \boxed{\delta=\frac{\pi}{3\sqrt{2}}M_2\sqrt{\frac{M_2}{M_4}}}$$

Method of Moments

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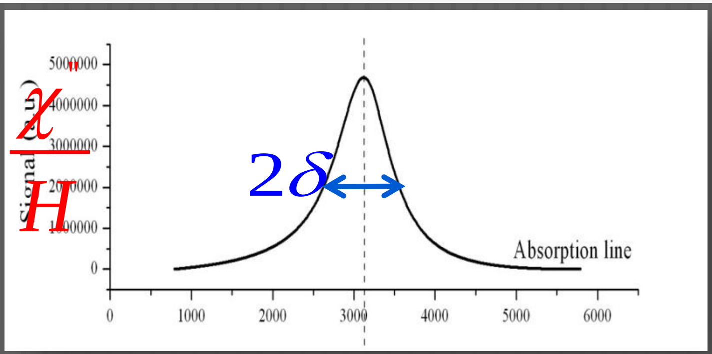
- What is the Hamiltonian of this compound?

Can we characterize it?



$$G_1 \equiv \left\langle \left\{ S_\alpha^I(t), S_\beta^I(0) \right\} \right\rangle; \; S_\alpha^I = e^{-\frac{i\mathcal{H}_{\mathrm{int}} t}{\hbar}} S_\alpha e^{\frac{i\mathcal{H}_{\mathrm{int}} t}{\hbar}}$$

$$\zeta \frac{\chi''(H,\omega)}{\omega} = \int_{-\infty}^{\infty} G_1(t) \exp(i\omega t)\, dt$$

We measure: $\chi''(\mathrm{H},\omega) \rightarrow \dfrac{\chi''(\mathrm{H},\omega)}{\mathrm{H}}$

We did fit to: $$\frac{\chi''(\mathrm{H},\omega)}{\mathrm{H}} = \frac{2A}{\pi} \frac{\delta}{(H-H_0)^2 + \delta^2}$$

We assume that: $$\frac{\chi''(\mathrm{H}_0,\omega)}{\omega} = \frac{2A}{\pi} \frac{\gamma\delta}{(\omega-\gamma H_0)^2 + (\gamma\delta)^2}$$

Method of Moments

$$G_1 \equiv \left\langle \left\{ S_\alpha^I(t), S_\beta^I(0) \right\} \right\rangle; \; S_\alpha^I = e^{-\frac{i\mathcal{H}_{\text{int}}t}{\hbar}} S_\alpha e^{\frac{i\mathcal{H}_{\text{int}}t}{\hbar}}$$

$$\zeta \frac{\chi''(H,\omega)}{\omega} = \int_{-\infty}^{\infty} G_1(t) \exp(i\omega t)\, dt$$

$$G_1 = \frac{1}{2\pi\zeta} \int_{-\infty}^{\infty} \frac{\chi''(0,\omega)}{\omega} \exp(-i\omega t)\, d\omega$$

$$M_n \equiv \int_{-\infty}^{\infty} \frac{\chi''(0,\omega)}{\omega} \omega^n d\omega \rightarrow M_{2n} = \frac{(-1)^n \left(\frac{d^{2n} G_1(t)}{dt^{2n}} \right)_{t=0}}{G_1(0)}$$

$$\left(\frac{d^n G_1(t)}{dt^n} \right)_{t=0} = (i)^n \left\{ \left[\underbrace{\mathcal{H}_1, \left[\mathcal{H}_1, \left[\ldots, \left[\mathcal{H}_1, S_x \right] \ldots \right] \right]}_{n \;\; times}, S_x \right] \right\}$$

Method of Moments

- Does it have anisotropy behavior ?
- What is the Hamiltonian of this compound?

Can we characterize it?

We obtain the M_2 and M_4 :

$$M_2^{\perp,\parallel} = -\frac{Tr\left(\left[\mathcal{H}, S_{z,x}\right]^2\right)}{Tr\left(S_{z,x}^2\right)}, \quad M_4^{\perp,\parallel} = \frac{Tr\left(\left[\mathcal{H},\left[\mathcal{H}, S_{z,x}\right]\right]^2\right)}{Tr\left(S_{z,x}^2\right)}$$

Let us consider the simplest anisotropy Hamiltonian:

$$\mathcal{H} = \sum_{j<k}\left(J_{jk}\vec{S}_j \cdot \vec{S}_k + D_{jk}S_{jz}S_{kz} + E_{jk}\left(S_{jx}S_{kx} - S_{jy}S_{ky}\right)\right)$$

Method of Moments – kagome lattice

Therefore, the 2nd and 4th moments for kagome lattice is given by:

$$M_2^{\parallel} = 4\left(D^2 + E^2\right)$$

$$M_2^{\perp} = 16E^2$$

$$M_4^{\parallel} = \frac{7D^2}{4} + 2D^3J - DE^2J + D^2\left(4E^2 + 3J^2\right) + \frac{1}{2}E^2\left(5E^2 + 9J^2\right)$$

$$M_4^{\perp} = 2E^2\left(4D^2 + 9DJ + 8\left(E^2 + J^2\right)\right)$$

- Acknowledgement to Dr. Ravi Chandra and Dr. Daniel Podolsky for the calculations.

- Does it have anisotropy behavior ?
- **What is the Hamiltonian of this compound?**
  Can we characterize it?

Method of Moment

The Method of Moments allows us to connect the susceptibility and ESR measurements along with theoretical calculations in order to get the Hamiltonian parameters J, D and E .

$$\boxed{\begin{aligned}
&-2\cdot\theta_{CW}^{\parallel} = J + D = -2\cdot 4.03K \\
&\frac{2\pi f}{20.048\cdot H_{\perp}}\cdot\delta_{\perp} = \frac{\pi}{2\sqrt{3}}M_2^{\perp}\cdot\sqrt{\frac{M_2^{\perp}}{M_4^{\perp}}} \\
&\frac{2\pi f}{20.048\cdot H_{\parallel}}\cdot\delta_{\parallel} = \frac{\pi}{2\sqrt{3}}M_2^{\parallel}\cdot\sqrt{\frac{M_2^{\parallel}}{M_4^{\parallel}}}
\end{aligned}}$$

Solving numerically these three equations yiel

$$\left(J = -9.273K,\ D = 1.213,\ E = \pm 0.477K\right)$$

$$\left(J = -6.975K,\ D = -1.085K,\ E = \pm 0.449K\right)$$

Ferromagnetic interactions ???

Discussion – ferromagnetic interactions?

1. ESR method depends on the cut-off selection, the cut-off affects the moments calculations. Wrong selection of the cut-off. In order to get AFM interactions we need a robust change in this cut-off.

2. In our model we consider simplest anisotropy Hamiltonian which takes into account only n.n. interactions. Maybe n.n.n. interactions or/and DMI interactions needs to be considered.

3. If the cut-off selection is correct we found new phase of matter. Kagomé with ferromagnetic interactions with no long-range order and dynamic ground state.

Summary

- Does this compound behave as expected from a kagome compound?
  - ✓ There is no long range order
  - ✓The state at the lowest temperature is quasi-static with extremely slow spin fluctuations.
  - ✓A distinct anisotropy behavior detected by magnetization and ESR measurements.
  - ? We are able to characterize the spin Hamiltonian??

    FM interactions???

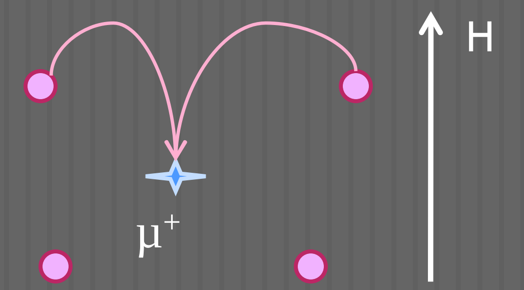
Thank you for your attention!

Muon shift

The field at the muon site given by: $B = H - \sum_k \tilde{A}_k(r)\langle S_k\rangle$

Assuming a distribution field in the z direction:

$$\tilde{A}_k = \bar{A}_k + \delta A_k$$



Using the distribution: $\rho(\delta A_k) = \frac{1}{\pi}\frac{\sigma_k}{(\delta A_k)^2 + \sigma_k^2}$

$$B = \frac{\omega_k}{\gamma}\ ,\ H = \frac{\omega_s}{\gamma}$$

$$\langle S\rangle = M = \chi \mathrm{H}$$

$$\Rightarrow B = H - M\cdot\underbrace{\sum_k \bar{A}_k}_{A_{eff}} \quad \Rightarrow \frac{A_{eff}\cdot M}{H} = \frac{H-B}{H} \Rightarrow A_{eff}\cdot\chi = \underbrace{\frac{\omega_s - \omega_k}{\omega_s}}_{K}$$

$$\bar{P}_\mu(t) = P_0\cos\left[\gamma_\mu\left(1+\bar{A}\chi\right)H_{TF}t\right]\underbrace{\int\cos\left(\gamma_\mu\delta\mathrm{A}\chi\mathrm{H}_{TF}t\right)\rho(\delta\mathrm{A})\,d(\delta\mathrm{A})}_{R_2}$$

$$\Rightarrow R_2 = \gamma_\mu\chi\mathrm{H}_{TF}\sum_k\sigma_k$$

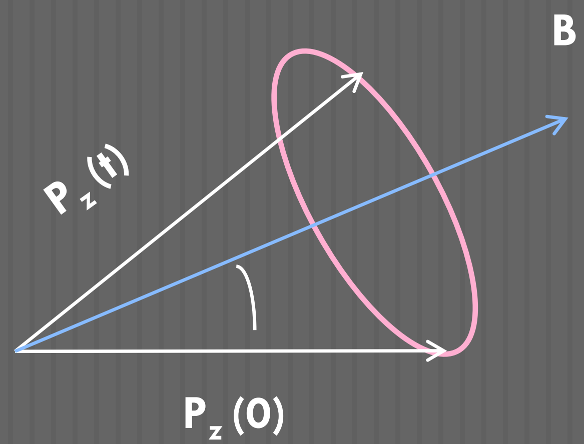
Static Gaussian Kubo Toyabe

$B - local\ \ magnetic\ \ field$

$$P_z(t) = \cos^2\theta + \sin^2\theta\cos\left(\gamma_\mu |B| t\right)$$

$$\bar{P}_z(t) = \frac{1}{3} + \frac{2}{3}\cos\left(\gamma_\mu |B| t\right)$$

$$\bar{P}_z(t) = \frac{1}{3} + \frac{2}{3}\left(1 - \Delta^2 t^2\right)\exp\left(-\frac{1}{2}\Delta^2 t^2\right)$$



Dynamical Gaussian Kubo Toyabe

The static Gaussian LF KT:

$$\bar{P}_z(0,\mathrm{H},\Delta,t)=1-\frac{2\Delta^2}{(\omega_L)^2}\left[1-\exp\left(-\frac{1}{2}\Delta^2t^2\right)\cos(\omega_L t)\right]+\frac{2\Delta^4}{(\omega_L)^3}\int_0^t\exp\left(-\frac{1}{2}\Delta^2\tau^2\right)\sin(\omega_L\tau)\,d\tau$$

The dynamic Gaussian KT:

$$\bar{P}_z(\nu,\mathrm{H},\Delta,t)=\underbrace{e^{-\nu t}\bar{P}_z(0,\mathrm{H},\Delta,t)}+\underbrace{\nu\int_0^t dt'\,\bar{P}_z(\nu,\mathrm{H},\Delta,t-t')e^{-\nu t'}\bar{P}_z(0,\mathrm{H},\Delta,t')}$$

The polarization at time t due to muons that did not experience any field change.

Contribution from those muons that experienced their first field change at time t’.

Filling factor and Q factor

Filling factor

$$\eta = \frac{\int\limits_{V_s} B_1^2 dV}{\int\limits_{V_C} B_1^2 dV}$$

B_1 – the microwave magnetic field intensity.

Quality factor

$$Q = 2\pi \frac{\text{Maximum microwave energy stored in the resonator}}{\text{Energy dissipated per microwave cycle}}$$

Change in the reflected power

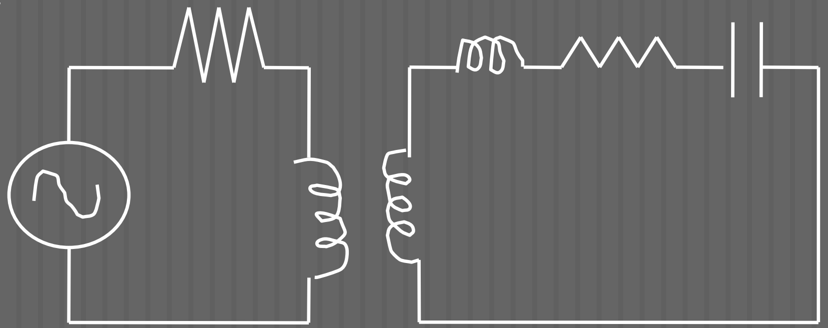
$$B = B_1 \sin(\omega t) \rightarrow M' = \chi' \sin(\omega t), M'' = -\chi' \cos(\omega t)$$

$$E = \vec{B} \cdot \vec{M} \ ; \ P = B \cdot \frac{dM}{dt} \Rightarrow \frac{1}{T} \int_0^T B \cdot \left(\frac{dM}{dt} \right) dt = \frac{1}{2} \omega \chi'' B_1$$

$$\frac{1}{Q} = \frac{1}{Q_u} + \frac{1}{Q_{\chi''}}$$

$$Q_{\chi''} = 2\pi \frac{\int_{V_C} \frac{1}{2} \omega_0 B_1^2}{\int_{V_S} \frac{1}{2} \omega_0 B_1^2 \chi''} = \frac{1}{\eta \chi''} \quad ; \quad \Delta\left(\frac{1}{Q}\right) = \frac{1}{Q^2} \Delta Q$$

$$\frac{\Delta Q}{Q} = \frac{\Delta P}{P} \cong \pm \eta \chi'' Q_u$$



g-factor calculation

$$g_{\perp,\|} = \frac{\Delta \mathrm{H}}{H_r} \cdot 2.0023 + 2.0023$$