2D Hard Core Bosons Paradigm for Cuprates Superconductivity

"How wonderful that we have met with a paradox. Now we have some hope of making progress." **~Niels Bohr**

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Under the guidance of *prof.* Amit Keren

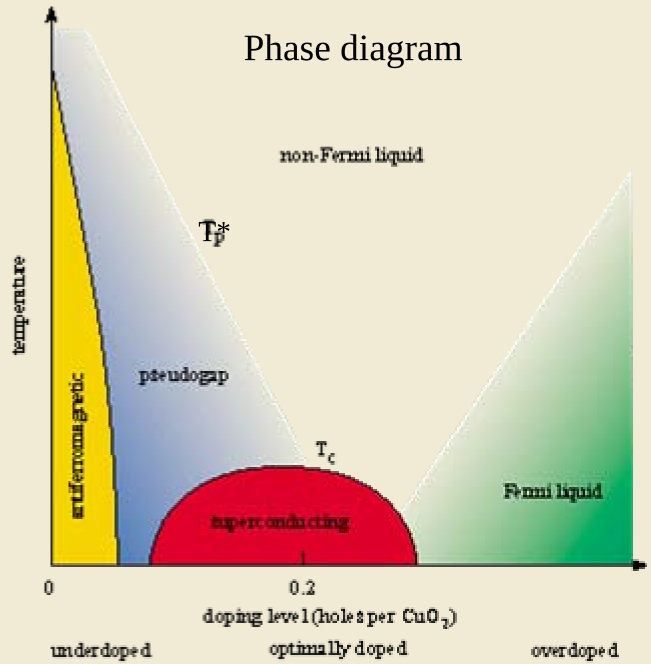
Collaborators – Dr. Meni Shai

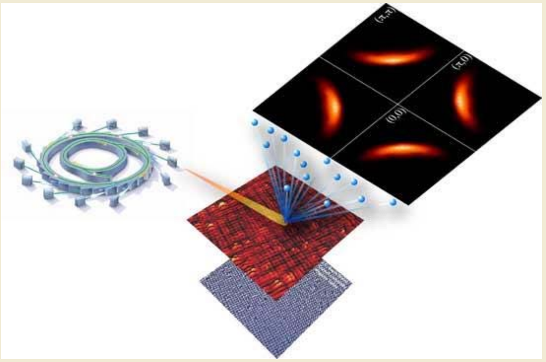
Basic concepts in SC

- Pseudogap
- Meissner effect
- Homes’ Law – Uemura Relations
- Coherence Length

The Cuprates

In the pseudogap state, the DOS around the fermi energy decreases with temperature drop.





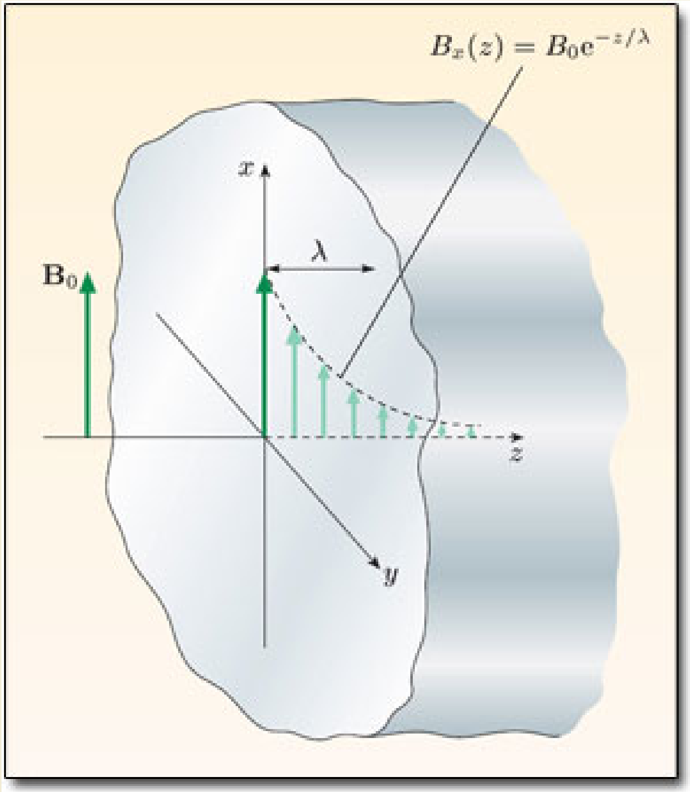
$(7 - \delta)$	T_c, K	T^*, K	T_{sqr}, K	Δ^*_m		Q, $K^{1/2}$	k
				K	meV		
6.93	91.2	133	133	180	15.5	72.2	4.62
6.88	90.8	189	189	602	52	148	3.38
6.85	89.5	203.3	203.2	524	45	95.5	2.6
6.78	80.5	220	198	298	25.7	37.4	1.86
6.68	58.7	268	231	299	25.7	33.8	1.72

D. D. Prokof'ev, M. P. Volkov, and Yu. A. Boĭkov, "*Pseudogap and Its Temperature Dependence in YBCO from the Data of Resistance Measurements*" Ioffe Physicotechnical Institute, Russian Academy of Sciences, Politekhnicheskaya ul. 26, St. Petersburg, 194021 Russia

The London Eq.

To explain the *Meissner Effect*, F. and H. London (brothers) came

up with the relations $\vec{J} = -\frac{\rho_s e^2}{m^* c}\vec{A}$



Along with Maxwell's Eq.

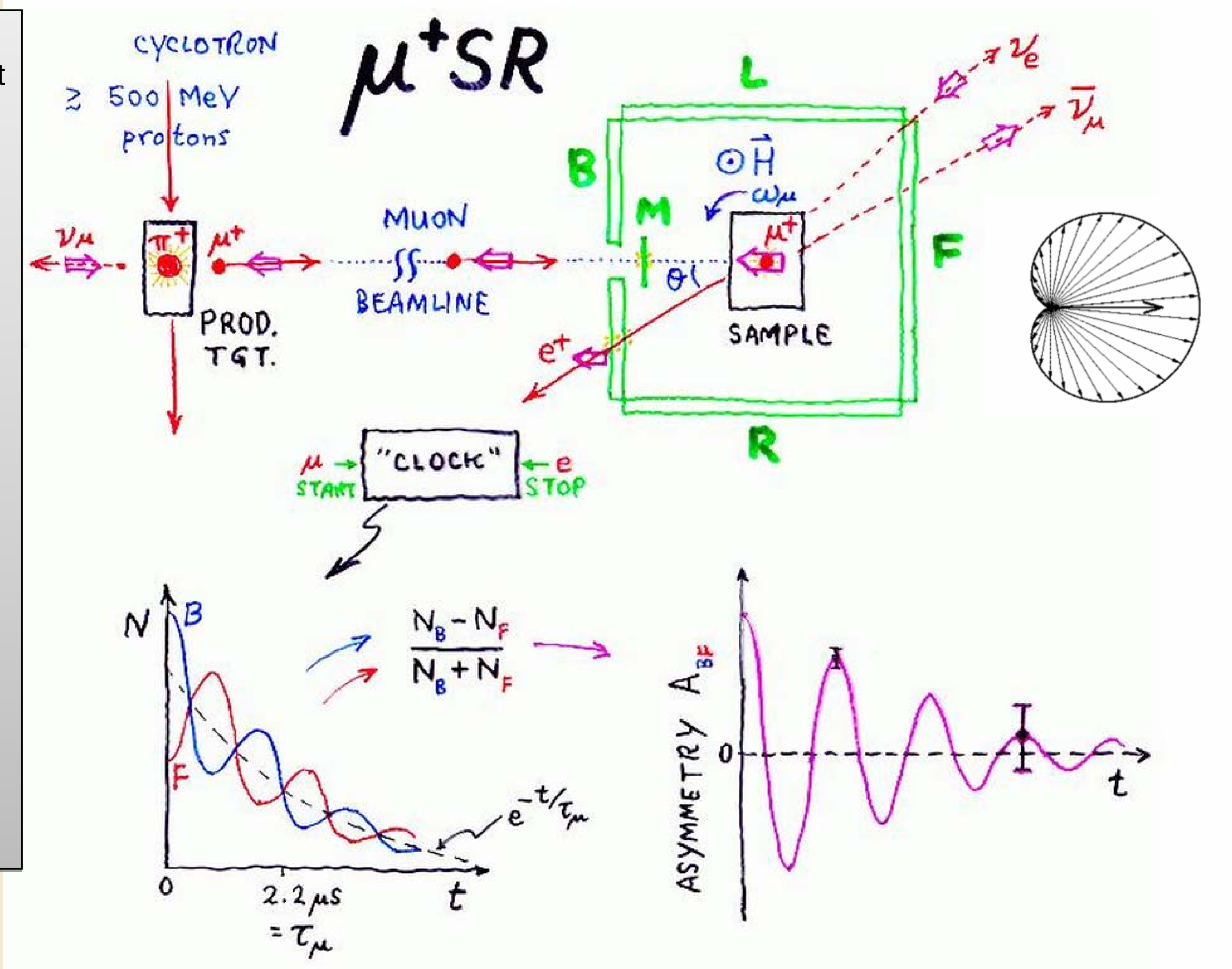
$$\vec{\nabla} \times J = \frac{\rho_s e^2}{mc}\vec{B} \qquad \vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J}$$

We get magnetic field exclusion that is governed by

$$\nabla^2 \vec{B} = \frac{1}{\lambda^2}\vec{B} \quad \Rightarrow \quad \frac{1}{\lambda^2} = \frac{4\pi e^2 \rho_s}{m^* c^2}$$

Penetration Depth Measurement μSR

- The Muon ray arrives to the target material nearly 100% spin polarized.
- The Muon decays inside the target and emits a positron favorably in it's spin direction.
- The anisotropy of the positron distribution around the sample tells the story of the Muon spin interaction with the bulk's local field, hence is deduced once external classical field is applied.



Penetration Depth Measurement LE-μSR

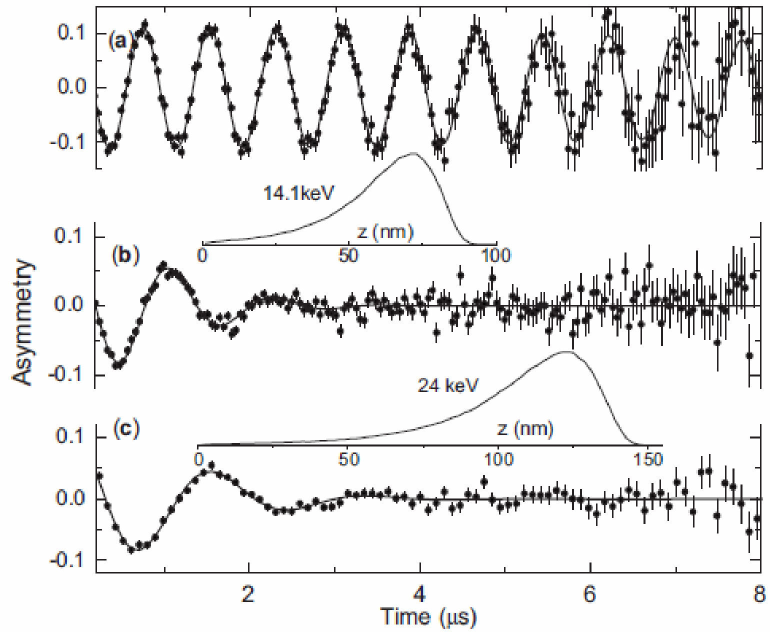


FIG. 1. (a) The muon spin precession signal in the normal state of $YBa_2Cu_3O_{6.92}$ at 110 K in an external field of 9.46 mT applied parallel to the *a* direction. The mean implantation energy is $E = 14.1$ keV which corresponds to a mean implantation depth of 62.8 nm. (b) The same conditions as (a) except in the superconducting state at $T=8$ K. The inset shows the calculated stopping distribution. (c) The same conditions as (b) except the energy of implantation is increased to 24 keV.

$$\omega = \gamma H$$

$$A(t) = \exp(-\sigma^2 t^2/2)\cos(\omega t + \varphi)$$

$$B(z) = \begin{cases} B_0 \exp[-(z-d)/\lambda_{a,b}] & z \geq d \\ B_0 & z < d, \end{cases}$$

Kiefl, R. F. and Hossain, M. D. and Wojek, B. M. and Dunsiger, S. R. and Morris, G. D. and Prokscha, T. and Salman, Z. and Baglo, J. and Bonn, D. A. and Liang, R. and Hardy, W. N. and Suter, A. and Morenzoni, E., "*Direct measurement of the London penetration depth in YBa_{2}Cu_{3}O_{6.92} using low-energy ÎŒSR*", *Physical Review B* 81, 18 (2010), pp. 180502.

Homes' Law

- From Reflectance amplitude measurements over a wide range of frequencies ($50-8000\left[cm^{-1}\right]$), the Kramers–Kronig relation is used to obtain the phase (see:Kramers-Kronig constrained variational analysis of optical spectra), therefore, dielectric function and optical conductivity are achieved.

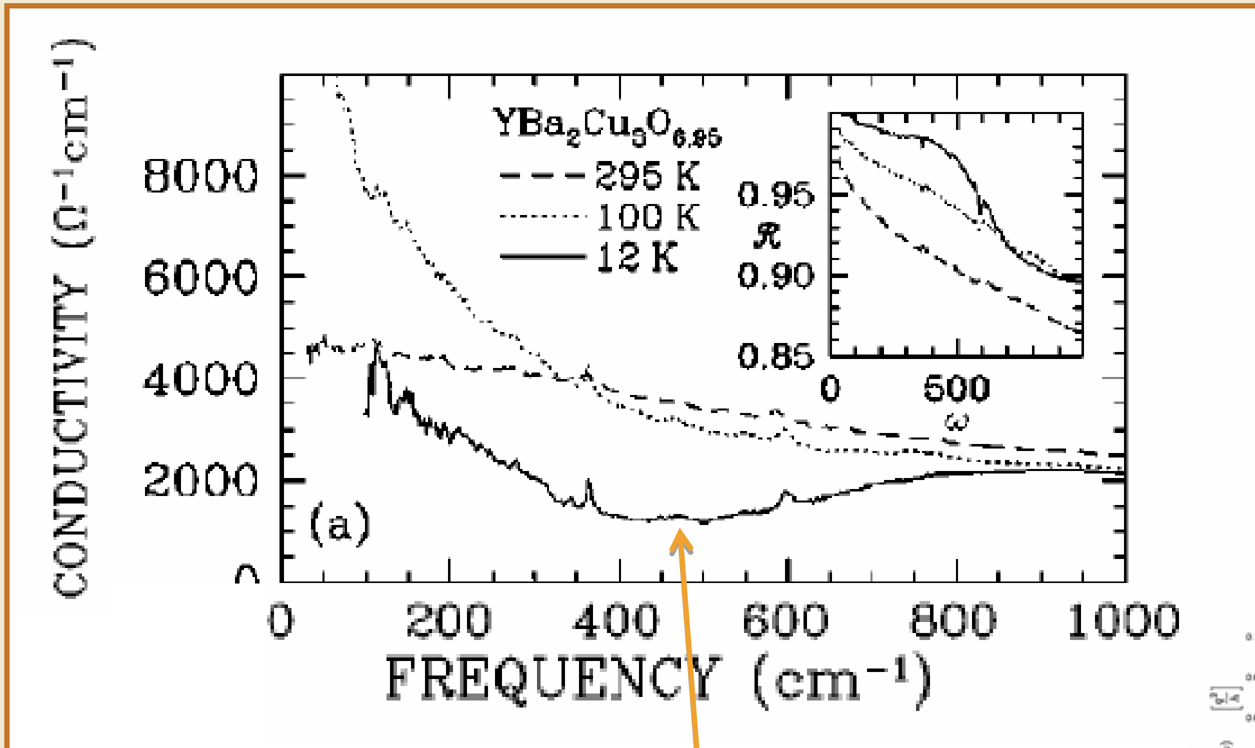
S.C. state

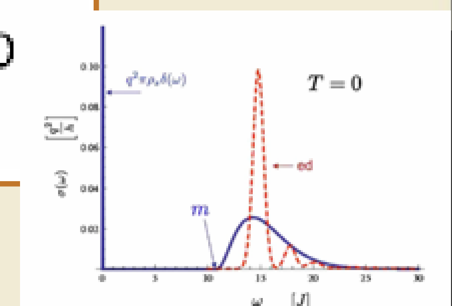
- The Superconducting state plasma frequency was obtained using the dielectric function (assuming all the carriers collapse into a delta function under the transition $\varepsilon_1(\omega)=\varepsilon_\infty-\frac{\omega_{p,s}^2}{\omega^2}$ where $\rho_s\equiv\omega_{p,s}^2\propto\lambda^{-2}$
Homes 1 Homes 2

- The D.C. conductivity was extrapolated from the real conductivity (Hagen-Rubens) $\sigma_1(\omega\to 0)=\sigma_{D.C.}$

$$\rho_s(0)\propto\sigma_{D.C.}(T_c)T_c$$

Homes raw data



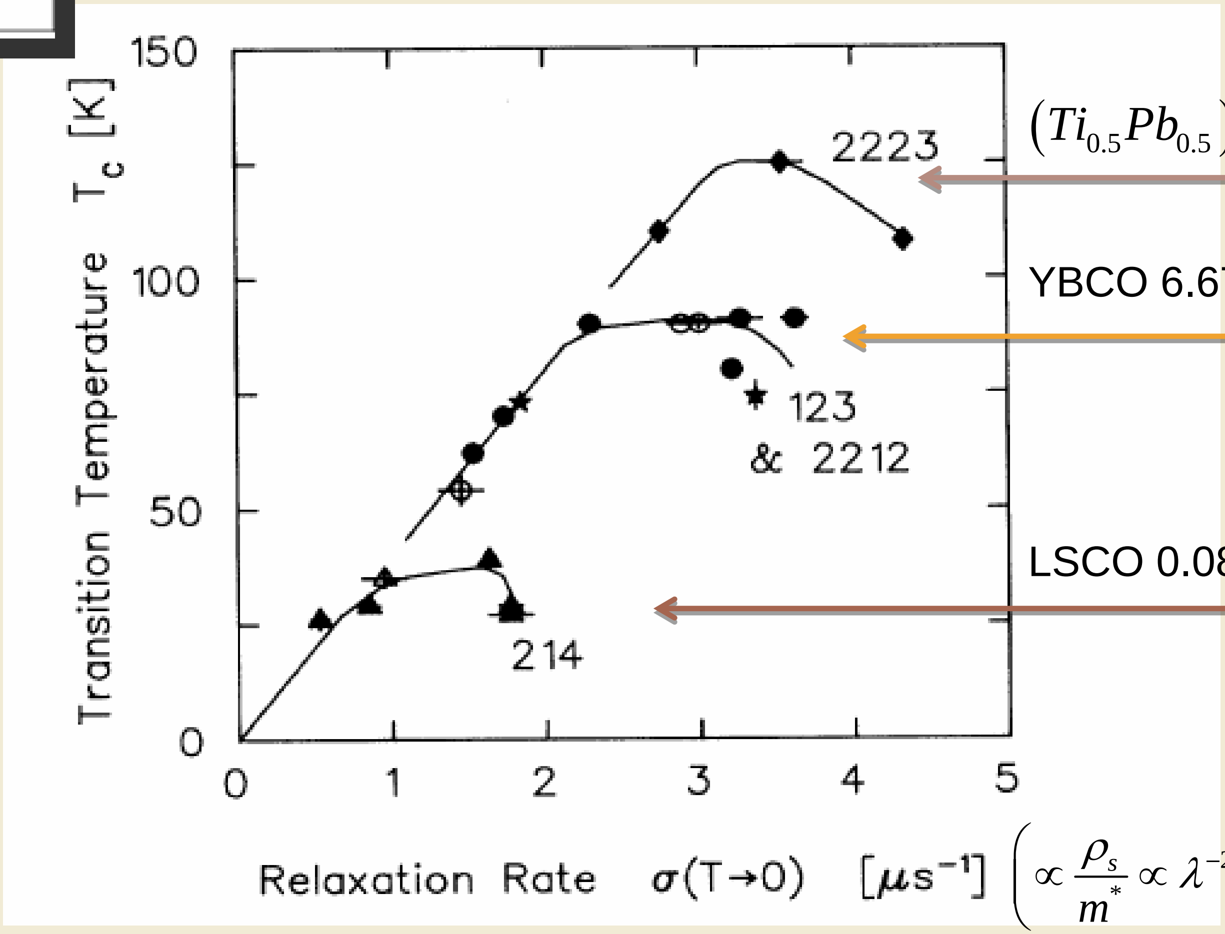


~60meV

C.C Homes *Effect of Ni impurities on the optical properties of YBa2Cu3O61y* Phys. Rev. B 60, 9782–9792

$(7-\delta)$	T_c, K
6.93	91.2
6.88	90.8
6.85	89.5
6.78	80.5
6.68	58.7

Uemura



$\left(Ti_{0.5}Pb_{0.5}\right)Sr_2Ca_2Cu_3O_{10}$

YBCO 6.67,6.76,6.87

LSCO 0.08,0.10,0.15,0.20,0.21

$$\left(\propto \frac{\rho_s}{m^*} \propto \lambda^{-2}\right)$$

Phys. Rev. Lett. 62, 2317–2320 (1989), YJ Uemura-
Universal Correlations between T_c and n_s/m^* (Carrier Density over Effective Mass) in High-T_c Cuprate Superconductors

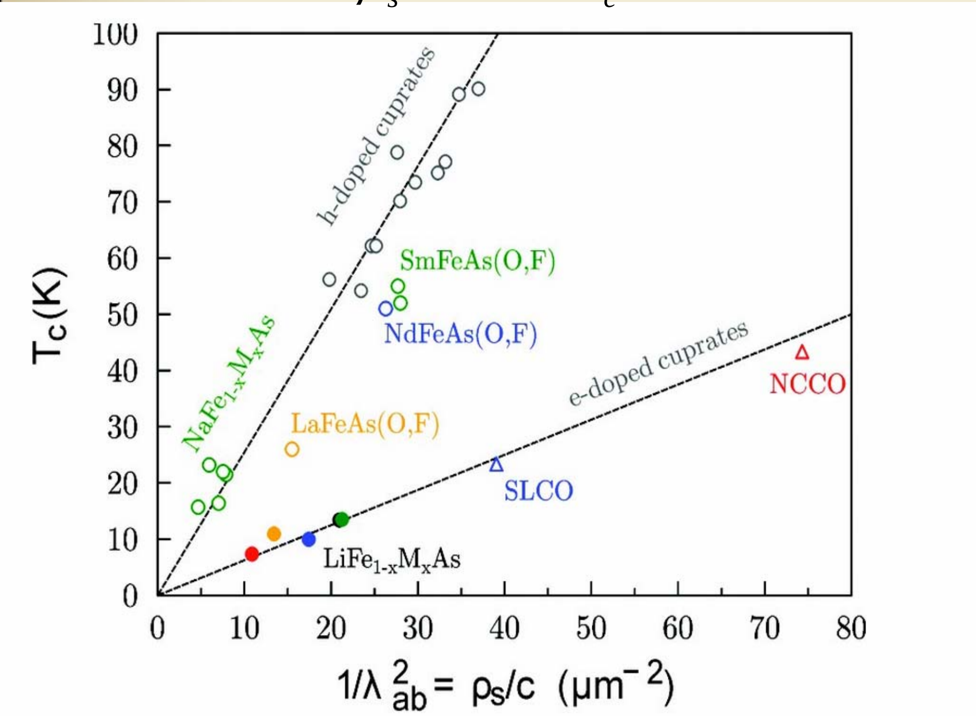
Uemura relations - Homes' Law

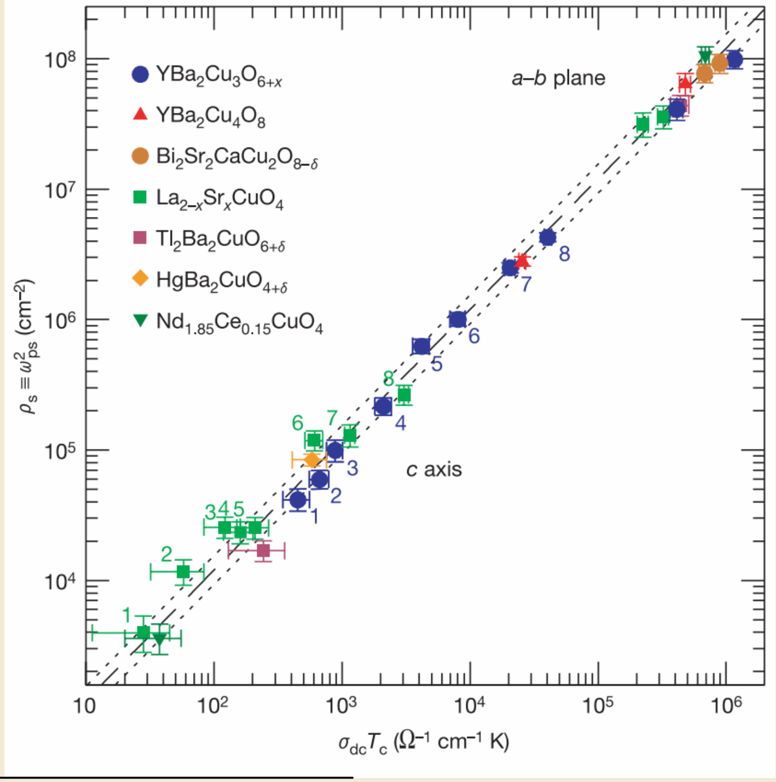
$$Uemora\ Plot - Underdoped\,[1]$$

$$\rho_s \propto \lambda^{-2} \propto T_c$$

$$Homes'\,Law\,[2]$$

$$\rho_s(0) \propto \sigma(T_c)T_c$$





[1] http://www.diamond.ac.uk/Home/Beamlines/I11/casestudies/casestudy10.html

[2]Homes, C. C. and Dordevic, S. V. and Valla, T. and Strongin, M., "*Scaling of the superfluid density in high-temperature superconductors*", *Physical Review B* 72, 13 (2005), pp. 134517.

Uemura relations - Homes' Law

$$T_c \propto \lambda^{-2} \qquad \sigma_{D.C.}(T_c)T_c \propto \lambda^{-2}$$

If both statements are true for Underdoped cuprates, the D.C. conductivity must be universal at T_c !

The G-L Coherence Length

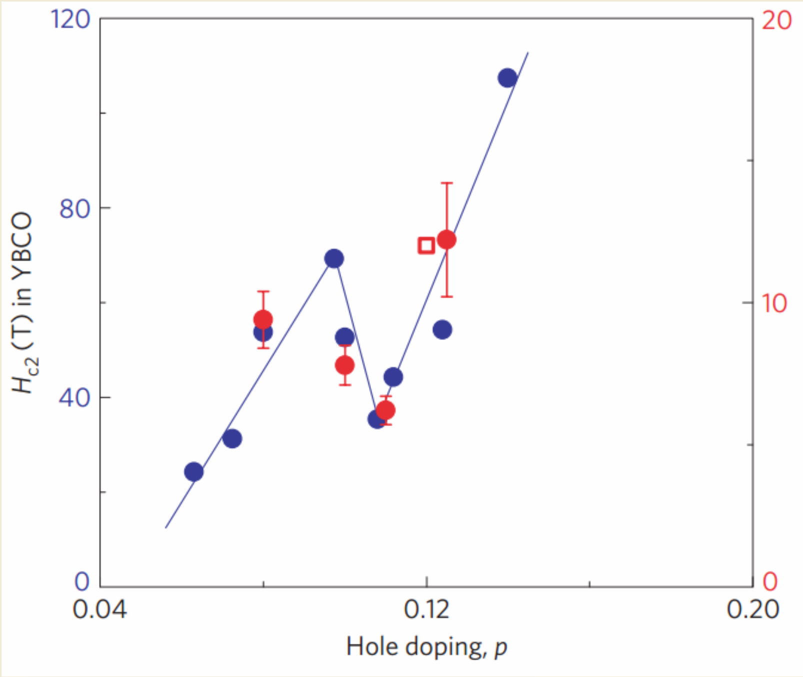
Ginzburg-Landau (GL) in the framework of their theory for 2nd order phase transitions, have introduced a complex order parameter

$$\psi = |\psi(r)| e^{i\varphi(r)}$$

from the GL Eq. a characteristic length scale arose, any variation/ disturbance of the value of $\psi(r)$ from the value it takes deep in the bulk, will typically decay according to

$$\xi^2(T) = \frac{\phi_0}{2\pi H_{c2}}$$

Where H_{c2} is the applied field that destroys the superconductivity, and $\phi_0 = \frac{h}{2e}$ is the Flux quanta.



Distance within a Cooper pair

Why HCB?

- Impurity/disorder based models cannot explain cuprates s.a. YBCO in the dirty limit of the BCS theory – the resistivity at optimal doping extrapolates to zero.
- Recent measurements have shown short coherence length in Cuprates in comparison to the classical Type I - SC (i.e. Tables for YBCO, LSCO)
- the spatial separation of paired electrons/holes (Cooper pair) in HTS is within a few lattice-constant scale (if one would like to keep the microscopic image of the mechanism)
- There is an effective attractive interaction between 2 charge carriers of similar polarity.

Intuitive picture :

the average distance within a cooper pair bound state is much smaller than the average distance between cooper pairs. Effectively a boson (between T^* and T_c)

Yang, H.-B. and Rameau, J. D. and Johnson, P. D. and Valla, T. and Tsvelik, A. and Gu, G. D., *"Emergence of preformed Cooper pairs from the doped Mott insulating state in Bi2Sr2CaCu2O8+ÎŽ"*, Nature 456, 7218 (2008), pp. 77--80.

Hard Core Bosons, as a model for cuprates superconductivity

The Bose-Hubbard lattice model Hamiltonian

$$H = -2J \underbrace{\sum_{\langle i,j \rangle} \left(b_i^{\dagger} b_j + h.c. \right)}_{Kinetic} - \mu \sum_i \hat{n}_i$$

The Holstein-Primakoff transformation to a spin model is possible under the assumption of **low temperatures**, thus low probability for high excitations/occupancies to exist.

Lindner, Netanel H. and Auerbach, Assa, "*Conductivity of hard core bosons: A paradigm of a bad metal*", Physical Review B 81, 5 (2010), pp. 054512.

The Holstein-Primakoff mapping

$$S_i^+ \to (2s)^{1/2}\left(1-\frac{\hat{n}_i}{2s}\right)^{1/2} b_i^\dagger \qquad S_i^- \to (2s)^{1/2} b_i \left(1-\frac{\hat{n}_i}{2s}\right)^{1/2} \qquad S_i^z \to s-\hat{n}_i$$

Hence, the Kinetic term manifests Quantum XY model

$$H = -2J\sum_{\langle i,j\rangle} S_i^x S_j^x + S_i^y S_j^y - \mu \sum_i \underbrace{S_i^z}_{i^{th}\ \text{site occupation}}$$

Lindner-Auerbach conductivity

The Current in HCB

- Using the HCB current operator

Carrier Charge!

$$J^x = 4\frac{J(qe)}{\hbar}\sum_r S_r^x S_{r+x}^y - S_r^y S_{r+x}^x$$

- From Fluctuation-Dissipation relations

$$\frac{2}{\hbar\omega} tanh\left(\frac{\beta\hbar\omega}{2}\right)\int\langle\left\{\mathbf{J}_\alpha^I(\mathbf{q},t),\mathbf{J}_\beta^I(-\mathbf{q},0)\right\}\rangle e^{i\omega t}dt = \sigma_{\alpha\beta}(\mathbf{q},\omega)+\sigma_{\beta\alpha}^\dagger(\mathbf{q},\omega)$$

Where

$$\mathbf{J}^I(\mathbf{q},t) = e^{\frac{i}{\hbar}H_0 t}\mathbf{J}(\mathbf{q})e^{-\frac{i}{\hbar}H_0 t}$$

The conductivity is calculated in the HCB model framework

Lindner-Auerbach conductivity

In the framework of HCB, LA have calculated the high temperature 2d resistivity between T_c T^* and

$$\frac{d\rho^{2D}}{dT} = 0.245 \frac{R_Q}{\rho_s^{2D}(0)} \qquad R_Q = \frac{h}{(qe)^2}$$

When q is the boson charge, equals exactly 2 in the original model.

We define: q The *Mean Effective Charge*

Lindner, Netanel H. and Auerbach, Assa, "*Conductivity of hard core bosons: A paradigm of a bad metal*", Physical Review B 81, 5 (2010), pp. 054512.

Lindner-Auerbach conductivity

Therefore, rearranging our main term

$$\frac{d\rho^{3D}}{dT} = 0.245\frac{R_Q}{\rho_s^{3D}(0)} \quad \rightarrow \quad q^2 = 77.378\frac{K_B}{\hbar c^2}\lambda^2(0) / \frac{d\rho^{3D}}{dT}$$

From transitivity and magnetic field exclusion measurements, a *mean effective charge* for the conductance carriers may be realized!

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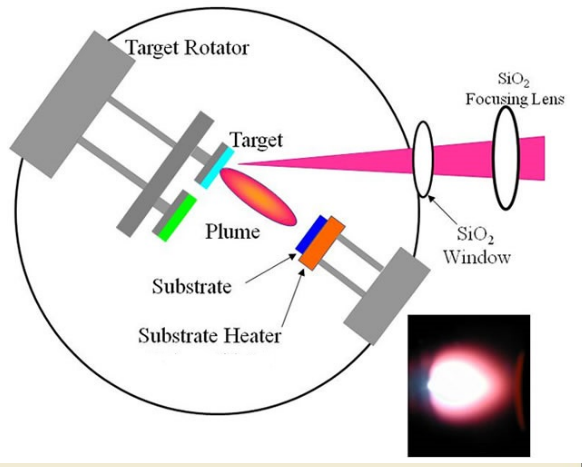
Experimental Method

- Sample preparation – PLD*
- Every geometrical shape was made using photo lithography*.
- Residuals riddance – Wet etching procedure*
- The probe, with the sample was

  placed in a Mu metal** covered chamber
  background minimization.
- 4 point probe

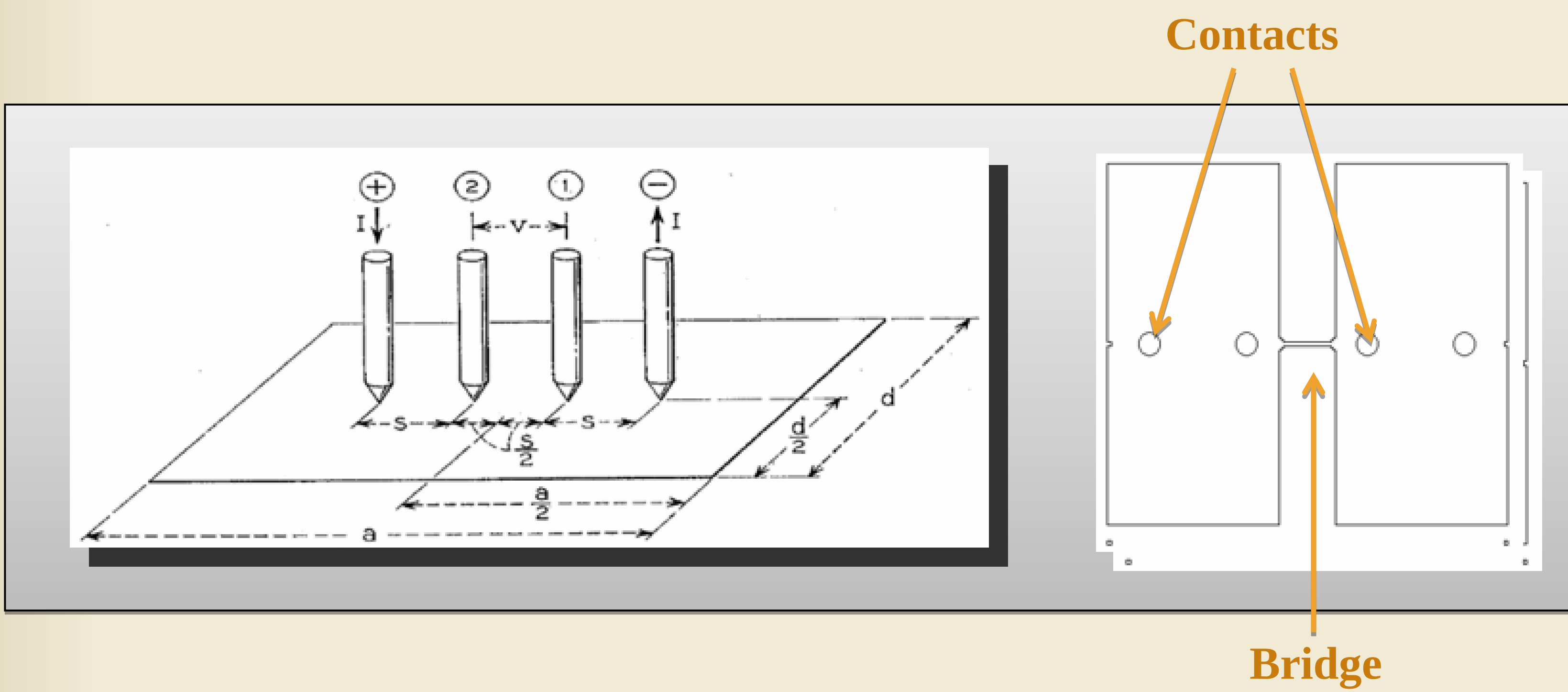


* Done in *prof*. Gad Koren's lab

**Mu metal - 77% nickel, 16% iron, 5% copper and 2% chromium or molybdenum

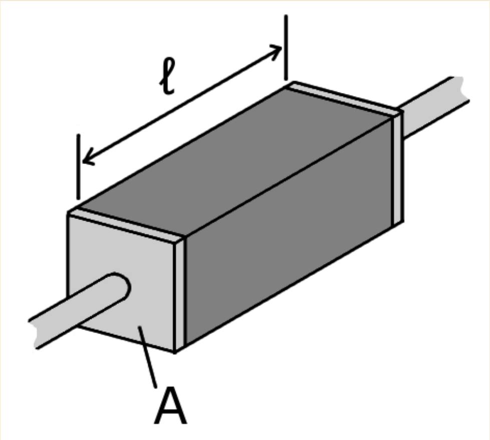
Four Point Probe Measurement Method

- Minimizes contact resistance bias



Bridge resistivity Measurements

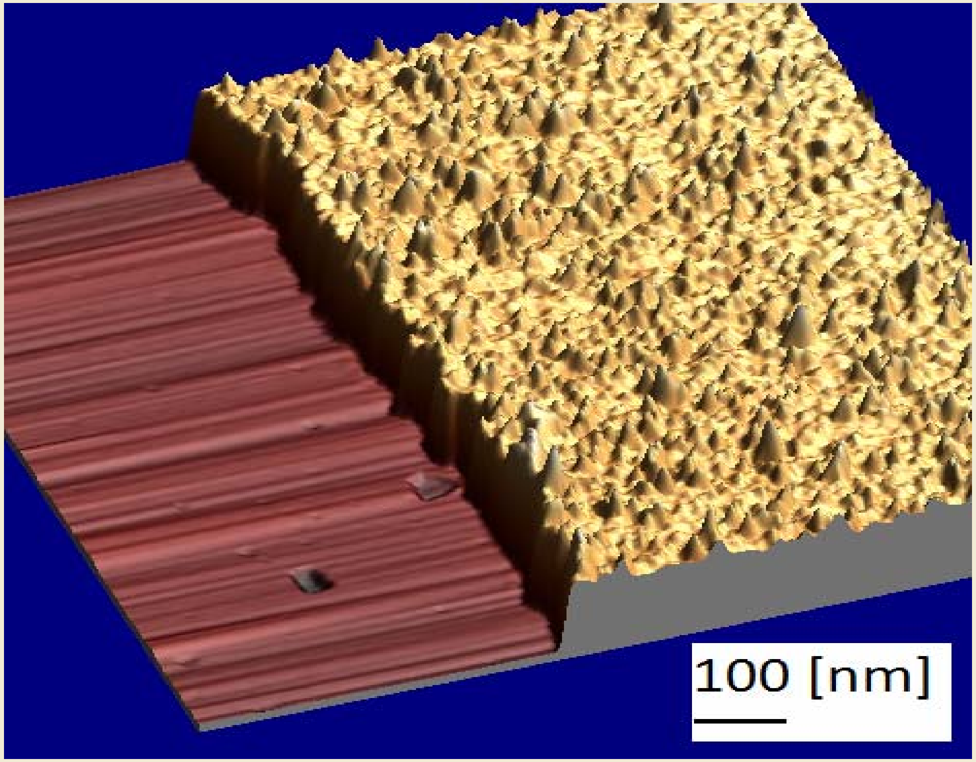
- In a bridge measurement, the resistivity is simple for extraction, $R = \rho \frac{l}{A}$



- 4 point probe technique is used
- The geometrical correction factor reduces to length to cross-section area ratio.

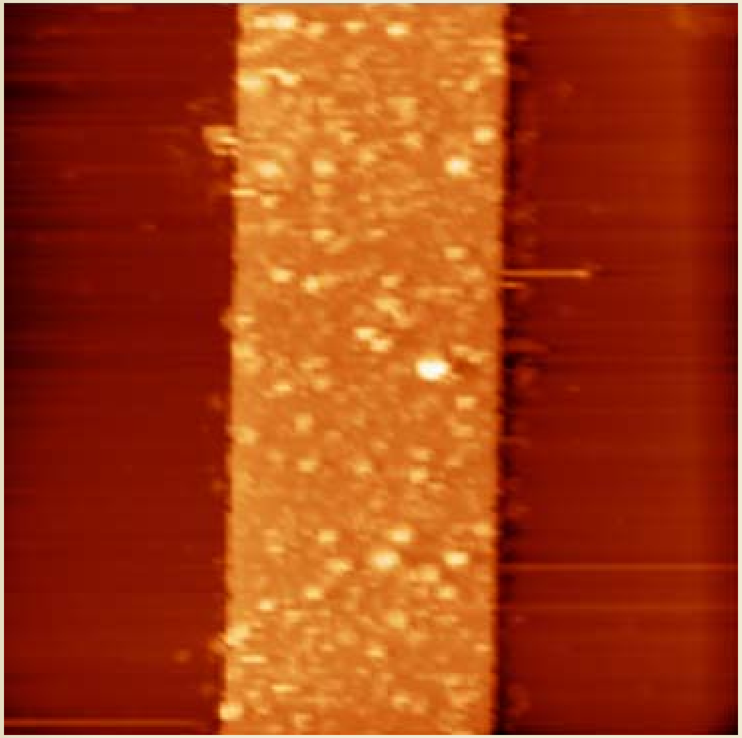
Film AFM Imaging

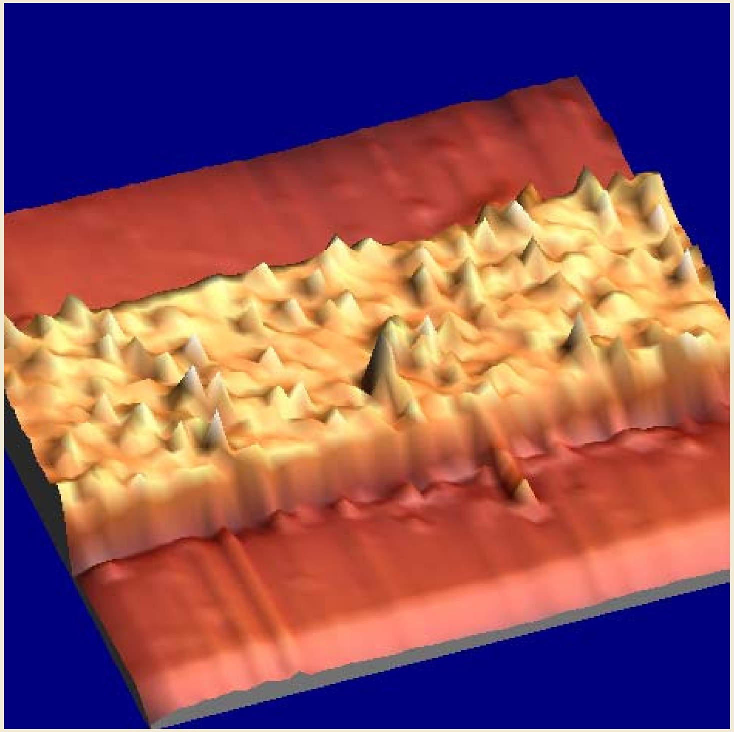
The sample's Height is measured via AFM in *Prof.* Gad Koren's laboratory.



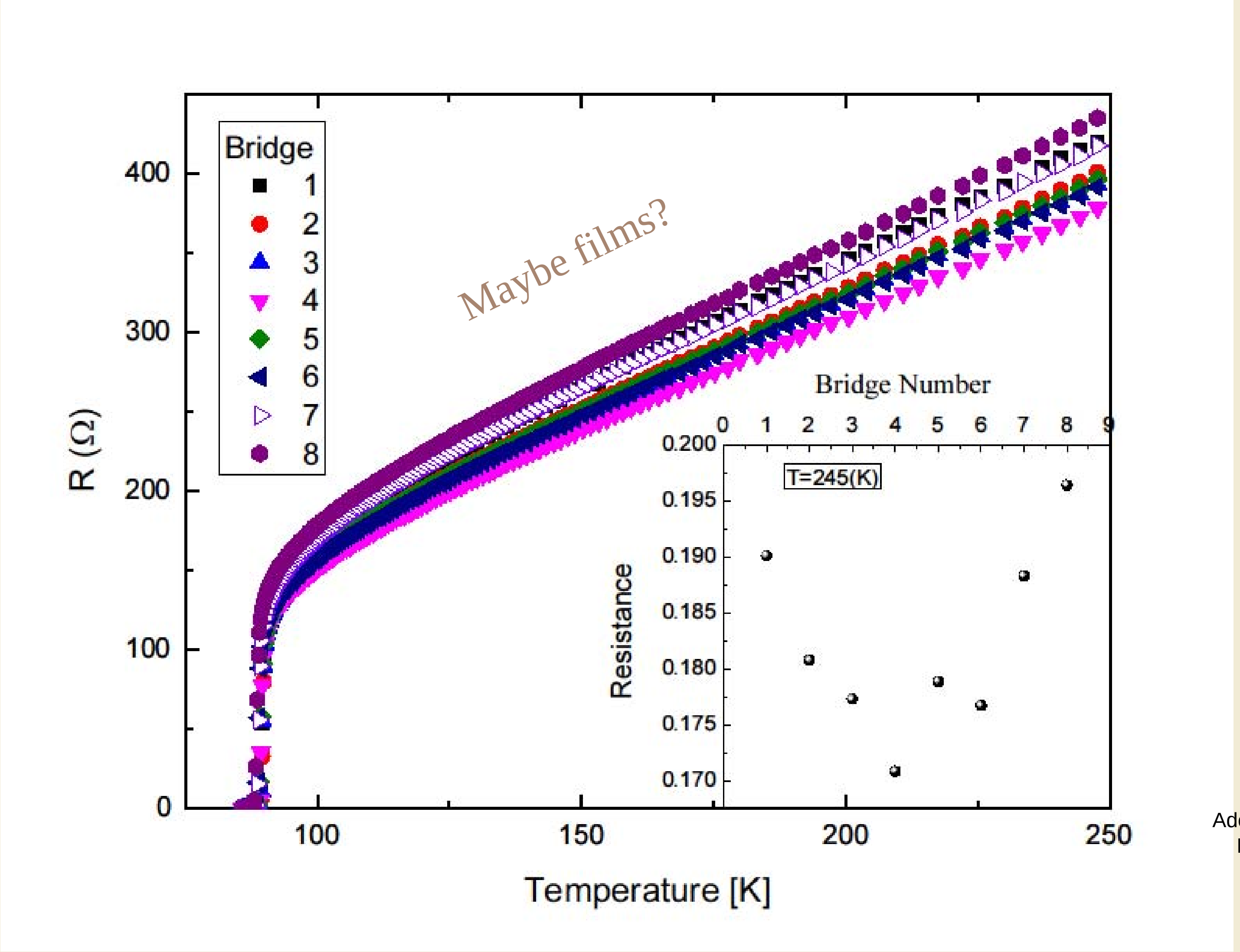
Height homogeneity throughout the sample

Bridge AFM Imaging





Bridges



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*The presented bridges measurement is courteously contributed by *Prof*. Gad Koren

Resistivity Measurement

Rectangular Films

Resistance for "Ohmic" materials is defined as the ratio between the voltage applied and the response current.

$$R = \frac{V}{I} = \underset{Geometry}{C} \cdot \underset{\text{" friction"}}{\rho} = \underset{Geometry}{C} \cdot \frac{1}{\underset{transitivity}{\sigma}}$$

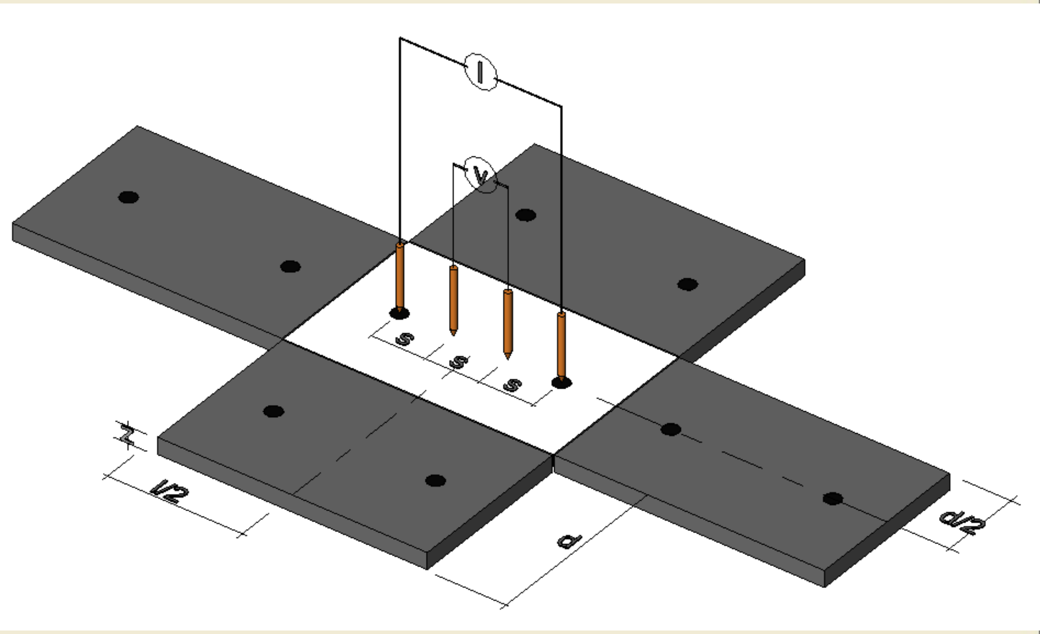
the geometrical factor is dependent on the sample geometry, but also on the measurement setup.

To avoid contact resistance, Four Point Probe technique is applied.(especially for relatively low resistance samples)

Four Point Probe sheet resistivity Measurement Method

To eliminate the geometric dependency of the resistance and measure the material dependent property – resistivity, <u>an infinite system of images </u>is considered as previously done by Smits

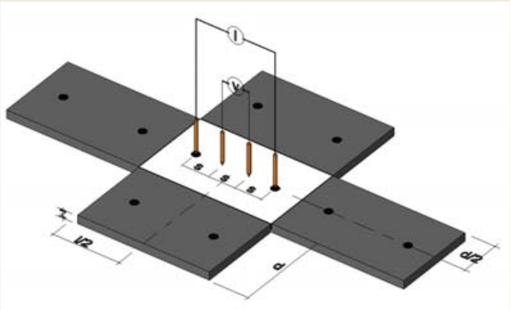
The perpendicular electric field and current cancel completely in t rectangular boundaries, thus the resistance measured has only physical boundaries contribution.



Four Point Probe sheet resistivity Measurement Method

To calculate $\Delta\varphi_{12}$, we consider first the contribution due to a single dipole current source on a 2D infinite plane

$$\varphi_2 - \varphi_1 = \frac{I\rho}{2\pi} ln\left(\frac{r_1}{r_2}\right)$$



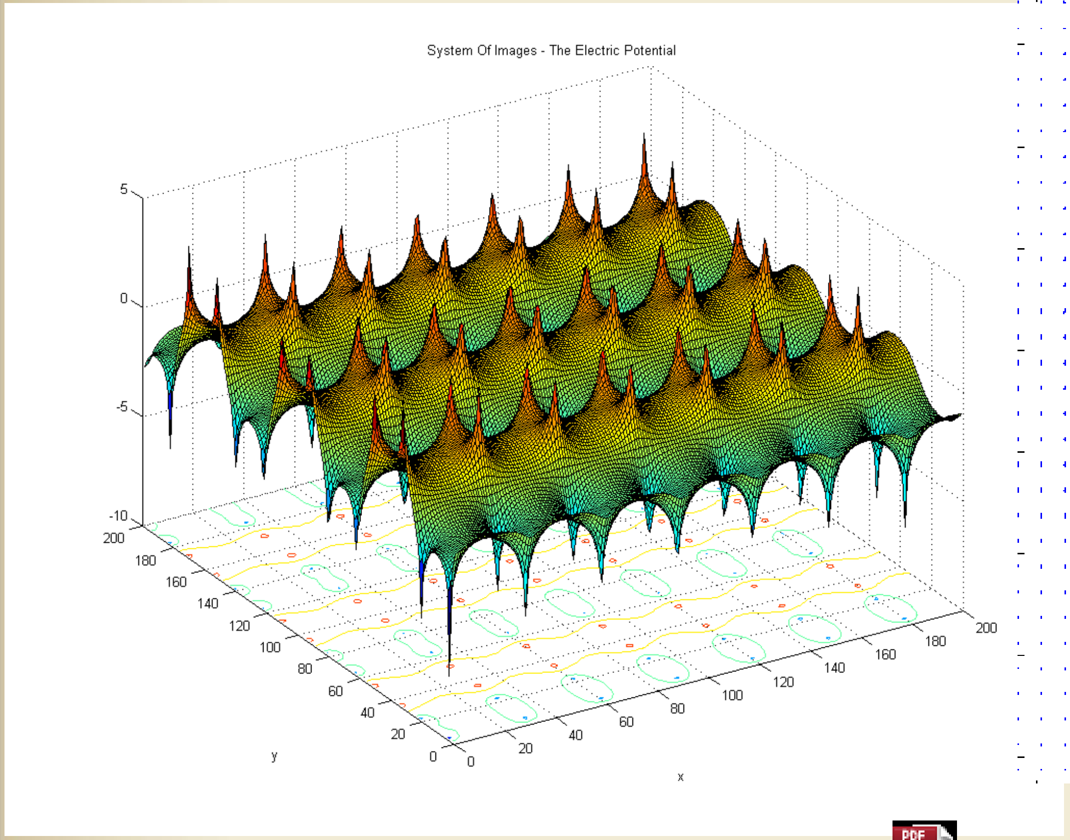
Sum, using the sources coordinates

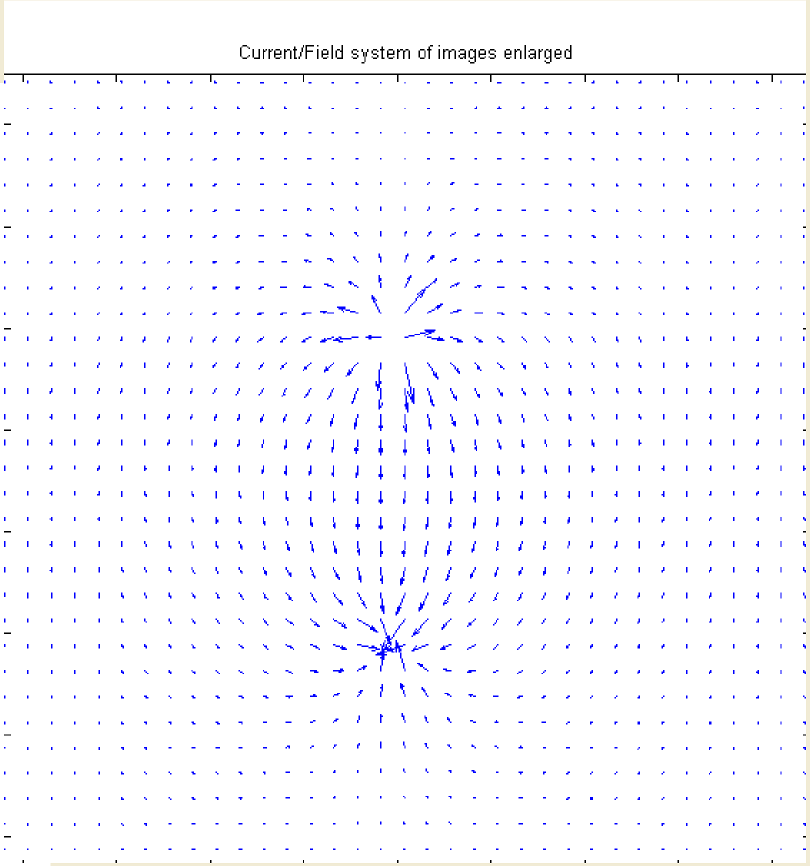
$$r^a_{nm} = (md, s + na) \qquad r^b_{nm} = (md, na - 2s)$$

All contributions to the potential difference $\Delta\varphi_{12}$ ends in

$$\Delta\varphi_{12} = \frac{I\rho}{2\pi} \sum_{n,m} (-1)^n \, ln\left(\frac{(md)^2 + (s + na)^2}{(md)^2 + (na - 2s)^2}\right)$$

Four Point Probe sheet resistivity Measurement Method





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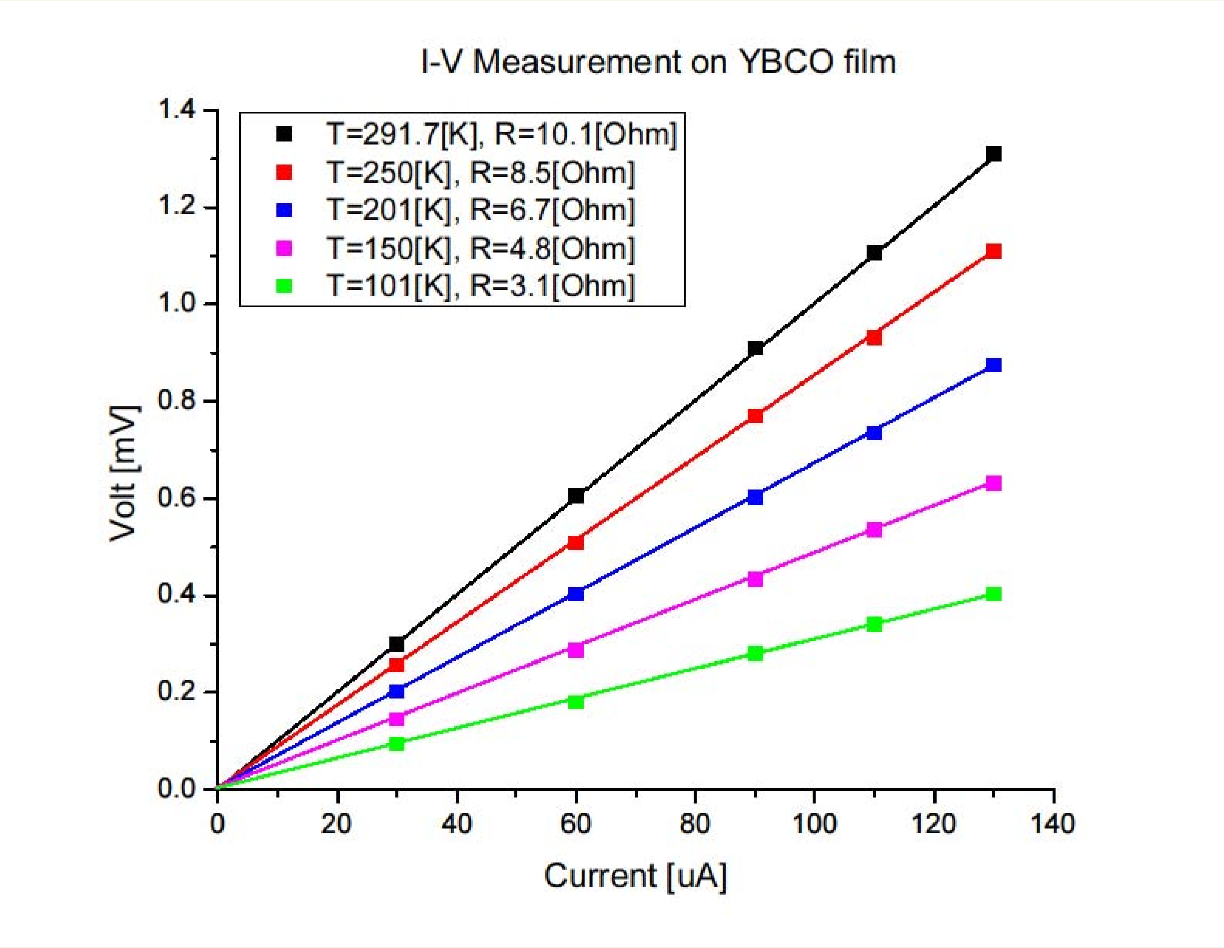
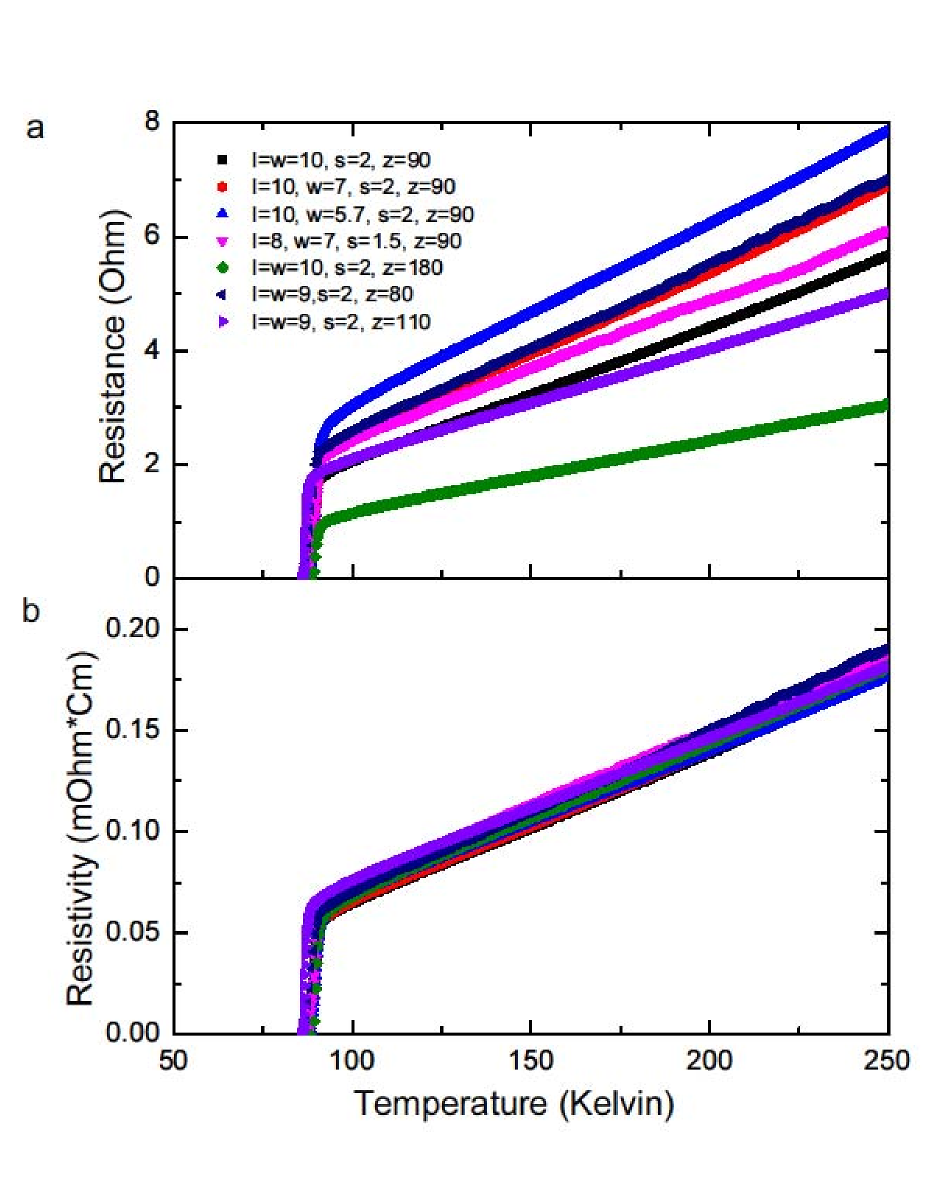
I-V Measurements

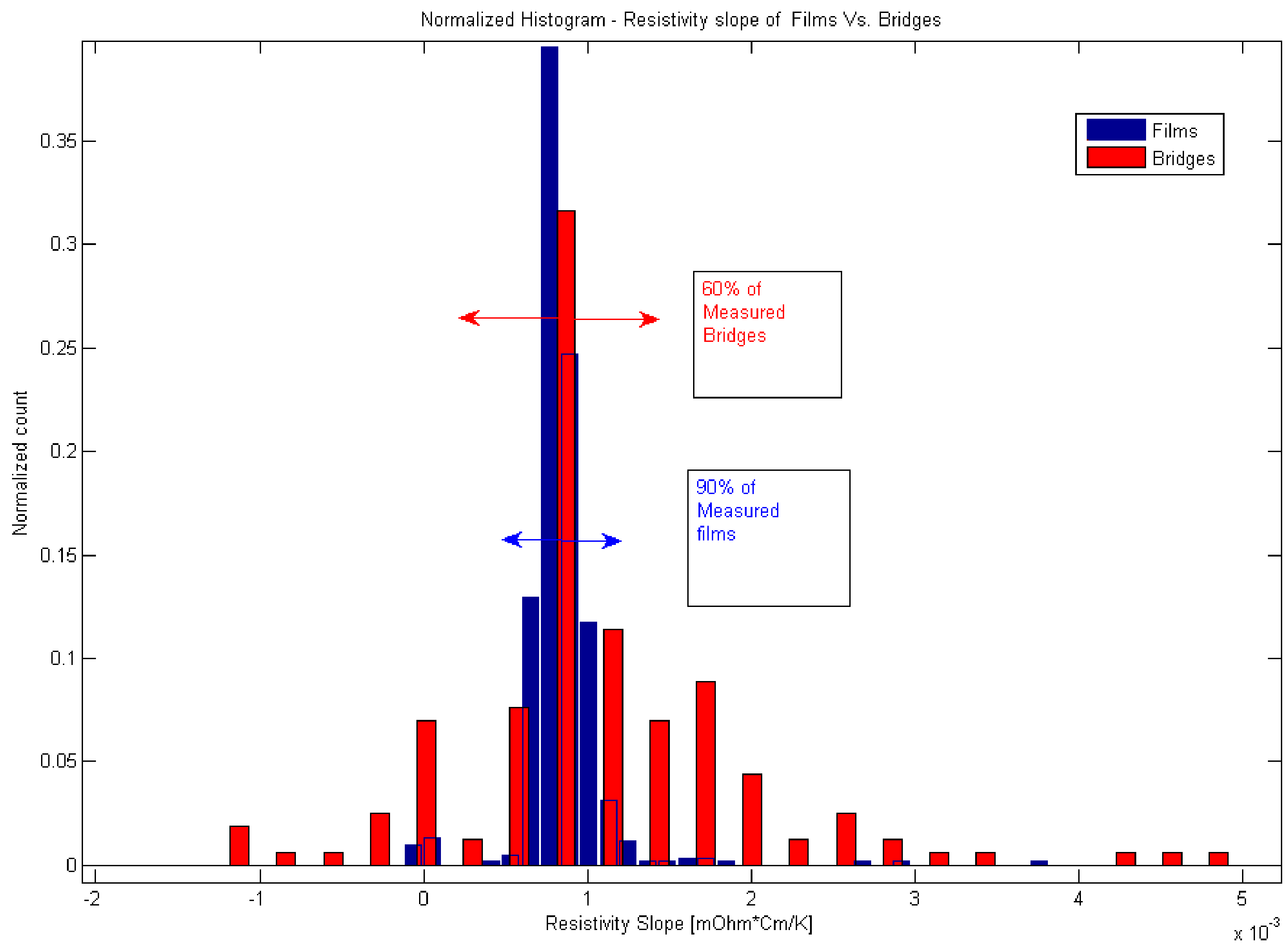
Normal state YBCO measurements



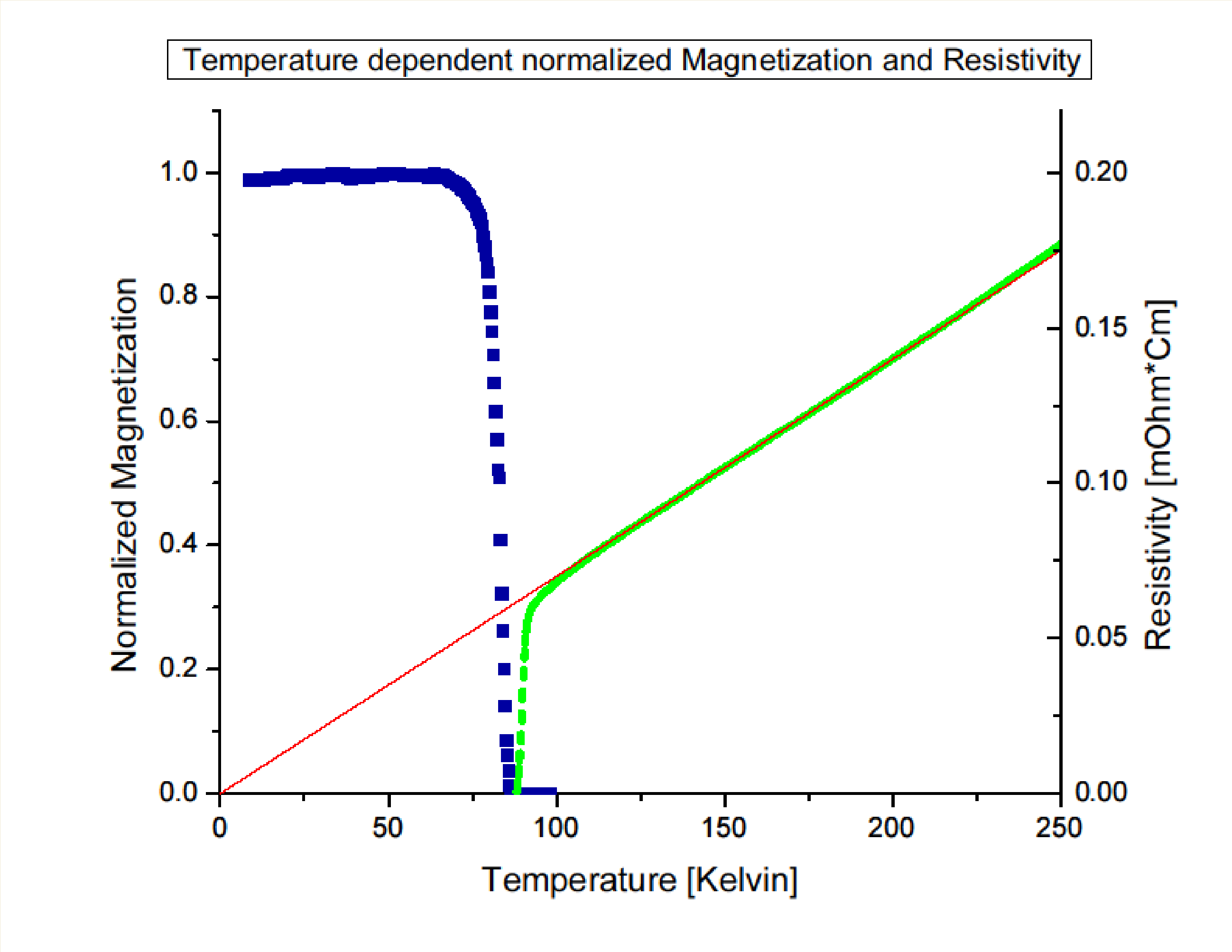
YBCO Films of various Geometries



Resistivity slope Bridges Vs. Films

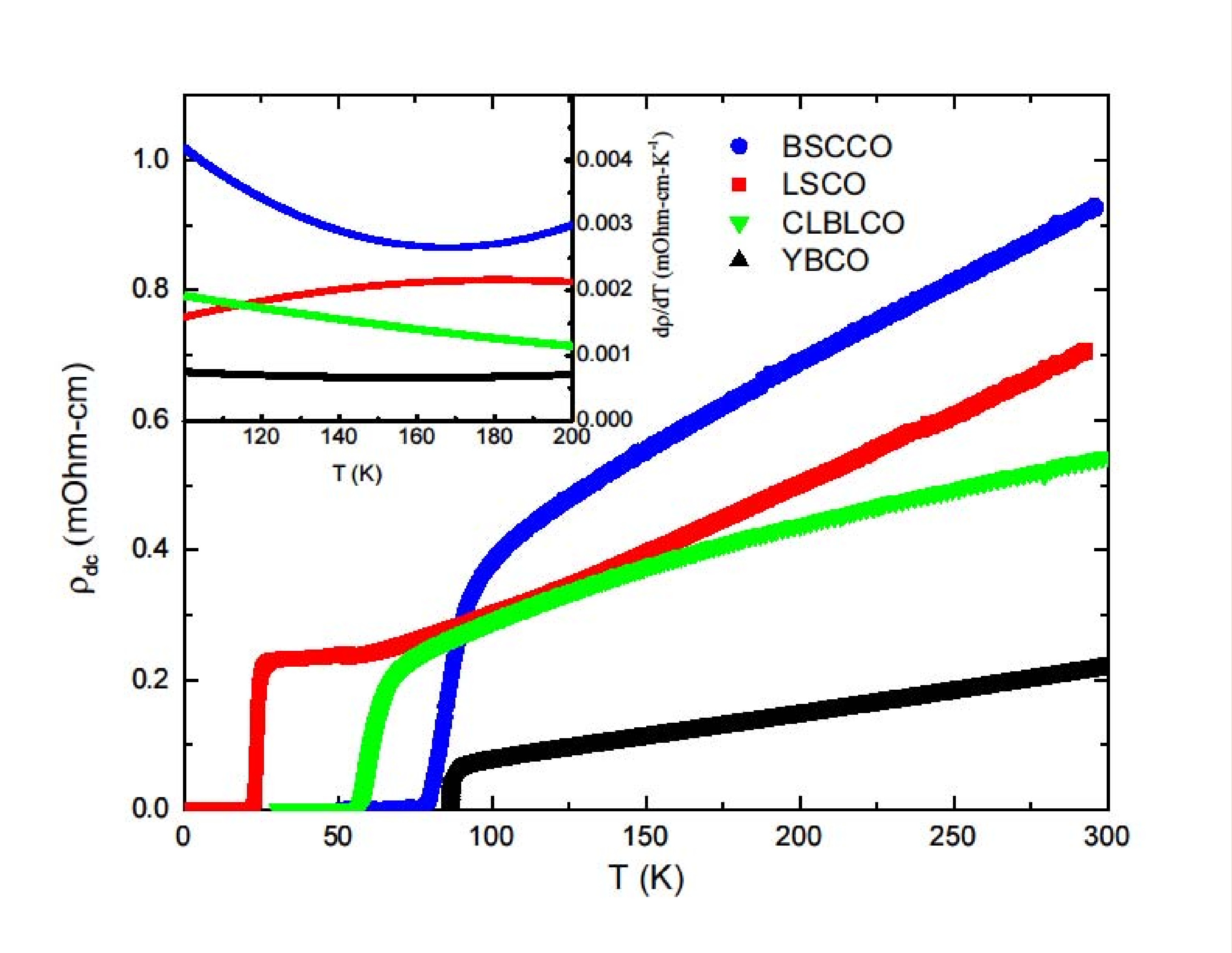


T-Dependent Resistivity Vs. Magnetization



Transition width ~4K and resistivity line extrapolated to 0.
Exclusive features in YBCO samples.

Resistivity Measurements



Penetration Depth sources

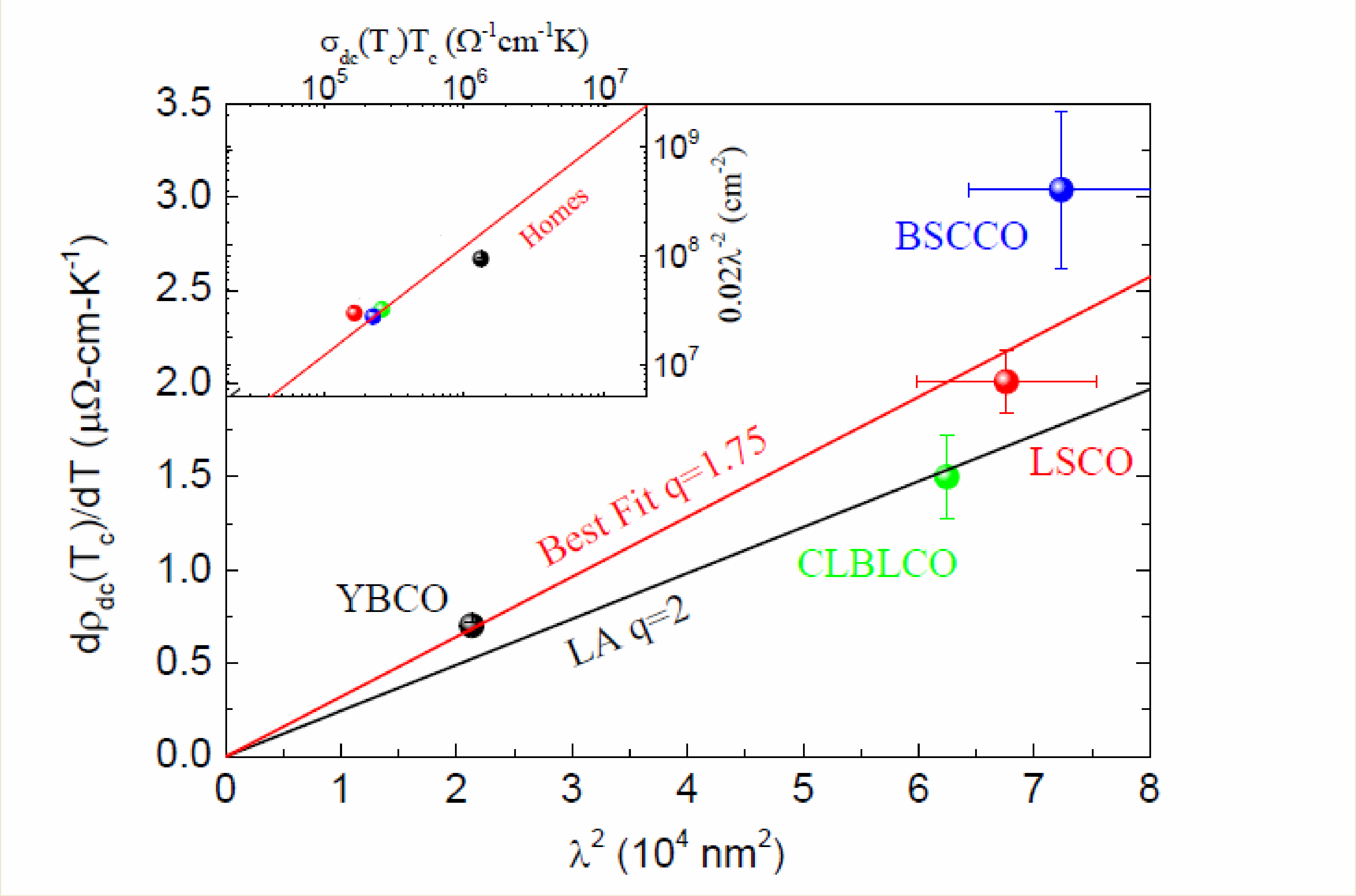
Remembering the main Eq. for *mean effective charge*

$$q^2 = 77.37\lambda_{ab}^2(T=0)\frac{K_B}{\hbar c^2}\frac{dT}{d\rho_{3D}^{\infty}}$$

The penetration depth is an input parameter taken from

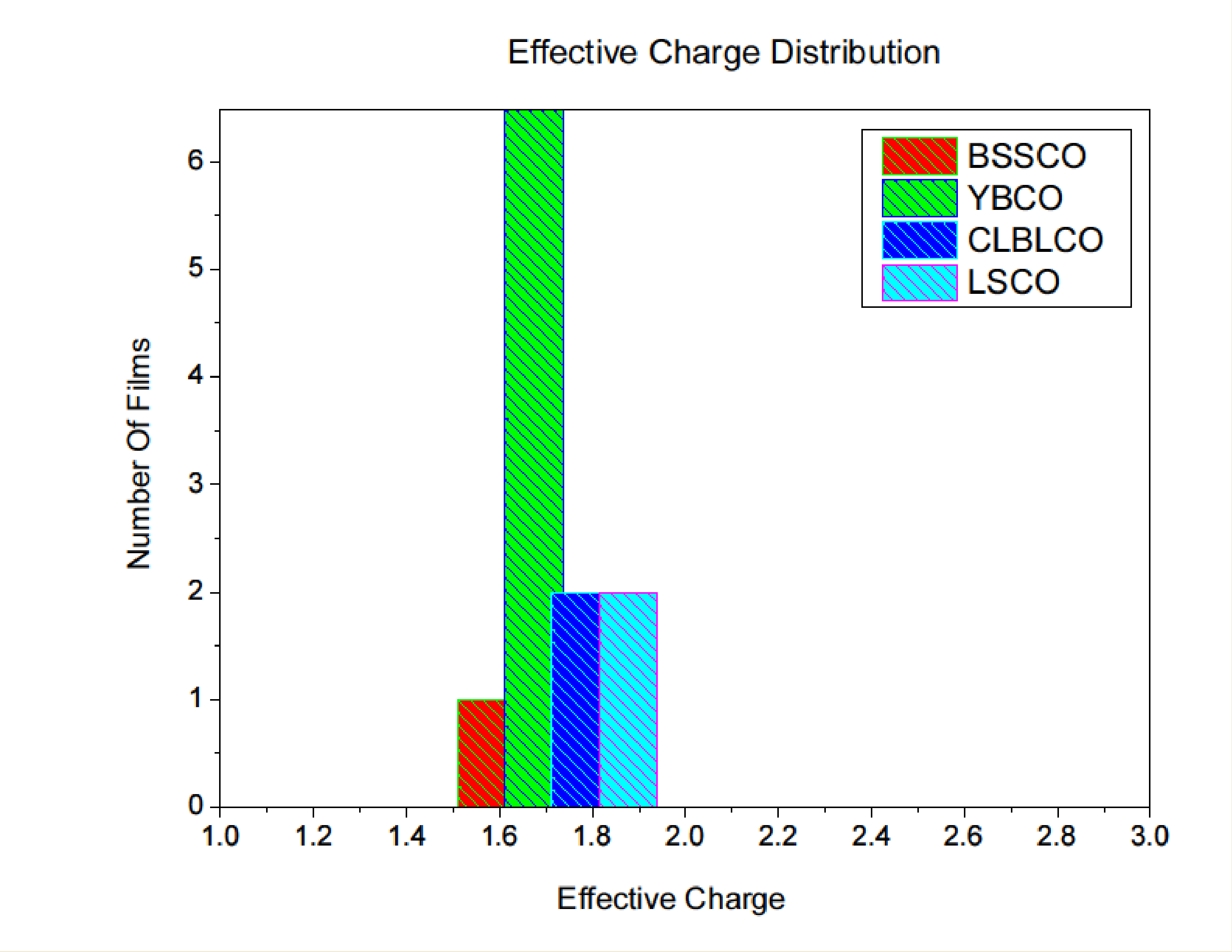
Material	$\lambda[nm]$	**Source**
YBCO	146	1. Journal of Applied Physics, 73(10):58655867, May 1993. 2. Physical Review B, 81(18):180502, May 2010
LSCO	250	Phys. Rev. B 47, 2854–2860 (1993)
BSCCO	270	Applied Physics Letters, 77(25):4202 4204, December 2000
CLBLCO	250	A.Keren, A.Kanigel ,*Solid State Communications* 126, 1â2 (2003), pp. 39--46.

LA Prediction close to the transition temperature



Mean Effective Charge

$$q^2 = 77.378 \frac{K_B}{\hbar c^2} \lambda^2(0) / \frac{d\rho^{3D}}{dT}$$



Summary & Conclusions

- The resistivity under T^* and above T_c is a linear function of the temperature for YBCO, curves for the other materials examined.
- The LA version of the Homes' law $\lambda^2(0) \propto \frac{d\rho_{dc}}{dT}(T_c)$ is also verified on **linear scale** and compared with Homes' results.
- If the model would have describe the materials perfectly, we should have found $q=2$, our research concludes $q=1.75(15)$

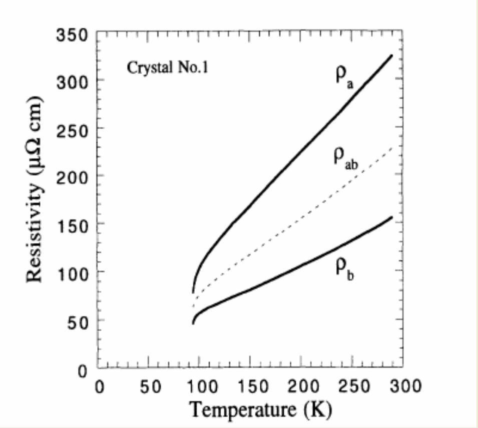
  The proximity to the HCB initial assumption points to self consistency.

Summary & Conclusions

- This conclusion supports the growing belief and other experimental data in the existence of preformed pairs (cooper pairs) at temperatures above T_c and under T^*

Possible modifications to the HCB model

- It is a disorder free model – What would disorder change?
- does not take into account possible fermionic excitations above T_c
- Anisotropy, which is common among the cuprates is not considered
- doping variations



Thanks

- Prof. *Amit Keren* for his guidance, support, patience and encouragement during my work in his group as well as teaching the essence of scientific work.
- *Gil Drachuk* for helping me with my first steps in the research as well as endless, fruitful and inspiring conversations throughout the experience.
- *Meni Shay* for his help with the measurement system and sample preparation, with high level of expertise, in a pleasant and patient way .
- Prof. *Gad Koren*, *Tal Kirzhner* and *Montaser Naamneh* for their guidance, help and usage in their laboratory facilities.
- I thank the lab technicians, Dr. *Leonid Iomin* and *Shmuel Hoida*, for their help.

Thank you