Boundary force effects exerted on solitons in highly nonlocal nonlinear media

Barak Alfassi, Carmel Rotschild, Ofer Manela, and Mordechai Segev
Physics Department and Solid State Institute, Technion, Haifa 32000, Israel

Demetrios N. Christodoulides
College of Optics/CREOL, University of Central Florida, Florida 32816-2700, USA

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We study the effects caused by remote boundaries on soliton dynamics in nonlinear media with a large range of nonlocality, and demonstrate theoretically and experimentally how asymmetric boundary forces can lead to soliton steering and oscillation in predetermined trajectories. © 2006 Optical Society of America
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Spatial solitons have been investigated extensively for decades.[1] An interesting aspect that has received little attention is the interaction of solitons with boundaries. Several theoretical[2–8] and experimental[9–12] studies addressed soliton reflection/transmission at an interface between a nonlinear and a linear material,[9,10] and between regions of different phase matching,[11,12] but thus far, all studies on soliton-boundary interactions have considered only local nonlinearities (Kerr,[9] quadratic,[11,12] and the photorefractive screening nonlinearity,[10] which is effectively local for solitons much wider than Debye length[13]). To our knowledge, soliton-boundary interactions have never been explored in nonlocal nonlinear media.

Here, we study interactions between solitons and far-away boundaries. We show that solitons in nonlinear media with a very large range of nonlocality can be controlled through interaction forces exerted by remote boundaries, leading to oscillations and steering in predetermined trajectories.

As a typical system for a long-range nonlocal nonlinearity, we use the thermal self-focusing nonlinearity of lead glass.[14] In this medium, a light beam is slightly absorbed and heats the glass, acting as a heat source. The heat diffuses with a thermal conductivity κ, leading to a nonuniform temperature distribution T induced by the intensity I, satisfying the heat equation in (temporal) steady state[14]:

$$\kappa\nabla^2 T(x,y,z) = -\alpha I(x,y,z), \qquad (1)$$

with α being the absorption coefficient. The change in temperature, ΔT, results in a proportional increase in refractive index $\Delta n=\beta\Delta T=\beta(T-T_0)$, with β being a (real) coefficient.[14] Here, T_0 is the temperature for $I=0$, which is imposed by the temperature at the boundaries of the finite sample [the boundary temperatures are the boundary conditions for Eq. (1)]. These boundary conditions directly affect T everywhere in the sample, thus affecting Δn induced by I. Denoting the optical field as $E=A(x,y,z)e^{i(\omega t-kz)}+c.c.$, A being the slowly varying amplitude, $k=\omega n_0/c$, ω the frequency, n_0 the unperturbed refracting index ($|\Delta n|\ll n_0$), c the vacuum light speed, and $I=|A|^2$, the lossless paraxial nonlinear wave equation is

$$(\partial_x^2+\partial_y^2)A + 2ik\partial_z A + 2k^2(\Delta n/n_0)A = 0. \qquad (2)$$

Equations (1) and (2) can be solved self-consistently, yielding solitons, subject to boundary conditions.[14] When the boundary conditions are symmetric $T=T_0$ on all boundaries, and A and its derivatives vanish at the boundaries, Eqs. (1) and (2) yield a soliton centered at the origin, propagating strictly along z, of the form $A(x,y,z)=U(x,y)e^{i\Gamma z}$.[14] Here, we are interested in the dynamics of solitons when they are subjected to forces exerted by asymmetric boundary conditions. The asymmetry can arise from unequal distances to the boundaries or from different boundary temperatures.

We first study the dynamics of a soliton launched off center with unequal distances to the boundaries [Fig. 1(a)]. Consider a sample with a square cross section of width $2d\times 2d$, and equal temperatures at all boundaries $T(x=\pm d,y,z)=T(x,y=\pm d,z)=T_0$. The

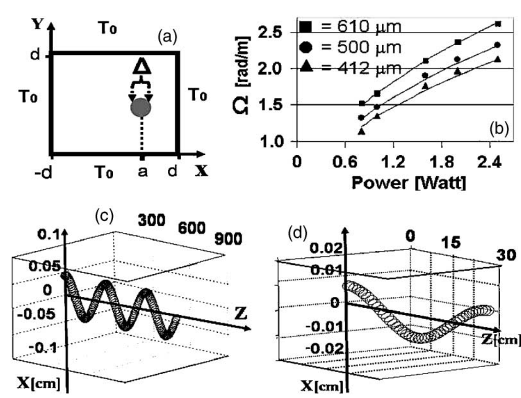

Fig. 1. (a) Sketch of a soliton of width Δ launched at an offset a in a square sample of cross section $2d\times 2d$. (b) Calculated longitudinal oscillation frequency of the soliton as a function of its optical power at various launch offsets; the curves are best fit to $\sqrt{P_{\text{tot}}}$. (c) Simulated oscillation under the experimental parameters of Fig. 2 (neglecting absorption) for a soliton launched off center. (d) Simulated oscillation, including absorption, for $P=10$ W and $2d=0.4$ mm.

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soliton is launched with its center at $(a, 0, 0)$. Denoting the width of the soliton as Δ, when $\Delta \ll (d-a)$, the initial wavefunction $U^a(x-a, y, 0)$ of the off-center soliton is practically identical to that of a soliton launched at the origin, $U(x,y)$. We simulate the (2+1)-dimensional [(2+1)-D] propagation by solving Eqs. (1) and (2) together (as in Ref. 14.) under the experimental parameters $\lambda = 488$ nm, $\beta = 1.4 \times 10^{-5}\ \mathrm{K}^{-1}$, and $\kappa = 0.00637\ \mathrm{W\,K^{-1}\,cm^{-1}}$. The results are displayed in Fig. 1(c). The soliton launched off center exhibits oscillations about the origin, while maintaining its shape to within a good approximation. The propagation dynamics is fully periodic. Under our experimental parameters, with $2d = 2$ mm and laser power $P = 1.2$ W, the soliton width is 50 μm FWHM, and the oscillation period is ~ 3 m, which is too long to be observed in our experiments owing to absorption. To assess whether it is possible to observe soliton oscillations in our particular material, we repeat the simulation with the experimentally measured absorption ($\alpha = 0.01\ \mathrm{cm}^{-1}$) included in Eq. (2) but with $2d = 0.4$ mm and $P = 10$ W [Fig. 1(d)]. With this higher power and thinner sample, the oscillation period is 30 cm, which is experimentally accessible despite the absorption.

As shown in Fig. 1, the problem is inherently (2+1)-D. Nonetheless, because the oscillation is only in the direction of the launch offset (x direction in Fig. 1), it is possible to derive a qualitative (1+1)-D model. The model assumes that the initial soliton wavefunction is identical to that of the on-center soliton, i.e., $U^a(x-a, 0) \cong U(x)$, and that the z evolution is adiabatic. We recast the example of Fig. 1 as a (1+1)-D problem, with a soliton U^a of width Δ, launched at a small offset a, where $\Delta \ll (d-a)$, with $T(x = \pm d) = T_0$. The heat generated at the hottest point (beam center) diffuses toward the colder regions. However, one boundary is closer to the beam center (by offset a); hence the temperature gradient in the region $a < x < d$ is higher than that for $-d < x < a$. This introduces asymmetry in Δn, acting on the beam as a restoring force toward $x = 0$ (as the boundaries exert unequal repulsion forces on the beam). This scenario occurs for all z: the beam is always pushed toward $x = 0$, where the repulsion forces exerted by the boundaries cancel one another. When the beam is launched off center, it moves toward $x = 0$ but never settles there; instead, it oscillates about $x = 0$ as it propagates. We first evaluate the temperature distribution at plane $z = 0$. $I = |U^a(x)|^2$ acts as a (heat) source in Eq. (1); hence $\partial^2 T/\partial^2 x = -\alpha I/\kappa$, leading to a formal solution:

$$T(x) = -\frac{\alpha}{\kappa}\int_{-d}^{X}\left\{\int_{-d}^{x'} I(x'')\mathrm{d}x''\right\}\mathrm{d}x' + c_1 x + c_0$$
$$\equiv -\frac{\alpha}{\kappa}Q(x) + c_1 x + c_0, \qquad (3)$$

where c_1, c_0 are integration constants, $Q(x) \equiv \int_{-d}^{X} P(x')\mathrm{d}x'$, and $P(x) \equiv \int_{-d}^{X} I(x')\mathrm{d}x'$ is the power in the interval $\{-d, x\}$. For $x = -d$, $Q(-d) = 0$; thus $T(x) = -\alpha Q(x)/\kappa + (x+d)c_1 + T_0$. For $x = d$, $P_{\mathrm{tot}} \equiv P(d)$ is the total power. We now need to evaluate $Q(d) \equiv \int_{-d}^{d} P(x')\mathrm{d}x'$. Most of the soliton power (say, 98%) is contained in a narrow region Δ, i.e., $\int_{-a-\Delta}^{-a+\Delta} I(x)\mathrm{d}x = 0.98 P_{\mathrm{tot}}$; thus some algebra yields $Q(d) \cong (d-a)P_{\mathrm{tot}}$, leading to $T(d) \cong -\alpha[(d-a)P_{\mathrm{tot}}/\kappa] + (2d)c_1 + T_0$. Since $T(d) = T_0$, we get $C_1 \cong (a/2\kappa d)[(d-a)P_{\mathrm{tot}}]\alpha$. Thus

$$T(x) \cong -[\alpha Q(x)]/\kappa + \alpha[(d-a)P_{\mathrm{tot}}](x+d)/2\kappa d + T_0. \qquad (4)$$

From the Eikonal equation, the acceleration of a light ray is proportional to $\bar{\nabla} n = \beta(\partial T/\partial x)$. Here,

$$\partial T/\partial x = (\alpha/\kappa)[0.5P_{\mathrm{tot}} - P(x)] - \alpha P_{\mathrm{tot}} a/(2\kappa d). \qquad (5)$$

Because $I(x)$ is symmetric about the beam center ($x = a$), and $P(x)$ is the integral over $I(x)$, the first two terms are antisymmetric about $x = a$. These terms act to balance diffraction, not to affect the beam trajectory. The trajectory is affected only by the last term, giving an acceleration (of the beam center) that is proportional to the displacement of the beam center

$$\partial^2(a)/\partial^2 z = \bar{\nabla} n/n_0 = \beta(\partial T/\partial x)/n_0$$
$$= -\beta\alpha P_{\mathrm{tot}} a/(n_0\kappa 2d) \equiv -\Omega^2 a,$$

where $\Omega = \sqrt{\beta\alpha P_{\mathrm{tot}}/(n_0\kappa 2d)}$. Hence the beam displacement experiences harmonic oscillation about $x = 0$ with a period $\Lambda = 2\pi/\Omega$.

Both the (1+1)-D analytical model and the (2+1)-D simulation reveal that a soliton launched off center follows an oscillatory trajectory. It is instructive to compare the two. Figure 1(b) shows results obtained from the (2+1)-D simulations for several values of optical power and initial displacements (the solid curves represent best fit). Just as the analytical (1+1)-D model predicts, the simulations show that the oscillation frequency Ω depends on $\sqrt{P_{\mathrm{tot}}}$. However, Fig. 1(b) also indicates that Ω is weakly increasing with displacement a, unlike the (1+1)-D analytical model that yields Ω independent of a. This discrepancy is natural: heat-flow dynamics is different in 1-D and 2-D, and hence one cannot expect to have full agreement between the models. Nevertheless, the 1-D analytical model gives intuition on the expected dynamics of the system, predicting the spatial oscillation and its dependence on the optical power.

In the experiment we launch a 50 μm FWHM beam into a lead-glass sample of dimensions 2 mm $\times$ 2 mm $\times$ 170 mm. We set the boundary temperatures to room temperature, and first launch the beam at the center of the input plane of the sample. The beam forms a soliton at $P = 1.2$ W, exiting the sample at the center of the output plane. (At low power the beam broadens to 450 μm.) We then vary the launch point in the transverse direction (x), while keeping the initial trajectory parallel to the z axis, and monitor the soliton exiting the sample [Fig. 2(a)]. Figure 2(b) displays the results of a series of experiments with various input and output positions. As evident from Fig. 2(b), the larger the input offset, the greater

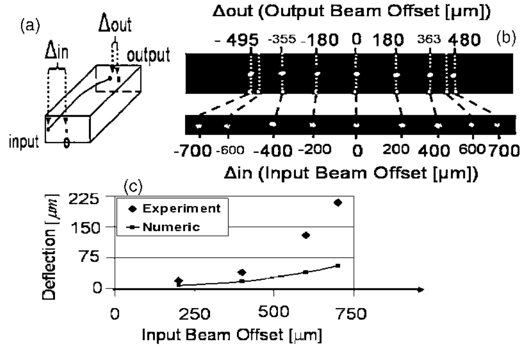

Fig. 2. Soliton steering arising from off-center launch. (a) Experimental scheme. (b) Photographs displaying the output displacement as a function of input offset. (c) Comparison between experimental and numerical results.

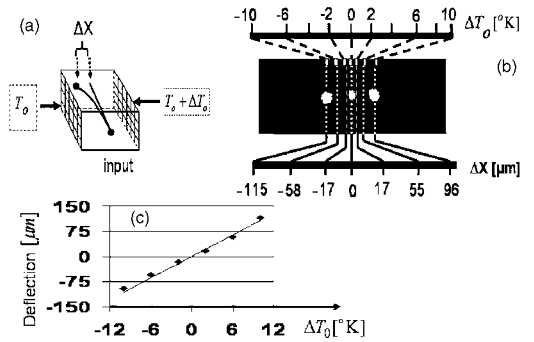

Fig. 3. Soliton steering arising from unequal boundary temperatures. (a) Experimental scheme. (b) Photographs displaying the output displacement as a function of boundary temperature difference. (c) Comparison between experimental and analytical results.

the output displacement of the soliton. The experimental results are compared with the simulations in Fig. 2(c). The comparison shows qualitative agreement of the dependence of the output displacement on the input offset, after finding the value of $\alpha\beta/\kappa$ from the measured optical power and soliton width. However, the observed displacement values are larger than those calculated by a factor of ~ 3. Such discrepancy has been observed in the past,[14] suggesting the existence of an additional mechanism in lead-glass, giving rise to an increased Δn. Under our experimental parameters, the expected oscillation period Λ is ~ 2 m. Our samples are 17 cm long; hence we can observe only the initial part of the oscillation. This limitation is not a fundamental: using higher optical power P, materials with higher β, and thinner samples reduces the oscillation period to experimentally accessible propagation distances [Fig. 1(d)].

Next we demonstrate how different boundary temperatures provide a means of controlled steering of solitons. The setup is sketched in Fig. 3(a), along with the experimental results [Fig. 3(b)] displaying the displacement of the output 50 μm soliton in a 83 mm long sample, as a function of temperature difference ΔT_0 between two boundaries separated by $2d=25$ mm. The net heat flow from the hot boundary to the cold one diverts the trajectory of the soliton. The propagation dynamics can be modeled analytically, with the acceleration of the beam center approximated by $\partial^2 a/\partial z^2 = \bar{\nabla} n/n_0 = \beta \bar{\nabla} T/n_0$. ΔT has an antisymmetry term balancing diffraction but not affecting the trajectory, and a constant term arising from ΔT_0. Solving Eq. (1) with $T(d,y,z)=T_0+\Delta T_0$, $T(-d,y,z)=T_0$, $\partial T(x,\pm d,z)/\partial y=0$ yields a constant temperature gradient across the sample, $\bar{\nabla} T = \Delta T_0/2d$, diverting the beam toward the hotter boundary. The displacement of the beam center after a distance z is $a(z)=\beta\Delta T_0 z^2/(2n_0 d)$, with a launch beam at $x=0$. Thus there is a linear relation between the displacement $a(z)$ and ΔT_0. Figure 3(c) compares this analytical result with our experiments, showing an excellent agreement with no fitting parameters.

To conclude, we studied interactions between solitons and far-away boundaries, and demonstrated how asymmetric boundary forces can control soliton dynamics by remote, leading to oscillations and steering. These ideas of controlling localized beams from afar can be utilized in various nonlocal nonlinearities (liquid crystals,[15,16] thermal,[17,18] etc.), offering a new tool for remote-control light manipulation and opening new possibilities for interactions among multiple solitons mediated by boundary force effects.

References

1. G. I. Stegeman and M. Segev, Science **286**, 1518 (1999).
2. A. D. Capobianco, C. De Angelis, A. Laureti Palma, and G. F. Nalesso, J. Opt. Soc. Am. B **14**, 1956 (1997).
3. I. V. Shadrivov and A. A. Zharov, J. Opt. Soc. Am. B **19**, 596 (2002).
4. A. B. Aceves, J. V. Moloney, and A. C. Newell, Phys. Rev. A **39**, 1809 (1989).
5. Y. S. Kivshar, A. M. Kosevich, and O. A. Chubykalo, Phys. Rev. A **41**, 1677 (1990).
6. J. Scheuer and M. Orenstein, J. Opt. Soc. Am. B **19**, 732 (2002).
7. C. B. Clausen and L. Torner, Opt. Lett. **24**, 7 (1999).
8. G. I. Stegeman, D. J. Hagan, and L. Torner, Opt. Quantum Electron. **28**, 1691 (1996).
9. P. Dumais, A. Villeneuve, A. Saher-Helm, J. S. Aitchison, L. Freidrich, R. A. Fuerst, and G. I. Stegeman, Opt. Lett. **25**, 1282 (2000).
10. E. Alvarado-Mendez, R. Rojas-Laguna, J. G. Avina-Cervantes, M. Torres-Cisneros, J. A. Andrade-Lucio, J. C. Pedraza-Ortega, E. A. Kuzin, J. J. Sanchez-Mondragon, and V. Vysloukh, Opt. Commun. **193**, 267 (2001).
11. F. Baronio, C. De Angelis, P. Pioger, V. Couderc, and A. Barthelemy, Opt. Lett. **29**, 986 (2004).
12. L. Jankovic, H. Kim, G. Stegeman, S. Carrasco, L. Torner, and M. Katz, Opt. Lett. **28**, 2103 (2003).
13. M. Segev, G. C. Valley, B. Crosignani, P. D. Porto, and A. Yariv, Phys. Rev. Lett. **73**, 3211 (1994).
14. C. Rotschild, O. Cohen, O. Manela, M. Segev, and T. Carmon, Phys. Rev. Lett. **95**, 213904 (2005).
15. M. Peccianti, C. Conti, G. Assanto, A. De Luca, and C. Umeton, Appl. Phys. Lett. **81**, 3335 (2002).
16. J. F. Henninot, M. Debailleul, and M. Warenghem, Mol. Cryst. Liq. Cryst. Sci. Technol., Sect. A **375**, 631 (2002).
17. A. Dreischuh, D. Neshev, D. E. Petersen, O. Bang, and W. Krolikowski, Phys. Rev. Lett. **96**, 043901 (2006).
18. M. Bertolotti, R. Li Voti, S. Marchetti, and C. Sibilia, Opt. Commun. **133**, 578 (1997).