## Negative radiation pressure and negative effective refractive index via dielectric birefringence

Jonathan Nemirovsky,<sup>1</sup> Mikael C. Rechtsman,<sup>1</sup> and Mordechai Segev<sup>1</sup>\* <sup>1</sup>Physics Department, Technion, Haifa 32000, Israel \*msegev@tx.technion.ac.il

Abstract: We show that light guided in a planar dielectric slab geometry incorporating a biaxial medium has lossless modes with group and phase velocities in opposite directions. Particles in a vacuum gap inserted into the structure experience negative radiation pressure: the particles are pulled by light rather than pushed by it. This effectively one-dimensional dielectric structure represents a new geometry for achieving negative radiation pressure in a wide range of frequencies with minimal loss. Moreover, this geometry provides a straightforward platform for experimentally resolving the Abrahams-Minkowski dilemma.

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Negative radiation pressure (NRP), the pulling of particles using light, was proposed by Veselago [1] in his pioneering paper on materials with negative refractive index. Veselago showed that when the dielectric permittivity and magnetic permeability of a material are simultaneously negative, its refractive index is negative. As a consequence, the directions of phase velocity and energy flow are opposite to each other, implying the striking and counterintuitive possibility of pulling particles via negative radiation pressure [2–5]. Unfortunately, though, the current realizations of bulk optical negative index media (or those exhibiting negative modes) [6–13] involve solids and metamaterials that would impede the motion of particles inserted into them. Furthermore, structures with negative modes that contain absorptive components, thermal heating and losses hamper conclusive observation of the NRP effect [14].

Perhaps the ideal system for demonstrating NRP would be a large volume where small particles could move freely and be "pulled back" by the light. In principle, this can be done using "backward modes" in waveguides [15-17], provided that there is empty space for particles to be inserted within the structure. Backward modes are those with group velocities that point in a direction opposite their phase velocities, similarly to the scenario in negativeindex bulk materials. Consequently, in such waveguides the electromagnetic energy flows in the direction of the group velocity, whereas the momentum imparted to free particles (given by the phase velocity) is in the opposite direction. Small particles [16] are therefore pulled (and can move freely) in a direction opposite to the energy flow. In principle, NRP is achievable in waveguides with small wavelength-sized holes [17]. Here, we propose a slabwaveguide structure exhibiting perhaps the four most important criteria to achieve the NRP effect: (1) the structure contains backward modes within a large range of frequencies; (2) to accommodate the particles, a large (possibly tens of wavelengths wide) air gap can be fabricated within the structure without eliminating the backward modes; (3) the structure can be fabricated from a wide range of material constituents; and (4) the structure contains only transparent dielectric materials, meaning that it exhibits very low losses. The components of the structure are birefringent materials (both uniaxial and biaxial). As we explain below, it is the rich nature of the wave dispersion in biaxial birefringents that gives rise to the negative modes; such modes cannot exist in isotropic materials [18]. In fact, negative modes have been proposed to exist in a slab waveguide whose core is made of uniaxial dielectrics with a metal serving as the cladding [19].

The waveguide geometry we propose comprises both *uniaxial* and *biaxial* dielectric materials. The waveguide has a hollow gap within it, with a large width (even tens of wavelengths in size) and it supports many backward modes in a wide range of frequencies. Unlike metamaterials, it contains no subwavelength structures, which simplifies its fabrication. Perhaps the most important feature is the fact that small particles can be inserted

in the gap, pulled backward and viewed (in experiments) directly from the side. This NRP effect is highly robust to fabrication imperfections and the gap may be made almost arbitrarily large (or small). As such, our proposed structure is experimentally accessible with today's technology; it is perhaps the best avenue to experiment with NRP on small particles.

Our proposed dielectric NRP structure has another aspect: it is an excellent experimental avenue to resolve the century-old Abrahams-Minkowski dilemma [20–22]. Recently, a theoretical resolution for this dilemma has been proposed [23,24], however, there are only a small number of experiments that have been done, with some supporting the Abrahams momentum [25] while other supporting the Minkowski momentum [23,24,26]. The translation-invariant waveguide geometry proposed here provides an ideal experimental setup for studying the momentum imparted by light. As we show below, we find that small particles will be pulled by light opposite to the direction of total energy flow - which is in the direction of the phase velocity. Moreover, in our proposed scheme the particles are placed in vacuum, so that momentum to (or from) surrounding material. Such an experiment, if successful, would prove that the Minkowski momentum is indeed the canonical momentum of the light [23, 24].

The essence of the mechanism by which biaxially birefringent materials can give rise to eigenmodes with NRP can be understood by considering the isofrequency surface of a homogeneous biaxial material, as depicted in Fig. 1(a) [27]. The eigenvalues of the dielectric tensor are  $\varepsilon_1 < \varepsilon_2 < \varepsilon_3$  corresponding to the principal axes of the crystal (marked by  $\vec{\varepsilon}_1$ ,  $\vec{\varepsilon}_2$ ,  $\vec{\varepsilon}_3$  in the figure) and the surface shown is the isofrequency surface at a given frequency,  $\omega$ . The same surface is shown in Fig. 1(b), rotated in a different orientation and cut by the  $\hat{\mathbf{k}}_y \cdot \hat{\mathbf{k}}_z$  plane. Finally, the cross-section of the isofrequency surface with the  $\hat{\mathbf{k}}_y \cdot \hat{\mathbf{k}}_z$  plane is shown in Fig. 1(b) and 1(c) that the  $\hat{\mathbf{k}}_z$ -component of the wavevector  $\mathbf{k}_1$  (as depicted in Fig. 1(c)) points in the positive  $\hat{\mathbf{k}}_z$ -direction, but the  $\hat{\mathbf{k}}_z$ -component of the associated Poynting vector (denoted by **S** in Fig. 1(b)) points in the negative  $\hat{\mathbf{k}}_z$ -direction. This suggests that a propagating mode that is a superposition of plane waves with wavevectors  $\mathbf{k}_1$  and  $\mathbf{k}_4$  would thus have no energy flow in the  $\hat{\mathbf{k}}_y$ -direction, but it will have a net Poynting vector opposite to that of the phase velocity (both along the z-axis).



Fig. 1. (a) Sketch of the three-dimensional isofrequency surface of a homogeneous biaxial medium, where the axes are the principal axes of the index ellipsoid, and  $\sqrt{\varepsilon_i} \omega / c = \sqrt{\varepsilon_i} 2\pi / \lambda_0$  mark the intersections of the ellipsoid with these axes. Here,  $\mathbf{k} = (k_x, k_y, k_z)$  designates the wavevector of a plane wave propagating in this medium. (b) Rotated version of (a), showing the electric field **E** and magnetic field **H** of the plane wave at a particular point on the isofrequency surface; note that the vectors  $\mathbf{k}$ ,  $\mathbf{E}$ ,  $\mathbf{D}$  (electric displacement), and S (Poynting vector) lie in the same plane. (c) Two-dimensional intersection of the isofrequency surface with the yz-plane. For a given  $k_z$  there are four possible plane waves in the yz-plane. Two of them ( $\mathbf{k}_1$  and  $\mathbf{k}_4$ ) have group velocities with negative z-components.

In the previous paragraph, we described the possibility of achieving negative modes in the context of the dispersion relation of a bulk biaxially birefringent material. However, as we now explain, it is essential to use a waveguide geometry, in combination with Bragg mirrors as cladding, in order to confine the electromagnetic energy and achieve true NRP inside a gap (a hole) within the medium, such that the light could pull particles. In Fig. 2(a) we schematically depict what would happen to light if we simply drill a hole in a biaxial medium, and hope to achieve a negative mode, and thus NRP. For simplicity, for the sake of illustrating the physical mechanism behind negative modes, we use the ray-optics picture, and ignore all reflected beams and positively-refracting beams within the biaxial material. As shown in Fig. 2(a), when light is incident on the biaxial medium at an angle appropriate for exciting a negative mode, it follows the following path: it negatively-refracts at the first interface, then again negatively refracts at the boundary of the gap within the structure, and then follows a similar, parallel path out the other side of the biaxial medium. Therefore, the particles placed in the gap are only affected within the area of the illuminating beam. The light is not confined within this structure (even if a grating or prism is used to in-couple the light) because negative refraction may only occur when  $\hat{\mathbf{k}}_{z} \ll \hat{\mathbf{k}}_{y}$  (as made clear above), implying that total-internalreflection cannot confine the light. Thus, a simple biaxial material with a slab-hole is not suitable for achieving NRP inside a gap within the structure.



Fig. 2. (a) Cross-section of a block of biaxial medium with a slab-hole within it, for the insertion of small particles. As shown, and discussed in the text, NRP cannot be achieved in this geometry. (b) Cross-section of a waveguide in which negative modes and NRP can be realized. The ray diagram shows that while light flows in the positive z-direction (the direction of the group velocity), radiation pressure in the particle gap is in the negative z-direction (the direction of the phase velocity). (c) Full three-dimensional depiction of the waveguide geometry. The sphere and the corresponding arrow indicate the direction in which a small particle is forced within the gap.

Instead of gap in bulk biaxial dielectrics, we propose the slab-waveguide geometry sketched in Fig. 2(b), in which NRP can be achieved inside the hole in the structure. The waveguide consists of a biaxial slab (with a hole inserted inside for small particles), two uniaxial slabs, and Bragg mirrors on both sides of the waveguide, acting as cladding. The uniaxial layers are needed to match the boundary conditions to realize the negative modes. The width of these uniaxial layers is designed so that they act as waves plates that rotate the polarization mode of the waves as they propagate to and from the Bragg mirrors. To achieve a negative mode, the  $\mathbf{k}_1$  polarization (see Fig. 1(c)) must be reflected onto the  $\mathbf{k}_4$  polarization mode (see Fig. 2(b)). Vice versa, the  $\mathbf{k}_4$  mode must be reflected onto the  $\mathbf{k}_1$  mode (see Fig. 2(b)). The uniaxial layers rotate the  $\mathbf{k}_1$  ( $\mathbf{k}_4$ ) modes twice: first when the wave passes through the uniaxial coating towards the Bragg mirror, and second after the wave is reflected by the (Bragg) mirror as it goes back into the biaxial layer. If the width of the uniaxial layers is properly chosen, the total rotation is such that the  $\mathbf{k}_1$  mode is reflected (by the combined structure: uniaxial layer + Bragg mirror) onto the  $\mathbf{k}_4$  mode, and vice versa (the  $\mathbf{k}_4$  mode is reflected back onto the  $\mathbf{k}_1$  mode), as shown in Fig. 2(b). The light rays shown in Fig. 2(b) depict how the negative mode arises from the superposition of the  $\mathbf{k}_1$  and  $\mathbf{k}_4$  modes): while the total energy moves in the positive z-direction (to the right), the  $\hat{\mathbf{k}}_z$  vector is pointing at the negative z-direction and therefore this is a negative mode. The particles are therefore "pulled" by radiation pressure over the entire length of the waveguide, experiencing NRP. Pulling the particles in this way is consistent with the Minkowski momentum (as opposed to the Abrahams), wherein the phase velocity gives the direction of the force on particles (an exact expression for this force as a function of the wavevector is given in [17]). Figure 2(c) depicts the full three-dimensional nature of the slab-waveguide. Note that the principal axes of the biaxial crystal are rotated around their  $\vec{\mathbf{\epsilon}}_2$ -axis (and  $\vec{\mathbf{\epsilon}}_2$  is parallel to the z-axis).

We perform frequency-domain numerical calculations to confirm the presence of negative modes, using the MIT photonic bands package [28]. For each  $k_z$  we calculate the frequencies  $\omega_n(k_z)$  of the guided modes in units of  $2\pi c / \lambda_0$ . In the calculations, we use a geometry that corresponds to the one depicted in Fig. 2(b), where the particle gap has width of  $31.5 \lambda_0$ , the

biaxial birefringent (sodium nitrite) layers have a width of  $112.5 \lambda_0$ , the uniaxial birefringent (lithium niobate) layers are of width  $17.5 \lambda_0$ , and these are enclosed by a Bragg mirror cladding used to confine the modes. The orientations of the birefringent materials are as shown in Fig. 2. The tilt angle of the biaxial material axes is as shown in Fig. 1. The specific values of the dielectric tensors are given as follows: (a) Sodium nitrite  $-\varepsilon_{xx} = 2.038$ ,  $\varepsilon_{yy} = 2.494$ ,  $\varepsilon_{zz} = 1.991$ ,  $\varepsilon_{xy} = \varepsilon_{yx} = 0.399$  and  $\varepsilon_{xz} = \varepsilon_{zx} = \varepsilon_{yz} = 0$ , (b) Lithium niobate  $-\varepsilon_{xx} = \varepsilon_{yy} = 5.3$ ,  $\varepsilon_{zz} = 4.9$ , and  $\varepsilon_{xy} = \varepsilon_{yx} = \varepsilon_{xz} = \varepsilon_{zy} = 0$ , (c) the Bragg mirror in our calculations is made of 8 Silicon layers of  $0.5 \cdot \lambda_0$  ( $\varepsilon_{ij} = 12.12 \cdot \delta_{ij}$ ) separated by gaps of  $1.5 \cdot \lambda_0$  ( $\varepsilon_{ij} = 1 \cdot \delta_{ij}$ ). The results of the dispersion calculations are shown in Fig. 3. In them, we plot the frequency of the waveguide modes as a function of wavevector in the z-direction. The slope of these curves gives the group velocity in the z-direction, and therefore the fact that some of the curves have negative slope indicates that they are negative modes, where the group and phase velocities propagate in opposite directions. All frequencies shown are within the band gap of the Bragg-mirror-cladding, meaning that the modes are confined in the waveguide.



Fig. 3. Dispersion diagram for the slab-waveguide arrangement depicted in Fig. 2 and described in the text. In the frequency range shown, roughly half of the modes have negative slope, corresponding to negative group velocities: these are the negative modes that act to produce NRP. These dispersion curves are for the configuration with a "particle gap," as depicted in Fig. 2(b). The presence of the gap does not qualitatively effect the presence of the negative modes. Negative modes are indicated with arrows.

It is instructive to show the Poynting vector (i.e., the energy flux) of both a positive mode and a negative mode. In Fig. 4(a) and 4(b) respectively, we plot the Poynting vector of two of the modes in the center of Fig. 3, as a function of position across the waveguide structure. Here, one mode is at frequency 0.1589  $(2\pi c / \lambda_0)$ , a negative mode; and the other at frequency 0.1594  $(2\pi c / \lambda_0)$ , a positive mode; both have  $k_z = 0.008 (2\pi / \lambda_0)$ . Here,  $\lambda_0$  is the wavelength of the incident beam in vacuum. Note that the spatial frequency is very high because these are necessarily higher-order modes of the waveguide, as mentioned previously. In both Figs. 4(a) and 4(b), the positive direction is set to be the direction of total Poynting flux (and therefore the group velocity). Figure 4(a) depicts a negative mode in which the phase velocity is in the negative direction. In this case particles will be pulled in a direction

opposite to the direction in which the light is traveling through the waveguide. By contrast, Fig. 4(b) shows a positive mode, where the total flux and phase velocity point in the same direction, resulting in the particles being pushed by the light in the direction that it propagates. For a concrete example, consider a polarized laser beam at  $\lambda_0 = 532$ nm and of diameter 100  $\mu$ m, propagating in this structure. An optical power of 100mW will create a negative radiation pressure of ~-70mPa, which is more than enough for various particles' manipulation [29].



Fig. 4. (a) Time-averaged Poynting flux (in the z-direction) of a negative mode with an air gap in the center. (b) Same as (a) but for a positive mode (i.e. positive phase and group velocity). In the gap, particles experience a force opposite that of the direction of energy transport of the wave. In both plots the positive direction is defined as the direction of propagation of the group velocity.

It is important to make clear how the appropriate negative modes in the waveguides should be excited in the context of an experiment. To this end, a section of the Bragg mirror should be removed to allow for in-coupling of light (as depicted in Fig. 2(b)). Therefore, the z-component of the beam's input wavevector will correspond to the waveguide mode's propagation constant, and thus phase velocity. Following the logic of the ray diagram of Fig. 2(b), the energy flow (and group velocity) of the light will propagate in the opposite direction. Thus, we will have selected the negative mode with the appropriate frequency and propagation constant.

Before closing, it is instructive to compare the biaxial dielectric structure proposed here to the uniaxial dielectric metal cladding proposed by Podolskiy and Narimanov [19]. First, it is unclear if NRP would exist inside a hole made in the structure of [19], to allow small particles to be pulled by light. Second, the metallic cladding in [18] would induce strong loss, and at the same time would give rise to thermal heating. Third, that structure offers little flexibility in design tolerance. Altogether, the 2005 proposition in [18] was indeed pioneering and inspired our work, but it is most likely unrealistic for experimenting with NRP.

We believe this approach to "pulling" particles with light is experimentally practical and can be easily implemented using modern techniques of crystal growth and beam shaping. Indeed, biaxially birefringent materials have been grown with their principal axes oriented in directions highly conducive to the geometry described here [30]. Moreover, we have found our scheme for creating negative modes to be robust, so that a certain amount of experimental

uncertainty in the orientation (or width) of the biaxial material is permissible. The high-order waveguide modes that give rise to the NRP effect can be excited by interfering two plane waves to coincide with the spatial frequency of the desired mode. In conclusion, we have proposed a novel method to achieve negative radiation pressure on particles by taking advantage of the ornate dispersion properties of biaxially birefringent materials. It is a method that is both robust and highly realistic to implement experimentally.

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