$$-\frac{1}{l^2}\frac{\Delta n}{n_0}+\frac{\Delta n''}{n_0}=-c|u|^2 \qquad (9)$$

Where c is defined above, and l describes the nonlocality range. We use Eq. (9) that describes a nonlocal nonlinearity with an exponential response function, instead of Eq. (5). When coming to explore the effects of the finite range of nonlocal response, the interesting domain is when l is comparable to the width of the lobes of the beam. If the range of the nonlocality is much greater than the width of the main lobe, the nonlinearity is highly nonlocal, and the propagation dynamic of the beam is similar to that displayed in the previous sections. If the range of the nonlocality is smaller than the width of the narrowest lobes, then the nonlinearity is effectively local and the propagation dynamic of the beam is known from previous work [10]. We therefore study the propagation of beams under nonlinearity with a varying range of nonlocality, in the interesting regime where the nonlocality range is comparable to the width of the beam lobes. Under self-focusing, we find that as the range of the nonlocality is larger the acceleration of the beam is smaller. This can be understood by the fact that the larger range of the nonlocality allows the boundaries to exert stronger forces on the beam, resisting the original tendency of the beam to accelerate. Additionally, the relaxation term in Eq. (9) reduces the maximum nonlinear index change, which also reduces the forces exerted by the sample boundaries. Figures 2c and 2d demonstrate the propagation dynamics of a beam entering a sample with a nonlinear response with a finite range of nonlocality. The parameters of the beam (power, wavelength) are equal to those of Fig. 2a. The range of the nonlocality is $20\,\mu m$, which is smaller than the width of the main lobe yet greater than the width of the lobes far from the main lobe. The propagation dynamics is similar to that in an infinite-range nonlocal nonlinearity, except that the acceleration here is stronger due to lack of the forces exerted by the boundaries.

6. Conclusion

We have found self-accelerating self-trapped beams supported by the highly nonlocal thermal nonlinearity. We demonstrated the complex dynamic of an accelerating beam propagating under non-accelerating boundary conditions that exert forces on the beam. We showed that changing the nonlinearity strength allows control over the trajectories of the beam, to the extent that it gives rise even to trajectories that are completely different from those of the linear self-accelerating Airy beam. Moreover, we found a new surprising phenomenon created by the presence of high focusing nonlinearity: a very broad ("infinite") launch beam decomposes into two beams, one of which becoming a localized spatial soliton, whose trajectory oscillates around a straight line, while the other beam is shedding power and eventually dies out. This picturesque phenomenon brings about a variety of intriguing questions: can the interaction between nonlinear accelerating beams produce spatial solitons? What would be their trajectories? Can they be pre-designed? We leave these ideas for future research.

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