Disorder-Enhanced Transport in Photonic Quasicrystals
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Science **332**, 1541 (2011);
DOI: 10.1126/science.1202977

also can be applied to TEGO (figs. S11 and S12), which is already being manufactured in ton quantities (*13*). By use of this type of simple activation process already commercially demonstrated for ACs, scaled a-MEGO and a-TEGO production for advanced energy/power electrochemical electrical energy storage devices may be realized in a short period.

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Acknowledgments: We appreciate funding support from NSF under award DMR-0907324, the U.S. Department of Energy (DOE) under award DE-SC001951, and the Institute for Advanced Technology. The research by E.A. and D.S. has been carried out at the Center for Functional Nanomaterials, Brookhaven National Laboratory, which is supported by the DOE, Office of Basic Energy Sciences, under contract DE-AC02-98CH10886. We thank P. Stephens for XRD data collection, which is supported by DOE under contract DE-AC02-98CH10886, and P. Ercius for data collection on the TEAM instrument, supported by DOE contract DE-AC02-05CH11231. We thank M. Nilges for help with EPR. We thank J. Potts for providing graphite oxide samples. We appreciate use of equipment in K. Johnston's lab. R.M.W. and A.P. acknowledge the partial support of the GRC-NRI SWAN Center for the XPS data collection and analysis. R.S.R., Y.Z., M.D.S., and S.M. have filed a U.S. patent application (application no. PCT/US2011/036164) regarding work in this paper.

Supporting Online Material
www.sciencemag.org/cgi/content/full/science.1200770/DC1
SOM Text
Figs. S1 to S12
References
Movie S1

22 November 2010; accepted 2 May 2011
Published online 12 May 2011;
10.1126/science.1200770

Disorder-Enhanced Transport in Photonic Quasicrystals

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Quasicrystals are aperiodic structures with rotational symmetries forbidden to conventional periodic crystals; examples of quasicrystals can be found in aluminum alloys, polymers, and even ancient Islamic art. Here, we present direct experimental observation of disorder-enhanced wave transport in quasicrystals, which contrasts directly with the characteristic suppression of transport by disorder. Our experiments are carried out in photonic quasicrystals, where we find that increasing disorder leads to enhanced expansion of the beam propagating through the medium. By further increasing the disorder, we observe that the beam progresses through a regime of diffusive-like transport until it finally transitions to Anderson localization and the suppression of transport. We study this fundamental phenomenon and elucidate its origins by relating it to the basic properties of quasicrystalline media in the presence of disorder.

Anderson localization (*1*), a fundamental concept in solid-state physics, describes how introducing disorder can transform a conducting crystal into an insulator. This prediction and subsequent experiments have shown that, generally, disorder works to arrest transport in periodic systems containing disorder (*2*–*5*), as well as in fully random potentials (*6*–*10*). However, some systems still pose fundamental challenges to this concept—most notably, quasicrystals. Quasicrystals (QCs) (*11*, *12*) constitute an intermediate phase between fully periodic and fully disordered media: They do not have a unit cell and do not exhibit translation symmetry; nevertheless, they possess noncrystallographic rotational symmetry and long-range order and display Bragg diffraction. Although many of the properties of QCs are now well understood, some fundamental questions remain. Perhaps one of the most intriguing questions related to QCs has to do with transport. Opposite to crystals containing disorder, which exhibit Anderson localization, it has been suggested that disorder can enhance transport in QCs (*13*, *14*). Indirect experiments have indicated that in some regimes, increasing disorder could enhance transport (*14*).

The electronic structure of atomic QCs has been shown to have multifractal eigenstates (*15*, *16*), which may or may not be normalizable (thus, localized), depending on the critical exponent associated with the given state. The transport properties of QCs are directly related to the critical nature of their eigenstates, in particular, in the presence of disorder (*17*). QCs have been shown to exhibit counterintuitive transport properties, including extremely low conductivity that increases with both temperature (inverse Matheisen rule) and spatial disorder arising from structural defects (*14*). Both of these effects have been attributed (*16*, *18*) to hopping between critical states of different spatial extents near the Fermi energy (due to inelastic electron-phonon scattering for the former and elastic scattering from structural defects for the latter). This increase in transport with disorder is directly opposite to the characteristic behavior of crystals, wherein transport is reduced with increasing disorder.

Thus far, experiments on transport in atomic QCs were carried out by the study of macroscopic conductivity. However, conductivity experiments are problematic for addressing some basic questions on QCs. First, the mechanisms proposed to explain the unusual transport in QCs assume noninteracting electrons; however, conductivity measurements inevitably incorporate electron-electron interactions. Second, conductivity measurements do not allow direct observation of wave packets, which could be a key property in unraveling the mechanisms underlying transport. With the recent progress in photonic lattices (*19*), manifesting analogies between light propagating in a waveguide

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array and an electron in an atomic lattice, it is natural to expect experimental studies of transport in photonic QCs. Indeed, experiments have studied the QC band structure (*20*), the dynamics of phasons (*21*), and propagation though QCs without disorder (*22*, *23*). However, the fundamental issue of transport in photonic QCs in the presence of disorder has not previously been studied experimentally.

Here, we study photonic quasicrystals containing disorder and present the first direct experimental observation of disorder-enhanced transport in QCs by directly imaging wave packets propagating through the photonic QC containing disorder. We show that disorder considerably enhances the transport of wave packets associated with eigenstates in the proximity of a pseudogap (a sharp reduction in the density of states), the region in which the Fermi energy is found in electronic systems. Enhanced transport occurs because disorder acts to couple highly localized states near the pseudogap, and as a result, states become more extended. When disorder is further increased, we experimentally demonstrate finite-time, diffusive-like transport, as predicted for weakly disordered QCs (*24*). Upon increasing the disorder even further, Anderson localization prevails: the width of the wave packet shrinks, and its tails display exponential decay. Our photonic system is equivalent to a two-dimensional (2D) Penrose QC containing disorder, and the wave packet we image is analogous to the probability amplitude of an electron propagating in it; hence, our findings are relevant to conduction electrons in quasicrystalline electronic systems.

We work with photonic lattices, in the transverse localization scheme (*25*), described by the paraxial equation for monochromatic light

$$i\frac{\partial\Psi}{\partial z} = \hat{H}\Psi \triangleq \left[-\frac{1}{2k}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)-\frac{k}{n_0}\Delta n(x,y)\right]\Psi \quad (1)$$

Here z is the propagation coordinate, x and y are the transverse coordinates, $\hat{H}$ is the Hamiltonian (defined by Eq. 1), Ψ is the slowly varying envelope of an optical field $E(x,y,z,t) = \mathrm{Re}[\Psi(x,y,z)e^{i(kz-\omega t)}]$ (t is the time coordinate) of frequency ω and wave number $k = \omega n_0/c$, n_0 is the bulk refractive index, Δn is the local change in the refractive index (lattice plus disorder), and $i^2 = -1$. Equation 1 has the form of the Schrödinger equation: the equivalence emerges when $z \to t$ and $-\Delta n \to V$ (where V is the potential). Hence, the evolution of a light beam behaves like the wave packet of a quantum particle in a 2D potential, but with the coordinate z replacing time. Solving for the eigenstates $\Psi_\beta = A_\beta(x,y)\exp(-i\beta z)$ (A_β is the z-independent eigenmode), for which $\hat{H}\Psi_\beta = \beta\Psi_\beta$ (β is the energy), is the equivalent to solving for the eigenmode of a particle, where the propagation constant corresponds to the eigenenergy E. Our photonic QC containing disorder is a Penrose-tiled 2D refractive index structure on which we superimpose 2D random disorder. The refractive index structure $\Delta n(x,y)$ corresponds to an array of parallel waveguides, ordered as a Penrose tiling, with random transverse variations whose strength is controlled at will.

Figure 1 shows the experimental scheme. We use the induction technique (*26*) to transform an optical intensity pattern into a z-independent refractive index structure $\Delta n(x,y,z)$, which includes both the QC lattice and the disorder (*27*). We study transport by launching a weak probe beam and monitoring the beam exiting the sample (*28*). Meaningful results are obtained by repeating experiments multiple times with many realizations of the disorder (same parameters) and ensemble-averaging over the intensity patterns at the exit face.

We now describe the results on enhanced transport in disordered QCs. The simulation results, shown in Fig. 2A, display the ensemble-averaged width, W_{eff} (*3*, *29*), of the beam propagating through the QC, without disorder (lower curve) and with 20% disorder (upper curve). We numerically launch a narrow Gaussian beam at a center of local 10-fold symmetry in the QC structure (center of a "flower"; see two example points marked by arrows in Fig. 2B). Without disorder, transport through the pure QC displays a "bumpy ride," with irregular oscillations occurring because of the presence of states of very different spatial extent within small energy ranges. On the other hand, transport through the QC containing disorder is considerably enhanced (Fig. 2A, upper curve): Throughout propagation, the width of the beam propagating in the disordered QC is larger than the width of the beam propagating in the pure QC. Moving the launch point to any

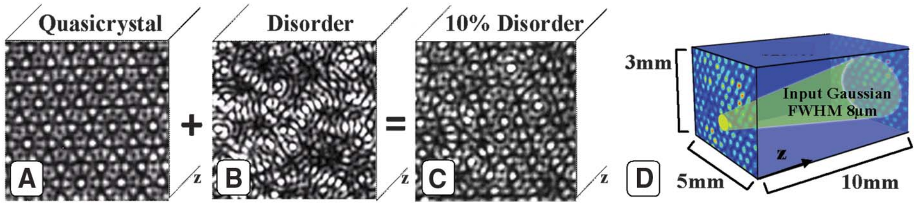

Fig. 1. Experimental scheme for transverse localization in photonic QCs containing disorder. A narrow optical beam is launched at the input face of a 2D quasicrystal lattice (**A**) containing disorder (**B** and **C**). The output intensity (**D**) pattern is monitored and ensemble-averaged over many realizations of disorder. FWHM, full width at half maximum.

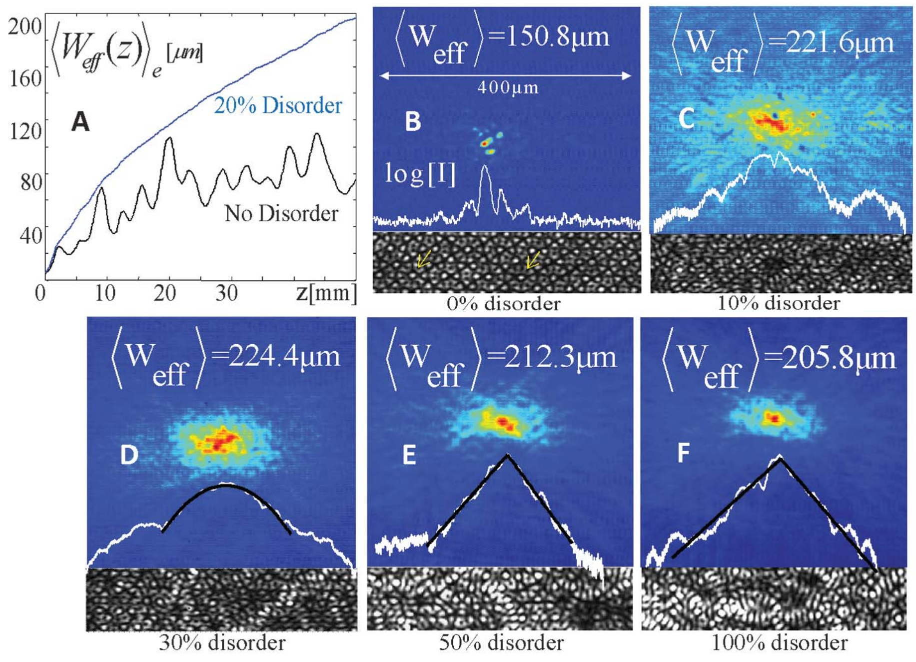

Fig. 2. Experimental and simulation results of transport through photonic quasicrystals, demonstrating disorder-enhanced transport and Anderson localization. (**A**) Simulation: beam width (ensemble-averaged) versus propagation distance for a pure QC (black) and a QC containing 20% disorder (blue). Transport is always higher in the disordered QC. (**B** to **F**) Experimental results: output intensity after z = 10 mm (ensemble-averaged), the log of its cross section (shown in white), and samples of the refractive-index profile (below each panel). Disorder-enhanced transport is apparent from the transition from 0% to 10% disorder [(B) and (C)]. For higher disorder levels, parabolic and linear fits [in (D) to (F)] indicate diffusive-like transport and the transition to Anderson localization, respectively. Yellow arrows in (B) indicate points of local 10-fold symmetry.

other center of local 10-fold symmetry yields virtually identical results. The experimental results are depicted in Fig. 2, B to F. Figure 2B shows the beam exiting the pure QC after 10 mm of propagation. The exiting beam, which has a mean width (W_{eff}) of ~150 μm, is always fractured and varies depending on the launch point, because the QC has no translational symmetry. Figure 2, C to F, depicts the ensemble-averaged output beam and its log-plot cross section, for increasing strength of disorder. The ensemble averaging is taken over 100 realizations of the disorder for Fig. 2, B to D, and over 50 realizations for Fig. 2, E and F, for each value of disorder strength. At 30% disorder (Fig. 2D), the wave packet is diffusive-like, as indicated by the parabolic cross section near the center of (the log of) the ensemble-averaged beam. A parabolic fit to these data in the central region, where the signal is strongest, gives a R^2 goodness-of-fit value of 96% (as shown in the figure). Notice that the width of the averaged beam in Fig. 2, C and D (221.6 and 224.4 μm, with standard deviations of 47 and 31 μm, respectively) is greater than the width of the output beam in the pure QC (150.8 mm in Fig. 2A), indicating enhanced transport. On the other hand, further increasing the disorder strength (50 and 100% disorder) makes the beam more localized (Fig. 2, E and F), while displaying the exponential tails characteristic of the transition to Anderson localization, with an average beam width of 212 and 206 μm (standard deviations of 28 and 35 μm), respectively. The linear fits to Fig. 2E give R^2 values of 96% (for both the left and right sides); the fits in Fig. 2F gives R^2 values of 86 and 98% (left and right sides). The distribution of W_{eff} in all these experiments shows no significant outliers on the high end, meaning that the decay properties of the wave functions in Fig. 2, B to F, are not skewed by a small number of measurements.

Our results on disorder-enhanced transport call for a direct comparison between crystals and quasicrystals. We therefore simulate transport in triangular and QC lattices of the same mean lattice spacing. Figure 3A shows the simulated W_{eff} exiting the hexagonal lattice after propagating in it (z = 30 mm), as function of disorder strength. Clearly, for the hexagonal lattice, transport decreases monotonically with increasing disorder strength. In sharp contrast, for the QC lattice (Fig. 3B) increasing disorder first enhances transport, and only after reaching a pronounced peak transport begins to decline with increasing disorder. To examine the expansion rate of the beam, we follow W_{eff} while propagating through the QC (Fig. 3C) for different levels of disorder and calculate the derivative of log(W_{eff}) with respect to log(z) to deduce its characteristic exponent. Figure 3D reveals that the exponent of the expanding beam, for a wide range of disorder levels, is close to 0.5, indicating a diffusive-like expansion. Increasing the disorder past the level causing maximal transport (between 5 and 10% in Fig. 3B) shows that the exponent converges toward 0.5. By fitting W_{eff} versus z, the diffusion constants for 10, 30, 50, and 100% disorder are found to be 0.038, 0.029, 0.029, and 0.027 μm, respectively, giving mean free paths of 8.1, 6.3, 6.3, and 5.7 μm [derived as in (*3*)]. As clearly shown in Fig. 3C, the wave-packet widths greatly exceed these mean free paths, indicating that we are in the multiple-scattering regime. This fact, together with the characteristic exponent of 0.5, strongly suggests diffusive-like transport. We examine the log of the ensemble-averaged and azimuthally averaged beam intensities (over 100 realizations) in Fig. 3, E to H, as a function of the transverse radial coordinate, r. In these plots, for 0, 10, 30, and 50% disorder, respectively, we fit parabolas (Gaussian wave packets; i.e., diffusion) or lines (exponential wave packets; i.e., a signature of localization) only where the fit is highly appropriate (R^2 > 97%). We find that for 10% disorder (Fig. 3F), some features of the original QC remain; thus, parabolic and linear fits are not appropriate. For 30% disorder (Fig. 3G), a Gaussian wave packet is observed at z = 5 mm, whereas the wave packet shows the exponential tails signifying the start of localization by z = 20 mm. At 50% disorder (Fig. 3H), the wave packet quickly reaches exponential decay. As explained below, we use a narrow beam selected specifically to excite pseudogap states to demonstrate disorder-enhanced transport [unlike in (*3*) where a broader beam was used]. Consequently, the beam is a superposition of many eigenmodes, some of which have high energy and extremely large localization lengths, larger than the simulation box. Therefore, we do not observe the wave function coming to an absolute halt. It is well known that in two dimensions, some localization lengths can be extremely large and out of reach of any simulation. That said, the beam in Fig. 3, G and H, exhibits exponential decay (for $z \geq$ 10 mm),

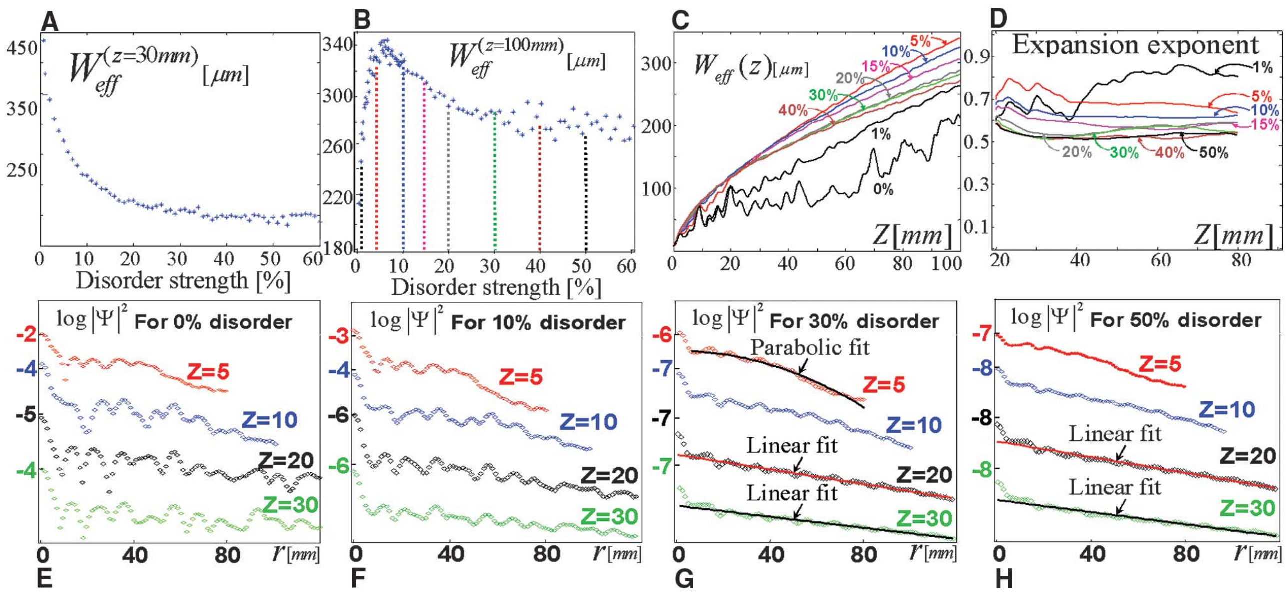

Fig. 3. Simulation results comparing transport through a hexagonal lattice and a QC for increasing disorder, showing disorder-enhanced and diffusive-like transport, as well as signatures of Anderson localization. (**A** and **B**) Ensemble-averaged beam width versus disorder strength for the hexagonal and QC lattices (same characteristic lattice spacing), showing disorder-enhanced transport for weak disorder (0 to 10%), then transport declines with further increase of disorder. (**C**) Beam width versus propagation distance in the QC for disorder levels indicated in (B). (**D**) The derivative with respect to log(z) of the log-log plot of (C); i.e., the characteristic expansion exponent. Convergence to 0.5 with increasing disorder indicates diffusive-like transport. (**E** to **H**) Logarithm of the ensemble- and azimuthally averaged wave function at 0, 10, 30, and 50% disorder, for various propagation distances. Parabolic fits indicate diffusive-like transport, and linear fits indicate the transition to Anderson localization for sufficiently large distances and disorder levels.

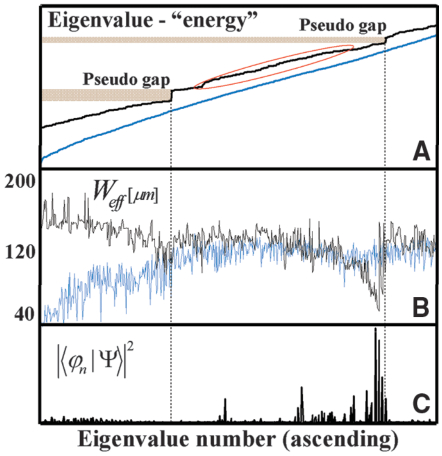

Fig. 4. Revealing the mechanism underlying disorder-enhanced transport in quasicrystals. (**A**) Band structure of a pure QC (black) and of a QC containing 20% disorder (blue). The fractured self-similar structure of the spectrum characteristic of a pure QC is smeared by the disorder, leading to a continuous density of states, which increases transport. The red oval denotes the fractal-like band. (**B**) W_{eff} of each eigenstate of a pure QC state (black) and in the QC containing 20% disorder (blue). Adding 20% disorder to a pure QC results in the broadening of all eigenfunctions associated with the vicinity of a large pseudogap (A). It is therefore expected that the expansion rate of a wave packet made up of such eigenfunctions will be higher in the disordered QC than in the pure QC. (**C**) Projection of a Gaussian wave packet (of Fig. 1A) on the eigenfunctions of the pure QC; φ_n represents the eigenfunctions. This wave packet, associated with the vicinity of a pseudogap, expands faster in a disordered QC, due to both the increase in the density of states (A) and to the broadening of the width, W_{eff}, of the eigenfunctions (B) in this region.

which is a clear signature that the wave packet is undergoing a transition to Anderson localization (*1*). The experimental and theoretical results displayed in Figs. 2 and 3 unequivocally show that disorder enhances transport in QCs; they demonstrate diffusive-like transport and show exponentially localized wave packets, a signature of the transition to Anderson localization.

These results raise the natural question: What is the underlying mechanism responsible for disorder-enhanced transport in QCs? Transport in atomic crystals is closely related to the density of states around the Fermi energy. In aperiodic systems, higher density of states is generally associated with broader eigenfunctions, which support higher transport. This is the case for potentials where the eigenmodes are localized (as in any potential containing disorder) or for critical states (as in QCs). To examine this point in our system, we solve Eq. 1 for a QC potential and find its eigenfunctions and eigenvalues β. Figure 4A shows a comparison between the band structure of a pure QC (black) and a QC containing 20% disorder (blue). The eigenvalues (energies) are presented in ascending order, because the notion of Brillouin zone does not exist for a QC. Nonetheless, there are regions (pseudogaps) in the band structure (gray in Fig. 4A) where the density of states is considerably lower. The band structure of the quasiperiodic potential is fractal-like; hence, higher-order pseudogaps exist on any scale (*12*). It is this fractured structure that is responsible for the low density of states, especially around pseudogaps, which in turn leads to highly localized states and thus to low conductivity/transport in QCs.

When disorder is introduced in a QC, the highly localized states near the pseudogap couple to one another, together forming eigenstates that are broader and less localized (at other energies, disorder acts to "smooth out" the fractal band structure, but the effect on transport is less pronounced). In other words, disorder mediates "hopping" between localized quasicrystalline eigenstates near the pseudogap (*30*). In turn, this coupling between localized states results in smoothing of the density of states, reducing the pseudogap until it altogether disappears (blue curve in Fig. 4A).

We calculate and plot W_{eff} of each of the eigenfunctions (Fig. 4B) for pure QC (black) and for the QC containing 20% disorder (blue). Going back to the plot of the eigenvalues (Fig. 4A), we find that the eigenstates near the two large pseudogaps tend to be more localized (Fig. 4B). At the same time, simple initial wave functions (e.g., Gaussian) are found to easily excite the localized eigenstates near the higher pseudogap, an experimentally indispensable condition. Figure 4B shows that adding 20% disorder to a pure QC results in the broadening of all eigenfunctions with energies in the vicinity of a large pseudogap. It is therefore expected that the expansion rate of a wave packet made up of such eigenfunctions will be higher in the disordered QC than in the pure QC. That is, the phenomenon of disorder-enhanced transport is expected to be most pronounced for wave packets associated with the pseudogaps in QCs. With this in mind, we analyze the wave packets launched in our simulations and experiments and examine their transport. Consider the Gaussian wave packet of Fig. 2A whose propagation displays enhanced transport in the disordered QC. Figure 4C displays the projection of this wave packet on the eigenfunctions of the pure QC. The underlying mechanism for disorder-enhanced transport in QCs is therefore due to the increase in the density of states near its pseudogaps. This explanation holds well for any launch point of a high 10-fold symmetry, which always excites mostly localized states from the vicinity of the pseudogap. Finally, we emphasize that the Fermi energy in atomic QCs resides in a pseudogap (similar to crystals where it resides in the gap), hence our initial wave packet represents conduction electrons residing in a $k_B T$-sized stripe (where k_B is the Boltzmann constant and T is temperature) around the Fermi energy.

This article was devoted to providing a direct experimental demonstration that transport in quasicrystals is enhanced by virtue of disorder, while displaying features associated with diffusion and localization. We studied this fundamental phenomenon and elucidated its origins, relating it to the basic properties of quasicrystalline media in the presence of disorder.

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29. For each realization of disorder, the confinement of the beam at the output plane is quantified by the inverse participation ratio $P \equiv \left[\int I(x,y,L)^2 dxdy\right] / \left[\int I(x,y,L)dxdy\right]^2$ (where I is the intensity and L is the propagation distance), having units of inverse-area and an average effective width $W_{eff} = \langle P \rangle^{-1/2}$; the averaging is over multiple realizations of the disorder (same statistics).
30. We emphasize that the disorder-enhanced transport in QCs observed here is fundamentally different from transport effects associated with the Urbach tail. See detailed discussion in (*27*).

Acknowledgments: This work was supported by an Advanced Grant from the European Research Council, the Israel Ministry of Science, the German-Israeli Foundation, and the Israel Science Foundation. M.R. gratefully acknowledges the generous support of an Azrieli Postdoctoral Fellowship. All authors thank S. Fishman, B. Shapiro, and Y. Krivolapov for useful discussions.

Supporting Online Material

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References

18 January 2011; accepted 28 April 2011
Published online 12 May 2011;
10.1126/science.1202977