Domain Walls in Super-QCD

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Avant-garde methods for QFT and gravity

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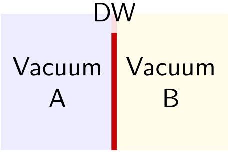
Progress in our understanding of 3D and 4D QFT dynamics:

- (Bosonization) dualites in 3D
- Anomalies of 0-form and higher-form symmetries
- Topological sectors
- ...

3D/4D connection:

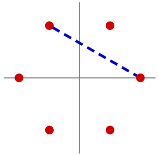


4D theory with multiple gapped vacua $\Rightarrow$ Domain Walls separating them with 3D worldvolume theory

- Relation between 4D and 3D dynamics, their anomalies, ...
- Changing "bulk" parameters $\rightarrow$ transitions in the worldvolume theory
  - ⋆ 3D transition with no bulk transition
- Different descriptions in the bulk $\longleftrightarrow$ Different phases or dualities in 3D
- Beautifully applied to YM and QCD [Gaiotto, Kapustin, Komargodski, Seiberg 17]

[Gaiotto, Komargodski, Seiberg 17; di Vecchia, Rossi, Veneziano, Yankielowicz 17]

We look at 4D $\mathcal{N} = 1$ (massive) SQCD:



- Multiple vacua for generic parameters
- BPS domain walls host a 3D $\mathcal{N} = 1$ worldvolume theory
- SUSY $\Rightarrow$ 2^{nd} order phase transitions on DWs
- A missing piece in the SQCD story

A lot has been done, but not complete classification

(*e.g.* Witten index not saturated)

[Acharya, Armoni, de Carlos, Dvali, Giveon, Hindmarsh, Hollowood, Israel, Kaplunovsky, Kovner, McNair, Moreno, Niarchos, Ritz, Shifman, Smilga, Sonnenshein, Vafa, Vainshtein, Vaselov, Witten, Yankielowicz, . . .]

4D $\mathcal{N} = 1$ (massive) Super-QCD

$$\boxed{SU(N) \text{ SQCD with } F \text{ flavors } Q, \widetilde{Q}}$$

- Restrict to $F < N$: only mesons (not baryons) are relevant

  $\rightarrow$ gauge-invariant meson superfields $M = \widetilde{Q}Q$

- Parameters: $\Lambda^{3N-F} = \mu^{3N-F}\, e^{-\frac{8\pi^2}{g(\mu)^2} + i\theta}$, mass matrix m_{4d} $(F \times F)$

Without mass m_{4d}: *classical* mesonic moduli space [Affleck, Dine, Seiberg 84]

Dynamically generated superpotential $W_{\text{ADS}} = (N - F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}$

$\rightarrow$ runaway behavior

Add flavor-invariant diagonal mass $m_{\text{4d}} \propto \mathbb{1}_F$

4D $\mathcal{N} = 1$ (massive) Super-QCD

$$\boxed{W_{\text{eff}} = m_{4d}\,\mathrm{Tr}\,M + (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}}$$

- Symmetries: $\mathbb{Z}_{2N} \times SU(F) \times U(1)_B$

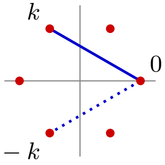
N gapped vacua from spontaneous R-symmetry breaking $\mathbb{Z}_{2N} \to \mathbb{Z}_2$

$$\langle M \rangle = \widetilde{M}\,\mathbb{1}_F \qquad \langle \lambda\lambda \rangle = m_{4d}\,\widetilde{M} \qquad \widetilde{M} = \left(\frac{\Lambda^{3N-F}}{m_{4d}^{N-F}}\right)^{1/N}$$

- Domain walls from j^{th} vacuum to $(j+k)^{\text{th}}$ vacuum

Broken R-symmetry: k-walls are all equivalent

- An $(N-k)$-wall is parity reversal of a k-wall



- 4D $\mathcal{N} = 1$ superalgebra admits a two-brane charge:

[Azcarraga, Gauntlett, Izquierdo, Townsend 89; Dvali, Shifman 96]

$$\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}\, P_\mu$$

$$\{Q_\alpha, Q_\beta\} = \sigma^{\mu\nu}_{\alpha\beta}\, Z_{\mu\nu} \qquad \text{and c.c.}$$

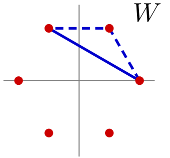
$\Rightarrow$ BPS domain walls with protected tension

- Write $Z_{\mu\nu} = Z\,\mathrm{vol}(\Sigma)\,\widehat{d\mathsf{vol}}_{\mu\nu}$ in terms of "central charge" Z

[Abraham, Townsend 91; Cecotti, Vafa 92]

$$T = |Z|\,, \qquad\qquad \boxed{Z = 2\,\Delta W}$$

Tension is controlled by the shift of superpotential



$\Rightarrow$ Parallel multiple walls decay to a bound state

3D phase transitions

- Large mass $m_{4d} \gg \Lambda$: $SU(N)$ Super-Yang-Mills

- Small mass $m_{4d} \ll \Lambda$:

  almost-weakly-coupled Wess-Zumino description on mesonic space

- In the two regimes we will find different walls:

  phase transition at $m_{4d} = m_*$

We seek a 3D $\mathcal{N} = 1$ worldvolume description

Domain walls in Super-Yang-Mills

$$\boxed{SU(N)\ \mathsf{SYM}}$$

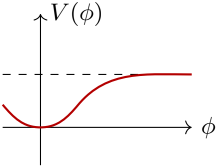
N gapped vacua with superpotential

$$W_{\mathsf{SYM}} = N(\Lambda^{3N})^{1/N} = N\,\Lambda^3\, e^{\frac{2\pi i}{N}k} \qquad k = 0, \ldots, N-1$$

Gaugino condensate $\quad \langle\lambda\lambda\rangle = \Lambda^3\, e^{\frac{2\pi i}{N}k} \quad$ breaks R-symmetry $\quad \mathbb{Z}_{2N} \to \mathbb{Z}_2$

- Acharya-Vafa proposal for 3D worldvolume theory on k-wall: [Acharya, Vafa 01]

$$\text{3D} \quad \mathcal{N}=1 \quad U(k)_N \quad \text{with (traceless) adjoint } \Phi \ + \ \text{singlet } \Phi_0$$



$$\Big\downarrow \quad \Phi \text{ has “negative” mass}$$

[Armoni, Hollowood 05; *ibid.* 06]

[Bashmakov, Gomis, Komargodski, Sharon 18]

$$\boxed{\begin{array}{c} \mathcal{N}=1 \quad U(k)_{N-\frac{k}{2},N} \\ \Big\downarrow \\ U(k)_{N-k,N} \end{array}}$$

- k-wall $\xleftrightarrow{\text{parity reversal}}$ $(N-k)$-wall from level-rank duality

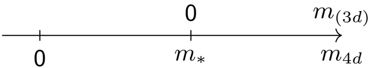
Domain walls in Super-QCD

$SU(N)$ massive SQCD with F flavors

Our proposal for 3D worldvolume theory:

$$\text{3D} \quad \mathcal{N}=1 \quad U(k)_{N-\frac{k}{2}-\frac{F}{2},N-\frac{F}{2}} \quad \text{with } F \text{ flavors of } X\text{'s}$$

$$\mathcal{W} = \mathrm{Tr}\, X^\dagger X X^\dagger X + \alpha \big(\mathrm{Tr}\, X^\dagger X\big)^2 + m\, \mathrm{Tr}\, X^\dagger X$$



(From adding flavors to AV brane construction, and integrating adjoint Φ out)

⋆ k-walls and $(N-k)$-walls related by parity reversal: [Choi, Roček, Sharon 18]

$$U(k)_{N-\frac{k+F}{2},N-\frac{F}{2}} \text{ with } F \text{ flavors} \quad \leftrightarrow \quad U(N-k)_{-\frac{N+k-F}{2},-N+\frac{F}{2}} \text{ with } F \text{ flavors}$$

Analysis of vacua

- $m > 0$: unique vacuum at $X^\dagger X = 0$ $\quad\rightarrow\quad$ $\mathcal{N}=1 \quad U(k)_{N-\frac{k}{2},N}$ (AV)
- $m < 0$: multiple vacua $\quad X^\dagger X \propto \begin{pmatrix} \mathbb{1}_J & \\ & 0 \end{pmatrix}$ flavor symmety breaking

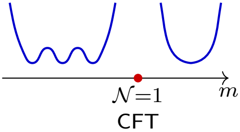
$$\boxed{\begin{array}{c} \mathcal{N}=1 \qquad U(k-J)_{N-\frac{k}{2}-F+\frac{J}{2},\,N-F} \quad \times \quad \text{NLSM} \quad \dfrac{U(F)}{U(J)\times U(F-J)} \\[2ex] \text{with} \quad \max(0, F+k-N) \leq J \leq \min(k,F) \end{array}}$$

⋆ Some of the classical vacua are lifted non-perturbatively, because

$\mathcal{N}=1 \quad U(n)_{j-\frac{n}{2},j}$ breaks SUSY ($V>0$) for $\quad 0 < j < n$

- SUSY $\quad\Rightarrow\quad$ 2^{nd} order phase transition (CFT)

  Multiple vacua coalesce into one at a single point



- Enhanced 3D $\mathcal{N}=2$ SUSY (SCFT) for special values

  $F=1$ or $k=1$: only one quartic superpotential term

  At large CS level can be seen perturbatively [Avdeev, Grigorev, Kazakov 92]

  [Avdeed, Kazakov, Kondrashuk 93]

  Duality $\quad\Rightarrow\quad$ also for $k=N-1$ [Benini, Closset, Cremonesi 11]

- Enhanced 3D $\mathcal{N}=4$ SUSY for $SU(2)$, $F=1$, $k=1$ [Gang, Yamazaki 18]

$$\mathcal{N}=1 \quad U(1)_{\pm\frac{3}{2}} \text{ with 1 flavor} \quad\longleftrightarrow\quad \mathcal{N}=1 \quad SU(2)_{\pm\frac{3}{2}} \text{ with 1 flavor}$$

Checks

- Reproduces Acharya-Vafa for $m > 0$
- Reproduces 4D constructions for $m < 0$

4D construction of domain walls

$$\text{Vacua:} \qquad M = \left(\frac{\Lambda^{3N-F}}{m_{4d}^{N-F}}\right)^{1/N} \mathbb{1}_F$$

- For $m_{4d} \ll \Lambda$: vacua lie in the weakly-coupled Higgsed phase

Low energy effective theory: Wess-Zumino (WZ) model on mesonic space

$$\mathcal{K} = 2\,\mathrm{Tr}\sqrt{\overline{M}M} \qquad\qquad W = m_{4d}\,\mathrm{Tr}\,M + W_{\mathsf{ADS}}(M)$$

$\Rightarrow$ Construct WZ-type walls

Valid as long as we remain in the Higgsed phase throughout the wall

⋆ *Caveat*: unbroken $SU(N-F)$ SYM theory on mesonic space

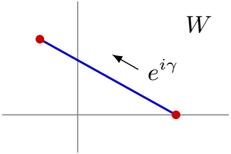
Domain walls in WZ theory [Fendley, Mathur, Vafa, Warner 90; Cecotti, Vafa 92]

BPS domain wall equation (central charge $Z = 2\,\Delta W = e^{i\gamma}|Z|$):

$$\mathcal{K}_{a\bar{b}}\,\partial_x \Phi^a = e^{i\gamma}\,\frac{\partial \overline{W}}{\partial \overline{\Phi}^{\bar{b}}}$$

Orbit in W-space is a straight line:

$$\mathbb{I}\mathrm{m}\left(e^{-i\gamma} W(\Phi(x))\right) = \text{const}$$



Interested in orbit $\Phi^a(x)$

- Single variable: solve by inverting $W(\Phi)$
- Multiple variables: solve the ODEs

$$W = m\,\mathrm{Tr}\,M + (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}$$

- $F = N-1$: WZ model on mesonic space (no SYM sector)

- $F < N-1$: unbroken $SU(N-F)$ SYM on mesonic space

$$\Lambda^{3(N-F)}_{\text{unbroken}} = \tfrac{\Lambda^{3N-F}}{\det M} \qquad\qquad W_{\text{unbroken}} = (N-F)\big(\Lambda^{3(N-F)}_{\text{unbroken}}\big)^{1/(N-F)}$$

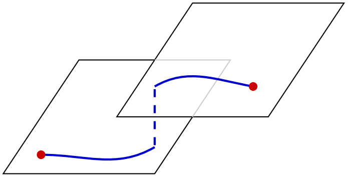
Multivaluedness of W_{ADS} from $N-F$ vacua of unbroken gauge theory

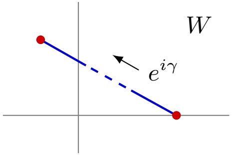
$\Rightarrow$ $N-F$ sheets over each point on mesonic space

Size of walls: $$\ell_{\text{WZ}} \sim \frac{M}{\partial_x M} \sim \frac{1}{m_{4d}} \quad \gg \quad \ell_{SYM} \sim \frac{1}{\Lambda_{\text{unbroken}}} \sim \frac{1}{m_{4d}^{F/3N}\Lambda^{1-F/3N}}$$

"Hybrid" SQCD walls

Hybrid walls: WZ type + "instantaneous" jumps from one sheet to another





⋆ 3D theory:

AV topological sector associated to jump Δ in unbroken SYM

$$\mathcal{N}=1 \quad U(\Delta)_{N-F-\frac{\Delta}{2},N-F}$$

$\times$

Goldstone bosons for broken flavor symmetry

NLSM

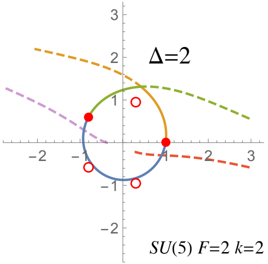
Symmetry preserving walls

Restrict to $$M(x) = \widetilde{M}(x)\, \mathbb{1}_F$$

Automatic for $F = 1$

- Solved by inverting $W(\widetilde{M})$

3D theory: trival or TQFT $\quad \mathcal{N} = 1\ U(\Delta)_{N-F-\frac{\Delta}{2},N-F}$ $\quad$ (no Goldstone bosons)



★ All cases analyzed match with 3D prediction

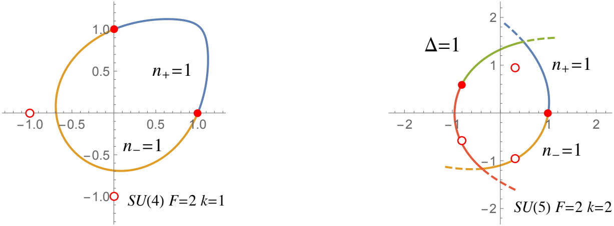
Symmetry breaking walls

Analyzed by numerically solving ODEs

3D predictions:

- eigenvalues form at most two groups
- at most one jump on a domain wall

$$\mathcal{N}=1 \quad U(\Delta)_{N-F-\frac{\Delta}{2},\,N-F} \quad \times \quad \text{NLSM} \quad \frac{U(F)}{U(n_+)\times U(n_-)}$$



⋆ All cases analyzed match with 3D prediction

Summary

- Proposed 3D description of DWs in 4D $SU(N)$ SQCD with $F < N$ flavors
- Predicts 2^{nd} order phase transition for $m_{4d} = m_*$ and intricate zoo of walls
- Predicts supersymmetry enhancement
- Large and small m_{4d} limits reproduced by 4D QFT arguments

Some open questions

- More flavors: $F \geq N$ (baryons, Seiberg duality, ...)
- Other gauge groups
- Domain wall junctions: 2D $\mathcal{N} = (0, 1)$ [*cfr.* Gaiotto 13]