

# Comments on $T\bar{T}$ , $J\bar{T}$ and String Theory

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Perturbative String Theory

on solvable worldsheet  $CFT_2$



Solvable irrelevant deformations

of spacetime  $CFT_2$

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$$-t\bar{T}\bar{\bar{T}} + M\bar{J}\bar{\bar{T}}$$

$$ER = n + \frac{1}{2A} [-B - \sqrt{B^2 - 4AC}]$$

$$PR = n = h - \bar{h}$$

$$A = \frac{\pi M^2 - 8t}{16\pi R^2}$$

$$B = -1 + \frac{M^2}{2R} - \frac{t n}{\pi R^2}$$

$$C = 2\left(h - \frac{c}{24}\right)$$



$$ds^2 = k \left( d\phi^2 + f d\gamma d\bar{\gamma} - \epsilon^2 f^2 d\gamma^2 \right)$$

$$f^{-1} = f_1 = \lambda + e^{-2\phi}$$

$$e^{2\Phi} = g^2 e^{-2\phi} f$$

$$A_\gamma = 2\sqrt{k}EF$$

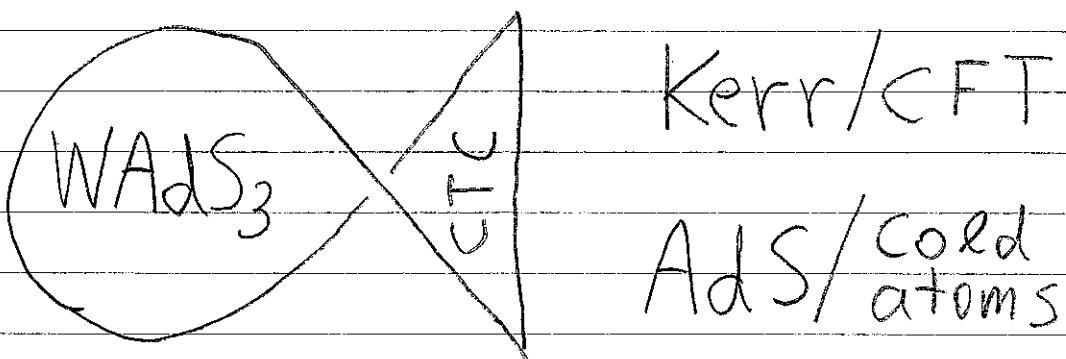
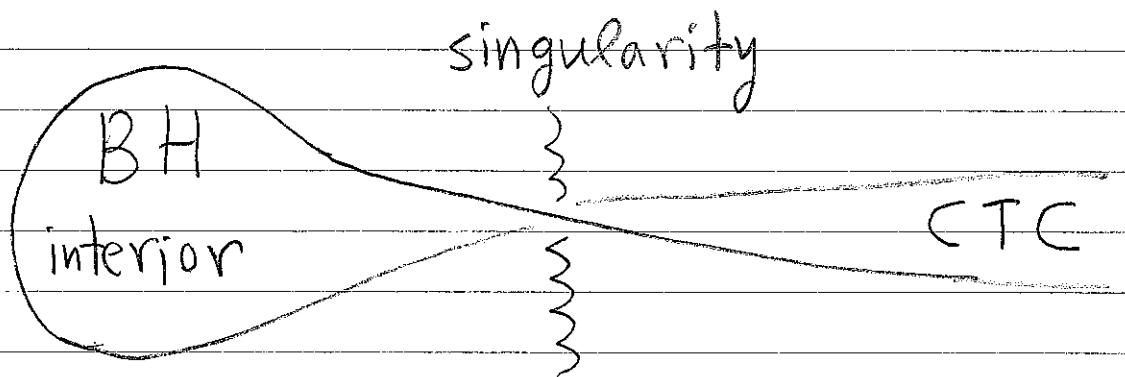
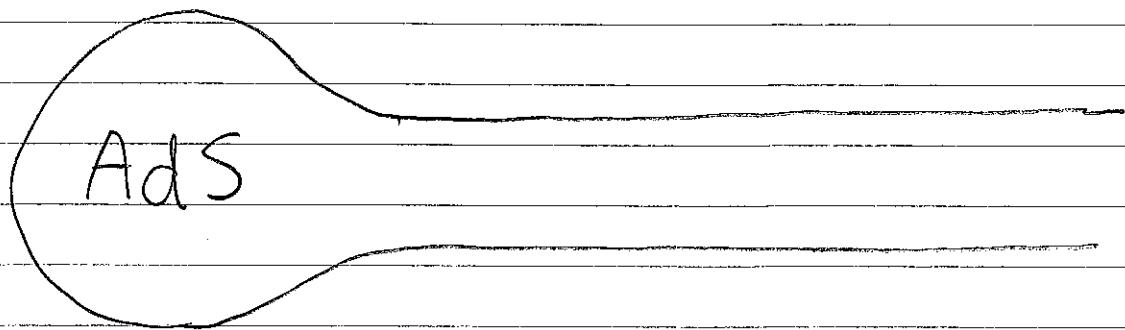
$$B_{\gamma\bar{\gamma}} = \frac{1}{2}kF$$

$$t = \pi \alpha' \lambda \quad M = 2\sqrt{2} l_s \epsilon$$

$$\lambda J^- \bar{J}^- + \epsilon K \bar{J}^- \quad \text{deformed } AdS_3 \times S^1$$

$$M_3 \equiv J^- \bar{J}^- \quad \text{deformed } AdS_3 \longleftrightarrow ST - T \bar{T}$$

$$K \bar{J}^- \quad \text{deformed } AdS_3 \times S^1 \rightarrow WAdS_3 \longleftrightarrow J \bar{T}$$



- \* Holography in asym. flat spacetime
- \* Horizons, Singularities, CTC

# $T\bar{T}$ in perturbative string theory

- heuristic

Suppose that we have a consistent string theory,  
say, a perturbative type II superstring,  
on a worldsheet CFT<sub>2</sub> background  
that has the following properties:

(1) It looks asymptotically like

$$R_\phi \times S_R^1 \times R_t \times N$$

$$\text{w/ } \phi = -\frac{Q}{2}\phi$$

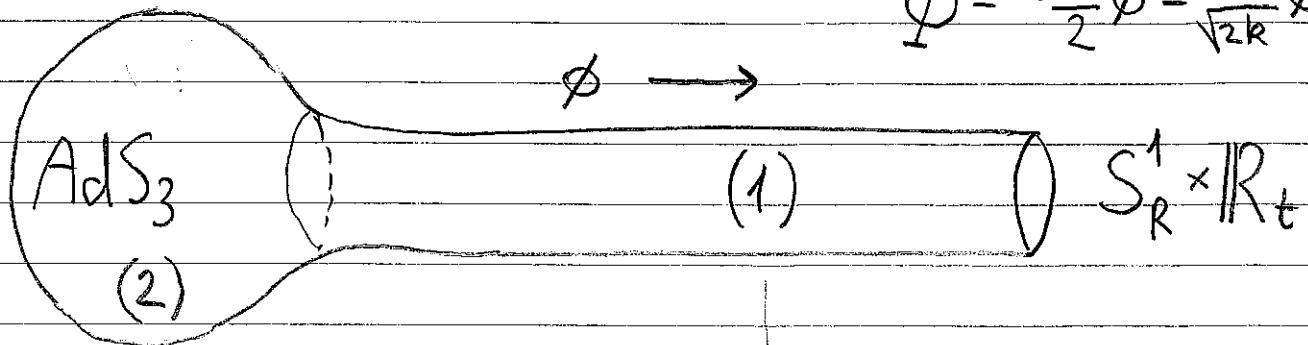
(2) In the IR it is AdS<sub>3</sub> × N

w/ the bdry the cylinder  $R_t \times S_R^1$

Example:  $M_3 \times N$

$$\mathbb{R}_\phi \times S^1_R \times \mathbb{R}_t \times N$$

$$\tilde{\phi} = -\frac{Q}{2}\phi = \frac{1}{\sqrt{2k}}\phi$$



$$W \left( h_w^{L,R} - \frac{Rw}{4} \right) = -\frac{J(J+1)}{R} + N_{L,R} - \frac{1}{2} = \frac{\alpha'^1}{4} (E_t^2 - P_{L,R}^2)$$

$$V_{N_{L,R}} e^{-iE_t t} e^{iP_L x_L + iP_R x_R} e^{QJ\phi}$$

$$P_{L,R} = \frac{WR}{\alpha'^1} \pm \frac{n}{R} \quad E_t = E + \frac{WR}{\alpha'^1}$$

$$\left( E + \frac{Rw}{\alpha'^1} \right)^2 - \left( \frac{Rw}{\alpha'^1} \right)^2 = \frac{2}{\alpha'^1} \left( h_1 + \bar{h}_1 - \frac{k}{2} \right) + \left( \frac{n_w}{Rw} \right)^2$$

$$Rw = WR \quad h_1 - \bar{h}_1 = n_w$$

$$n_w = w n$$

$$(M_{t\neq T})^N / S_N$$

$$t = \pi \alpha'^1 \quad C_M = 6k$$

↓ IR

$$h_w = \frac{h_1}{w} + \frac{k}{4} \left( w - \frac{1}{w} \right) \quad \mu^N / S_N$$

$$N \sim \frac{1}{g_s^2}$$

w=1, n=0:

$$E(\alpha'; R) = \frac{R}{\alpha'} \left[ -1 + \sqrt{1 + \frac{4\alpha'}{R^2} \left( h - \frac{k}{4} \right)} \right]$$

$$t \equiv \frac{\pi}{\alpha'} \quad w \quad t = \pi \alpha'$$

of CFT<sub>2</sub> w/ C=6R

Example:  $M_3 \equiv \bar{T}\bar{T}$  deformed CFT<sub>2</sub>

$$\int d^2z \bar{T}\bar{T} = \int d^2x D(x, \bar{x})$$

$\underbrace{\phantom{D(x, \bar{x})}}_{(2,2)}$

trans. like  $T\bar{T}$  under  $T(x), \bar{T}(\bar{x})$

but "single-trace"

e.g.  $\sum_{i=1}^N (\bar{T}\bar{T})_i$  in  $M^N/S_N$

# Superstring on $M=0$ BTZ $\times N$

$$\mathcal{L} = k (\partial\phi\bar{\partial}\phi + e^{2\phi}\partial\bar{\phi}\bar{\partial}\phi) \quad \begin{aligned} \gamma &= \gamma_1 + \gamma_0 \\ \bar{\gamma} &= \gamma_1 - \gamma_0 \end{aligned}$$

integrating  
out  $\beta, \bar{\beta}$

$$\gamma \approx \gamma_1 + 2\pi R$$

$$\mathcal{L}_W = \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} + \underbrace{\partial\bar{\phi}\bar{\partial}\phi}_{\mathcal{L}_\phi} - \sqrt{\frac{2}{k}}\hat{R}\phi - \beta\bar{\beta}e^{-\sqrt{\frac{2}{k}}\phi}$$

$$\gamma = i\phi_-$$

$$\beta = i\partial\phi_+ = J^-$$

$$\phi_+(z)\phi_-(w) \sim \ln(z-w)$$

$$\mathcal{L} = -\partial\phi_+\bar{\partial}\phi_- - \partial\phi_-\bar{\partial}\phi_+ + \mathcal{L}_\phi \quad t^w = e^{iw(\phi_+ + \bar{\phi}_+)}$$

$$V_{BTZ} = e^{\sqrt{\frac{2}{k}}J\phi} e^{iw\phi_+ + iE_L\phi_-} e^{i w\bar{\phi}_+ + iE_R\bar{\phi}_-}$$

$$E_{L,R} = \frac{1}{2}(ER \pm n)$$

$$\Delta_{L,R} = -WE_{L,R} - \frac{\jmath(j+1)}{k} \quad \Delta_R - \Delta_L = wn$$

$$\jmath = -\frac{1}{2} + is$$

$$V_{BTZ} V_N \quad \Delta_{L,R} + N_{L,R} - \frac{1}{2} = 0$$

$$E_{L,R} = \frac{1}{w} \left[ -\frac{\jmath(j+1)}{k} + N_{L,R} - \frac{1}{2} \right]$$

$AdS_3/CFT_2 \longrightarrow$

$$n_w^{L,R} - \frac{k w}{4} \neq$$

$$h_w = \frac{h_1}{w} + \frac{k}{4} \left( w - \frac{1}{w} \right) \quad \mathbb{Z}_w \text{ twisted sector of } M^N/S_N$$

# Superstring on $M_3 \times N$

$$SL \simeq \lambda J^- \bar{J}^-$$

$$ds^2 = k(d\phi^2 + f d\gamma d\bar{\gamma}) \quad e^{2\phi} = g^2 e^{-2\phi} f$$

$$J^- \simeq i \partial \phi_+$$

$$f^{-1} = \lambda + e^{-2\phi}$$

$\phi_+ \quad \phi_-$

$$-2\phi_+ \bar{\partial} \phi_- - 2\phi_- \bar{\partial} \phi_+ + \lambda 2\phi_+ \bar{\partial} \phi_+$$

$$G = \begin{pmatrix} \lambda & -1 \\ -1 & 0 \end{pmatrix}$$

$$\Delta_{L,R} = \frac{1}{2} P_{L,R}^2 - \frac{j(j+1)}{R}$$

$$P_{L,R} = (n^t + m^t(B \mp G)) e^*$$

$$\frac{1}{2} P_{L,R}^2 = -\omega E_{L,R} - \frac{\lambda}{2} E_L E_R$$

$$e^* (e^*)^t = \frac{1}{2} G^{-1}$$

$$h_w^{LR} - \frac{kW}{4} = E_{L,R} + \frac{\lambda R^2}{8W} (E^2 - P^2) \quad h_w - \bar{h}_w = n$$

$$(M_{t+\bar{T}})^N / S_N \quad t = \frac{\pi}{2} \lambda R^2$$

$w=1, n=0:$

$$E(t, R) = \frac{\pi R}{t} \left[ -1 + \sqrt{1 + \frac{4t}{\pi R^2} \left( h - \frac{c_M}{24} \right)} \right]$$

$$IR: \quad ER \simeq 2 \left( h - \frac{c_M}{24} \right)$$

$$UV \quad t > 0: \quad E \simeq \sqrt{\frac{4\pi}{t} \left( h - \frac{c_M}{24} \right)}$$

$$S(E(h)) \simeq 4\pi \sqrt{\frac{c_M}{6} \left( h - \frac{c_M}{24} \right)} \simeq \beta_H E$$

$$\beta_H = \sqrt{\frac{2\pi c_M t}{3}} \equiv 2\pi R_H \quad \text{where the g.s. } E(h=0) \text{ becomes complex}$$

$$t < 0: \quad E \text{ develop imaginary piece above } E_{\max} = \frac{\pi R}{|t|}$$

## More comments:

- \* The high energy behavior of the entropy of the deformed symm. product CFT,  $(M_t)^N/S_N$ , was shown to agree w/ the entropy of black holes in the deformed geometry induced by the single-trace deformation,  $M_3^{(+)}$
- \* From the holographic duality, the Hagedorn behavior is clear, since the UV completion provided by string theory on  $M_3^{(+)}$  is a 2d Little String Theory
- \*  $M_3^{(+)}$  is smooth and the spectrum has no pathology; in particular, the spectrum does not receive non-perturbative corrections
- \* On the other hand, for  $t < 0$ , the partition sum has non-perturbative ambiguity; it is tempting to speculate that states w/ real  $E$  in some sense live in the interior of the black hole while those w/ complex  $E$  (above  $E_{\max} \approx R/|t|$ ), and those which decouple at  $t \rightarrow 0$ , live in the region beyond the singularity, where  $M_3^{(+)}$  also has CTC's

# Superstring on WAdS<sub>3</sub>

$$E\bar{K} \quad K \approx i\partial y$$

$$\mathcal{L} = k(\partial\phi\bar{\partial}\phi + e^{2\phi}\partial\bar{\phi}\bar{\partial}\phi + \partial y\bar{\partial}y) + \delta\mathcal{L}$$

↓ KK reduction

$$ds^2 = k(d\phi^2 + e^{2\phi}drd\bar{\phi} - \epsilon^2 e^{4\phi}dy^2) \quad A_y \approx \epsilon e^{2\phi}$$

$$B_{y\bar{\phi}} = \frac{1}{2}ke^{2\phi}$$

in  $\phi_+, \phi_-, y$  variables

$$G + B = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 2\epsilon & 0 & 1 \end{pmatrix}$$

$$\frac{1}{2}P_{L,R}^2 = -WE_{L,R} + \epsilon q_L E_R + \frac{1}{2}\epsilon^2 E_R^2 + \frac{1}{2}q_{L,R}^2$$

$$h_w^{L,R} - \frac{kW}{4} = E_{L,R} - \frac{\epsilon}{w} q_L E_R - \frac{\epsilon^2}{2w} E_R^2$$

$$w=1: \quad h_1 - \frac{c}{24} - \frac{1}{2}q_L^2 = E_L(\epsilon) - \frac{1}{2}(q_L(\epsilon))^2 \quad c=6k$$

$$q_L(\epsilon) = q_L + \epsilon E_R(\epsilon)$$

$$\text{MT deformed } M_{6k} \quad \mu = 2\epsilon R$$

$$n=q=0: \quad E(\mu, R) = \frac{8R}{\mu^2} \left[ 1 - \sqrt{1 - \frac{\mu^2}{2R^2} \left( h - \frac{c\mu}{24} \right)} \right]$$

$$E \text{ develop imaginary piece above } E_{\max} = \frac{8R}{\mu^2}$$

associated w/ CTC's when  $e^{2\phi} > \frac{1}{\epsilon^2}$

and there are non-perturbative ambiguities in  $\mathbb{Z}$

More comments:

$$\int d^2z K(z) \bar{J}(\bar{z}) \simeq \underbrace{\int d^2x A(x, \bar{x})}_{(1,2)}$$

trans. like  $J\bar{T}$  under  $T(x), \bar{T}(\bar{x}), J(x)$   
but single-trace

e.g.  $\sum_{i=1}^N (J\bar{T})_i$  in  $M^N/S_N$

\* Does the dual theory tell us if/how string theory  
resolves singularities/CTC's?

\* Applications to AdS/cold atoms and Kerr/CFT correspondence?

Combining the two: geometry and spectrum (on p. 1.2)

$$G + B = \begin{pmatrix} \hat{\lambda} & -1 & 0 \\ -1 & 0 & 0 \\ 2\hat{\varepsilon} & 0 & 1 \end{pmatrix} \quad \frac{(M_{-t\bar{T}+M\bar{T}})^N}{S_N}$$

$$\{\lambda, \varepsilon^2\} = \frac{R^2}{2\alpha'} \{\hat{\lambda}, \hat{\varepsilon}^2\}$$

↑ (was sloppy about this previously..)

$$h_w^{LR} - \frac{c w}{24} = E_{LR} - \frac{\hat{\varepsilon}}{w} q_L E_R + \frac{1}{2w} (\hat{\lambda} E_L - \hat{\varepsilon}^2 E_R) E_R \quad c=6k$$

$$w=1, n=q=0:$$

$$E(\lambda, \varepsilon; R) = \frac{R}{\alpha'(\varepsilon^2 - \lambda)} \left[ 1 - \sqrt{1 - \frac{4\alpha'}{R^2} (\varepsilon^2 - \lambda) \left( h_1 - \frac{c}{24} \right)} \right]$$

\* Led to conjecture that this is the spectrum of  $-t\bar{T} + M\bar{T}$  deformed CFT<sub>2</sub>; for  $n=0$  this was verified (w/ reasonable assumptions) CDGJK (work in progress)

\* Complex E in the UV  $\Leftrightarrow \varepsilon^2 - \lambda > 0 \Leftrightarrow$  CTC's

\* Led to conjecture that complex energies in the UV are associated w/ Closed Timelike Curves in the holographic String-Theory geometry