Emergent space time from the algebra of CFT modular Hamiltonians

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Introduction

- AdS/CFT our best model for QG
- AdS/CFT seems in some sense to be a geometric realization of quantum information
- How does bulk manifold arise and what CFT QI structure does it encode.
- How is a particular metric related to CFT QI (RT formula is not totally satisfactory)

Modular Hamiltonian

Given a density matrix one can define a modular Hamiltonian which generates a modular flow

$$H_{mod} = -\log \rho$$

For a state $|\Psi>$ and a region A in the CFT we can define a density matrix by tracing over the compliment region, so we get $H_{mod,A}$

One can similarly do for the compliment region of A, and get $H_{mod,\bar{A}}$

One defines the total modular Hamiltonian as

$$\tilde{H}_{mod} = H_{mod,A} - H_{mod,\bar{A}} \qquad \tilde{H}_{mod}|\Psi >= 0$$

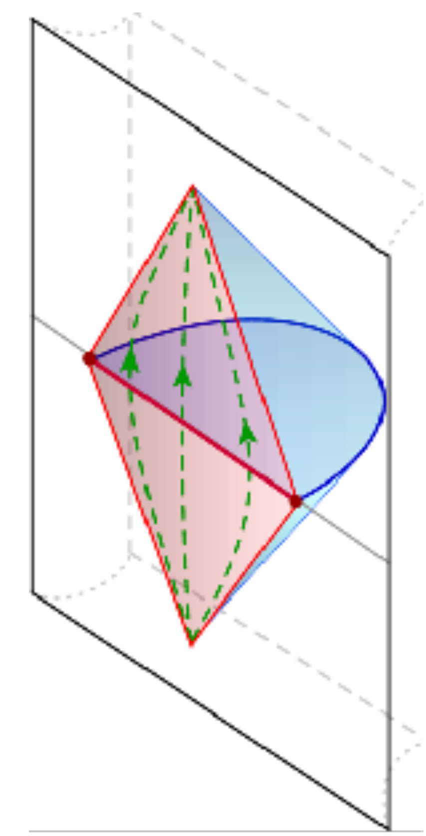
For spherical regions in the CFT ground state the expression for the modular Hamiltonian is known (Casini, Huerta, Myers)

Given a gravity dual and a region A on the boundary, one has an RT surface in the bulk which is the minimal surface in the bulk whose boundary is A.

The RT surface separates the bulk into two, and one can define a bulk modular Hamiltonain by tracing over one of the bulk regions.

In this way a bulk total bulk modular Hamiltonian can be constructed.

The action of the bulk and boundary modular Hamiltonians should be identified (JLMS)



The RT surface serve as a horizon for the modular evolution which is a fixed point of the modular flow, so one has for scalar bulk objects on the RT surface

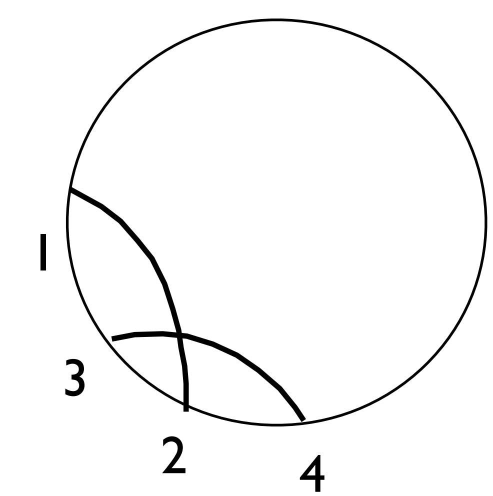
$$[\tilde{H}^{bulk}_{mod}, \Phi] = 0$$

Thus a CFT operator which represents a local scalar bulk objects inside the bulk should obey

$$[\tilde{H}^{CFT}_{mod}, \Phi] = 0$$

For any modular Hamiltonian whose RT surface goes through that point.

In AdS_3 it is enough to look at two surfaces



$$[\tilde{H}^{12}_{mod}, \Phi] = 0 \qquad [\tilde{H}^{34}_{mod}, \Phi] = 0$$

$$\tilde{H}_{mod} = \frac{2\pi}{y_2 - y_1}(Q_0 + y_1 y_2 P_0 + (y_1 + y_2)M_{01})$$

A solution to these equation is given for AdS_3, up to a space time dependent constant (which can be fixed) by (KL2017)

$$\Phi^{(0)}(Z,X,T) = \frac{\Delta - 1}{\pi} \int_{y'^2+t'^2<Z^2} dt'dy' \left(\frac{Z^2 - y'^2 - t'^2}{Z}\right)^{\Delta-2} \mathcal{O}(T+t', X+iy')$$

where $Z^2 = (y_2 - X)(X - y_1)$ $\quad$ $X = \frac{y_1 y_2 - y_3 y_4}{y_1 + y_2 - y_3 - y_4}$

for any scalar primary with any dimension, and any linear combination of such expressions.

This recovers the bulk operator from purely CFT considerations.

As we see one can get the form of the RT surface

Given a scalar and vector bulk operator for empty AdS one can compute ($H_{mod} = Q_0 - R^2 P_0$)

$$\frac{1}{2i\pi}[H_{mod}, \Phi(Z, \vec{X}, T)] = \xi^{\mu}_{R,0}\partial_{\mu}\Phi(Z, \vec{X}, T).$$

$$\frac{1}{2i\pi}[H_{mod}, V_{\nu}(Z, \vec{X}, T)] = \xi^{\mu}_{R,0}\partial_{\mu}V_{\nu} + V_{\mu}\partial_{\nu}\xi^{\mu}_{R,0} \equiv (\mathcal{L}_{\xi}V)_{\nu}.$$

$$\xi^{\mu}_{R,Y_i} = \Big(\frac{1}{R}TZ, \frac{1}{R}T(\vec{X} - \vec{Y}), \frac{1}{2R}(Z^2 + (\vec{X} - \vec{Y})^2 - R^2 + T^2)\Big)$$

On the RT surface, the modular Hamiltonian acts as a boost

in the two dimensional perpendicular to the RT surface.

If we include gauge and gravity interactions, then the action of the modular Hamiltonian on matter fields is more complicated.

However the parametrization of the solutions of the equations

$$[\tilde{H}^{12}_{mod}, \Phi] = 0 \qquad\qquad [\tilde{H}^{34}_{mod}, \Phi] = 0$$

<u>still parametrizes the bulk space-time</u>

Bulk from algebra

$$AdS_{d+1} = SO(d,2)/SO(d,1)$$

From modular Hamiltonians point of view the first factor is generated by the algebra of all modular Hamiltonians of spherical regions.

The second factor is generated by all such modular Hamiltonians that generate a maximal subgroup (they commute with a particular local bulk scalar operator)

Bulk space time is the parameter space of the maximal subgroup inside the larger group.

Can we generalize this ?

Clearly not all space time are quotient manifolds

The action of a modular Hamiltonians is generally not geometric

What is still true, holographic theories, is that each bulk point is related to a weakly maximal sub algebra of modular Hamiltonians.

$$[H_{mod}(Y_i, R), \Phi] = 0$$

Look for a family of (d-1)-parameter of modular Hamiltonians obeying this. Such phi’s should be labeled by (d+1) parameters labeling the bulk.

The sub-algebra generated by this set of modular Hamiltonian is then associated with each point in the bulk.

Lesson

The QI structure of a particular CFT state that the dual space time manifold encodes, is the structure of the maximal sub-algebras of modular Hamiltonians.

A Pure CFT state is invariant under modular flow, what is the corresponding bulk statement ?

It is then natural to expect that the metric be invariant under modular flow.

Modular flow is generally not geometric, but it is geometric on the extremal surface associated to the particular modular Hamiltonian. There as we saw it induces a boost,

$$[H_{mod}, V_\mu(\vec{X}, Z, T)]|_{\text{extremal surface}} = \Lambda_\mu{}^\nu V_\nu(\vec{X}, Z, T)|_{\text{extremal surface}}$$

$$\Lambda_\mu{}^\alpha(\vec{X}, Z, T) g_{\alpha\nu}(\vec{X}, Z, T) + \Lambda_\nu{}^\alpha(\vec{X}, Z, T) g_{\mu\alpha}(\vec{X}, Z, T) = 0$$

This fixes the metric up to a conformal factor-can be fixed with the knowledge of extremal surfaces.