On the Horizon of the $SL(2,\mathbb{R})_k/U(1)$ 2D Black Hole

Lior Liram

Tel Aviv University

February 17, 2019

Based on:
1702.03583 with R. Ben-Israel, A. Giveon, N. Itzhaki
1801.04939 with N. Itzhaki
1808.02036 with R. Basha, N. Itzhaki

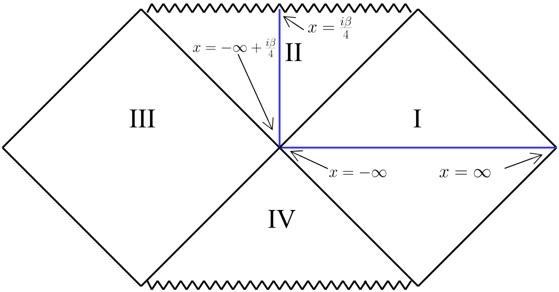
Introduction

What can be learned about the interior and horizon of a black hole from string theory?

- Observables in string theory (S-Matrix, boundary correlation functions) correspond to external observers $\Longrightarrow$ no access to interior
- Low energy effective theory $+$ perturbative corrections is smooth at the horizon
- Do non-perturbative corrections modify this? Do they render the horizon singular?
- Usually can't answer that in string theory. But for the $SL(2,\mathbb{R})_k/U(1)$ 2D black hole we know the α'-exact reflection coefficient of scattering modes.

The Tortoise Coordinate

The tortoise coordinate x or (r_*) is related to the Kruskal coordinates: $U = -e^{\frac{2\pi}{\beta}(x-t)}, \quad V = e^{\frac{2\pi}{\beta}(x+t)}.$



- It's a rescaled radial coordinate: The horizon is mapped to $x \to -\infty$ and the asymptotic region to $x \to \infty$.
- "Switch" between regions by taking $x \to x + i\beta/4$ (and t accordingly) – this places the singularity at $x = \pm i\beta/4$
- Everything should be invariant under $x \to x + i\beta$ (in fact $x \to x + i\beta/2$)

Scattering from Black Holes

A massless classical scalar field obeys, $\Box\phi = 0$.

In a spherically symmetric background we take, $\phi = r(x)^{-\frac{d-2}{2}} Y_{\ell m}(\Omega) e^{-i\omega t} \psi(r(x))$ and end up with a Schrödinger like equation for ψ,

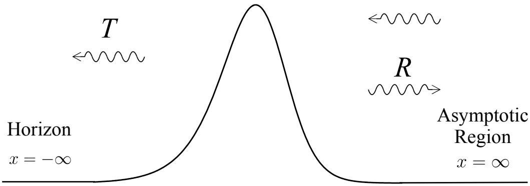
$$\left(-\frac{d^2}{dx^2} + V(x) - \omega^2\right)\psi(x) = 0$$

$V(x)$ is a function of the metric and background fields and therefore $V(x + i\beta/2) = V(x)$. It has poles where the background is singular.

Scattering from Black Holes

Simple QM scattering $\Rightarrow$ We take scattering solutions ($p = \omega$ – massless)

$$\psi(x) \sim \begin{cases} T(p)e^{-ipx}, & x \to -\infty, \\ e^{-ipx} + R(p)e^{ipx}, & x \to \infty, \end{cases}$$



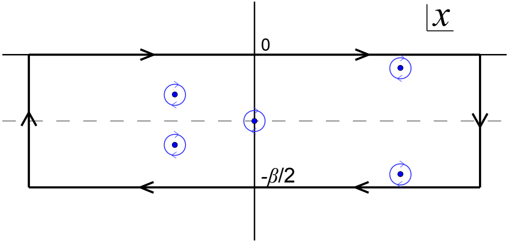
At high energies, the reflection coefficient is given by the Born approximation,

$$R(p) = \frac{1}{2ip} \int_{-\infty}^{\infty} e^{-2ipx} V(x) dx,$$

Calculating the Reflection Coefficient

$$R(p) = \frac{1}{2ip} \int_{-\infty}^{\infty} e^{-2ipx} V(x) dx,$$

The Born integral is evaluated in the complex x-plane by taking advantage of the periodicity $V(x + i\beta/2) = V(x)$.



The bottom line contributes $R(p)e^{-\beta p}$ so it can be omitted. The vertical lines do not contribute $(V(\pm\infty) \to 0)$
This gives,

$$R(p) \sim -\frac{\pi}{p} \sum_{\text{poles}} \text{Res}\left(V(x)e^{-2ipx}\right)$$

Calculating the Reflection Coefficient

The position of the singularities (poles) in the x-plane is encoded in a simple way in the reflection coeffcient!

$$R(p) = -\frac{\pi}{p} \sum_{\text{poles}} e^{-2ipx_i} f_i(p), \text{ with } f_i(p) \text{ a polynomial of } p$$

The position along the *imaginary* axis gives the rate of exponential suppression

The position along the *real* axis gives a phase linear in p

The $SL(2,\mathbb{R})_k/U(1)$ 2D Black Hole

The $SL(2,\mathbb{R})_k/U(1)$ 2D Black Hole has a description in terms of a low energy effective background
The reflection coefficient is known exactly (in α') on the sphere ($g_s \to 0$),

$$R_{exact}(p) = R_0(p) R_{non-pert}(p)$$

R_0 is accounted for by the effective background.

We want to understand how $R_{non-pert}$ modifies the effective background (the potential)

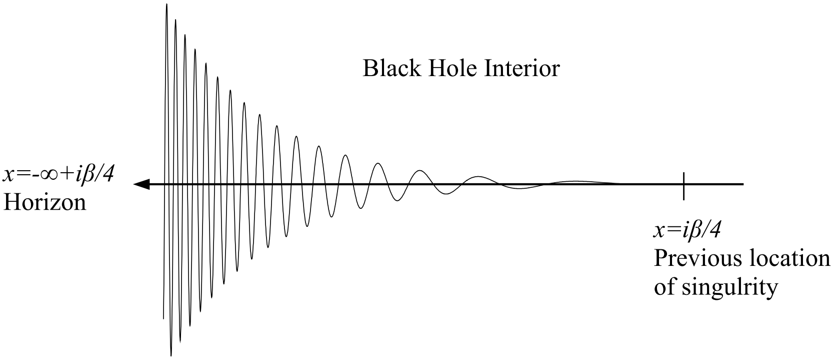
The $SL(2,\mathbb{R})_k/U(1)$ 2D Black Hole

For high energies,

$$R_0(p) \sim e^{-\frac{\beta}{2}p}, \quad R_{non-pert}(p) = e^{i\theta(p)}, \text{ with } \theta(p) \sim p\log(p)$$

The phase is not linear in $p \Rightarrow$ as we increase the momentum, the singularity is pushed further towards the horizon

A more careful calculation gives: $V_{\text{int.}}(x) \sim e^{-\frac{3\beta}{8\pi}x}\cos\left(2\,e^{-\frac{\beta}{4\pi}x}\right)$



Summary

- There is a simple relation between the location of the singularity (in the x-plane) and the reflection coefficient at high energies.
- Due to non-perturbative effects, the region just behind the horizon becomes singular.
- What is the physical reason for this divergence?
  Classically: Strings that fill the interior of the black hole.