Probing black hole microstates

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ICTP

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-Connection to information paradox (Mathur, AMPS)

-Emergence of space-time in AdS/CFT, do we need new principles to describe black hole interior?

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Review:
-Some reasons why question difficult

-Tomita-Takesaki modular theory and interior reconstruction (work with S.Raju)

-More recent techniques to (indirectly) probe interior:

i) Traversable wormhole protocol: Gao-Jafferis-Wall
ii) State-dependent perturbations of H_{SYK}: Kourkoulou-Maldacena (relevant for atypical states), see also Spenta's talk

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based on earlier work with S. Raju and more recent work [KP 1708.06328], [J. de Boer, R. van Breukelen, S. Lokhande, E. Verlinde, 1804.10580, 1901.08527]

Comments on bulk reconstruction

-large N, large λ

-HKLL construction

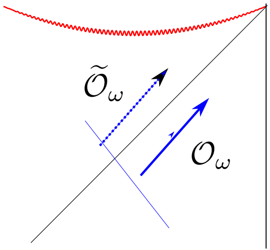
$$(\Box_{\mathrm{AdS}} - m^2)\phi = 0 \qquad \lim_{z\to 0} z^{-\Delta}\phi(x,z) = \mathcal{O}(x)$$

$$\phi_{\mathrm{CFT}}(t,x,z) = \int d\omega dk\ \mathcal{O}_{\omega,k} e^{-i\omega t + ikx} f_{\omega,k}(z) + h.c.$$

-On-shell, uses bulk EOMs

-perturbative in 1/N

Local analysis near the horizon

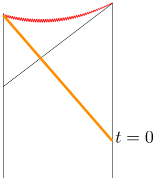


Demanding that low-point correlators of local fields at late times look locally like flat space we find some conditions which must hold at large N

$$[\mathcal{O}_\omega, \mathcal{O}_\omega^\dagger] = 1, \qquad [\widetilde{\mathcal{O}}_\omega, \widetilde{\mathcal{O}}_\omega^\dagger] = 1$$
$$[\mathcal{O}_\omega, \widetilde{\mathcal{O}}_{\omega'}] = 0$$

$$\langle \mathcal{O}_\omega^\dagger \mathcal{O}_\omega \rangle = \langle \widetilde{\mathcal{O}}_\omega^\dagger \widetilde{\mathcal{O}}_\omega \rangle = \frac{1}{e^{\beta\omega} - 1}$$

Collapsing vs typical black holes



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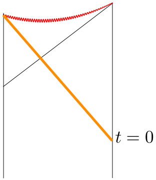

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Typical black hole microstates are defined by "microcanonical measure"

$$|\Psi\rangle = \sum_i c_i |E_i\rangle$$

where $E_i \in E_0 \pm \delta E$ and c_i selected randomly by Haar measure

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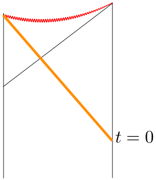

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$$\langle\Psi|\frac{dA}{dt}|\Psi\rangle = \sum_{ij} c_i^* c_j A_{ij} \frac{d}{dt} e^{iE_{ij}t} = O(e^{-S/2})$$

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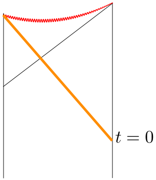

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Typical states are *equilibrium states*.

[S.Lloyd]

Define $\langle A\rangle_{\text{micro}} = \mathrm{Tr}(\rho_{\text{micro}} A)$

We also define the average over pure states in $\mathcal{H}_E$

$$\overline{\langle\Psi|A|\Psi\rangle} \equiv \int [d\mu_\Psi]\langle\Psi|A|\Psi\rangle$$

where $[d\mu_\Psi]$ is the Haar measure. Then for **any** observable A acting on $\mathcal{H}_E$, and **independent of the Hamiltonian**, we have

$$\overline{\langle\Psi|A|\Psi\rangle} = \langle A\rangle_{\text{micro}}$$

and

$$\text{variance} \equiv \overline{(\langle\Psi|A|\Psi\rangle^2)} - (\overline{\langle\Psi|A|\Psi\rangle})^2 = \frac{1}{e^S+1}\left(\langle A^2\rangle_{\text{micro}} - (\langle A\rangle_{\text{micro}})^2\right)$$

Observables have the same expectation value in most pure states, up to exponentially small corrections. Comments on 1) projectors 2) state-dependent observables

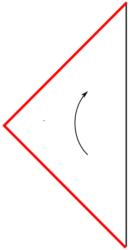
Exterior geometry of typical state in AdS/CFT

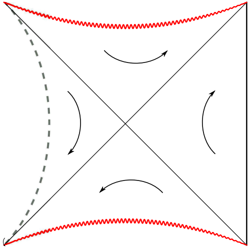
-low point functions of single-trace correlators on typical state are close to thermal correlators

-suggests that the dual exterior geometry is AdS-Schwarzchild

- fuzzball-like proposals are **significantly** constrained by previous theorem on typicality

Geometry of typical state





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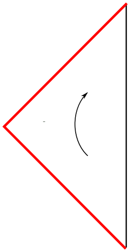

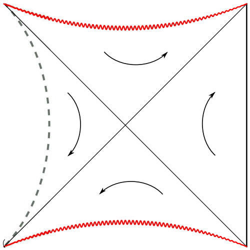

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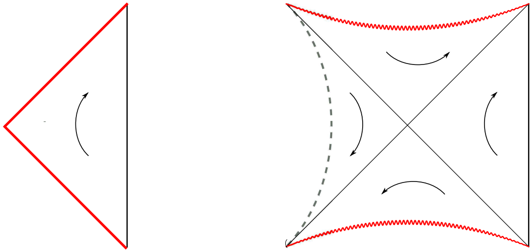
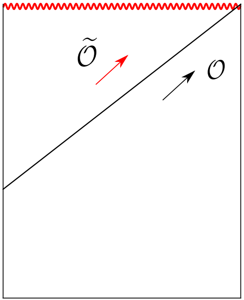


If future horizon is smooth, we expect interior region to be consistent with (approximate) Killing isometry.

Typical state paradox in AdS/CFT

- Large black holes in AdS are holographically dual to QGP states of $\mathcal{N} = 4$ SYM in deconfined phase
- These black holes are in equilibrium with their Hawking radiation and do not evaporate
- Nevertheless the analogue of the firewall paradox has been formulated even for these stable black holes [Almheiri, Marolf, Polchinski, Stanford, Sully], [Marolf, Polchinski]
- It suggests that big AdS black holes may have a singular horizon and no geometric interior.
- Most precise formulation of the paradox.

Firewall paradox for large AdS black holes



$$[\mathcal{O}, \mathcal{O}^\dagger] = 1$$
$$[H, \mathcal{O}^\dagger] = \omega \mathcal{O}^\dagger$$

$$[\widetilde{\mathcal{O}}, \widetilde{\mathcal{O}}^\dagger] = 1$$
$$[H, \widetilde{\mathcal{O}}^\dagger] = -\omega \widetilde{\mathcal{O}}^\dagger$$

- [AMPSS, MP] paradox: if typical black hole states have smooth horizon, using $[H, \widetilde{\mathcal{O}}^\dagger] = -\omega \widetilde{\mathcal{O}}^\dagger$ we find
$$Tr[e^{-\beta H} \widetilde{\mathcal{O}}^\dagger \widetilde{\mathcal{O}}] < 0$$
which is inconsistent.
- This suggests that there are no operators $\widetilde{\mathcal{O}}$ in the CFT with the desired properties, hence the BH has no interior and horizon is singular (?).

Using entanglement to go behind the horizon

[KP, S. Raju]

The quantum fields outside the horizon appear to be in an entangled state. They are entangled with certain CFT d.o.f. which can play the modes of the interior.

There is a natural mathematical construction allowing us to identify those.

Tomita-Takesaki modular theory

Consider a state $|\Psi\rangle$ and an algebra $\mathcal{A}$ with the properties:
1) The state is *cyclic* wrt the algebra $\mathcal{A}$ i.e.

$$\mathcal{H} = \mathrm{span}\mathcal{A}|\Psi\rangle$$

2)The state is *separating* wrt the algebra $\mathcal{A}$ i.e.

$$a|\Psi\rangle \neq 0 \qquad \forall a \in \mathcal{A}, a \neq 0$$

Then the Tomita-Takesaki theorem says (among other things) that:

The representation of the algebra $\mathcal{A}$ on $\mathcal{H}$ is reducible, and the algebra has a non-trivial commutant $\mathcal{A}'$ also acting on $\mathcal{H}$. Moreover $\mathcal{A}'$ is isomorphic to $\mathcal{A}$. Finally, the algebras $\mathcal{A}, \mathcal{A}'$ are entangled in a particular way.

Tomita-Takesaki modular theory

We define an antilinear map

$$S a|\Psi\rangle = a^\dagger|\Psi\rangle \qquad a \in \mathcal{A}$$

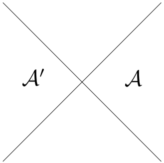
Consider the polar decomposition

$$S = J\Delta^{1/2} \qquad \Delta = S^\dagger S$$

where $\Delta = e^{-K}$ and K=modular Hamiltonian. Then we have:

1. $\mathcal{A}' = J\mathcal{A}J$: the commutant $\mathcal{A}'$ is isomorphic to $\mathcal{A}$ (notice $J^2 = 1$).
2. $\Delta^{is}\mathcal{A}\Delta^{-is} = \mathcal{A}, \qquad \Delta^{is}\mathcal{A}'\Delta^{-is} = \mathcal{A}' \qquad s \in \mathbb{R}$
3. KMS-like condition: $F(z) \equiv \langle\Psi|a\Delta^{iz}b\Delta^{-iz}|\Psi\rangle$, then $F(-i) = \langle\Psi|ba|\Psi\rangle$

Example:Rindler space



Consider a general, possibly strongly coupled, relativistic QFT in the Minkowski ground state $|0\rangle$. Suppose we have only access to right Rindler wedge. How can we use the entanglement to recover the rest of space-time?

Reeh-Schlieder theorem: The Minkowski vacuum $|0\rangle$ is a cyclic and separating state for the algebra $\mathcal{A}$:

1. States of form $a_1 ... a_n|0\rangle \qquad a_i \in \mathcal{A}$, span dense subspace of $\mathcal{H}$
2. There is no $a \in \mathcal{A}$ such that $a|0\rangle = 0$.

Example:Rindler space

Consider Lorentz boost $U = e^{iKs}$ on $t - x$ plane

$$t' = t\cosh s + x\sinh s$$
$$x' = t\sinh s + x\cosh s$$

A complexified Lorentz boost by $s = i\pi$ maps $(t, x, \vec{y}) \to (-t, -x, \vec{y})$

$$e^{-\pi K}\phi(t, x, \vec{y})|0\rangle = \phi(-t, -x, \vec{y})|0\rangle$$

Combine this with a rotation R_1 by π around x which takes $\vec{y} \to -\vec{y}$ and finally CPT transformation Θ which maps $(-t, -x, \vec{y})$ back to $(t, x, \vec{y})$. All in all we find

$$\Theta R_1 e^{-\pi K}\phi(t, x, \vec{y})|0\rangle = \phi^\dagger(t, x, \vec{y})|0\rangle$$

Generalizing to more operators (Bisognano-Wichmann thm.) it follows that the desired modular conjugation implementing $Sa|0\rangle = a^\dagger|0\rangle$ is

$$S = \Theta R_1 e^{-\pi K}$$

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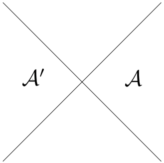

We have $S = \Theta R_1 e^{-\pi K}$. From this follows that

$$\Delta = S^\dagger S = e^{-2\pi K}$$

The modular Hamiltonian is the Lorentz boost generator with effective temperature $\frac{1}{2\pi}$. The antiunitary operator J mapping $\mathcal{A}$ to $\mathcal{A}'$ and allowing us to recover the left wedge is

$$J = \Theta R_1$$

The fact that each of the algebras $\mathcal{A}, \mathcal{A}'$ remain invariant under conjugation by Δ^{is} is obvious in this example. The KMS condition implies the Unruh temperature (even at strong coupling).

Tomita-Takesaki and the black hole

We do not have a decomposition of the algebra in **physical space**, but rather in the space of operators.
Introduce a "small algebra" $\mathcal{A}$ of simple operators (single trace + small products).

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The algebra $\mathcal{A}$ probes the typical pure state $|\Psi\rangle$ as a thermal state

$$\langle\Psi|\mathcal{O}(x_1)...\mathcal{O}(x_n)|\Psi\rangle = Z^{-1}\mathrm{Tr}[e^{-\beta H}\mathcal{O}(x_1)...\mathcal{O}(x_n)] + O(1/N)$$

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An analogue of the Tomita-Takesaki construction applies.
Using large N factorization and the KMS condition, we find the modular Hamiltonian for the small algebra

$$\Delta \equiv S^\dagger S = e^{-\beta(H-E_0)} + O(1/N)$$

The mirror operators

This leads to the “mirror operators”

$$\widetilde{\mathcal{O}}_\omega|\Psi\rangle = e^{-\frac{\beta H}{2}}\mathcal{O}_\omega^\dagger e^{\frac{\beta H}{2}}|\Psi\rangle$$

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- $[\mathcal{O}, \widetilde{\mathcal{O}}] = 0$ only inside $\mathcal{H}_\Psi$, not as operator equation
- Due to Boltzman factors $\langle \mathcal{O}_\omega^\dagger \mathcal{O}_\omega \rangle \propto e^{-\beta\omega}$, we define these operators for $\omega < \omega_*$, where ω_* does not grow too fast with N

The mirror operators

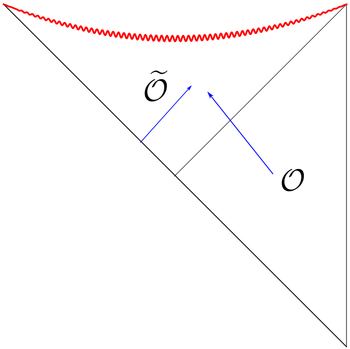
The small algebra $\mathcal{A}$ is not an exact algebra, hence the Tomita-Takesaki theorem can not be applied *exactly*. Hence $\mathcal{A}'$ is not an exact commutant.

From a physical point of view this is a **desirable feature** of the construction. It realizes the idea of black hole complementarity in a precise setting.

It also naturally implies that there is some non-locality in the construction of the interior.

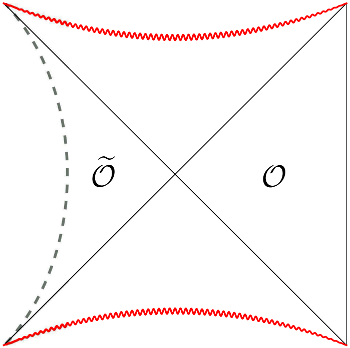
Finally, notice the operators $\widetilde{\mathcal{O}}$ defined by the Tomita-Takesaki constrcution are state-dependent, since they are "defined by the enganglement".

Infalling observer



$$\phi(t,r,\Omega) = \int_0^\infty d\omega \Big[\mathcal{O}_\omega \, f_\omega(t,\Omega,r) + \widetilde{\mathcal{O}}_\omega g_\omega(t,\Omega,r) + \mathrm{h.c.} \Big]$$

Extended geometry



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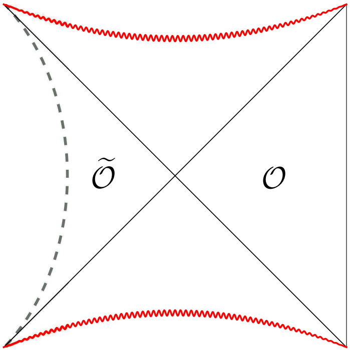
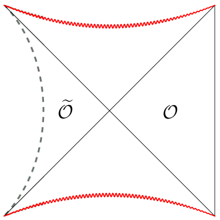


The cutoff on the left is determined by ω_*.

Since $\widetilde{\mathcal{O}}$ do not fundamentally commute with $\mathcal{O}$, left region should not be though as a fundamentally independent part of the Hilbert space (BH complementarity)

Summary

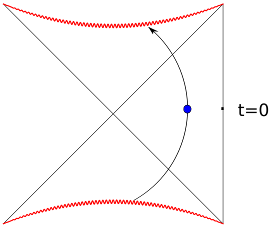
Conjecture: typical state should be associated to the following geometry:



In general we can characterize the geometry of a state by classifying possible ways to excite it.

We will identify perturbations of the CFT state corresponding to excitations of left region

Standard non-equilibrium states



$U(\mathcal{O})|\Psi\rangle$

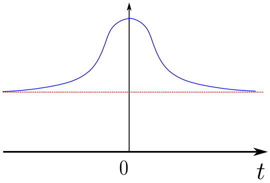
Excited (somewhat a-typical) state

$$|\Psi\rangle = U(\mathcal{O})|\Psi_0\rangle = e^{i\theta\mathcal{O}(0)}|\Psi_0\rangle$$

Correlators

$$\langle\Psi|\mathcal{O}(t)|\Psi\rangle$$

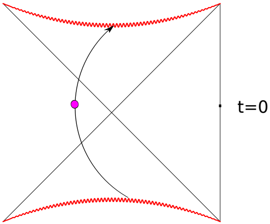
are t-dependent



State prepared to undergo a spontaneous fluctuation out of equilibrium at $t \approx 0$.

Exciting the left region

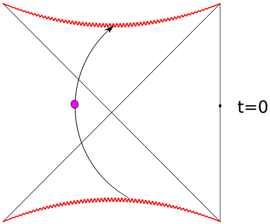
[KP 1708.06328]



$$U(\widetilde{\mathcal{O}})|\Psi\rangle$$

Exciting the left region

[KP 1708.06328]



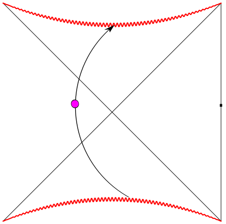
$$U(\widetilde{\mathcal{O}})|\Psi\rangle$$

But we can also write this as

$$U(\widetilde{\mathcal{O}})|\Psi\rangle = e^{-\frac{\beta H}{2}} U(\mathcal{O})^{\dagger} e^{\frac{\beta H}{2}} |\Psi\rangle$$

Exciting the left region

[KP 1708.06328]



$$e^{-\frac{\beta H}{2}} U(\mathcal{O})^\dagger e^{\frac{\beta H}{2}} |\Psi\rangle$$

$\boxed{\text{Existence and properties of these states } \textbf{independent} \text{ of } \widetilde{O}\text{-operator construction}}$

Unusual type of non-equilibrium state, excitation not visible in single-trace correlators

Acting with $e^{-\frac{\beta H}{2}} U(\mathcal{O})^\dagger e^{\frac{\beta H}{2}}$ *lowers* CFT energy

Properties of the states

At large N state

$$|\Psi\rangle = e^{-\frac{\beta H}{2}} U(\mathcal{O}) e^{\frac{\beta H}{2}} |\Psi_0\rangle$$

seems to be in equilibrium wrt algebra $\mathcal{A}$

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$$\langle\Psi|A|\Psi\rangle = \langle\Psi_0|e^{\frac{\beta H}{2}} U(O)^\dagger e^{-\frac{\beta H}{2}} A e^{-\frac{\beta H}{2}} U(O) e^{\frac{\beta H}{2}} |\Psi_0\rangle$$

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$$= \frac{1}{Z} \mathrm{Tr}[e^{-\beta H} e^{\frac{\beta H}{2}} U(O)^\dagger e^{-\frac{\beta H}{2}} A e^{-\frac{\beta H}{2}} U(O) e^{\frac{\beta H}{2}}] + O(1/S)$$

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$$= \frac{1}{Z} \mathrm{Tr}[e^{-\beta H} e^{\frac{\beta H}{2}} U(O)^\dagger e^{-\frac{\beta H}{2}} A e^{-\frac{\beta H}{2}} U(O) e^{\frac{\beta H}{2}}] + O(1/S)$$

$$= \frac{1}{Z} \mathrm{Tr}[e^{-\beta H} A] + O(1/S)$$

Including H in correlators. We define $\hat{H} = H - E_0$ and to be concrete consider the state

$$|\Psi\rangle = e^{-\frac{\beta H}{2}} e^{i\theta \mathcal{O}(t_0)} e^{\frac{\beta H}{2}} |\Psi_0\rangle \tag{1}$$

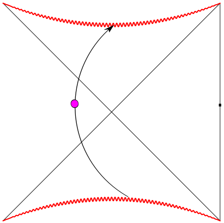
and compute

$$\langle\Psi|\mathcal{O}(t)\hat{H}|\Psi\rangle = i\theta \left[\langle\Psi_0|\mathcal{O}(t)\hat{H}\mathcal{O}(t_0 + i\frac{\beta}{2})|\Psi_0\rangle - \langle\Psi_0|\mathcal{O}(t_0 - i\frac{\beta}{2})\mathcal{O}(t)\hat{H}|\Psi_0\rangle \right] + O(\theta^2)$$

$$\langle\Psi|\{\mathcal{O}(t), \hat{H}\}|\Psi\rangle \approx \theta \langle\Psi_0|\mathcal{O}(t)\frac{d\mathcal{O}}{dt}(t_0 + i\frac{\beta}{2})|\Psi_0\rangle \tag{2}$$

This correlator decays exponentially as $|t - t_0|$ becomes very large, but it is nonzero and $O(1)$ around the time $t = t_0$.

Properties of the states



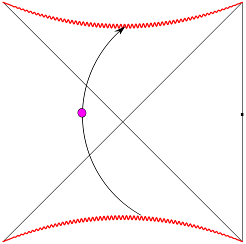
- They seem to be in equilibrium in terms of single-trace correlators

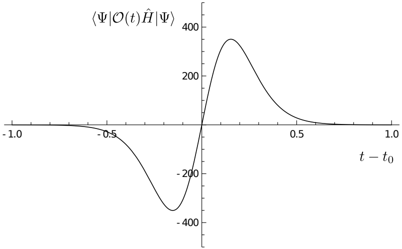
$$\frac{d}{dt}\langle\Psi|\mathcal{O}(t)|\Psi\rangle = 0$$

- It can be seen that they are out of equilibrium by incuding H in the correlator

$$\frac{d}{dt}\langle\Psi|\{\mathcal{O}(t), H\}|\Psi\rangle \neq 0$$

Example





Consider a 2d CFT on $\mathbb{S}^1 \times R$ on a state $|\Psi\rangle = e^{-\frac{\beta H}{2}} U(\mathcal{O}) e^{\frac{\beta H}{2}} |\Psi_0\rangle$, with $U = e^{i\theta\mathcal{O}(t_0)}$. Then at large c we find

$$\langle\Psi|\{\mathcal{O}(t), \hat{H}\}|\Psi\rangle = \theta\, 2\Delta \left(\frac{2\pi}{\beta}\right)^{2\Delta+1} \sum_{m=-\infty}^{+\infty} \frac{\sinh\left(\frac{2\pi(t-t_0)}{\beta}\right)}{\left[2\cosh\left(\frac{4\pi^2 m}{\beta}\right) + 2\cosh\left(\frac{2\pi(t-t_0)}{\beta}\right)\right]^{\Delta+1}}$$

Notice that

$$e^{-\frac{\beta H}{2}} U(\mathcal{O}) e^{\frac{\beta H}{2}}$$

is not a unitary, however the state $e^{-\frac{\beta H}{2}} U(\mathcal{O}) e^{\frac{\beta H}{2}} |\Psi_0\rangle$ has norm 1 up to $1/S$ corrections.

Also notice that

$$e^{-\frac{\beta H}{2}} U(\mathcal{O}) e^{\frac{\beta H}{2}}$$

lowers the energy for *typical states*. How is this possible given that this is an invertible operator and that there are fewer states at lower energies?

Also notice that

$$e^{-\frac{\beta H}{2}} U(\mathcal{O}) e^{\frac{\beta H}{2}}$$

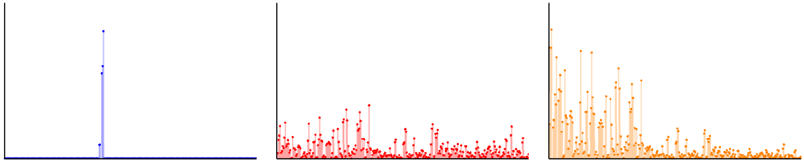
lowers the energy for *typical states*. How is this possible given that this is an invertible operator and that there are fewer states at lower energies?

Yes. This operator only lowers the **expectation value** of the energy. The states

$$e^{-\frac{\beta H}{2}} U(\mathcal{O}) e^{\frac{\beta H}{2}} |\Psi_0\rangle$$

have spread in energy, and are borrowing "phase space" from higher energies. However their low energy components are enhanced, thus decreasing the expectation value of the energy.

Non-equilibrium states in SYK



Distribution of $|\langle E_i|\Psi\rangle|^2$ in SYK for

a) left: typical state $|\Psi_0\rangle$

b) middle: usual non-equilibrium state $U(\mathcal{O})|\Psi_0\rangle$

c) right: non-equilibrium state of form $e^{-\frac{\beta H}{2}}U(\mathcal{O})^\dagger e^{\frac{\beta H}{2}}|\Psi_0\rangle$

- We identified a class of non-equilibrium states present in any statistical system. In holographic CFTs these states may correspond to excitations behind the black hole horizon.

- The number of such states is in correspondence with possible ways to excite the region behind the horizon in EFT assuming the conjectured geometry for a typical state

- The existence of these states is motivated by, but logically independent from state-dependent operators $\widetilde{\mathcal{O}}$.

- This shows that the CFT contains in its Hilbert space a class of states which can be naturally identified with excitations of the left region

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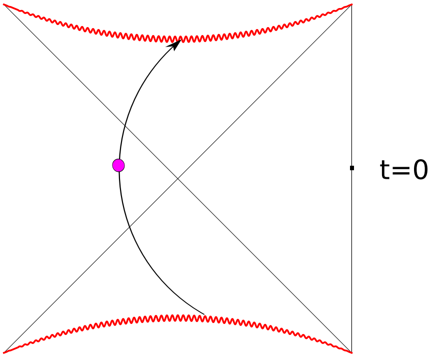
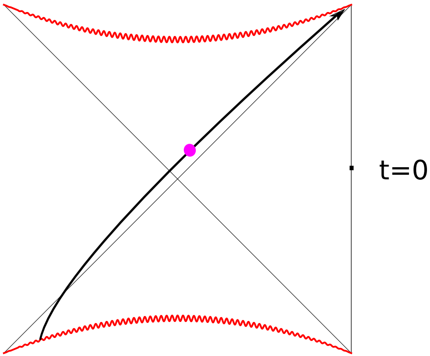
- This shows that the CFT contains in its Hilbert space a class of states which can be naturally identified with excitations of the left region

Can we find more evidence for the interpretation of these states?

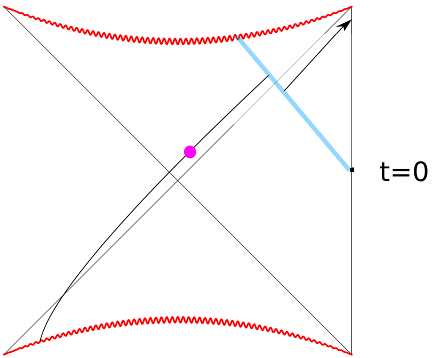

t=0


t=0

Extracting the particle

Following Gao-Jafferis-Wall we will try to create a negative energy shockwave by perturbing the CFT with

$$H = H_0 + g\mathcal{O}\widetilde{\mathcal{O}}$$



see also [Kourkoulou, Maldacena],[Almheiri, Mousatov, Shyani] for somewhat related constructions

Comments on using state-dependent operators on the boundary

1. The use of state-dependent operators on the boundary fits within the standard framework of quantum mechanics
2. We can imagine many identically prepared systems all in state $|\Psi\rangle$.
3. The boundary observer can use these systems to perform many measurements and identify the state $|\Psi\rangle$
4. Then the observer can prepare a device acting with $\widetilde{\mathcal{O}}$ on one of the remaining (un-measured) systems which is still in the state $|\Psi\rangle$.

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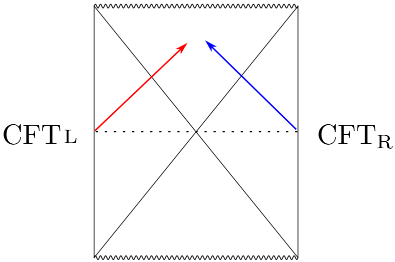
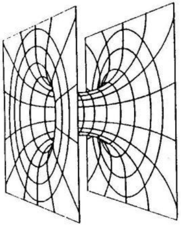
4. Then the observer can prepare a device acting with $\widetilde{\mathcal{O}}$ on one of the remaining (un-measured) systems which is still in the state $|\Psi\rangle$.

It remains a non-trivial question to understand better how the infalling bulk observer can use state-dependent operators to perform quantum measurements.

Eternal AdS black hole





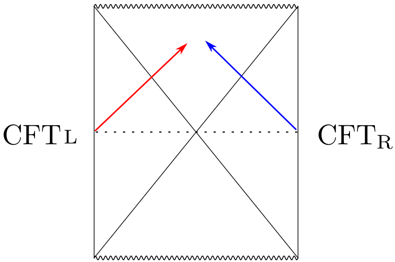
Two identical non-interacting CFTs

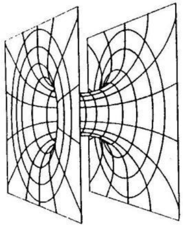
$$H = H_L + H_R$$

in an entangled state

$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_E e^{-\frac{\beta E}{2}} |E\rangle_L \otimes |E\rangle_R$$

Eternal AdS black hole

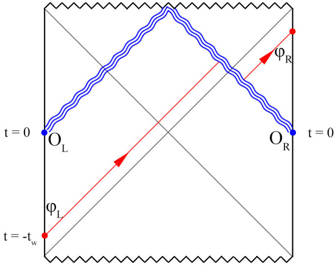




In the bulk they are connected by a wormhole (Einstein-Rosen bridge).

It is not traversable, consistent with the fact that CFTs are non-interacting

Gao-Jafferis-Wall protocol



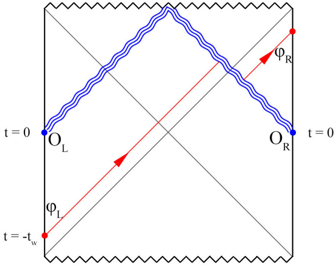
at $t = 0$ we briefly couple the CTFs by a double-trace interaction

$$H = H_L + H_R + g f(t) \mathcal{O}_L \mathcal{O}_R$$

For given sign of g this creates negative energy shockwaves in the bulk. Probe undergoes time advance when crossing shockwaves

Wormhole becomes traversable

Gao-Jafferis-Wall protocol

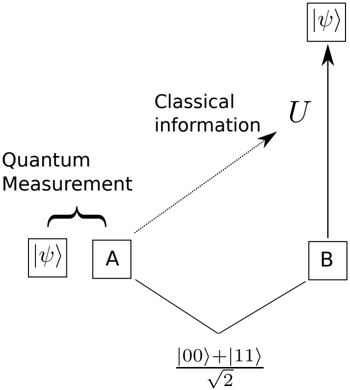


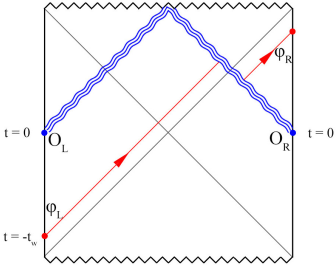
Change of CFT energy

$$\delta\langle H_R\rangle \propto g\langle \mathcal{O}_L \mathcal{O}_R\rangle + O(g^2)$$

Black hole horizon shrinks somewhat, probe can cross the wormhole
CFTs briefly interacted via $O_L O_R$ at $t = 0$, so information can be exchanged
Notice ϕ vs O

Quantum Teleportation Interpretation





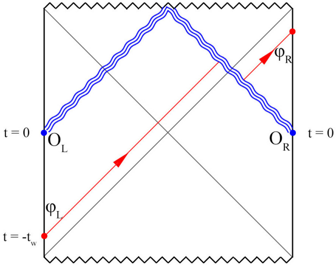
Measure O_L on CFT$_L$, then apply

$$e^{igo_L\mathcal{O}_R}$$

on CFT$_R$. The probe ϕ is teleported.

Gao-Jafferis-Wall protocol

analysis by [Maldacena-Stanford-Yang]



We create the probe on the left by

$$e^{i\epsilon\phi_L(-t)}|\mathrm{TFD}\rangle$$

At $t = 0$ we apply double-trace perturbation coupling the two CFTs

$$e^{igO_L O_R(0)} e^{i\epsilon\phi_L(-t)}|\mathrm{TFD}\rangle$$

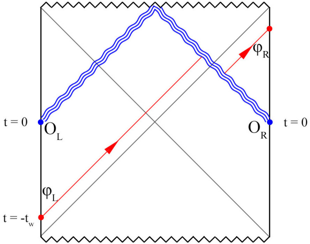
We measure the operator $\phi_R(t)$ on this state. To leading order in ϵ we need

$$\langle\mathrm{TFD}|[\phi_L(-t), e^{-igO_L O_R(0)}\phi_R(t)e^{igO_L O_R(0)}]|\mathrm{TFD}\rangle$$

Expanding in g

$$\langle\mathrm{TFD}|[\phi_L(-t), O_L(0)][\phi_R(t), O_R(0)]|\mathrm{TFD}\rangle$$

Traversable wormholes and quantum chaos



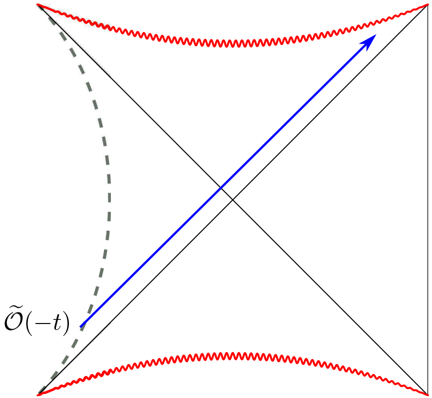
Growth of out-of-time-order-correlators (OTOC) due to **quantum chaos**

$$\langle \mathrm{TFD} | [\phi_L(-t), O_L(0)][\phi_R(t), O_R(0)] | \mathrm{TFD} \rangle \sim \frac{1}{N^2} e^{\frac{2\pi}{\beta} t}$$

Including higher orders in g, we find that the commutator is zero up to scrambling time $t \approx \beta \log S$, when it becomes nonzero and we get a nontrivial signal, corresponding to the probe appearing in the right CFT.

- Gao-Jafferis-Wall identified an S-matrix-like experiment which probes the interior of eternal black hole
- CFT correlators contain information about geometry inside horizon
- Computations provide evidence for smoothness of horizon of eternal black hole, dual to the TFD state, and ER/EPR proposal
- However, the real difficulty in reconciling unitarity with the smoothness of the black hole horizon is not for the TFD (which is a very special, atypical state), but rather for *typical black hole microstates*.
- Can we find a way of applying a similar protocol to (1-sided) typical black hole microstates, which will allow us to probe their interior?

Exciting the left region

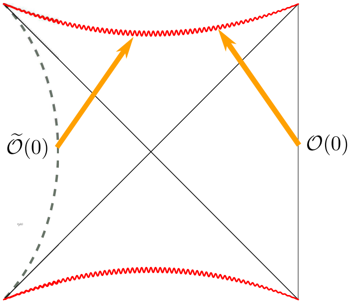


Mirror quench: we perturb the CFT Hamiltonian by $\widetilde{\mathcal{O}}$ at $-t$

Excitation is invisible by simple CFT operators

Creating negative energy shockwaves for 1-sided black hole

[J. de Boer, R. van Breukelen, S. Lokhande, KP, E. Verlinde, arXiv: 1804.10580, 1901.08527]

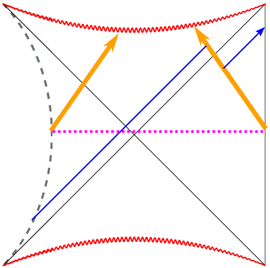


At $t = 0$ we perturb CFT Hamiltonian by

$$g f(t) \mathcal{O} \widetilde{\mathcal{O}}(0)$$

Compute effect on bulk correlators $\Rightarrow$ generates negative energy shockwaves for appropriate choice of g

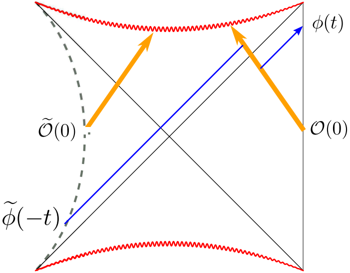
Some subtleties



Operators $\widetilde{\mathcal{O}}$ are gravitationally dressed wrt the right $\Rightarrow$ Wilson lines extending across geometry

Backreaction and Einstein equations at subleading order?

The experiment



We create a probe in the left region of the black hole by acting with $\widetilde{\phi}(-t)$.

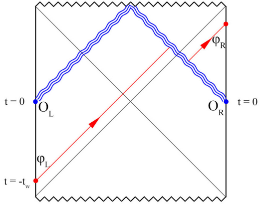
Then at $t = 0$ we perturb the CFT by $gf(t)\mathcal{O}(0)\widetilde{\mathcal{O}}(0)$. Finally we detect the probe by measuring $\phi(t)$.

The postulated Penrose diagram makes a prediction about CFT correlators (singal around $t = \beta \log S$)

$$\langle\Psi_0|[\widetilde{\phi}(-t), e^{-ig\widetilde{\mathcal{O}}\mathcal{O}(0)}\phi(t)e^{ig\widetilde{\mathcal{O}}\mathcal{O}(0)}]|\Psi_0\rangle$$

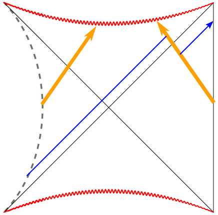
Comparison

Eternal BH



$$C = \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}\mathcal{X}(\phi, O)]$$

1-sided BH



$$C' = \langle\Psi_0|\mathcal{X}(\phi, O)|\Psi_0\rangle$$

Using properties of the TFD state and the mirror operators we find that both experiments are governed by the expectation value of **exactly the same** string of ordinary CFT operators $\chi(\phi, \mathcal{O})$. Moreover, in stat-mech we have

$$C' = \mathrm{Tr}[\rho_m \mathcal{X}(\phi, \mathcal{O})] + O(e^{-S})$$

Condition for CFT correlators

$$C = \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}\mathcal{X}(\phi, O)] \qquad C'' = \mathrm{Tr}[\rho_m \mathcal{X}(\phi, O)]$$

A necessary condition for horizon of typical BH mircostate to be smooth is

$$\boxed{\lim_{N\to\infty} C = \lim_{N\to\infty} C''}$$

keeping frequencies $\omega < \omega_*$.

- Not obvious, trace-distance $||\rho_\beta - \rho_m||$ between ensembles is almost maximal.
- $\mathcal{X}(\phi, O)$ is a complicated observable, product of operators at time separation $\Delta t \sim \beta \log S$
- Condition is related to whether $\mathcal{X}(\phi, O)$ obeys Eigenstate Thermalization Hypothesis (ETH)

$$\langle E_i|\mathcal{X}|E_j\rangle = f(E_i)\delta_{ij} + R_{ij}e^{-S/2} \tag{3}$$

with $\frac{df}{dE} \sim O(1/S)$

Condition for CFT correlators

- Interesting effect comes from subleading corrections of the form

$$\frac{1}{N^2} e^{\frac{2\pi t}{\beta}}$$

  At scrambling time they become O(1).

  Are these "chaos-enhanced" $1/N^2$ corrections the same in typical pure states and thermal ensemble?

- Our condition requires that correlators agree *even after analytic continuation* by $t \to t - i\frac{\beta}{2}$ (keeping frequencies up to ω_*)

Evidence

1. ETH holds for products of operators at small time separation. We can show that it also holds for very large time separations (when chaos saturates). It is natural to expect that it holds for intermediate times of order $\beta \log S$

2. In 2d CFTs with large c and sparse spectrum correlators are dominated by Virasoro identity block. In this case the conjecture is true.

3. Numerical evidence in SYK model

The SYK model

N-Majorana fermions in $0+1$d

$$\{\psi^i, \psi^j\} = \delta^{ij}$$

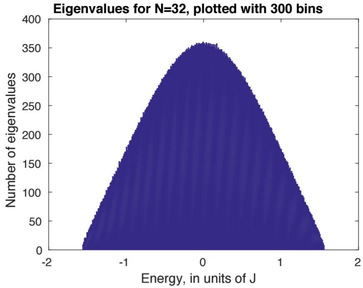
$$H = \sum_{ijkl} J_{ijkl} \psi^i \psi^j \psi^k \psi^l$$

where J_{ijkl} random couplings

$$dim\mathcal{H} = 2^{\frac{N}{2}}$$

Flows to strongly coupled CFT in IR

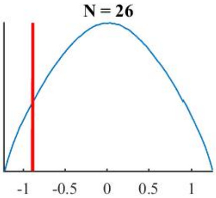
Model of black hole in AdS_2



[figure from Maldacena, Stanford]

The mirror operators in the SYK model

Typical state in SYK



$$|\Psi\rangle = \sum_i c_i |E_i\rangle$$

Introduce the spin operators [Kourkoulou,Maldacena]

$$S_k = 2i\ \psi_{2k-1}\psi_{2k}$$

The mirror operators in the SYK model

$$
\begin{aligned}
|1\rangle &= |\Psi_0\rangle\,, \\
|2\rangle &= S_{1,\omega_1}\,|\Psi_0\rangle\,, \\
|3\rangle &= S_{1,\omega_2}\,|\Psi_0\rangle\,, \\
&\;\vdots \\
|n\rangle &= S_{2,\omega_1}\,|\Psi_0\rangle\,, \\
|n+1\rangle &= S_{2,\omega_2}\,|\Psi_0\rangle\,, \\
&\;\vdots \\
|l\rangle &= S_{2,\omega_2}S_{1,\omega_1}\,|\Psi_0\rangle\,, \\
&\;\vdots
\end{aligned}
\tag{4}
$$

The mirror operators in the SYK model

To simplify the notation, we denote these states as

$$|I\rangle \equiv \mathcal{O}_I|\Psi_0\rangle, \tag{5}$$

where $\mathcal{O}_I$ is a combination of the spin operators introduced above. We define

$$G_{IJ} \equiv \langle I|J\rangle$$

and

$$B_{IJ,k\omega} \equiv \langle I|\, \widetilde{S}_{k,\omega}\, |J\rangle\,, \tag{6}$$

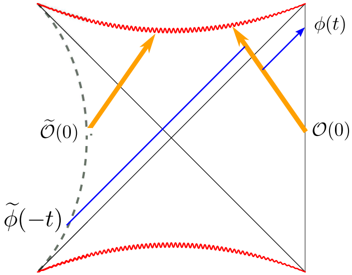
or using the equations for the mirror operators

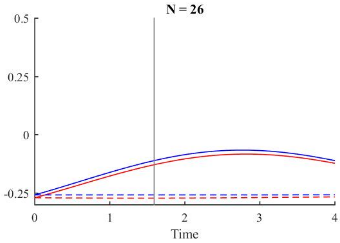
$$B_{IJ,k\omega} = \langle\Psi_0|O_I^\dagger O_J e^{-\frac{\beta H}{2}} S_{k,\omega} e^{\frac{\beta H}{2}}|\Psi_0\rangle. \tag{7}$$

Finally we can represent the mirror operators explicitly as

$$\widetilde{S}_{k,\omega} = G^{IJ} B_{JK,k\omega} G^{KL}\, |I\rangle\, \langle L|\,. \tag{8}$$

Extracting particle from behind the horizon





Relation to Kourkoulou-Maldacena

They consider a class of **a-typical**, non-equilibrium states in the SYK model

$$e^{-\frac{\beta H}{2}}|B_s\rangle \qquad \text{where} \qquad S_k|B_s\rangle = s_k|B_s\rangle$$

On these states they consider the (state-dependent) perturbation of the form

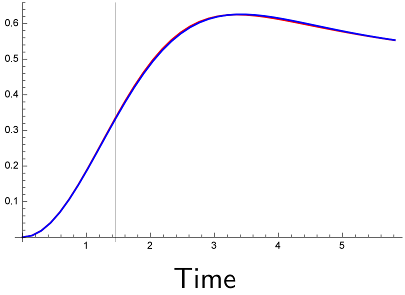
$$\delta H = g\sum_k s_k S_k$$

and they argue that this exposes part of the region behind the horizon.
In fact, this thought experiment is closely related to the perturbations

$$\delta H = g\mathcal{O}\widetilde{\mathcal{O}}$$

that we discussed earlier.

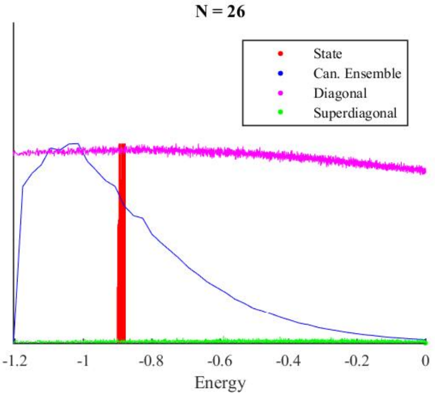
Pure vs thermal state OTOC in SYK



$$\langle \{\psi^i(t), \psi^i(0)\}^2 \rangle$$

on thermal state (red) vs typical pure state (blue).

ETH for chaotic observables in SYK



Matrix elements in SYK of

$$\{\psi^i(t), \psi^i(0)\}^2$$

for $t \approx \beta \log S$

Recovering information from a black hole

We throw a qubit into black hole. How long do we need to wait to recover the information from Hawking radiation?

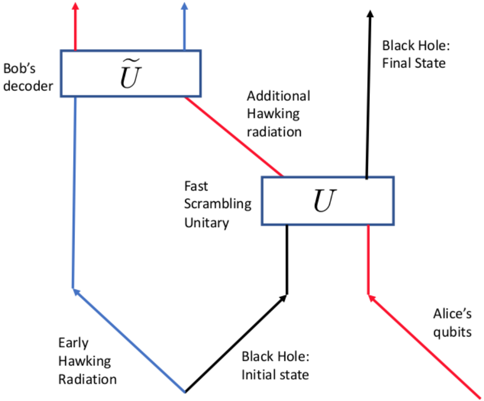
$$t_{evap} \sim G^2 M^3$$

Hayden Preskill (2007): if we have access to more than half of Hawking radiation we only need to wait scrambling time

$$t_S \sim GM \log S$$

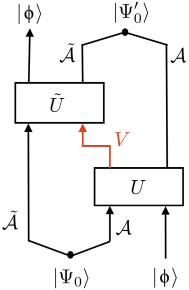
to recover information. For the protocol to work we need to know the initial state of the black hole.

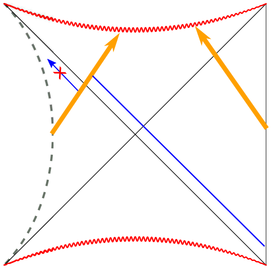
Hayden-Preskill protocol



Reformulated by Maldacena-Stanford-Yang in terms of traversable wormholes

A realization of Hayden-Preskill





We throw qubit $\phi(-t_s)$ into black hole

At $t = 0$ we act with $\mathcal{O}\widetilde{\mathcal{O}}$

After scrambling time we can extract the quantum information of the qubit my measuring operator $\widetilde{\phi}(t_s)$.

This provides an explicit decoding Hayden-Preskill protocol

Knowledge of the quantum state related to state-dependent $\widetilde{\mathcal{O}}$.

Summary

- The nature of space-time behind the horizon remains mysterious
- This question becomes particularly sharp for typical black hole microstates in AdS
- Presented a proposal for their geometry, by making use of state-dependent operators.
- Developments related to traversable wormholes: new calculational tools to probe BH interior
- Interesting connections with quantum teleportation, thermalization and quantum chaos in pure states.