Geometrization of relevance

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Technion

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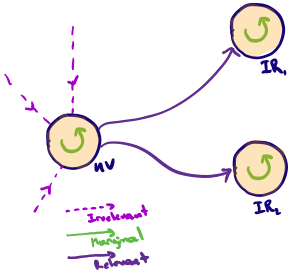
Deformations and CFTs

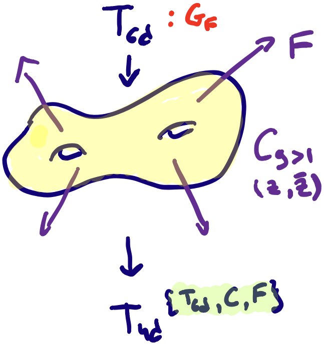
- CFTs have a variety of interesting operators
- Relevant trigger flow to a different fixed point
- Marginal change parameters of a CFT
- Irrelevant are irrelevant
- These operators serve as a road system in the space of theories



- Is there a way to understand the structure of this road system?

Geometric engineering of $\mathcal{N} = 1$ SCFTs

- This talk $4d$ $\mathcal{N} = 1$
- Many known and previously unknown models can be engineered by compactifications from six dimensions
- Working conjecture: ALL supersymmetric CFTs can be obtained in this kind of reductions for proper choice of six dimensional model and compactification



- The plan is to understand how we can deduce the spectrum of low lying, relevant and marginal, operators in $4d$ starting from $6d$

KK reduction for operators

- $6d$ theory on compact space flows to a $4d$ effective theory which might flow to interacting CFT
- Think about $6d$ CFT as yet another UV starting point to produce $4d$ IR CFT
- Dimensions of operators change along the flow
- Moreover, local operators in $4d$ can come from local in $6d$ or from surface operators wrapping C
- Any hope to match operators from $6d$ to $4d$ is to focus on protected operators
- We will make a prediction about a cohomology of some supercharge Q in $4d$ (index) starting from cohomology in $6d$: KK reduction of BPS operators

6d: $\mathcal{O}$

$\downarrow$

4d: $\mathcal{O}[\Omega] = \int \mathcal{O}\,\Omega$

KK for operators

Deformations from $6d$

- SCFTs in $6d$ have no relevant or marginal deformations (Cordova, Dumitrescu, Intriligator 16)
- However, they do have general interesting operators, energy-momentum tensor and conserved currents

- We will claim that in general compactifications a very robust class of relevant and marginal deformations comes from KK reductions of energy-momentum tensor and conserved currents

- There can be additional deformations coming from reduction of other operators for low genus and/or in some limits of the flux
- The derivation is elementary and echoes many localization results (See e.g. Benini, Zaffaroni 15)
- We will have a very simple but general set of physical claims

$(1,0)$ supersymmetry

- $Spin(6) = SU(4)$, R-symmetry is $su(2)$
- Supercharges Q^I_a $(I = 1, 2,\ a = 1, 2, 3, 4)$: $\{Q^I_a,\, Q^J_b\} \sim \epsilon^{IJ} P_{ab}$

- Reduction to four dimensions on surface $C_{g>1}$, twist $j'_C = j_C + \frac{1}{2}R$
- Supercharges split into four scalars on C, two $(1,0)$ forms and two $(0,1)$ forms

(Here $\mathbf{4} = \mathbf{2}^+_1 \oplus \mathbf{2}^-_2$ under $so(6) = so(4) \times so(2)_C$ decomposition)

	j_1	j_2	j_C	R	j'_C
Q_α	$\pm\frac{1}{2}$	0	$\frac{1}{2}$	-1	0
$\widetilde{Q}_{\dot\alpha}$	0	$\pm\frac{1}{2}$	$-\frac{1}{2}$	1	0
$Q^{(1,0)}_\alpha$	$\pm\frac{1}{2}$	0	$\frac{1}{2}$	1	1
$\widetilde{Q}^{(0,1)}_{\dot\alpha}$	0	$\pm\frac{1}{2}$	$-\frac{1}{2}$	-1	-1

- In particular

$$\{Q_\alpha,\, Q^{(1,0)}_\beta\} \sim \epsilon_{\alpha\beta} P_z, \quad \{\widetilde{Q}_{\dot\alpha},\, \widetilde{Q}^{(0,1)}_{\dot\beta}\} \sim \epsilon_{\dot\alpha\dot\beta} P_{\bar z}$$

BPS operators in $6d$

- We consider local operators in $6d$ $\mathcal{O}(x; z, \bar{z})$ such that $Q \equiv \widetilde{Q}_{\dot{-}}$ annihilates them but they are not Q exact
- The operators have some charge under the R-symmetry and the flavor symmetry G_F
- As we turn on non trivial bundles for the flavor symmetry we should think of the operators as taking value in some holomorphic vector bundle $\mathcal{V}_{\mathcal{O}}$ determined by its charges
- This in particular means that all the derivatives we consider are proper covariant derivatives $\partial \to \partial_{\mathcal{V}_{\mathcal{O}}}$ when acting on operator $\mathcal{O}$

$$[\{Q_\alpha, Q_\beta^{(1,0)}\}, \mathcal{O}] \sim \partial_{\mathcal{V}_{\mathcal{O}}} \cdot \mathcal{O}, \qquad [\{\widetilde{Q}_{\dot{\alpha}}, \widetilde{Q}_{\dot{\beta}}^{(0,1)}\}, \mathcal{O}] \sim \bar{\partial}_{\mathcal{V}_{\mathcal{O}}} \cdot \mathcal{O}$$

Primary reduction

- We consider smearing a BPS operator over the surface C

$$\mathcal{O}[\Omega](x) = \int_C \Omega \cdot \mathcal{O}(x; z, \bar{z})$$

- Here $\Omega \in \Omega^{1,1}(C, V^*_{\mathcal{O}})$ is an appropriate form on the surface
- Given a BPS operator in $6d$ we obtain a BPS operator in four dimensions labeled by a forms Ω on C

- Note that if $\Omega = \partial\eta$ then,

$$\mathcal{O}[\Omega](x) = -\int_C \eta \cdot \bar{\partial}_{\mathcal{V}_{\mathcal{O}}} \cdot \mathcal{O} \sim Q\left(\int_C \eta\, \widetilde{Q}^{(0,1)}_{+} \cdot \mathcal{O}\right)$$

- In particular exact forms give Q-cohomologically trivial operators
- The number of independent BPS operators obtained is counted by $h^0(C, \mathcal{V}_{\mathcal{O}})$

Secondary reduction

- Let us define for any BPS operator $\mathcal{O}$

$$\mathcal{O}^{(0,1)} = \widetilde{Q}^{(0,1)}_{\dot{+}} \cdot \mathcal{O}$$

- Using the supersymmetry relations

$$Q \cdot \mathcal{O}^{(0,1)} \sim \bar{\partial}_{\mathcal{V}_{\mathcal{O}}} \cdot \mathcal{O}$$

- From here smearing the new operator using a $(1,0)$ form ω

$$Q \cdot \mathcal{O}^{(0,1)}[\omega] = -\mathcal{O}[\bar{\partial}\omega]$$

- Thus for any closed ω we get an additional BPS operator in $4d$
- The number of independent BPS operators obtained is counted by $h^1(C, \mathcal{V}_{\mathcal{O}})$
- Note that operators obtained here have opposite fermion number to the ones obtained in the primary reduction

Counting in $4d$

- An useful way to count protected operators in $4d$ is to compute the supersymmetric index
- The index is just some measure defined on cohomology of some supercharge Q
- Taking the same supercharge as before,

$$\mathcal{I}(q,p,z_i) = Tr_{\mathbb{S}^3}(-1)^F q^{j_2-j_1+\frac{1}{2}R} p^{j_2+j_1+\frac{1}{2}R} \prod_{i=1}^{rank(G_F)} z_i^{\mathfrak{q}_i}$$

- Note $Q = \widetilde{Q}_{\dot{-}}$ has $(j_1, j_2, R) = (0, -\frac{1}{2}, 1)$ and thus the above charges vanish for it. We turn only chemical potentials for charges which commute with given supercharge
- Note also that $\widetilde{Q}_{\dot{+}}^{(0,1)}$ has $(j_1, j_2, R) = (0, \frac{1}{2}, -1)$ and thus the above charges vanish for it
- This means that $\mathcal{O}[\Omega]$ and $\mathcal{O}^{(0,1)}[\omega]$ contribute to the index with same weight but opposite sign

Prediction for the index

- Given a six dimensional BPS operator $\mathcal{O}$ with certain charges (j_1, j_2, R) and flavor charges $\mathfrak{q}_i$

- We can construct four dimensional BPS operators which will contribute to the index as

$$(-1)^{F_{\mathcal{O}}} \left(h^0(C, \mathcal{V}_{\mathcal{O}}) - h^1(C, \mathcal{V}_{\mathcal{O}})\right) q^{j_2 - j_1 + \frac{1}{2}R} p^{j_2 + j_1 + \frac{1}{2}R} \prod_{i=1}^{rank(G_F)} z_i^{\mathfrak{q}_i}$$

- Now we can use Riemann-Roch theorem to simplify this,

$$h^0(C, \mathcal{V}_{\mathcal{O}}) - h^1(C, \mathcal{V}_{\mathcal{O}}) = 1 - g + deg(\mathcal{V}_{\mathcal{O}})$$

- This is a very simple number to compute and it is solely determined by the charges of the operator and by the flux turned on the surface

Conserved currents

- Models with G_F have a BPS conserved current multiplet
- The conserved current multiplet has a scalar component annihilated by Q with charges $(j_1, j_2, R, j_C) = (0, 0, 2, 0)$
- This means that $\mathcal{V}_{\mathcal{O}} = K_C \otimes \mathcal{L}_{adj}$.
- From here taking the character of the adjoint of G_F to be

$$\chi_{adj}(z_i) = \sum_{h=1}^{dim G_F} \prod_{i=1}^{rank G_F} z_i^{\mathfrak{q}_i^h}$$

we obtain for each component

$$1 - g + deg(\mathcal{V}_{\mathcal{O}}) = 1 - g + 2g - 2 + \sum_{j=1}^{rank G_F} F_j \mathfrak{q}_j^h$$

- where $(F_1, \cdots, F_{rank G_F})$ are fluxes in $U(1)$ subgroups of G_F

Energy-Momentum tensor

- All models have a BPS energy-momentum tensor multiplet
- The energy-momentum tensor multiplet has a component annihilated by Q with charges implying that $\mathcal{V}_{\mathcal{O}} = K_C^{\otimes 2}$
- We obtain then

$$1 - g + deg(\mathcal{V}_{\mathcal{O}}) = 1 - g + 2(2g - 2) = 3g - 3$$

- For a general theory conserved currents and energy-momentum tensor will be the lowest BPS operators

General prediction for the index

- Prediction for the index using six dimensional R-symmetry,

$$1+qp\left(3g-3+\sum_{h=1}^{dimG_F}\left[g-1+\sum_{j=1}^{rankG_F}F_j\mathfrak{q}_j^h\right]\prod_{i=1}^{rankG_F}z_i^{\mathfrak{q}_i^h}\right)+\cdots$$

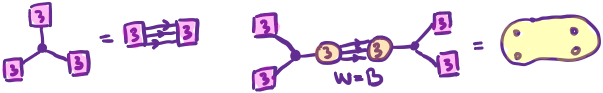
For marginals see also (SSR, Vafa, Zafrir 16)

- Using superconformal R-symmetry in $4d$ the index has the form (Beem-Gadde 2012)

$$1+(relevants)(qp)^{\#<1}+(Marginals-Currents)qp+\cdots$$

- Superconformal R-symmetry obtained by a maximization
- Generally operators not charged under any abelian symmetry will be (marginals-currents), the rest split to relevants and irrelevants
- $6d$ energy-momentum tensor and conserved currents predict the marginal and the relevant operators in a generic compactification

Example: $6d$ is Minimal $SU(3)$ SCFTs



- Consider a compactification of $6d$ theory with no flavor symmetry
- An example is pure glue $SU(3)$ $(1,0)$ SCFT

- $4d$ theories are quiver theories built from trifundamentals of $SU(3)$ (SSR, Zafrir 2018)

- From our general considerations $I = 1 + (3g - 3)qp + \cdots$

- As theories have simple Lagrangians can be verified that it is indeed the case (no chiral relevant operators)

- GL 1: No symmetry or zero flux implies no relevant deformations in a generic compactification: Supersymmetric dead-end models

Example: $6d$ is $(2,0)$

- Take $6d$ to be some $(2,0)$ SCFT. In $(1,0)$ language $G_F = su(2)$, have flux F for the Cartan $u(1)$

$$\chi_{adj}(x) = x^2 + x^{-2} + 1$$

$$I = 1+\left(3g-3+g-1+(g-1+2F)x^2+(g-1-2F)x^{-2}\right)qp+\cdots$$

- Matches perfectly computations of the index for general g and F
- Low g and F might have contributions from other operators (E.g. A_1, $g=2$, $F=1$)
- After a-maximization $g-1+2F$ rel, $3g-3+g-1$ marg−curr
- GL 2: The robust spectrum of relevant and marginal operators depends only on group theory and geometry, other details such as the type of $(2,0)$ do not enter
- GL 3: Turning on a robust relevant deformation breaks a $u(1)$ symmetry which has a flux and we flow to theory with that flux zero
- GL 4: $F=0$, $I = 1+(3g-3+dimG_F(g-1))qp+\cdots$, abelian symmetries do not lead to relevant deformations

Example: $6d$ is two $M5$ on $\mathbb{Z}_2$

- $G_F = so(7)(\supset su(2)_\beta \times su(2)_\gamma \times u(1)_t)$

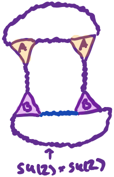
$$\chi_{adj} = 2 + \gamma^2 + \frac{1}{\gamma^2} + t + \frac{1}{t} + \mathbf{3}_\beta + (\gamma + \frac{1}{\gamma})(1 + t + \frac{1}{t})\mathbf{2}_\beta$$

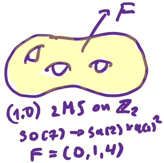
- Computed index $g = 3,\ F = (0, 1, 4)$ (SSR, Vafa, Zafrir 2016)

$$I = 1 + \left(10 + 2\ \mathbf{3}_\beta + (7t\gamma + 3\gamma - 3\frac{1}{t\gamma} + 5\frac{t}{\gamma} - \frac{\gamma}{t} + \frac{1}{\gamma})\mathbf{2}_\beta + 4\gamma^2 + 6t - 2\frac{1}{t}\right) pq + \cdots$$

- a-maximization: $R_{sc} = R_{6d} - 0.0657051 q_\gamma - 0.320467 q_t$

$2F_\gamma + g - 1 = 4$, $F_\gamma + 2F_t + g - 1 = 7$,
$-F_\gamma + F_t + g - 1 = 5$, $-F_t + g - 1 = -2$,
$-2F_\gamma + g - 1 = 0$: Perfect agreement





- GL 5: Charting the dictionary between $6d$ and $4d$ the match of relevant and marginal operators is extremely useful. For general flux these are $dimG_F - rankG_F$ numbers that need to match

Example: $6d$ is rank Q E-string

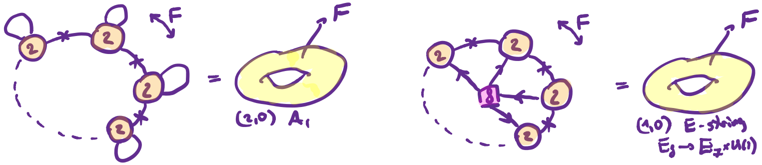
- Consider rank Q E-string. $G_F = su(2) \times e_8$ for $Q > 1$
- For $Q > 1$ no field theory construction is known for general compactification
- Can predict anomalies of $T_{4d}[T^{6d}, C, F]$ and also the index at low orders
- Say take flux $F > 0$ for Cartan of $su(2)$

$$I = 1 + ((g-1+2F)x^2 + (\mathbf{248}_{e_8}+1)(g-1) + 3g-3 + (g-1-2F)x^{-2})qp + \cdots$$

- This is a prediction that field theory constructions will have to satisfy. Note that the rank Q of the $6d$ SCFT does not enter as it is not effecting group theory or geometry.

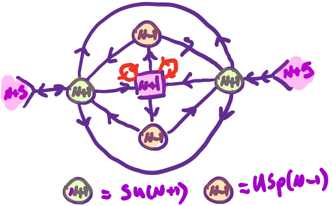
- GL 6: Can generate many predictions for deformations of $4d$ SCFTs

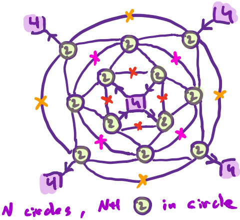
Example: torus



- In principle all we said can be applied to $g = 1$
- Comparing to explicit computation one typically finds that although we see the pattern of states predicted here, there are typically operators not coming from energy-momentum and currents, and some of the operators are missing
- For example, in the above the operators winding the circle have $6d$ R symmetry zero and their other charges scale with flux.
- GL 7: An explanation for these operators is that they might come from surface defects wrapping the torus, and such operators might cause extra cancelations removing some of the operators which we naively predict

Example: torus and duality





- D_{N+3} minimal conformal matter on torus, $G_F = so(4N+12)$ (Kim, SSR, Vafa, Zafrir 18)
- Flux breaking to $so(2N+10) \times su(N+1) \times u(1)_t$
- There are two different ways to construct the theory which leads to cute $4d$ duality (N odd and $=3$ below)

$$I = 1 + 3t^{-4}(\mathbf{1},\mathbf{1})(qp)^0 + \cdots + (2(\mathbf{6},\mathbf{1})(t^2 - t^{-2}) + (\mathbf{4},\mathbf{16})t - (\bar{\mathbf{4}},\mathbf{16})t^{-1})qp + \cdots$$

$$\chi^{so(24)}_{adj} = 1 + \chi^{so(16)}_{adj} + \chi^{su(4)}_{adj} + (\mathbf{4},\mathbf{16})t + (\bar{\mathbf{4}},\mathbf{16})t^{-1} + (\mathbf{6},\mathbf{1})(t^2 + t^{-2})$$

- $2(\mathbf{6},\mathbf{1})t^2$ come from free fields
- GL 8: Free fields are important

Summary and Outlook

Summary:

- $4d$ $\mathcal{N}=1$ SCFTs obtained from compactifications have typically a very robust set of relevant and marginal deformations
- This spectrum can be deduced from reduction of energy-momentum and flavor symmetry currents
- General physical lessons can be drawn from this

Outlook:

- Adding punctures (less generic but important)
- More operators (SSR, Sabag 18)
- Understanding defects
- Other dimensions
- Large N (Gaiotto, Rastelli, SSR unpublished)

Thank You

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