Dualities in $3d$ $\mathcal{N}=1$ Chern-Simons-Matter Theories

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Work with: V. Bashmakov, J. Gomis, Z. Komargodski (1802.10130)
C. Choi and M. Roček (1808.02184)
O. Aharony (to appear)

(Partial List of) References

[Gur-Ari, Jacoby, 2012]
[Armoni, Hollowood, 2006, 2006]
[Jain, Minwalla, Yokoyama, 2013]
[Avdeev, Grigoryev, Kazakov 1992; Avdeev, Kazakov, Kondrashuk, 1993]
[Inbasekar, Jain, Mazumdar, Minwalla, Umesh, Yokoyama, 2015]
[Aharony, Gur-Ari, Jacoby, 2011, 2012]
[Gomis, Komargodski, Seiberg, 2017]
[Gaiotto, Komargodski, Wu, 2018]
[Benini, Benvenuti, 2018, 2018]
[Aharony, Jain, Minwalla, 2018]

Motivation

Wide range of $3d$ bosonization dualities. Schematically:

$$U(N)_{k-N,k} + N_b\,\phi + N_f\,\psi \longleftrightarrow U(N')_{k'+N',k'} + N_f\,\phi + N_b\,\psi$$

plus additional dualities of the form $SU \leftrightarrow U$ and so on.

Question 1: Can they be proven? Not yet, but many checks:

- Many checks at large-N 't Hooft limit (e.g. matching of free energies and some correlation functions)
- Some finite-N checks (e.g. phases and anomaly matching)
- Exact checks at $\mathcal{N} = 2$ SUSY points

Question 2: Can they be made more precise? These are IR dualities, what are the fixed points?

Motivation

Focus on supersymmetric versions.

$$\mathcal{N} = 2$$

$$U(N)_k + N_f\ \Phi \leftrightarrow U(N')_{k'} + N_f\ \Phi$$

$$\mathcal{N} = 0$$

$$U(N)_{k-N,k} + N_f\ \phi, \psi\ \leftrightarrow U(N')_{k'+N',k'} + N_f\ \phi, \psi$$

Motivation

Focus on supersymmetric versions.

"boring"

$$\mathcal{N} = 2$$

$$U(N)_k + N_f\ \Phi \leftrightarrow U(N')_{k'} + N_f\ \Phi$$

"easy"

Interesting

$$\mathcal{N} = 0$$

$$U(N)_{k-N,k} + N_f\ \phi, \psi\ \leftrightarrow U(N')_{k'+N',k'} + N_f\ \phi, \psi$$

hard

Focus on supersymmetric versions.

“boring” **“easy”**

$$\mathcal{N} = 2$$

$$U(N)_k + N_f\ \Phi \leftrightarrow U(N')_{k'} + N_f\ \Phi$$

$$\mathcal{N} = 1$$

$$U(N)_{k-\frac{N}{2},k} + N_f\ \Phi \leftrightarrow U(N')_{k'+\frac{N'}{2},k'} + N_f\ \Phi$$

$$\mathcal{N} = 0$$

$$U(N)_{k-N,k} + N_f\ \phi,\psi \leftrightarrow U(N')_{k'+N',k'} + N_f\ \phi,\psi$$

Interesting **hard**

$3d$ $\mathcal{N} = 1$ SUSY

$3d$ $\mathcal{N} = 1$ theories have two supercharges $\Longrightarrow$ "not enough" SUSY. In particular:

- Superpotential isn't holomorphic, so no non-renormalization theorems.
- No localization techniques.

However, there are still some strong constraints:

- Still have some exact results, like Witten index.
- SUSY vacua must have zero energy $\Longrightarrow$ phase transitions must be second order (or higher).
- Recently, new exact results (exact moduli spaces and enhanced SUSY).

Outline

Study $3d$ $\mathcal{N}=1$ $SU(N)_k$ CS-matter theories with N_f fundamental flavors.
Parameters of the theory: k, N, masses m, superpotential terms W.

Focus on two regimes:

- **Part I**: weakly-coupled limits (large vev, large k, large m...). $W=0$, $m \neq 0$. Focus on phase diagram. [Benini's talk]
- **Part II**: Large-N 't Hooft limit. $W \neq 0$, $m=0$. Focus on beta functions. [Jain's talk]

Part I: Weakly-Coupled Limits

Take $\mathcal{N}=1$ $SU(N)_k$ with N_f fundamentals, and set $W=0, m\neq 0$. Study the phases as we vary the mass m.
Start with "easy part":

Note:

- Pure $\mathcal{N}=1$ CS breaks SUSY for small CS level, so vacua at large $|m|$ may be SUSY-preserving or SUSY-breaking.
- Witten index must jump.

For small $|m|$ semiclassical calculation isn't enough. Need heavier machinery: the Coleman-Weinberg (super)-potential.

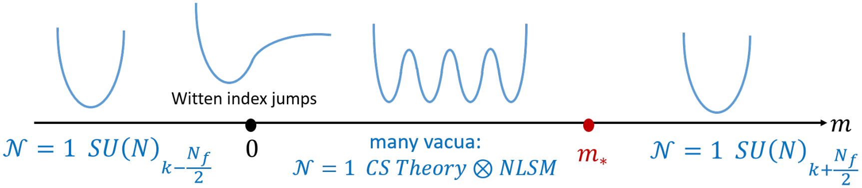
Phase Diagram

A 1-loop computation gives the Coleman-Weinberg superpotential:

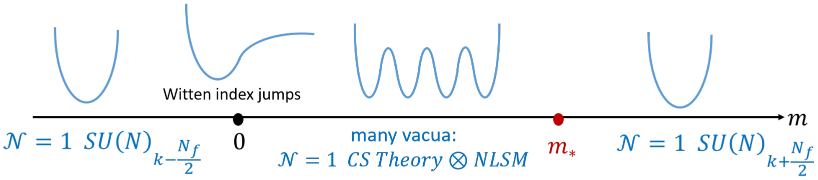
$$W \sim m|\Phi_i|^2 + \mathrm{Tr}\sqrt{\delta^{ab} + \bar{\Phi}_i T^{(a} T^{b)} \Phi_i}$$

This approximation is valid for large vev for ϕ.

Can now find vacua by solving the F-term equations $W' = 0$:



Phase Transition

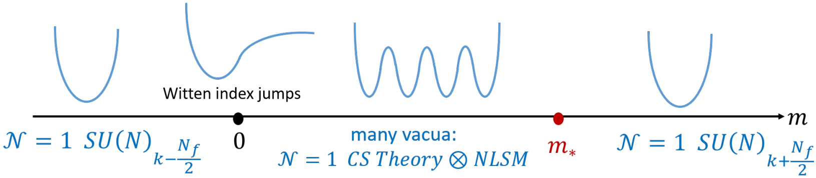


Focus on phase transition point m_*. Note:

- Surprising result: existence of a single phase transition where many vacua coalesce. A careful analysis of the symmetries of the problem shows that higher-order corrections to W cannot change this.
- Witten index now jumps only once, at $m = 0$, which can be expected.

General phase structure is thus exact at all loop order.

Dualities



Phase transitions between SUSY vacua must be second order (or higher), so can guess $\mathcal{N}=1$ dualities:

$$U(N)_{k-\frac{N+N_f}{2},k-\frac{N_f}{2}} + N_f\,\Phi \longleftrightarrow U(k-N)_{-\frac{k+N-N_f}{2},-k+\frac{N_f}{2}} + N_f\,\tilde{\Phi}$$

plus $SU \leftrightarrow U$ dualities and so on.

At $N_f = 1$ these coincide with the $\mathcal{N}=1$ bosonization dualities proposed using large-N 't Hooft limit techniques in [Jain, Minwalla, Yokoyama]. In special cases also reduce to well-known $\mathcal{N}=2$ dualities.

Dualities

$\mathcal{N}=1$ dualities:

$$U(N)_{k-\frac{N+N_f}{2},k-\frac{N_f}{2}}+N_f\ \Phi \longleftrightarrow U(k-N)_{-\frac{k+N-N_f}{2},-k+\frac{N_f}{2}}+N_f\ \tilde{\Phi}$$

Other checks:

- Many checks at large N for $N_f=1$
- Many checks when these reduce to $\mathcal{N}=2$ dualities
- Can analyze flows from $\mathcal{N}=2$ SUSY dualities at large k

Some interesting conclusions:

- Many examples of emergent symmetry and emergent SUSY
- Some examples with emergent time reversal ($k=2N$). Moduli spaces? [Gaiotto, Komargodski, Wu]

Part II: Large-N 't Hooft limit

Now work in the large-N 't Hooft limit:

$$N, \kappa \to \infty, \quad \lambda = \frac{N}{\kappa} = \text{fixed}$$

This time set all masses to zero.

For $N_f = 1$ there is one classically marginal superpotential couplings:

$$(\bar{\Phi}^a \Phi_a)^2$$

For $N_f > 1$ there are two:

$$(\bar{\Phi}^a_i \Phi^i_a)^2, \qquad (\bar{\Phi}^a_i \Phi^j_a)(\bar{\Phi}^b_j \Phi^i_b)$$

Goal: β functions for all marginal couplings.

Original Plan

At $N_f = 1$, there is a single classically marginal operator $\frac{\pi\lambda\omega}{N}(\bar{\Phi}^a\Phi_a)^2$.

Plan was to use effective action for $J_0 = \bar{\Phi}^a\Phi_a$ to calculate β function at large N [Aharony, Jain, Minwalla]. Requires the leading order results for:

- $\langle J_0 J_0 \rangle$ ✓
- $\langle J_0 J_0 J_0 \rangle$ ✓
- $\langle J_0 J_0 J_0 J_0 \rangle$ X

also studied by [Inbasekar, Jain, Nayak, Sharma]. Results are **duality-invariant**.

Not good enough for β functions (yet)...

β Function at $N_f = 1$

What can still be done?

Start with the beta function at weak coupling:

$$\lambda \to 0, \quad \tilde{\omega} = \pi\lambda\omega \text{ fixed}$$

Integrating out the gauge field we find a Φ^4 theory, with coupling $\frac{\tilde{\omega}}{N}(\bar{\Phi}^a\Phi_a)^2$. We find:

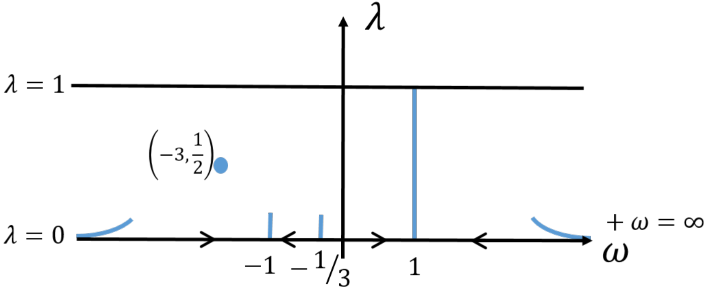
$$\beta_{\tilde{\omega}} = -\frac{1}{N}\frac{16\tilde{\omega}^3\left(\tilde{\omega}^2 - 48\right)}{\pi^2\left(\tilde{\omega}^2 + 16\right)^2}$$

In particular, there are six roots:

$$\tilde{\omega} = 0, 0, 0, \pm\sqrt{48}, \infty$$

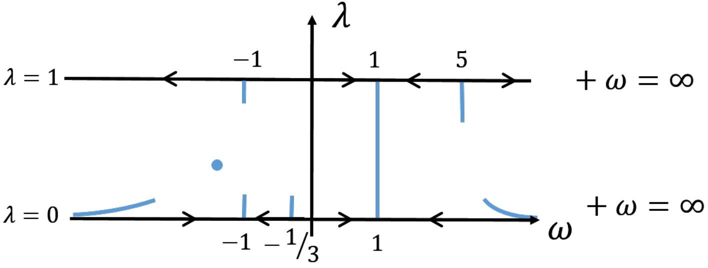
β Function at $N_f = 1$

With some help from [Avdeev, Kazakov, Kondrashuk], the results can be extended to ω and to leading order in λ:



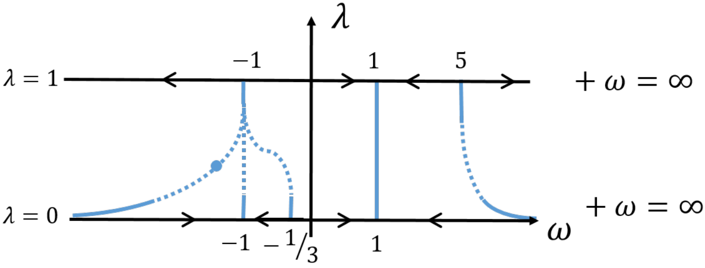
β Function at $N_f = 1$

And using the duality ($\lambda \to \lambda - \text{sign}\lambda$, $\omega \to \frac{3-\omega}{1+\omega}$):



β Function at $N_f = 1$

So the guess for general λ:



up to pairs of roots that can appear and annihilate.

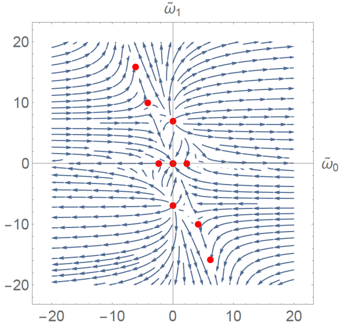
β Function at $N_f > 1$

Now take $N_f > 1$. Two couplings this time:

$$\omega_0(\bar{\Phi}_i^a \Phi_a^i)^2, \qquad \omega_1(\bar{\Phi}_i^a \Phi_a^j)(\bar{\Phi}_j^b \Phi_b^i)$$

Again, in the limit $\lambda \to 0$, we have Φ^4 theory (with two couplings):

$$\lambda \to 0, \quad \tilde{\omega}_i = \pi \lambda \omega_i \text{ fixed}$$



β Function at $N_f > 1$

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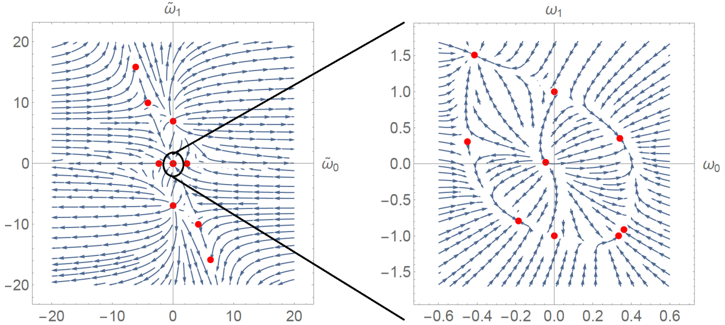
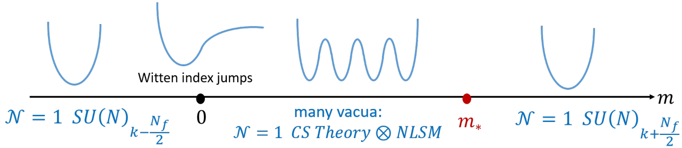


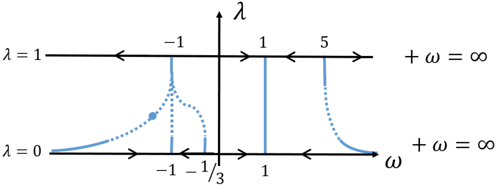
Summary

3d $\mathcal{N} = 1$ theories have a rich structure but are also simple enough to allow for computations. In particular we have calculated:

- The phase diagram



- Some gauge-invariant correlators of $J_0 = \bar{\Phi}\Phi$ ('t Hooft limit)
- The beta functions ('t Hooft limit)



These give further evidence for the $\mathcal{N} = 1$ bosonization dualities.

Thank You!