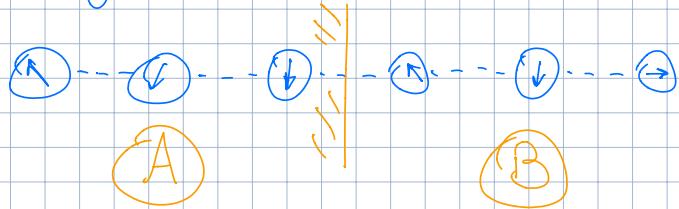


(1) Dynamics of MB systems.

Problem: too many (excessive) states!

(2) Strategy: Truncation. 1D.



Look at density matrix:

$$\rho = (\rho_{AB})_{2^n \times 2^n} = \sum_{\{\alpha_i\}, \{\beta_i\}} |\alpha_1 \rangle \langle \alpha_2 \dots \alpha_n \rangle \otimes_{i=1}^n |\beta_1 \rangle \langle \beta_2 \dots \beta_n| = |\Psi \rangle \langle \Psi|$$

Trace over A:

$$\rho_B = \sum_{\{\alpha_A\}} \langle \alpha_A | \rho_{AB} | \alpha_A \rangle$$

$$= |\alpha_{n+1} \rangle \langle \alpha_{n+2} | \dots \otimes_{i=n+1}^n |\beta_i \rangle \langle \beta_i|$$

$$= \sum_i |\Psi_i^R \rangle \langle \Psi_i^R|$$

$\{\rho_i\} \rightarrow$ entanglement spectrum

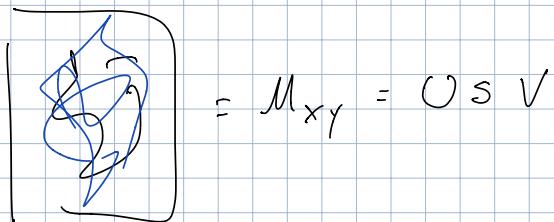
Familiar?

$$-\sum_i \rho_i \ln \rho_i = S$$

Von Neumann Entropy.

What can we cut? Small ℓ_i .

Picture:



Where are the ℓ_i coming from?

$$|\Psi\rangle = \sum s_i |L_i\rangle + |R_i\rangle$$

Can be chosen orthonormal.

SVD interlude.

$$H = H^+ \quad H = U D V^+$$

↑
diagonal

Hermitian Matrix
→ Diagonalize.

Non hermitian ?

$$\begin{array}{c} A \neq A^+ \\ \xrightarrow{\quad} A A^+ = U \Delta_1^2 V^+ \\ \xrightarrow{\quad} A^+ A = V^+ \Delta_2^2 V \end{array}$$

U, V
unitaries

Δ_1^2 and Δ_2^2 have same diagonal 1

entries.

$$\Delta^2 = \begin{pmatrix} d_1^2 & & & \\ & d_2^2 & & \\ & & d_3^2 & \\ & & & \ddots \\ & & & d_n^2 \\ & & & \ddots \end{pmatrix}$$

$$\text{if: } A \in \mathbb{C}^{n,m} \quad \Delta_1^2 \in \mathbb{R}^{n,n} \quad U \in \mathbb{C}^{n \times n}$$

$$\Delta_2^2 \in \mathbb{R}^{m,m} \quad V$$

And:

$$A = U \Delta V$$

$\Delta = \begin{pmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_n \\ & 0 & \dots & 0 \end{pmatrix}$

$\left[\Delta = \begin{pmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_n \\ & 0 & \dots & 0 \end{pmatrix} \right]_n \quad \text{if } m < n,$

A as wave function

$$\psi_{s_1 \dots s_r} = \boxed{1 \quad | \quad 1 \quad | \quad 1 \quad | \quad r \quad | \quad 1 \quad | \quad r+1 \quad | \quad s_r}$$

$$= \psi_{(S_1 \cup S_n), (S_{r+1} \cup S_n)} \rightarrow = \bigcup_{(S_1 \cup S_n)} x \in \bigcup_{x \in V_x(S_r \cup S_n)}$$

The diagram shows two horizontal timelines representing states. The left timeline has tick marks labeled 1, 1, 1, 1, 1. The right timeline has tick marks labeled 1, 1, 1, 1, 1. A curved arrow connects the two timelines. Between them is a circular state node labeled q_x . A horizontal arrow points from the left timeline to the state node, and another horizontal arrow points from the state node to the right timeline. Below the left timeline is the label S_r , below the right timeline is S_{r+1} , and below the state node is ψ_x .

Example : Singlet :

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow_1\rangle |\downarrow_2\rangle - |\downarrow_1\rangle |\uparrow_2\rangle \right) \Rightarrow = \frac{1}{\sqrt{2}} |\uparrow\rangle <\downarrow| - \frac{1}{\sqrt{2}} |\downarrow\rangle <\uparrow|.$$

Truncate x  $\rightarrow d_{\ln}(x) = 2^{\frac{m}{n_1, n_2}}$

\rightarrow MRC

so side to side.

But then:

$$- \boxed{A_1} \otimes \boxed{A_2} \otimes \boxed{A_3} \dots$$

Example:

$$\begin{array}{c}
 \text{AKLT State.} \\
 \begin{array}{ccccc}
 & \text{no spin} & \text{spin } \frac{1}{2} & \text{spin } \frac{1}{2} & \text{spin } \frac{1}{2} \\
 |0\rangle & \xrightarrow{a^+} & |1\rangle & \xrightarrow{b^+} & |1\rangle \\
 & \xleftarrow{b^+} & |0\rangle & \xleftarrow{a^+} & |0\rangle \\
 & & 0 & & -1 \\
 & & \downarrow & & \\
 & & + & & - \\
 & & - & & + \\
 & & & & \\
 & & \prod_i (a_i^+ b_{i+1}^+ - b_i^+ a_{i+1}^+) & &
 \end{array}
 \end{array}$$

Schwingr - Bosons $a^+ |0\rangle = \frac{1}{\sqrt{2}} |1\rangle$
 $\int_2 a^+ a^+ |0\rangle = |1,1\rangle$

$A_0(s_1, s_2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
 $A_{-1} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$
 $A_{+1} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

How to find? Minimize!

or:

TEBD

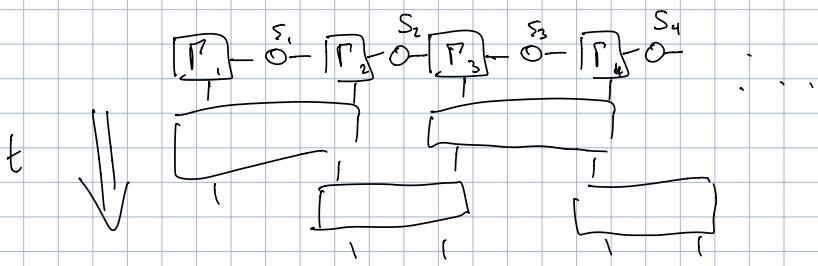
Evolve to Solve

Idea:

$$\prod_{i=1}^n e^{-\beta t H} |\psi_0\rangle = e^{-n\beta t E_{GS}} \cdot |\text{GS}\rangle$$

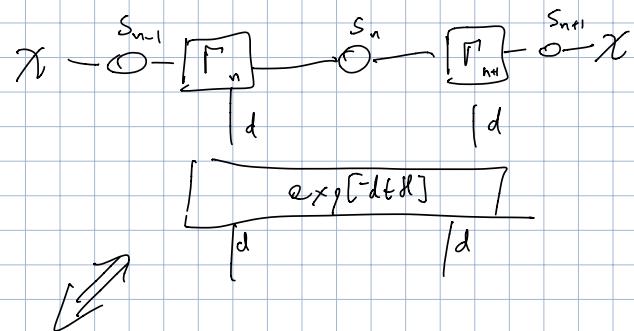
$$e^{-\beta t H} \iff \frac{s_1 | \quad | s_2}{\frac{|e^{-\beta t H}|}{s_1 | | s_2}}$$

DMRG equivalent:



problem: flow down brings back full Hilbert space.

Solution: SVD step.



$$= \bigcup_{\substack{\text{dim: } x_L \\ x_R}} (x_L, s_L) ; (x_R, s_R) = \bigcup_{\alpha} \alpha \cdot s_\alpha \sqrt{\beta} \quad \begin{matrix} \text{all } \alpha, \beta \\ \text{are } \text{real} \\ \text{dim} \end{matrix}$$

$x \uparrow \left(\begin{matrix} s_1 & s_2 \\ \vdots & \vdots \\ s_{xd} & \end{matrix} \right) \uparrow x_{xd}$

truncate!

after truncation:

$$\bigcup_{\alpha} (x_L, s_L) ; (x_R, s_R) \rightarrow \bigcup_{\alpha} (x_L, s_L) ; x_m \cdot D_{x_m} \cdot \sqrt{x_R, s_R}$$

Or:

$$A_n \rightarrow A_n^1 = S_n^{-1} \bigcup_{\alpha} (x_L, s_L) ; x_R$$

$$S_n \rightarrow S_n^1 = D_{x_m}$$

Canonical Form Note

$$A_{n+1} \rightarrow A_{n+1}' = V_{x_m j(S_R X_R)} \cdot S_{n+1}'$$

that:

$$\begin{aligned} & \left(\begin{array}{c} S_{n+1} \\ \oplus \\ S_{n+1}' \end{array} \right) \left(\begin{array}{c} A_n \\ | \\ A_n' \end{array} \right) \left(\begin{array}{c} X_R \\ | \\ X_R' \end{array} \right) = \sum_{X_L, S_L} U_{(X_L, S_L); X_R} \cdot U_{X_R'; (X_L, S_L)}^* \\ & = \delta_{X_R, X_R'} \end{aligned}$$

and:

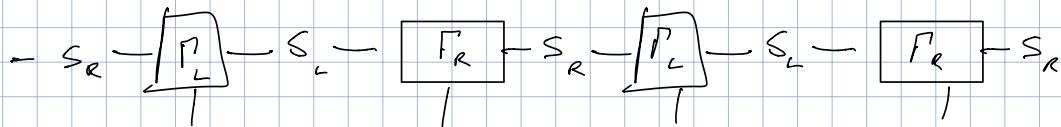
$$\begin{aligned} X_L - \left[\begin{array}{c} A_{n+1} \\ | \\ A_{n+1}' \end{array} \right] - S_{n+1} &= \delta_{X_L, X_L'} \\ X_L' - \left[\begin{array}{c} A_{n+1}' \\ | \\ A_{n+1} \end{array} \right] - S_{n+1} & \end{aligned}$$

This is the canonical form of odd/even ops
in the process.

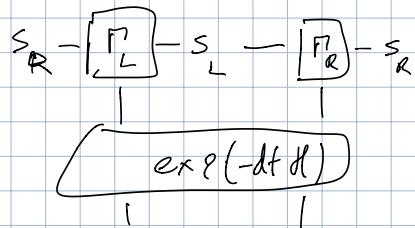
[time evolution block decimation]

DMRG App: iTEBD

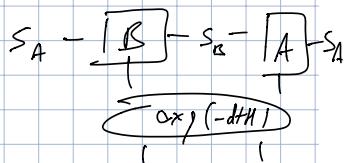
Assume:



Only two steps necessary!



and



even Better: $\boxed{A} = \boxed{F_L} \circ \boxed{S_L}$

So. Steps:

① Initiate: A $X \times X$ } rand.
 B $X \times X$ } num.

$\boxed{B} = -\boxed{F_R} \circ \boxed{S_R}$

S_L diagonal $X \times X$.
diag (0.9, 0.1 ...)

$$H = \sum_i \sigma_i^2 \sigma_{i+1}^{-2} + \frac{1}{2} (G_i^X + G_{i+1}^X)$$

② Construct $\exp[-dtH]$ as $\boxed{h}(\varepsilon_1, \varepsilon_2)(\varepsilon_1, \varepsilon_2)$

$$h = \begin{pmatrix} J & h/2 & h/2 \\ h/2 & -J & h/2 \\ h/2 & h/2 & J \end{pmatrix}$$

\downarrow
 $\exp(-dtH)$, reshape ($\varepsilon_1, \varepsilon_2$)

③ Make $\boxed{H}_0 = x_L - \boxed{A} - \boxed{B} - x_R$

④ Make $\boxed{H}_1 = x_L - \boxed{H}_0 - x_R$

h.c.: tensor dot.
transpose.

$\rightarrow \boxed{H}_1(x_L, S_L)^\dagger (x_R, S_R)$

⑤ SVD: $\rightarrow \bigcup_{(x_L, S_L)} x_m \in x_m \vee_{x_m; x_R, S_R}$

$$x_L - \boxed{U} - \boxed{S} - \boxed{V} - x_R$$

$A \rightarrow \tilde{A} = \left(S_L^{-1}\right)_{x_L} \bigcup_{x_L, S_L; x_m} (S_m)_{x_m}$

$B \rightarrow \tilde{B} = \bigvee_{x_m; S_R, x_R}$

end of day:

$$\begin{array}{c} S_K \\ \downarrow \\ \boxed{B} - \boxed{A} - \boxed{B} - \boxed{A} - \boxed{B} - \boxed{A} \\ S_L^{-1} \cup_{x_{\epsilon_L x_K}} S_K \quad \cup_{x_{K_m} x_K s_K} \end{array}$$

But:

$$\begin{array}{c} \boxed{B} - \boxed{A} -) = \boxed{\overline{S_L^{-1} \cup S_K}} - ? \\ - \boxed{B^*} - \boxed{A^*} - \end{array}$$

$$\begin{array}{c} A - B -) = \boxed{V - S_L^{-1} - V} - ? \\ A^* - B^* - \end{array}$$

$$x - \cup = d$$

$$p - \cup^d = s$$

$$= 1 \cdot |S_{\alpha\beta} < S_{\gamma\delta}| + \lambda' | > < | \dots$$

$$\cup S_K^2 \cup^+ = \left(\begin{array}{c} \equiv \\ \equiv \\ \equiv \end{array} \right) \left(\begin{array}{c} S_1^2 \\ \vdots \\ S_K^2 \end{array} \right) \left(\begin{array}{c} | \\ | \\ | \end{array} \right)$$

$$= \int \left(\begin{array}{c} S_1 \\ | \\ S_2 \\ | \\ \vdots \\ | \\ S_K \end{array} \right) \left(\begin{array}{c} \leftarrow dx \\ \downarrow \\ \left(\begin{array}{c} S_1 \\ S_2 \\ \vdots \\ S_K \end{array} \right) \end{array} \right)$$

$$\begin{array}{c} V - S_L^{-1} - V - S_K \\ V - S_L^{-1} - V - S_K \end{array} \stackrel{\sim}{\rightarrow} \underbrace{|S_{\alpha\beta} < S_{\gamma\delta}| (S_{L\gamma}^{-1})^2 |S_{\beta\delta} < S_{\epsilon\eta}| S_{K\eta}^2}_{(\mathcal{E} \frac{1}{S_{L\gamma}^2})}$$