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Enhancement of Persistent Currents by Hubbard Interactions in Disordered 1D Rings: Avoided Level Crossings Interpretation

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Abstract. — We study effects of local electron interactions on the persistent current of one dimensional disordered rings. For different realizations of disorder we compute the ensemble average current as a function of Aharonov-Bohm flux to zeroth and first orders in the Hubbard interaction. We find that the persistent current is *enhanced* by on site interactions. Using an avoided level crossings approach, we derive analytic formulas which explain the numerical results at weak disorder. The same approach also explains the opposite effect (suppression) found for spinless fermion models with intersite interactions.

1. Introduction

Recent experiments have found that small isolated metallic rings, threaded by magnetic flux, carry persistent currents. Although the existence of this effect had been anticipated theoretically for many years [1–4], the magnitude of the observed current was found to be much larger than expected.

Experiments performed on moderately disordered rings with small transverse width have measured the magnetic response of an ensemble of copper rings [5], of isolated gold loops [6], and of a clean, almost one-dimensional, sample [7].

The ensemble averaged persistent current was found to have an amplitude of order $10^{-2}ev_f/L$ and periodicity $\Phi_0/2$, (where L is the circumference of the ring, v_f is the Fermi velocity, e is the electron charge, and $\Phi_0 = h/e$ is the flux quantum). The single loops experiment reported finding an unexpectedly large current of order ev_f/L, the free electron value, and displaying periodicity in flux with period Φ_0.

Theoretical approaches can be classified into non-interacting and interacting electrons approaches. Non interacting theories which take into account the effects of disorder and deviations from a perfect 1D geometry [8], predict values of the average persistent current that are smaller than observed, but still within one or two orders of magnitude of the experimentally determined value of the average current. More surprisingly, the theoretical estimates for the typical value of the persistent current, measured in the single loop experiment, are perhaps

two or three orders of magnitude smaller than the reported values. It seems that agreement between theory and experiment worsens with increasing disorder. This suggests that perhaps interaction corrections are quantitatively important in the moderately disordered regime.

A first guess would be that interactions enhance the impurity scattering and further *suppress* the persistent current, as happens for the conductance of the disordered luttinger liquid [9]. However, this intuitive analogy may be quite misleading. While the conductance depends on the Fermi surface properties, i.e. velocity and scattering rates, the persistent current is a sum of contributions from all occupied states. Also, by Galilean invariance, an interacting electron liquid without disorder carries the same current as the non interacting gas, even though its Fermi velocity may be renormalized. Thus it is plausible that in the disordered system, interactions could work both ways, i.e. either suppress or enhance the persistent currents under conditions which need to be explored.

Exact diagonalization studies of spinless fermions with intersite interactions on small rings [10,11], have found that weak interactions *reduce* the persistent current below its noninteracting value. The correlation between the non interacting persistent current and the first order (in interactions) corrections have been evaluated numerically for the Hubbard model with spin by Ramin, Reulet and Bouchiat (RRB) [13] for various disorder realizations. RRB found that the non interacting and interaction correction have the same sign, which agrees with exact results for the disordered Hubbard model found by Giamarchi and Shastry [12]. They also explain this effect by estimating the ground state energy correction. In this paper [14] we shall gain more insight into the puzzle of interaction corrections by arriving at an analogous expression for the interaction correction to the persistent current and its explicit dependence on disorder strength. In Section 2 we diagonalize the tight binding Hamiltonian numerically for different realizations of disorder, and compute the persistent current to zeroth and to first order in the Hubbard interactions. We find that interactions *enhance* the persistent current. In Section 3 we present the Avoided Levels Crossing (ALC) theory, which provides an analytic approximation to the numerical results. This is an expansion, at weak disorder, of nearly degenerate eigenstates at fluxes close to points of time reversal symmetry. The correlation between the zeroth and first order currents is explained by a common mechanism, i.e. the avoided level crossings. The opposite (suppression) effect for spinless fermions, is also explained by the ALC theory in Section 4. We conclude by a summary and future directions.

2. Numerical Perturbation Theory

We consider a tight-binding Hamiltonian on a periodic chain with repulsive on-site Hubbard interaction:

$$H = H_0 + U \sum_i n_{i\uparrow} n_{i\downarrow}, \tag{1}$$

$$H_0 = -\sum_{i\sigma} [e^{i(\frac{2\pi}{L}\frac{\Phi}{\Phi_0})} c^\dagger_{i+1\sigma} c_{i\sigma} + \text{h.c.}] + \sum_{i\sigma} \epsilon_i c^\dagger_{i\sigma} c_{i\sigma}. \tag{2}$$

where $c^\dagger_{i\sigma}$ creates an electron at site i with spin σ. Our unit of energy is the hopping energy. ϵ_i is the dimensionless on-site disorder energy uniformly distributed in the interval $[-W/2,W/2]$. The chain has L sites, N_e electrons, the flux through its center is given by Φ. With these conventions, the energy spectrum is periodic in the enclosed flux with period Φ_0, and the current is given by

$$I(\Phi) = -\frac{\partial}{\partial \Phi} E(\Phi), \tag{3}$$

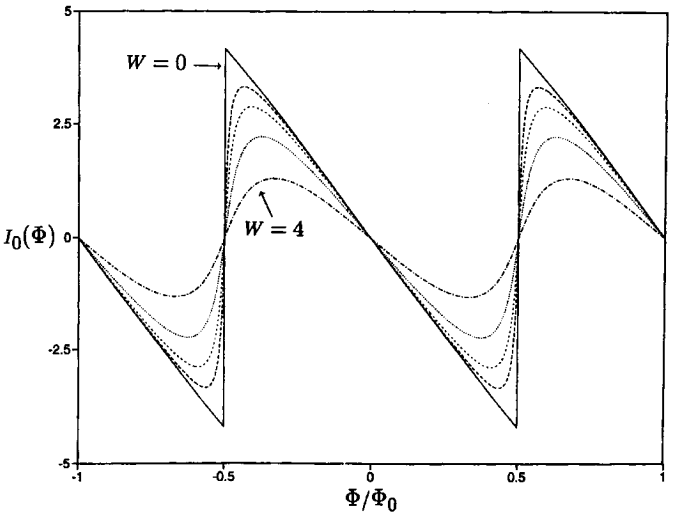

Fig. 1. — The ensemble averaged noninteracting current (I_0) as a function of the applied flux for several strengths of disordered potential: W=0, 0.5, 1.0, 2.0 and 4.0. All curves are for a half-filled lattice of six sites.

where

$$E(\Phi) = E_0(\Phi) + E_1(\Phi) + \mathcal{O}(U^2). \tag{4}$$

E_0 is the exact ground state energy of H_0, whose single electron eigenstates are determined numerically. E_1 is the first order correction in U, which is given by

$$\begin{aligned} E_1 &= U \sum_i n_{i\uparrow} n_{i\downarrow} \\ n_{i\sigma} &= \langle \Psi_0 | c^\dagger_{i\sigma} c_{i\sigma} | \Psi_0 \rangle \end{aligned} \tag{5}$$

where Ψ_0 is the Fock ground state of H_0. Thus by diagonalizing H_0 we can readily obtain

$$I(\Phi) = I_0(\Phi) + I_1(\Phi) + \mathcal{O}(U^2). \tag{6}$$

The effects of disorder on the ensemble averaged non interacting current $I_0(\Phi)$ can be seen in Figure 1, where it is shown as a function of flux for a half filled lattice of six sites, for different disorder strengths W. Disorder smoothens and reduces the magnitude of I_0, and for $W >> 1$ it is dominated by the first harmonic $\sin(2\pi\Phi/\Phi_0)$.

In Figure 2, the average value of $I_1(\Phi)$ is plotted. There are several features of the first order interaction correction that deserve comment. First, the most important observation is that it *generally* enhances the non interacting current for all values of disorder strength and flux. That is to say: there is a *positive* correlation between the non interacting current and the first order correction,

$$\langle I_0(\Phi) I_1(\Phi) \rangle_{\epsilon,\Phi} \geq 0. \tag{7}$$

which is in agreement with the numerical results obtained by RRB for the same model [13]. This result is found to hold for all realizations of disorder which we have used. Second, in the limit of weak disorder, $I_1(\Phi)$ becomes singular at discontinuity points of $I_0(\Phi)$, which are at

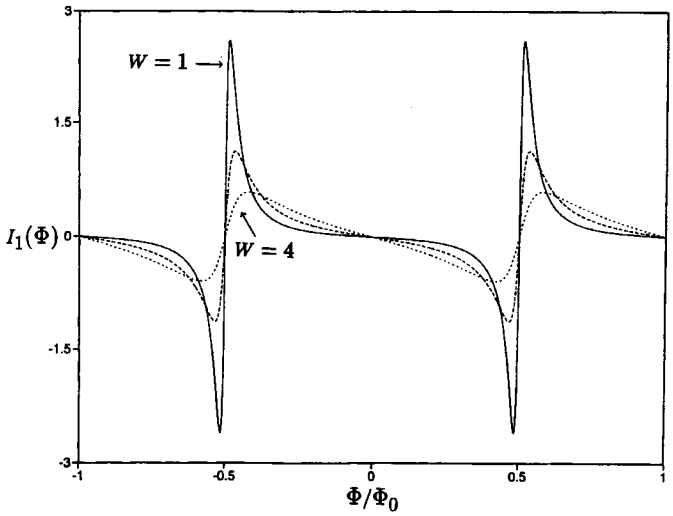

Fig. 2. — The first order ensemble averaged interacting current (I_1) as a function of the applied flux for disordered potential values: W= 1.0, 2.0 and 4.0. As before, all curves are for a half-filled lattice of six sites.

fluxes $(m+\frac{1}{2})\Phi_0$, $(m\Phi_0)$ for even (odd) number of filled orbitals, where m are integers. Finally, we see the first order current also becomes dominated by its first harmonic at large disorder.

To study the scaling properties of the current with system size and disorder strength, it is convenient to characterize the strength of both I_0 and I_1 by their amplitude at $\Phi = \Phi_0/4$. And while it does not capture the singular nature of $I_1(\Phi)$ near the regime of very weak disorder, this characterization is still useful to study the scaling properties of the first order current. Previous studies have found [4, 10] both numerically and analytically, that the amplitude of the noninteracting current, averaged over disorder, behaves like:

$$|I_0(\Phi_0/4)| \simeq \frac{1}{2}\frac{ev_f}{L}\exp(-L/\xi). \tag{8}$$

By fitting to this functional form we extracted the localization length ξ for different system sizes and disorder values. Figure 3 shows the values of the localization length obtained for sizes L=6,10,14,20 and 25, at half-filling, averaged over many (up to four thousand) impurity configurations for each value of W. We find good agreement between our inferred value of the localization length and the known asymptotic form in the weak disorder limit: $\xi = 105/W^2$, valid when $W \ll 2\pi$ for large systems at half-filling.

There is less agreement in the strongly localized limit where the localization length is known to behave like: $\xi = (\ln(W/2) - 1)^{-1}$. We find a better fit taking $\xi = 1/\ln(W^{0.6}/2.5)$ for the $L = 25$ data. This discrepancy could be due to the small sizes considered, or a breakdown of equation (8) when ξ is of order unity. We have calculated the amplitude $I_1(\Phi = \Phi_0/4)$ as function of strength of disorder, characterized by the scaling parameter L/ξ, for system sizes of L=6,10,14,20,25. At weak disorder (large ξ), the amplitude increases with the strength of disorder achieving its maximum value at some intermediate strength of disorder ($\xi \simeq L$). In this weakly disordered regime, a single scaling function could be used to describe the results:

$$\frac{|I_1|}{U} \sim f(L/\xi)/L. \tag{9}$$

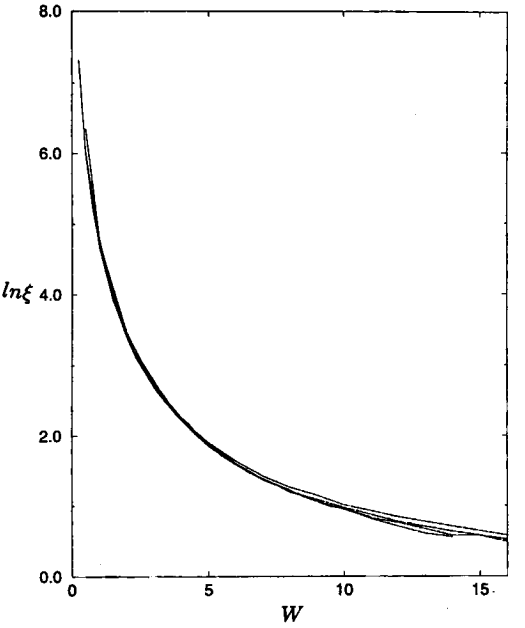

Fig. 3. — The logarithm of the localization length ($\ln \xi$), implied by equation (8), is plotted as a function of strength of disorder (W) for half-filled rings with N=6, 10, 14, 20 and 25 sites. Data for the different system sizes collapse well onto a single curve.

This behavior is similar to that of the amplitude of the noninteracting current given by (8). Upon increasing the impurity scattering further, the first order current decreases with disorder. When the single particle wavefunctions are sufficiently localized, I_1 can be described by a different scaling form:

$$\frac{|I_1|}{U} \sim g(L/\xi), \tag{10}$$

where g is a decreasing function of its argument. Equation (10) suggests that in the localized regime I_1 dominates I_0 for large system sizes. However for localized doubly occupied single electron states

$$\langle \psi_{\alpha\uparrow}\psi_{\alpha\downarrow}|U\sum_i n_{i\uparrow}n_{i\downarrow}|\ \psi_{\alpha\uparrow}\psi_{\alpha\downarrow}\rangle \sim \frac{U}{\xi} \tag{11}$$

Thus even for weak interactions, the interaction corrections may be larger than the non interacting level spacings which go as $1/L$. *This invalidates perturbation theory in U in the localized regime.*

3. Avoided Level Crossings Theory

Here we will discuss how the numerical results of the previous section can be understood in terms of avoided level crossings (ALC) at weak disorder. First, we derive the ALC approximation for the noninteracting current I_0.

3.1. ALC THEORY FOR I_0. — In Figure 4 we can see a typical spectrum for a tight-binding

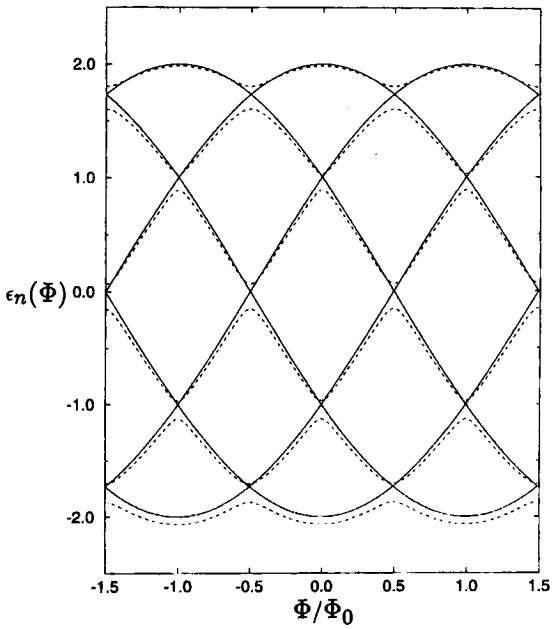

Fig. 4. — Energy level spectrum of the tight-binding Hamiltonian(12) as a function of applied flux for a N=6 ring in the absence of disorder (solid lines) and for weak disorder (dotted lines).

Hamiltonian of a 6 site ring, as a function of the applied flux. In the absence of disorder ($W = 0$) the eigenenergies are

$$\epsilon_n(\Phi) = -2\cos\left(\frac{2\pi}{N}(n + \frac{\Phi}{\Phi_0})\right), \tag{12}$$

with period $\Phi_0 = 2\pi$. At points of time reversal symmetry, i.e. where Φ is an integer multiple of $\Phi_0/2$, level crossings occurs between states of opposite angular momenta. The noninteracting persistent current is a sum over the currents carried by all occupied levels,

$$I_0(\Phi) = -\frac{\partial E_0}{\partial \Phi} = -\sum_{n,s}^{\text{occupied}} \frac{\partial}{\partial \Phi}\epsilon_n(\Phi), \tag{13}$$

where n,s are the orbital and spin index, respectively. The noninteracting current $I_0(\Phi)$ will be a smooth function of Φ away from the points of level crossings. By symmetry of $\Phi \to -\Phi$, any pair of levels cross with opposite slopes, and thus if they are both fully occupied their contribution to the total current cancels. The only nonvanishing contribution comes from a topmost level which is not compensated by its partner. Then the current changes sign abruptly as the occupation moves from one branch to another. Thus for an odd number of fully occupied orbitals the current discontinuity occur at $\Phi = (m+1/2)\Phi_0$, otherwise the discontinuities will occur at $\Phi = m\Phi_0$. In between the discontinuities the current varies linearly with the flux, which explains the periodic sawtooth shape for I_0 as seen in Figure 1. Introducing a small amount of disorder lifts the degeneracy at the crossings by opening small gaps. This reduces the persistent current I_0 since the occupied levels have smaller slopes near the former crossing points. This explains the behavior shown in Figure 1: weak impurity scattering softens the

discontinuities and leads to an overall reduction of the magnitude of current. We can quantify this observation by examining pairs of levels with momenta $\pm k$ which cross at $\Phi = 0$. We specialize to the case of an even number of filled levels with equal occupations for both spin directions. The unperturbed energies are given by

$$\epsilon_{\pm k}(\Phi) = -2\cos\left(\pm k + \frac{2\pi}{L}\frac{\Phi}{\Phi_0}\right) \tag{14}$$

with $k = 2\pi m/L$, for a positive integer m. Consider the effect of a weak random potential ϵ_i which lifts the degeneracy in the 2×2 subspace of $k, -k$,

$$H_0 \Longrightarrow \begin{bmatrix} \epsilon_k(\Phi) & \tilde{V}_{k,-k} \\ \tilde{V}_{-k,k} & \epsilon_{-k}(\Phi) \end{bmatrix}, \tag{15}$$

where

$$\tilde{V}_{k,-k} = \frac{1}{L}\sum_{i=1}^{L} \exp(-i2kx_i)\epsilon_i. \tag{16}$$

From now on we will omit the subcripts $k, -k$ off $\tilde{V}$.

The eigenvalues and normalized eigenfunctions of (15) are:

$$\begin{aligned} \epsilon_+(\Phi): &\qquad \psi_+ = \tfrac{1}{\sqrt{L}}\left(A_+ e^{ikx} + B_+ e^{-ikx}\right) \\ \epsilon_-(\Phi): &\qquad \psi_- = \tfrac{1}{\sqrt{L}}\left(A_- e^{ikx} + B_- e^{-ikx}\right) \end{aligned} \tag{17}$$

where:

$$\begin{aligned} A_- &= e^{i\delta_-}\left[1 + \frac{4|\tilde{V}|^2}{\left(\epsilon_k - \epsilon_{-k} + \sqrt{(\epsilon_{-k} - \epsilon_k)^2 + 4|\tilde{V}|^2}\right)^2}\right]^{-1/2}, \\ B_- &= \frac{-2\tilde{V}^*}{\epsilon_{-k} - \epsilon_k + \sqrt{(\epsilon_{-k} - \epsilon_k)^2 + 4|\tilde{V}|^2}} A_-, \end{aligned} \tag{18}$$

and,

$$\epsilon_\pm = \frac{\epsilon_k + \epsilon_{-k} \pm \sqrt{(\epsilon_k - \epsilon_{-k})^2 + 4|\tilde{V}|^2}}{2}, \tag{19}$$

which satisfy:

$$\epsilon_+(\Phi) + \epsilon_-(\Phi) = \epsilon_k(\Phi) + \epsilon_{-k}(\Phi). \tag{20}$$

We see that the contribution of the occupied orbitals to I_0 is unchanged by weak disorder since:

$$I_0^{k,-k} = -\frac{\partial(\epsilon_+ + \epsilon_-)}{\partial\Phi} = -\frac{\partial(\epsilon_k + \epsilon_{-k})}{\partial\Phi} \tag{21}$$

Let us now use (13) to calculate $I_0(\Phi)$ for free electrons with weak disorder by summing over occupied levels. We fill all levels up to $k_f = 2\pi m_f/L$, plus two electrons at k_f so that an even number of orbital levels is filled. Although we restrict our calculations to an even number of filled orbitals, the ALC theory can be easily extended to all other fillings as follows: we note that the non interacting wavefunctions have two-fold spin degeneracy. An unpaired

spin in an open orbital will not produce any interaction correction to the persistent current since its density interacts with an approximately uniform core density. For odd number of doubly occupied orbitals, the important level l crossings occur at $\Phi \approx \pm\frac{1}{2}\Phi_0$, rather than near $\Phi \approx 0$. The total noninteracting current is given by the contribution from the filled level pairs, together with the contribution of lowest $m = 0$ (nondegenerate) level and the topmost orbital level at m_f:

$$\begin{aligned} I_0(\Phi) &= \sum_m I_0^m + I_0^{m=0} + I_0^{m_f} \\ &= -2 \sum_{-(m_f-1)}^{m_f-1} \frac{4\pi}{L\Phi_0} \sin \frac{2\pi}{L}\left(m + \frac{\Phi}{\Phi_0}\right) - 2\frac{\partial \epsilon_-}{\partial \Phi} \end{aligned} \tag{22}$$

Neglecting corrections of order $1/L^2$, we obtain the expression:

$$I_0(\Phi) = \frac{8\pi \sin(k_f)}{L\Phi_0}\left[-2\frac{\Phi}{\Phi_0} + \left(\frac{\Phi}{\Phi_0}\right) \Big/ \sqrt{\left(\frac{\Phi}{\Phi_0}\right)^2 + \left(\frac{L|\tilde{V}|}{4\pi \sin(k_f)}\right)^2}\right], \tag{23}$$

for $\Phi \in [-\Phi_0/2, \Phi_0/2]$. This expression for I_0 is valid as long as the energy scale of the disorder is much smaller than the level spacing at the Fermi level:

$$|\tilde{V}| << 4\pi \sin(k_f)/L. \tag{24}$$

Using the relation between $|\tilde{V}|$ and the mean free path l_{el} defined by the one dimensional Born approximation,

$$l_{el} \equiv \frac{2\pi \sin^2 k_f}{|\tilde{V}|^2 L}. \tag{25}$$

According to (24), the ALC approximation is valid for

$$8\pi l_{el}/L \geq 1, \tag{26}$$

which is the ballistic, or delocalized regime([1]). In terms of l_{el} one can write

$$I_0(\Phi) = \frac{8\pi \sin(k_f)}{L\Phi_0}\left[-2\frac{\Phi}{\Phi_0} + \left(\frac{\Phi}{\Phi_0}\right) \Big/ \sqrt{\left(\frac{\Phi}{\Phi_0}\right)^2 + \frac{L}{8\pi l_{el}}}\right]. \tag{27}$$

In order to compare (27) to the numerical result for equation (2), one needs to determine the parameter l_{el}. Since fluctuations in I_0 for different disorder realizations are large, we determine l_{el} by fitting (27) to the numerical ensemble averaged I_0. Once l_{el} is determined for a particular disorder realization, we use it to evaluate I_1 as shown below.

3.2. ALC THEORY FOR I_1. — We shall now proceed to use the same approximation to explain the behavior of I_1([2]).

([1]) There is no intermediate "diffusive regime" between the localized and ballistic regimes in one dimension [15].

([2]) This approach is similar to the discussion of Aharonov-Bohm oscillations of the participation ratio in disordered rings [16].

According to (5), the first order energy is a sum of the density squared at all sites,

$$E_1(\Phi) = U \sum_i n_{i\uparrow} n_{i\downarrow} \tag{28}$$

where for the filling of an even number of orbitals per spin, the single spin density is,

$$n_{is}(\Phi) = |\psi_0(i)|^2 + |\psi_{m_f,-}(i)|^2 + \sum_{m=0}^{m_f-1} \sum_{\sigma=\pm} |\psi_{m,\sigma}(i)|^2. \tag{29}$$

We will now show that the first order energy,

$$E_1(\Phi) = U \sum_i n_{i\uparrow} n_{i\downarrow} \tag{30}$$

is enhanced near level crossings.

For a pair of fully occupied levels one has, by unitary

$$|\psi_{m,+}(i)|^2 + |\psi_{m,-}(i)|^2 = |\psi_k(i)|^2 + |\psi_{-k}(i)|^2 = \frac{2}{L}. \tag{31}$$

Consequently, for weak disorder, all occupied levels, except for the last one, contribute constant values to the total density:

$$n_{i\uparrow} = \frac{2m_f - 1}{L} + |\psi_-(i)|^2 \tag{32}$$

or in terms of the coefficients A, B (17)

$$n_{i\uparrow} = \frac{2m_f}{L} + \frac{A_-^* B_- e^{i2kx_i}}{L} + \frac{A_- B_-^* e^{-i2kx_i}}{L}. \tag{33}$$

The first order energy is thus given by (32),

$$E_1(\Phi)/U = \frac{4{m_f}^2}{L} + \frac{2|A_-|^2|B_-|^2}{L}. \tag{34}$$

Using (17) and (25) we can write

$$E_1(\Phi)/U \approx \frac{4{m_f}^2}{L} + \frac{1}{2L}\left[1 + \left(\frac{8\pi l_{el}}{L}\right)\left(\frac{\Phi}{\Phi_0}\right)^2\right]^{-1}, \tag{35}$$

Differentiating (35) with respect to flux yields

$$I_1(\Phi)/U = \frac{8\pi l_{el}\Phi}{(L\Phi_0)^2} \Bigg/ \left[1 + \left(\frac{8\pi l_{el}}{L}\right)\left(\frac{\Phi}{\Phi_0}\right)^2\right]^2. \tag{36}$$

In Figure 5, (36) is compared to the numerical ensemble averaged result for I_1 for various values of disorder. For each disorder realization, l_{el} is determined by fitting the numerical and ALC results for the average I_0. We see that for weak disorder there is a satisfying agreement between the ALC approximation and the numerical results. In Figure 5c, the disorder is too large, and the ALC approximation fairs badly.

Equation (36) explains both the positive correlation between I_0 and I_1 of (7). Since in one dimension the mean free path (l_{el}) and the localization length (ξ) differ by a proportionality factor of order unity [15], (36) agrees with the empirical scaling form (9) $|I_1|/U \sim f(L/l_{el})/L$ which was found numerically at weak disorder.

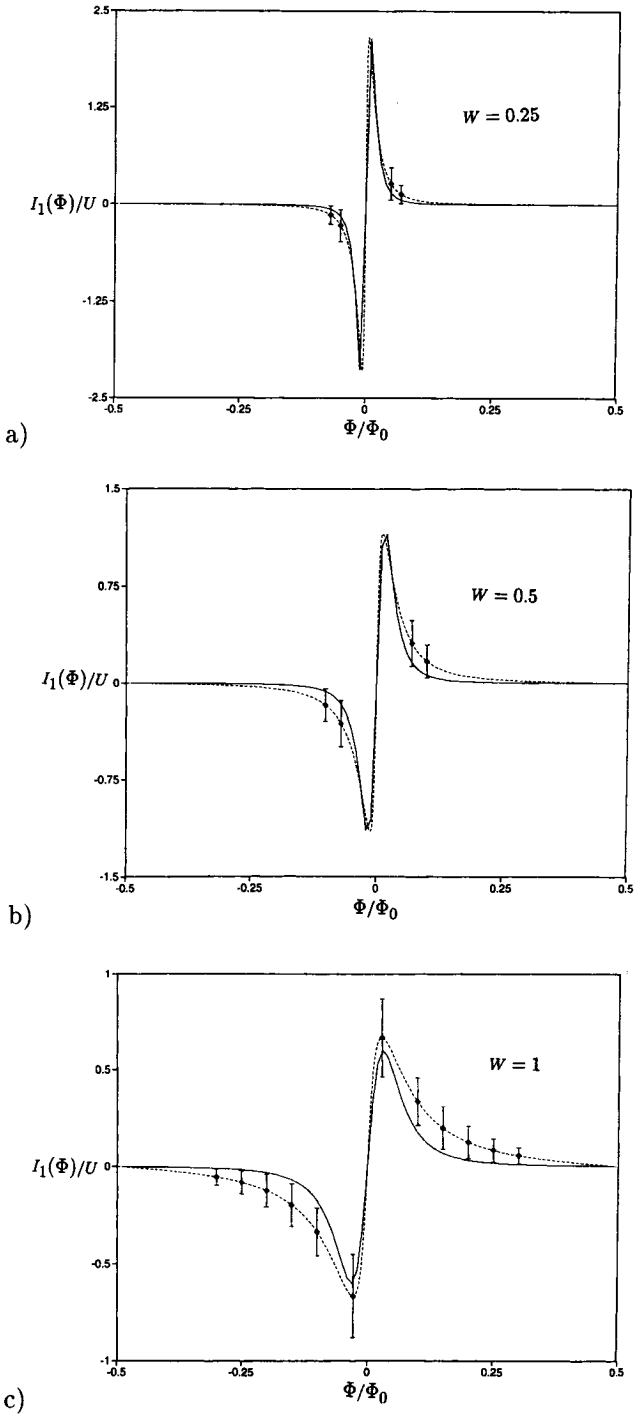

Fig. 5. — Comparison between numerically determined interactions correction I_1 (dashed lines) and the analytic ALC result (36) (solid lines). a) – c) show results for three values of disorder strength W. $l_{el}(W)$ are determined by fitting equation (27) to the numerical disorder averaged $I_0(\Phi)$. Error bars depict fluctuations of numerical I_1 for different disorder realizations.

4. Difference of Hubbard Model and Spinless Fermions

In recent papers [10, 11], small disordered rings of spinless electrons have been exactly diagonalized. The persistent currents have been computed as a function of disorder and interaction strength. In contrast to our result for the Hubbard model, interactions have been seen to *reduce* the persistent current at weak disorder. Here we apply the ALC approach to explain this apparent difference between the models.

The spinless Hamiltonian H^s is given by

$$\begin{aligned} H^s &= H_0^s + \sum_i U_{ij} n_i n_j, \\ H_0^s &= -\sum_i [e^{i(\frac{2\pi}{L}\frac{\Phi}{\Phi_0})} c_{i+1}^\dagger c_i + \text{h.c.}] + \sum_i \epsilon_i c_i^\dagger c_i. \end{aligned} \tag{37}$$

The non interacting current, I_0^s, is given by half the value of equation (13). In the ALC approximation, it is given by half the value of equation (27).

Following the analogous derivation of I_1, we use (33) to obtain:

$$E_1^s(\Phi) = 4{m_f}^2 \tilde{U}(0) + 2|A_-|^2 |B_-|^2 \tilde{U}(2k_f), \tag{38}$$

where

$$\tilde{U}(k) = \frac{1}{L} \sum_{j \neq i} \exp(-ikx_j) U_{ij}, \tag{39}$$

which implies

$$\begin{aligned} E_1^s(\Phi) &= 4m_f^2 \tilde{U}(0) + \tilde{U}(2k_f) \left[1 + \left(\frac{8\pi l_{el}}{L} \right) \left(\frac{\Phi}{\Phi_0} \right)^2 \right]^{-1} \\ I_1^s(\Phi) &= \frac{16\pi l_{el} \Phi}{\Phi_0^2 L} \tilde{U}(2k_f) \left[1 + \left(\frac{8\pi l_{el}}{L} \right) \left(\frac{\Phi}{\Phi_0} \right)^2 \right]^{-2} \end{aligned} \tag{40}$$

For the spinless nearest neighbor case studied in reference [11]:

$$E_1(\Phi) = U \sum_i n_i n_{i+1}, \tag{41}$$

the corresponding Fourier coefficient is

$$\tilde{U}(2k_f) = 2U \cos(2k_f). \tag{42}$$

Above a quarter filling, $k_f \geq \pi/4$, $\tilde{U}$ is negative. Consequently I_1^s has the opposite sign to that of I_1 in equation (36), and to I_0^s. That is to say, the persistent current of the spinless fermion model is *suppressed* by the interactions which is in agreement with the results obtained by reference [11, 13].

The difference between the models can be attributed to the different effects of *intersite* interactions (in the spinless model), *versus local* interactions (in the Hubbard model).

5. Remarks and Conclusions

We have investigated the first order effect of Hubbard interactions on the persistent current in one dimensional disordered rings.

The findings can be summarized as follows. I_1 was found to correlate in sign with I_0, a fact which goes contrary to the naive intuition that interactions suppress currents (e.g. as they do for the conductivity of the Luttinger model with an impurity [9]).

We can understand this result by observing that I_1 depends on charge fluctuations which vary strongly with flux near the degeneracy points, which also determine the sign and magnitude of I_0. Using the avoided level crossings theory, we obtain analytical expressions which fit the numerical results for both I_0 and I_1. We can use the same theory to explain why spinless fermions with intersite interactions exhibit suppression of currents rather than enhancement.

We have found numerically that $I_1(L/\xi)$ scales differently with the localization length than $I_0(L/\xi)$, it seems that effects of electron-electron interactions grow as disorder is increased.

However, first order perturbation theory in the weakly disordered case is expected to hold as long as the interaction matrix elements do not exceed the single particle level spacings. Thus we are restricted to the regime $U \leq 1$. We can draw on the exact diagonalization results [10,11] where for weak disorder, the numerical currents are found to vary linearly with interaction strength in a sizeable regime. Thus we believe that first order perturbation theory should be valid for physically interesting interaction parameters.

It would be very satisfying if a similar analysis could be applied to the experimentally relevant case of three dimensional rings. Preliminary results yield positive correlations between I_0 and I_1, but where a simple minded application of the ALC approach cannot describe the diffusive regime of $l_{el} \ll L$. It would also be important to understand the effects of true long range Coulomb interactions with the screening and exchange effects which are absent in the Hubbard model.

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