Andreev Peaks and Massive Magnons in Cuprate Superconductor-Normal-Superconductor Junctions

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The projected SO(5) theory [pSO(5)] is used to resolve the puzzle of two distinct energy gaps in high T_c superconductor-normal-superconductor junctions. Counter to the conventional theory of multiple Andreev reflections (MAR), the differential resistance peaks are associated with the antiferromagnetic resonance observed in neutron scattering, and not with Cooper pair breaking. The pSO(5) and MAR theories differ by the expected tunneling charges at the peaks. We propose that shot noise experiments could discriminate against the conventional interpretation.

In current transport through high T_c superconductor junctions, there seem to be *two* energy scales [1]. The upper energy is seen in tunneling conductance [2], and is identified with the "pseudogap" Δ_p which appears in magnetic resonance [3] and photoemission [4]. A lower gap, which scales differently with hole doping, manifests as peaks in the differential resistance of superconductor-normal-superconductor (SNS) Josephson junctions [5]. These peaks have been interpreted using the conventional theory of multiple Andreev reflections, following Klapwijk, Blonder, and Tinkham (KBT) [6].

KBT theory treats two conventional superconductors with a single s-wave BCS quasiparticle gap Δ, separated by a free electron metal. Electrons traversing the metal are Andreev reflected back as holes, gaining energy increments eV at each traversal (as depicted in Fig. 1). Peaks in the differential resistance appear at voltages $2\Delta/ne$, and are due to the $(E - \Delta)^{-1/2}$ singularity in the quasiparticles' density of states. However, in cuprate SNS junctions, such as YBa$_2$-Cu$_3$-O$_{6.6}$-YBa$_2$-Cu$_{2.55}$-Fe$_{0.45}$-O$_y$-YBa$_2$-Cu$_3$-O$_{6.6}$ examined by Nesher and Koren [5], application of KBT theory is problematic. A naive fit to KBT expression faces the two gaps puzzle, i.e., an "Andreev gap" is of the order of $\Delta \approx 16$ meV, while the tunneling gap is about three times $3\times$ larger [7], and scales differently with T_c. Without perfect alignment of the interfaces, it is hard to understand the observed sharpness of peaks [5] since the d-wave gap is modulated for different directions. Moreover, the barrier is by no means a "normal" metal devoid of interactions: it is an underdoped cuprate with antiferromagnetic correlations and strong pairing interactions as evidenced by a large proximity effect [8].

The purpose of this Letter is to provide an alternative explanation for the differential resistance peaks series [5], which takes into account the strong correlations in the pseudogap regime. Our analysis resolves the two energy scales puzzle. We employ the *projected* SO(5) [pSO(5)] model [9], which is a strong coupling effective Hamiltonian. It describes the dynamics and interactions of four primary bosonic modes of cuprates: *preformed hole pairs* and *massive spin one magnons*.

A differential resistance peaks series is found at bias voltages $V_n = \Delta_s/(en)$, $n = 1, 2, \ldots$ where Δ_s is the antiferromagnetic resonance energy. This resonance has been directly measured by inelastic neutron scattering. The peaks are thus associated with emission of magnon pairs at the resonance threshold, and *not* with pair breaking, as in KBT theory. We note that other predictions to observe magnons (also called π modes) in various cuprate junctions were made [10,11], but await experimental confirmation. We propose that measurement of the excess shot noise below the peaks could discriminate against the KBT interpretation. pSO(5) theory predicts tunneling charge $2ne$ below the nth peak, while KBT theory expects charge ne. We shall discuss this proposed experiment at the end of the paper.

Degrees of freedom.— At energies below the pseudogap Δ_p, preformed hole pairs (with internal d-wave symmetry), describe the primary charge degrees of freedom in the underdoped regime [12]. The hole pairs are bosons, and their phase fluctuations are controlled by the two dimensional superconducting stiffness ρ_c, as measured by
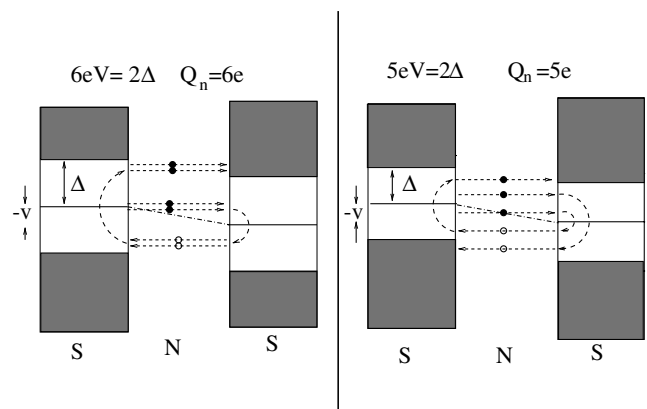

FIG. 1. KBT theory: Differential resistance peaks of $n = 6$ (left diagram), and $n = 5$ (right diagram), involve a cascade of n Andreev reflected charges traversing the normal metal. Singular dissipation is due to emission of quasiparticles above the s-wave gap. Filled (empty) circles denote electrons (holes) in the normal barrier.

the London penetration depth. At T_c, the pairs Bose condense and long range phase coherence is established. This scenario can explain [13] the empirical relations $T_c \propto \rho_c$, which have been observed in cuprates [12] at low doping concentrations. The other low energy charge excitations are fermionic quasiparticles near the d-wave nodes. These have a smooth density of states which decreases below Δ_p.

Additional bosonic excitations below the pseudogap energy scale, are antiferromagnetic spin fluctuations, i.e., magnons. Massive spin one magnons have been observed in inelastic neutron scattering [14] in YBa$_2$-Cu$_3$-O$_{6+\delta}$. They manifest as a sharp resonance in the spin correlation function $S_{\alpha\alpha'}$, which near the antiferromagnetic wave vector $\mathbf{q} \approx \vec{\pi}$ has the form

$$S_{\alpha\alpha'}(\omega,\mathbf{q}) \approx s_0 \frac{\delta_{\alpha\alpha'}}{\omega^2 - c^2(\mathbf{q}-\vec{\pi})^2 - \Delta_s^2}. \qquad (1)$$

Here c is the spin wave velocity, and s_0 is a normalization factor. The doping dependent resonance energy $\Delta_s(\delta)$ increases [14] between $\Delta_s(0.5) = 25$ meV (with $T_c = 52$ K) and $\Delta_s(1) = 40$ meV (with $T_c = 92$ K) [14].

The projected SO(5) theory.—The large onsite Hubbard repulsion between electrons is imposed by an *a priori* projection of doubly occupied states from the Hilbert space [15].

The undoped vacuum $|0\rangle$ is a half-filled Mott insulator in a quantum spin liquid state. The pSO(5) vacuum possesses short range antiferromagnetic correlations. A translationally invariant realization of $|0\rangle$ on the microscopic square lattice, is the short range resonating valence bonds state [16,17]. Out of this undoped vacuum, $b_h^\dagger$ create charge $2e$ bosons (hole pairs) with internal d-wave symmetry under rotations, and $b_\alpha^\dagger$, $\alpha = x, y, z$ create a triplet of antiferromagnetic, spin one magnons. The lattice pSO(5) Hamiltonian is

$$\mathcal{H}^{\text{pSO(5)}} = \mathcal{H}^{\text{charge}} + \mathcal{H}^{\text{spin}} + \mathcal{H}^{\text{int}} + \mathcal{H}^{\text{Coul}}$$
$$+ \mathcal{H}^{\text{ferm}},$$

$$\mathcal{H}^{\text{charge}} = (\epsilon_c - 2\mu) \sum_i b_{hi}^\dagger b_{hi}$$
$$- \frac{J_c}{2} \sum_{\langle ij \rangle} (b_{hi}^\dagger b_{hi} + \text{h.c.}), \qquad (2)$$

$$\mathcal{H}^{\text{spin}} = \epsilon_s \sum_{i\alpha} b_{\alpha i}^\dagger b_{\alpha i} - J_s \sum_{\alpha\langle ij \rangle} n_i^\alpha n_j^\alpha,$$

$$\mathcal{H}^{\text{int}} = W \sum_i :\left(b_{hi}^\dagger b_{hi} + \sum_\alpha b_{\alpha i}^\dagger b_{\alpha i}\right)^2:,$$

where $:(\,):$ denotes normal ordering, and $n_i^\alpha = (b_{i\alpha}^\dagger + b_{i\alpha})/\sqrt{2}$ is the Néel spin field. $\mathcal{H}^{\text{int}}$ describes short range interactions between bosons, and $\mathcal{H}^{\text{Coul}}$ describes the long range Coulomb interactions. H^{ferm} describes coupling to the nodal (fermionic) quasiparticles, which contribute to a large, but smooth, conductance background.

Here we will concentrate on the conductance singularities, and will not compute the fermionic background.

The mean field approximation to Eq. (2) is straightforward [9]. It amounts to replacing $b_{\gamma i}^\dagger \to \langle b_{\gamma i}^\dagger \rangle$, $\gamma = h, \alpha$, and minimizing $\mathcal{H}^{\text{charge}} + \mathcal{H}^{\text{spin}} + \mathcal{H}^{\text{int}}$ with respect to $\langle b_{\gamma i}^\dagger \rangle$. There is a first order transition between the two primary mean field phases on the square lattice at $\mu = \mu_c$, where

$$\mu_c = \frac{1}{2}(\epsilon_c - \epsilon_s) - (J_c - 2J_s), \qquad (3)$$

μ_c is of the order of the Hubbard interaction scale. At $\mu < \mu_c$ we have an undoped Mott insulator with no hole pair bosons, and where the magnons Bose-condense. The condensate supports a finite staggered magnetization

$$|\langle n^\alpha \rangle|^2 = \left(2J_s - \frac{1}{2}\epsilon_s\right)/W, \qquad \mu < \mu_c. \qquad (4)$$

There are two linear spin wave modes $\omega = c|\mathbf{q}|$, where $c = \sqrt{2}J_s/\hbar$ is the semiclassical spinwave velocity of the Heisenberg antiferromagnet. At $\mu > \mu_c$ the ground state becomes doped with hole pairs which Bose-condense into a superconducting phase with an order parameter

$$|\langle b_i^\dagger \rangle|^2 = (\mu - \mu_c + 2J_s - \epsilon_s/2)/W, \qquad \mu > \mu_c. \qquad (5)$$

Long range interactions in $\mathcal{H}^{\text{Coul}}$ frustrate the first order transition and create intermediate (possibly incommensurate) phases [9], which we shall not discuss here.

The mean field phase stiffness is given by $\rho_c = J_c\langle b_i^\dagger \rangle^2$, and therefore Eq. (5) explains why ρ_c increases with chemical potential (and doping) in the underdoped superconducting regime, as observed experimentally [12]. Analysis of the linear quantum fluctuations about mean field theory determines the magnon dispersion, i.e., the poles of Eq. (1). The mean field magnon gap is found to be

$$\Delta_s = 2\sqrt{(\mu - \mu_c)(\mu - \mu_c + 4J_s)}, \qquad (6)$$

which by Eq. (5) implies that $\Delta_s^2 \propto \rho_c, T_c$. Thus the pSO(5) mean field theory can explain the systematic increase of Δ_s with T_c which is observed by Fong *et al.* [14].

The cuprate SNS junction.—We consider a junction, where the barrier (N) has no superconducting or magnetic order $\langle b_h^\dagger \rangle = 0, \langle n^\alpha \rangle = 0$. We derive on general grounds the form of the effective tunneling Hamiltonian between superconductors as follows. An integration of the barrier's charged bosons b_h out of the path integral results in an effective action S^{tun} which couples the charges of the two superconductors. $S^{\text{tun}}[b_{h_L}, b_{h_R}, b_\alpha]$ explicitly depends on the hole pairs bosons on the left and right interfaces, and on the magnons in the barrier. By charge conservation, an expansion of S^{tun} as a power series leaves only terms with an equal number of b_h's and $b_h^\dagger$'s. By spin conservation, the magnon terms are singlets and hence at least bilinear

in n^α. This expansion leads to a series of tunneling terms. For the Andreev peaks we retain only the leading order terms (in $b^\dagger, b$) which are

$$\mathcal{H}^{\text{tun-mag}} = -\sum_n (\mathcal{A}_n + \mathcal{A}_n^\dagger)$$

$$\mathcal{A}_n = \sum_{y_1 \ldots y_{2n}, \mathbf{x}, \mathbf{x}'} T_n b_{h_L,1}^\dagger \ldots b_{h_L,n}^\dagger b_{h_R n+1} \ldots b_{h_R,2n}$$

$$\times \left[\sum_\alpha n^\alpha(\mathbf{x}) n^\alpha(\mathbf{x}') \right]. \quad (7)$$

$\mathcal{A}_n^\dagger$ describes a simultaneous tunneling of n hole pairs from the left to the right superconductor, coupled to a magnon pair excitation. T_n is the tunneling vertex function, which depends on the bosons' positions.

The energy transfer mechanism is depicted diagrammatically in Fig. 2. We do not compute T_n's which depend on the details of the barrier and the interfaces. A "good" N barrier is defined to have sizable T_n, if multiple pair tunneling terms are to be observed. This requires a thin barrier with slowly decaying spin and charge correlations [8]. It is important to note that multiple pair tunneling, i.e., the differential resistance peaks at $n > 1$, depends on strong anharmonic interactions between the hole pairs and magnons. *These interactions are an essential part of the pSO(5) theory* as modeled by $\mathcal{H}^{\text{int}}$ in Eq. (2).

The junction's conductance is calculated in the standard fashion [18]: The bias voltage V transforms the left bosons $b_{h_L} \rightarrow e^{i2eVt} b_{h_L}$, which yields time dependent operators $\mathcal{A}_n(t)$. The current is calculated by second order perturbation theory in $\mathcal{H}^{\text{tun-mag}}$ yielding
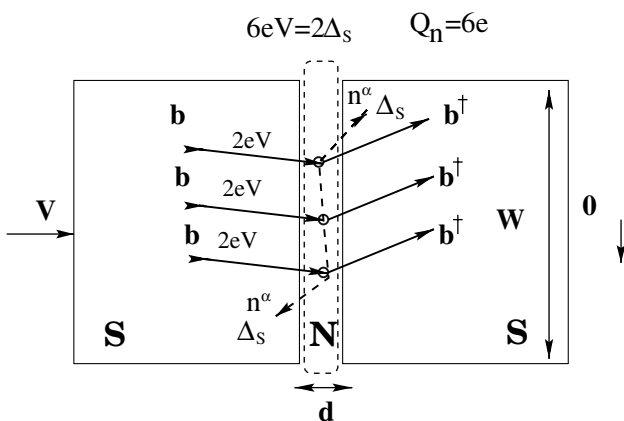

FIG. 2.   pSO(5) theory for Andreev peaks in cuprate SNS junctions: Three hole pairs cotunneling from left to right, generate a pair of magnons. At the antiferromagnetic resonance threshold $6eV = 2\Delta_s$, this process contributes to the $n = 3$ peak of the differential resistance. The diagram contains lowest order contributions of hole pairs-magnon interactions to the tunneling vertex T_3.

$$I = \sum_n 2ne X_n^{\text{ret}}(2eV),$$

$$X_n^{\text{ret}}(\omega) = i \int_0^\infty dt\, e^{i\omega t} \langle [A_n(t), A_n^\dagger] \rangle. \quad (8)$$

For singular contributions I^{sing}, we ignore superconducting condensate fluctuations $b_h^\dagger - \langle b_h^\dagger \rangle$, which have a smooth spectrum. Similarly, we ignore the frequency dependence of $T_n(\omega)$. Setting $b_R^\dagger \rightarrow \langle b_h^\dagger \rangle$ and $b_L^\dagger \rightarrow e^{i2eVt} \langle b_h^\dagger \rangle$ leads to

$$I^{\text{sing}} = \sum_n 2ne \sum_{|q_x| \leq \pi/d, |q_y| \leq \pi/W} \langle b_h^\dagger \rangle^{4n} |T_n[\mathbf{q}]|^2$$

$$\times \Im \sum_\omega S(\mathbf{q}, i\omega + 2neV + i0^+) S(-\mathbf{q}, i\omega), \quad (9)$$

where the barrier dimensions are $d \times W$ (see Fig. 2), and $\sum_\omega$ is a Matsubara sum.

For a nearly antiferromagnetic "N" barrier, $T_n(\mathbf{x} - \mathbf{x}')$ in (7) decays slowly with the distance between magnons. Thus for a narrow barrier $d \ll W$, the magnons are excited at $q_y \approx 0$, and the momentum sum reduces to a one dimensional sum over q_x. At zero temperature we obtain

$$I^{\text{sing}} = \sum_n 2ne \langle b_h^\dagger \rangle^{4n} |T_n[0]|^2$$

$$\times s_0^2 \int \frac{dq_x}{2\pi} \frac{\delta(2neV - 2\sqrt{c^2 q_x^2 + \Delta_s^2})}{2(\Delta_s^2 + c^2 q_x^2)}$$

$$\approx \sum_n t_n \frac{\theta(neV - \Delta_s)}{\Delta_s^{3/2} \sqrt{neV - \Delta_s}}. \quad (10)$$

The last expression emphasizes the singular form of $I^{\text{sing}}(V, \Delta_s)$ at the peaks. For a large background conductance $dI/dV \gg dI^{\text{sing}}/dV$, the inverse square root singularities in I^{sing} create peaks in the differential resistance dV/dI at voltages

$$V_n = \Delta_s/(ne), \qquad n = 1, 2, \ldots, \qquad Q_n = 2ne, \quad (11)$$

where Q_n is the excess tunneling charge below the nth peak. Note that Q_n changes in increments of $2e$. The differential resistance peak series is depicted in Fig. 3, for weak broadening of the singularities and an arbitrary set of coefficients t_n.

Discussion and proposed experiment.—We have seen that magnon pair creation induces peaks in the differential resistance which are similar *in appearance* to the Andreev peaks of the KBT mechanism. The crucial difference is that here *the singular dissipative process does not involve Cooper pair breaking*, but low energy antiferromagnetic excitations. In the KBT mechanism, a single sharp gaplike feature can be obtained in a d wave superconductor only by precise alignment of the a-b axes of the two superconductors. Here, one only requires the junction to be flat in the transverse direction, such that q_y
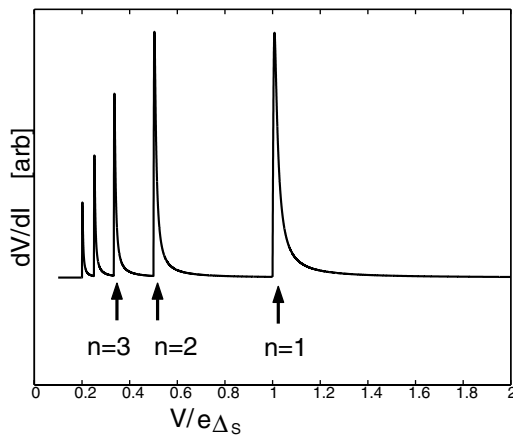
FIG. 3. pSO(5) Andreev peaks: Eq. (10) is plotted for a choice of $t_n/\Delta_s^{3/2} = 2^{-n}10^{-4}$, $n \leq 5$, and a background conductance of unity. Below the nth peak, the excess tunneling charge is $2ne$, rather than BTK's ne.

is conserved and the charge pairs are coupled mostly to the one dimensional singularity of the magnon density of states. This requirement is less stringent for weakly dispersive magnons near the resonance. In KBT theory for two identical superconductors, the peaks appear at voltages $V_n^{KBT} = 2\Delta/ne, n = 1, 2, \ldots$ which are the upper threshold for tunneling of charges $Q_n = ne$. Thus, KBT allows both an even and odd number of electron charges to participate in the multiple Andreev reflection process, as depicted in Fig. 1, while the pSO(5) theory expects only pair charges $Q_n = 2ne$. Observation of Andreev reflection enhanced shot noise $S(V)$ has been reported by Dieleman et al. [19] in a conventional SNS junction. They have measured the tunneling charge via the relation [20] $S = 2Q_nI(V_n)$. We propose that a similar measurement in YBCO junctions could provide a decisive discrimination between the processes of Figs. 1 and 2. The goal is to measure the charge increments $Q_n - Q_{n-1}$ at the peak positions V_n and see whether they are of magnitude $2e$ rather than e. The measurement would probably involve a careful subtraction of the large but smooth background quasiparticle contribution to the current and the noise spectrum. We eagerly look forward to results of such experiments.

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