Comment on "Optimal Stroke Patterns for Purcell's Three-Link Swimmer"

In their Letter "Optimal Stroke Patterns for Purcell's Three-Link Swimmer" Tam and Hosoi [1] describe strokes that optimize the distance and swimming efficiency of Purcell's three-link swimmer. The calculations are made in the framework of Cox's slender body theory [2], where the aspect ratio κ, also known as the slenderness, is small.

Tam and Hosoi find that the approximate square and circle at the center of Fig. 1 are optimizers of the distance and efficiency, respectively. Here we want to point out that these strokes are likely to be global optimizers only for κ that are moderately small. However, when κ is sufficiently small, there are other strokes that outperform them. For example, the dumbbell in Fig. 1 (with a narrow "corridor" of about 0.01 rad) will cover about 1.14 the distance covered by the approximate square, (in the *opposite* directions). The case of optimally efficient stroke is similar: The excenter wobbly oval on the top left of Fig. 1 is 1.53 times more efficient than the centered approximate circle of Tam and Hosoi. Neither are claimed to be optimal strokes in any sense, but are just examples of better strokes than the strokes proposed by Tam and Hosoi.

That large strokes outperform small strokes is known from the work of Koehler, Becker, and Stone [3] who considered square strokes. However, large square strokes are considered as physically deficient since the two arms collide. In contrast with these large square strokes, both the dumbbell stroke and the wobbly oval stroke have the property that the two arms move *out of each other's way* and never collide, even if the arms are larger than the body. These strokes are therefore legitimate strokes from a geometric point of view.

Cox's slender body theory is valid when the interaction forces between the links are small compared with the forces due to the motion of the links. Given κ, this then imposes a constraint on the angles: $(\pi - |\Omega_j|)\ln\frac{1}{\kappa} \gg 1$. In the dumbbell case where $\pi - |\Omega| \geq \frac{\pi}{4}$, the constraint is $\log\frac{1}{\kappa} \gg \frac{4}{\pi}$. The wobbly oval has $\pi - |\Omega| \geq 0.14$ and in here Cox's slender body theory imposes the more stringent constraint $\log\frac{1}{\kappa} \gg 7.2$. Therefore, the wobbly oval falls within Cox's theory only for sufficiently slender bodies and only then can we say that they outperform the strokes of Tam and Hosoi. When κ is only moderately small, then, presumably, the stokes of Tam and Hosoi are global optimizers.

We have shown that as κ decreases from moderate to small values, the optimizers found by Tam and Hososi are not global optimizers, and there are strokes that outperform them. It is an interesting open question to determine the values of κ where this transition happens. The answer to this question determines if the orbits other than those found by Tam and Hososi are relevant to applications or are only of academic and mathematical interest. Unfortunately, the determination of this threshold κ appears to be a rather difficult minimization problem.

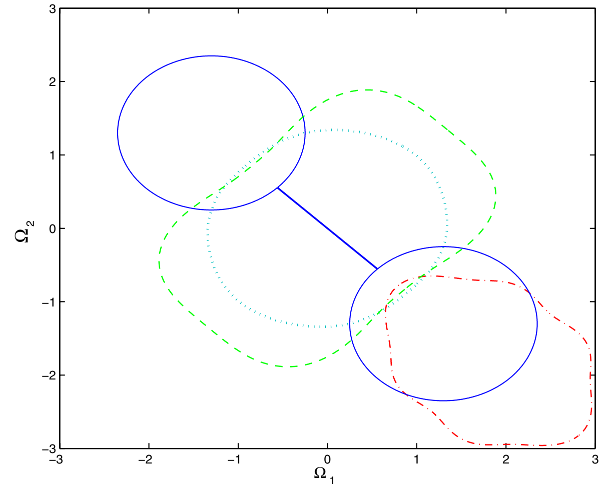

FIG. 1 (color online). Ω_1, Ω_2 are the exterior angles made by the center link and the two arms. The approximate rectangle (green) represents the distance optimizer of the moderate strokes of Tam and Hosoi. The (blue) dumbbell is a large stroke that outperforms the moderate stroke. The approximate (light-blue) circle in the center represents the efficiency optimizer of the moderate strokes found by Tam and Hosoi. The off-center (red) wobble oval is a large stroke with higher efficiency.

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