EXERCISE 6- DUE BY 04/01/05

1) **Characterization of Maximally Entangled States**

Consider the set of two-party pure states of a composite system AB defined on a product Hilbert space of dimension N^2 with the property

$$\rho_A = \rho_B = \frac{1}{N} I,$$

where $\rho_i, i = A, B$ denotes the reduced density operator on each subsystem and I is the identity operator in N dimensions. Show that this set coincides with the set of states generated from one of its elements by applying single-party unitary transformations $U_A \otimes I$, or equivalently $I \otimes U_B$.

2) **Two-qubit Fourier transform**

Consider the transformation representing the *quantum Fourier transform* for two qubits

$$U_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}.$$

Give a decomposition of this operator into a product of two-level unitary operators.

3) **Composite systems quantum measurements**

Let $|\Psi\rangle_{AB}$ be a pure state of a composite system AB, shared between two parties, Alice and Bob, defined on a Hilbert space $H_A \otimes H_B$, with $dim(H_A) = dim(H_B) = N$. Consider the following measurements on the system described by $|\Psi\rangle_{AB}$:

i) Alice performs a measurement described by a POVM $\{E_j^A\}_{j=1,\ldots J}$, getting outcome j and then Bob performs a measurement described by a POVM $\{E_k^B\}_{k=1,\ldots K}$, knowing the result of the first measurement.

ii) Alice and Bob perform a joint measurement described by the POVM $\{E_j^A \otimes E_k^B\}$.

iii) Alice performs a measurement described by $\{E_j^A\}_{j=1,\ldots J}$ and then Bob performs a measurement described by $\{E_k^B\}_{k=1,\ldots K}$, without knowing the result of the first measurement.

Compute in each case the probabilities that the sequence of measurements gives result (j,k) for initial state $|\Psi\rangle_{AB}$. Compute, in each case, the probability that Bob gets result k, conditional on the result of Alice. What is in each case the state after the measurement?

4) **Zero Fourier transform projector**

Consider the Fourier transform of the state $|0\rangle$ in N dimensions given by

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$$

and define the operator $U = 2|\psi\rangle\langle\psi| - I$.

Show that the action of U on a general state $|\alpha\rangle = \sum_k \alpha_k |k\rangle$ produces the state $\sum_k [-\alpha_k + 2\langle\alpha\rangle] \, |k\rangle$, where $\langle\alpha\rangle \equiv \sum_k \alpha_k / N$.

5) **Multiplicative inverse mod N**

Compute the multiplicative inverse of 17 modulo 24.