Landau-Zener tunneling for Lindbladians

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Outline

- Crash course on Linadbladians
- Adiabatic evolutions

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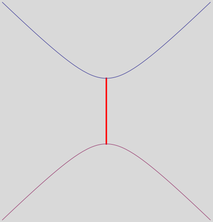
- Adiabatic evolutions
- Tunneling rate out of P for Lindbladian (matrix) $\mathcal{L}$ is

$$\dot{T} = -\operatorname{Tr}\left(\dot{P}\mathcal{L}^{-1}(\dot{P})\right) \geq 0$$

- Landau-Zener-Majornana tunneling with dephasing γ

$$T = \frac{2\epsilon}{3g^2}\frac{\gamma}{1+\gamma^2} + O(\epsilon^2)$$

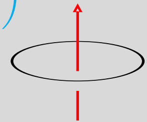


- Contrast with LMZ formula

$$T = e^{-\pi g^2/2\epsilon}$$

Motivating example

- Unitary evolution $U(t) = \begin{pmatrix} e^{ig(t)/2} & 0 \\ 0 & e^{-ig(t)/2} \end{pmatrix}$
- Evolution of density matrix



$$\rho(t) = \begin{pmatrix} \square & \square\, e^{ig(t)} \\ \square\, e^{-ig(t)} & \square \end{pmatrix}$$

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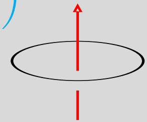

$$\rho(t) = \begin{pmatrix} \Box & \Box\, e^{ig(t)} \\ \Box\, e^{-ig(t)} & \Box \end{pmatrix}$$

- Suppose g_ω stochastic process

$$\langle \rho_\omega(t) \rangle = \begin{pmatrix} \Box & \Box \left\langle e^{ig_\omega(t)} \right\rangle \\ \Box \left\langle e^{-ig_\omega(t)} \right\rangle & \Box \end{pmatrix}$$

Gaussian process

- g_ω Gaussian $\left\langle e^{ig_\omega} \right\rangle = e^{i\langle g_\omega \rangle - \left\langle \delta^2 g_\omega \right\rangle}$
- g_ω Brownian motion with drift

$$\langle g_\omega(t) \rangle = \mu t, \quad \left\langle \delta^2 g_\omega \right\rangle = Dt$$

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- Average-state evolution generated by

$$\langle \rho \rangle = \rho, \quad \dot{\rho} = \mathcal{L}(\rho)$$

$$\mathcal{L}(\rho) = -i[H, \rho] + \left(2\Gamma\rho\Gamma - \rho\Gamma^2 - \Gamma^2\rho\right)$$

$$2H = \mu\sigma_z, \; \Gamma = \sqrt{D}\sigma_z$$

- A dephasing Lindbladian

Geometric interpretation

- Unitary: Rotation of Bloch sphere

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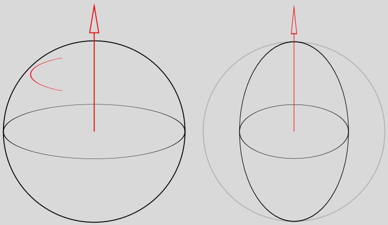
- Unitary: Rotation of Bloch sphere
- Average of unitaries: Contraction of Bloch sphere

Hamiltonian:
Rotation



Lindbladian:
Rotation+
contraction
about B axis

Crash course on Lindbladians

- Evolution of quantum state $\dot{\rho} = \mathcal{L}(\rho)$
- Lindbladian structure:

$$\mathcal{L}(\rho) = -i[H, \rho] + (2\Gamma\rho\Gamma^* - \rho\Gamma^*\Gamma - \Gamma^*\Gamma\rho)$$

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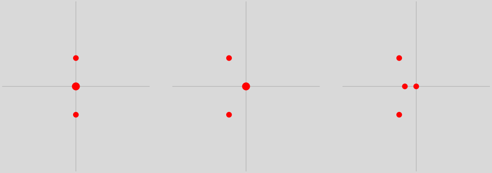
- $\Gamma = 0$ unitary (Heisenberg) evolutions
- $[\Gamma, H] = 0$ *Dephasing Lindbladians*:
- $\Gamma = \textit{anything}$ General Lindbladians

Spectral properties

- Dissipative
- Scenarios



Hamiltonian Dephasing Generic

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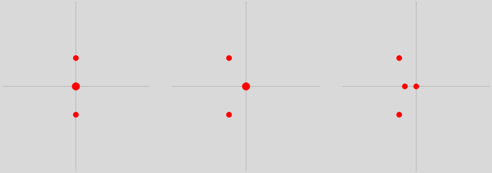

Hamiltonian Dephasing Generic

- $Spec(\mathcal{L})$ in Hamiltonian case

$$\mathcal{L}(\,|j\rangle\langle k|\,) = i(e_j - e_k)\,|j\rangle\langle k|$$

- Note: 0 multiply degenerate

Lindbladians: Physical meaning

- Stochastic time dependence
- Interaction with Markowian bath
- Quantum measurments; measurements of H dephasing

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- Interaction with Markowian bath
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- $\rho \geq 0$ Positivity preserving
- $\mathrm{Tr}\,\rho = 1$ preserving

Adiabatic evolutions

- Real time t; slow time $s = \epsilon t$; ϵ adiabaticity

$$\epsilon \dot{\rho} = \mathcal{L}_s(\rho)$$

- $\mathcal{L}_s$ depend on slow time; $\epsilon \to 0$ singular limit.

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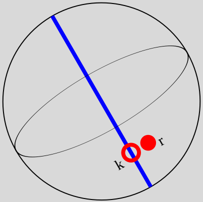
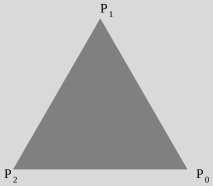
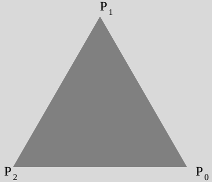
- $\mathcal{L}_s(\rho_s) = 0, \quad \rho_s =$ instantaneous stationary
- $\rho = k + r, \quad k \in Ker\ \mathcal{L}, \ \ r \in Range\, \mathcal{L}$

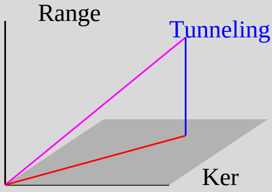




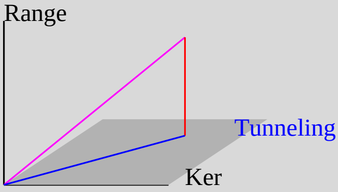
Dephasing Lindbladian vs Hamiltonians

- Tunneling: Motion in *Range* $(H - e)$
- Tunneling: Motion in *Ker* $\mathcal{L}$
- Adiabatic dynamics: Motion in *Ker* $(H - e)$
- Motion in *Range* $\mathcal{L}$: phasing





Hamiltonian



Lindbladian

Adiabatic expansion

$$\varepsilon \dot{\rho} = \mathcal{L}_s(\rho)$$

$$\rho(s) = \sum \varepsilon^n (k_n(s) + r_n(s)), \quad k \in Ker\, \mathcal{L}, \quad r \in Range\, \mathcal{L}$$

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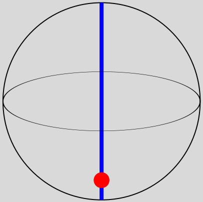
$$n = 0, \quad 0 = \mathcal{L}\big(k_0(s) + r_0(s)\big) = \mathcal{L}\big(r_0(s)\big) \Rightarrow r_0(s) = 0$$

0-th order lives in $ker\,\mathcal{L}$



Motion in Kernel: 1-st order

- $n = 1, \quad \dot{k}_0(s) = \mathcal{L}(k_1 + r_1) = \mathcal{L}(r_1)$
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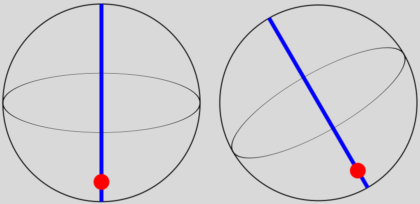
- Motion in $Ker\, \mathcal{L}_s$: $\quad \sum \dot{p}_j P_j \Rightarrow \dot{p}_j = 0$
- Motion frozen in $Ker\, \mathcal{L}_s$ to order ϵ^0



Tunneling: 1-st order

- Start at ground state $P_0 \Rightarrow k_0(s) = P_0(s)$
- $\dot{k}_0 = \mathcal{L}(r_1) \Rightarrow r_1(s) = \mathcal{L}^{-1}(\dot{P}_0)$, an algebraic equation
- All we need to compute tunneling

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- All we need to compute tunneling
- Tunneling rate out of P_0 to leading order:

$$\dot{T} = \frac{d\,\mathrm{Tr}\left((1 - P_0)\rho\right)}{dt} = -\epsilon^2\,\mathrm{Tr}\left(\dot{P}_0\mathcal{L}^{-1}(\dot{P}_0)\right) \geq 0$$

- Tunneling is irreversible

Dephasing Lindbladians: 2 level systems

- For 2-level $H = B \cdot \sigma$. Since $H^2 = B \cdot B$ any $f(H) = a + bH$
- Since $[\Gamma, H] = 0 \Rightarrow \Gamma = a + bH$ any dephasing Lindbladian

$$\mathcal{L}(\rho) = -i[H, \rho] + \gamma_f \left[[H, \rho], H\right]$$

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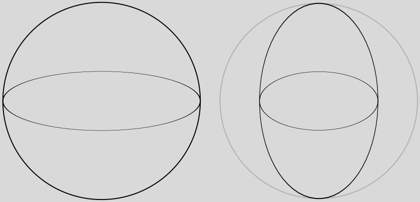
- $\Gamma = H \sim \sqrt{H} \sim P$ etc are equivalent up to redef γ.
- Action on Bloch sphere

Hamiltonian: Rotation



Lindbladian: Rotation+ contraction about B axis

Landau-Zener-Majorana model

- $2H(s) = \begin{pmatrix} s & g \\ g & -s \end{pmatrix}$
- Dimensionless adiabaticity ϵ/g^2
- Generic *local* behavior
  near avoided crossing

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- Dimensionless adiabaticity ϵ/g^2
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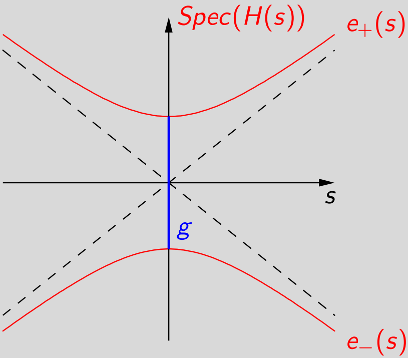
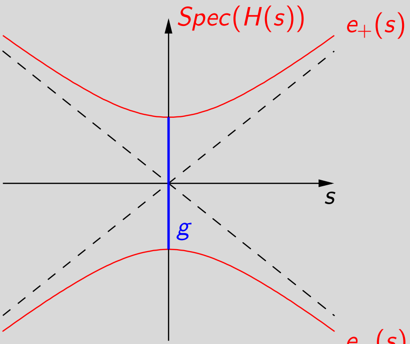
  near avoided crossing
- The tunneling $T = e^{-\pi g^2/2\epsilon}$
- $\pi/2$ not universal
  e.g $s \to s(1+\epsilon s^2)$



Dephasing Landau-Zener

- Write $H = e(s)\,(1 - 2P)$
- Energy scale $e^2 = s^2 + g^2$
- General dephasing
  $\mathcal{L}(\rho) = ie(s)[P, \rho] + \gamma\,[[P, \rho], P]$



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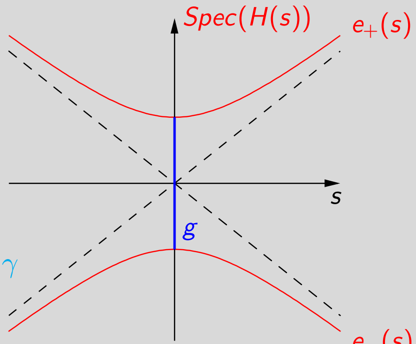
  $\mathcal{L}(\rho) = ie(s)[P, \rho] + \gamma\,\big[[P, \rho], P\big]$
- Constant *dephasing rate* $\gamma(s) = \gamma$
- Single energy scale $\gamma(s) = \gamma e(s)$



Tunneling formula

$$T = 2\epsilon \int \frac{\gamma(s)\ \mathrm{Tr}(\dot{P}^2)}{e^2(s) + \gamma^2(s)}\, ds \geq 0, \quad \mathrm{Tr}(\dot{P}^2) = \frac{g^2}{e^4(s)}$$

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- Single scale

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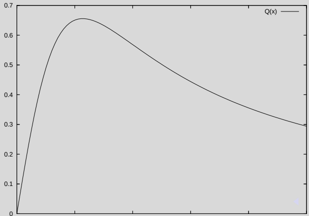
- Constant dephasing

$$T = \frac{\pi\varepsilon}{4g^2} Q\left(\frac{\gamma}{g}\right), \quad Q(x) = \frac{\pi}{2}\frac{x(2+\sqrt{1+x^2})}{\sqrt{1+x^2}(\sqrt{1+x^2}+1)^2}$$



Summary

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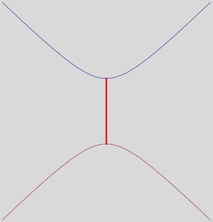
$$T = \frac{2\epsilon}{3g^2}\frac{\gamma}{1+\gamma^2} + O(\epsilon^2)$$

- Unitary LMZ formula

$$T = e^{-\pi g^2/2\epsilon}$$



- Open issue: Molecular magnets