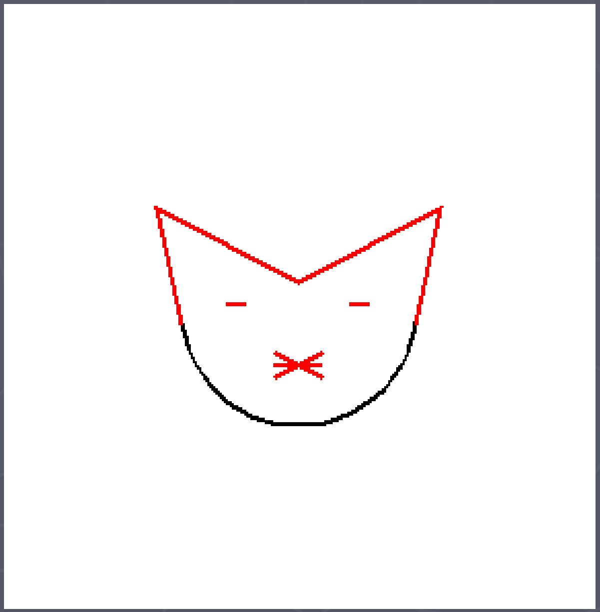
The Baron and the cat

Oded Kenneth

Baron and cat

- A cat can rotate with zero angular momentum
- Can Baron von Munchausen lift himself?
- Translate with zero momentum?



Movie: omri Gat

H. Knoerer: Cats always fall on their feet

Newton's law

If a system experiences no external force,
the center-of-mass of the system will remain at rest.

http://webphysics.davidson.edu/physlet_resources/bu_semester1/c12_cofm_motion.html

Newton:

Lex I: Corpus omne perseverare in statu suo quiescendi vel Movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed

Wikipedia

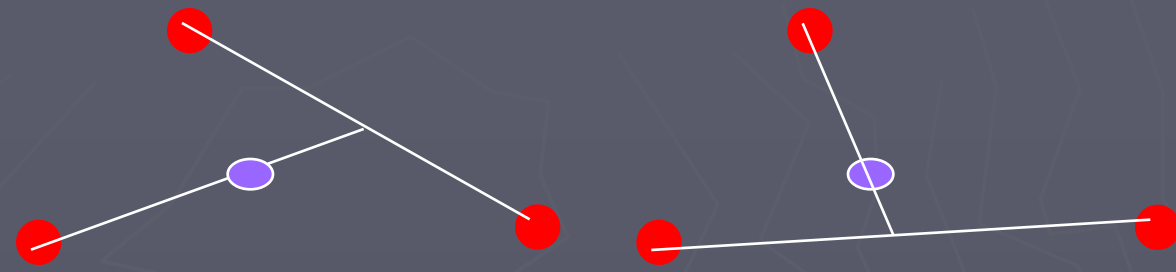
Respecting the third law

- Internal forces equal and opposite
- Guarantee conservation of total momentum

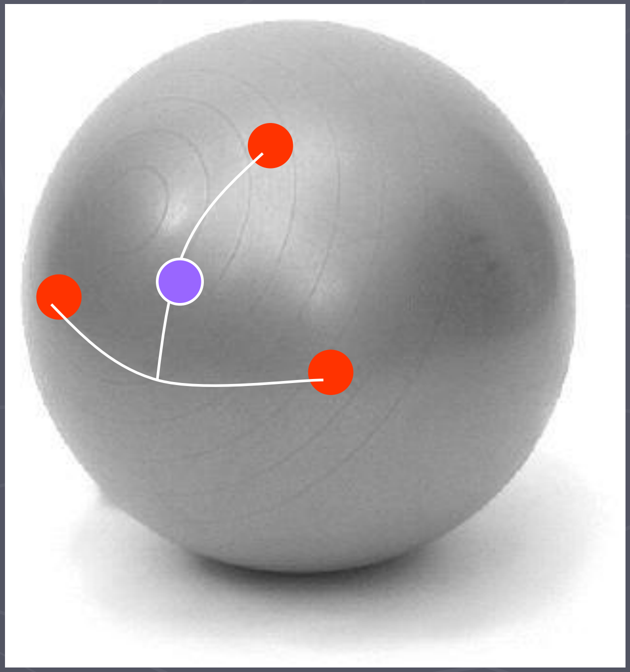
It is possible to swim in empty curved space at zero total momentum

Ambiguity in center of mass

Euclidean plane

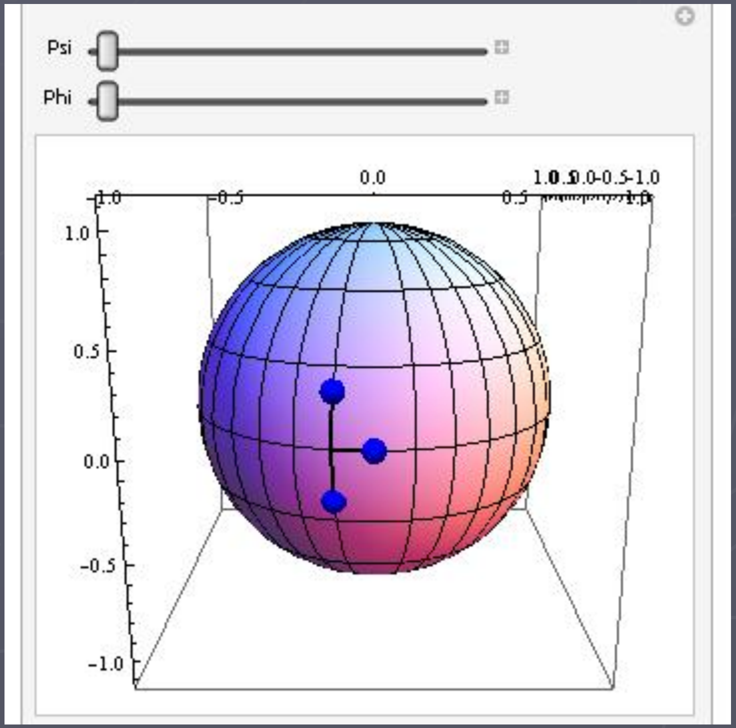


On a sphere



Swimming triangles

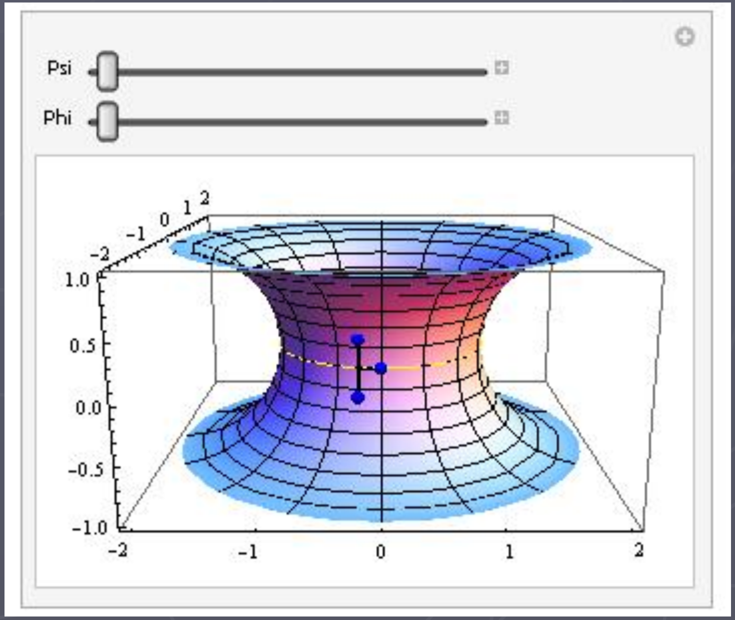
On sphere



Movies: Oren Raz

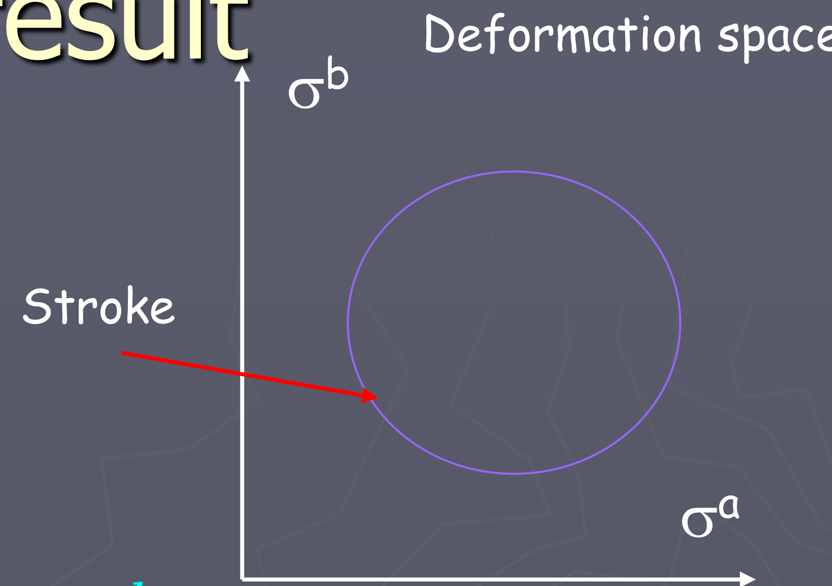
Swimming triangles

Inside a torus



Movies: Oren Raz

Main result



Swimming distance

$$\text{đ}\tau = \langle d\xi | \eta_a, \eta_b \rangle \; d\sigma^a \wedge d\sigma^b$$

Killing fields

Deformation fields

Why cats rotate while Baron lies

Translations $$\xi_t = dx \longrightarrow d\xi_t = 0$$

The Baron lies

Rotations

$$\xi_R = x dy - y dx \longrightarrow d\xi_R = 2 dx \wedge dy$$

The cat falls on its feet

Small swimmers: Wisdom's type formula

Property of space

How hard swims

$$\delta x_\mu \sim R_{\mu\nu\alpha\beta}\, x^\nu\, dx^\alpha \wedge dx^\beta$$

Wisdom
Swim away from Black hole
Relativistic motions

AK:
Symmetric spaces
Non relativistic bodies

Outline

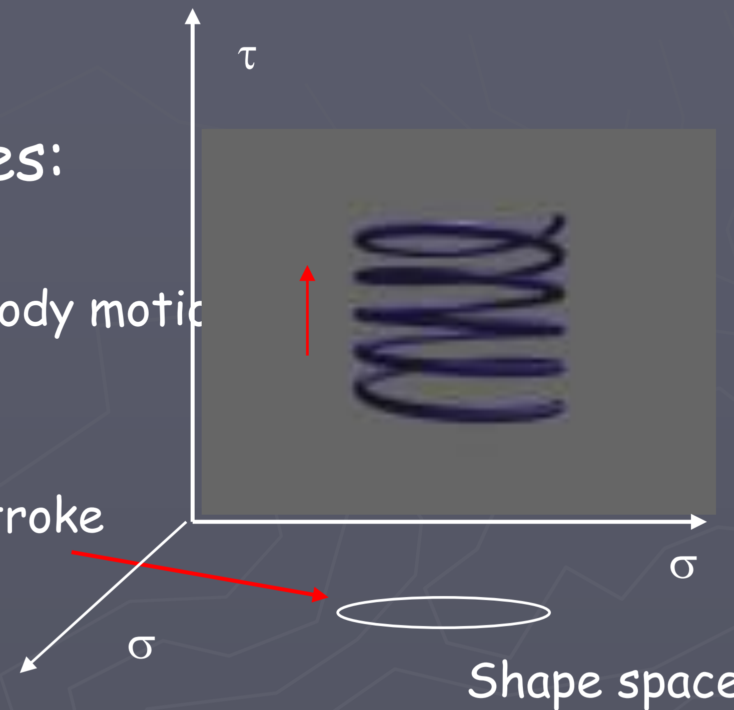
- Setting the problem
- Killing and deformation fields
- Swimming in empty space

Wisdom, Science

AK NJP

The swimming problem

Swimming with periodic strokes:

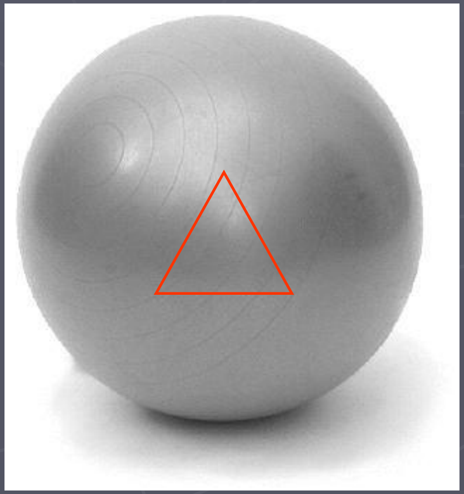


Shapes are hard to fit in a curved space

Periodic strokes bring you back home

Swimming in symmetric spaces

- Symmetric spaces: Isotropic and homogeneous
- Examples: Euclidean plane, Sphere, Hyperbolic plane

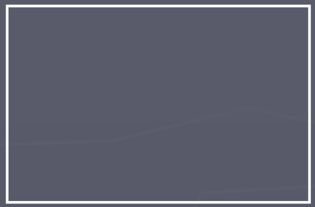


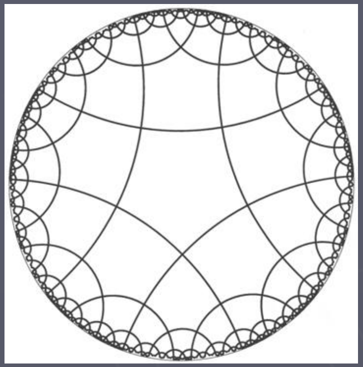
Killing fields: Euclidean case

Killing for Euclidean translation $\xi_k = \partial_k$

Killing for Euclidean rotation $\xi_\theta = x_1\partial_2 - x_2\partial_1$

Killing fields for Symmetric planes





Metric

$$(d\ell)^2 = \frac{|dz|^2}{(1+R|z|^2)^2}, \qquad R = 0, \pm 1$$

Killing fields

$$\xi_1 = \partial_x + R\left((x^2-y^2)\,\partial_x + 2xy\partial_y\right)$$

$$\xi_2 = \partial_y + R\left(2xy\,\partial_x + (y^2-x^2)\,\partial_y\right)$$

$$\xi_3 = x\partial_y - y\partial_x$$

Conservation of momentum

A system of particles

$$P_\xi = \sum_n p_n \cdot \xi(x_n), \quad p_n = \frac{\partial L}{\partial \dot{x}_n} = m\dot{x}_n$$

scalar

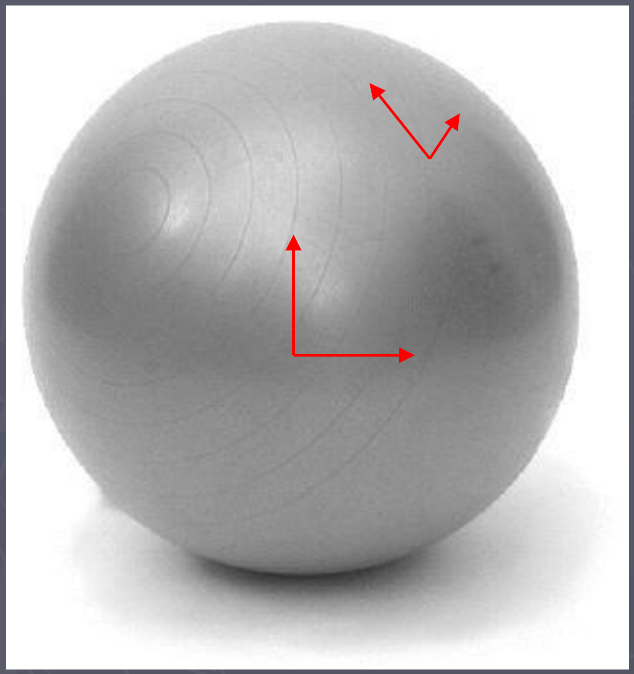
Swimming equations

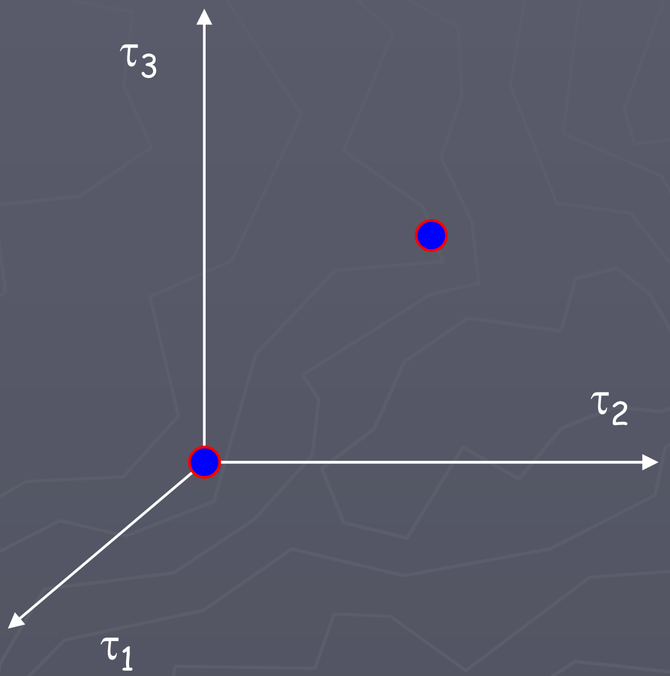
$$P_\xi = 0 \longrightarrow \sum m_n \xi(x_n) \cdot dx_n = 0$$

As many equations as Killing fields:
Can't move in other directions

Eq. independent of time parameterization: Geometric

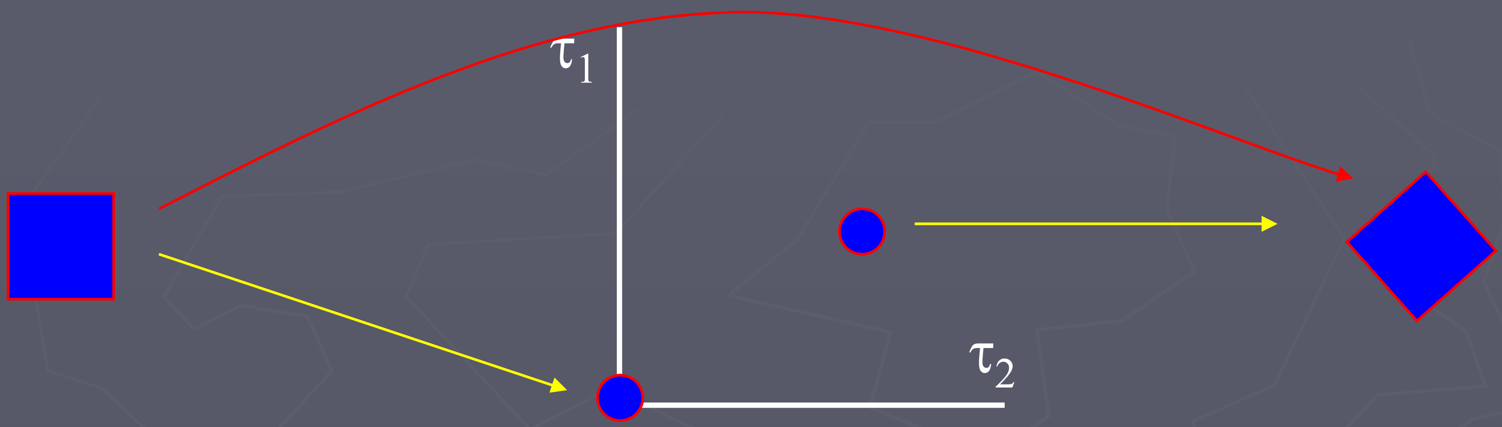
Rigid body motion: Coordinates





Putting coordinates for
The analog of Euclidean
motions

Coordinates for rigid body motion



$$\dot{x}(t) = \tau \cdot \xi\big(x(t)\big), \quad x(0) \to x_\tau(1)$$

$$S(\tau) = e^{\tau\xi} S$$

Symbolic notation for transported shape

Controls: Deformations

Example: Euclidean case

$$\eta_1 = x\partial_x - y\partial_y$$

$$\eta_2 = x\partial_y + y\partial_x$$

Vector field for deformations depend on choice of origin: Ambiguity in Killing

$$\eta_1 \rightarrow \eta_1 + a\xi_x - b\xi_y$$

Gauge choice instead of CM

$$\langle \xi | \eta \rangle = \frac{1}{M} \sum m_n \xi(x_n) \cdot \eta(x_n)$$

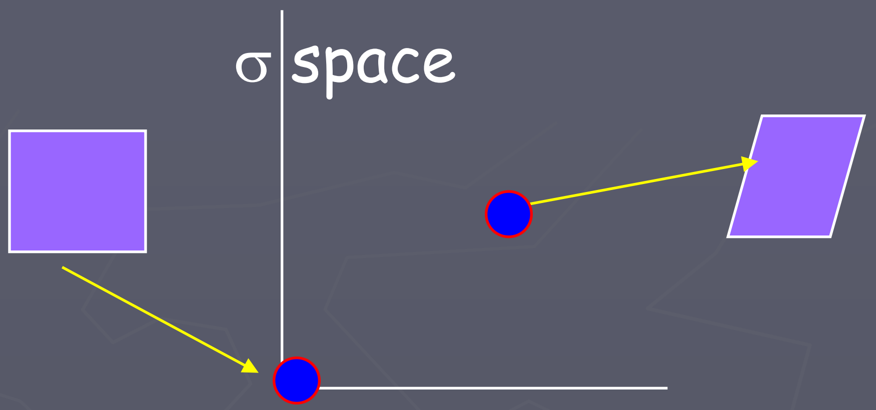
Euclidean example:

$$\langle \xi_1 | \eta_1 \rangle = \frac{1}{M} \sum m_n \, x_n$$

Gauge choice: Analog to choosing cm as fiducial pt for translations

$$\langle \xi_j | \eta_a \rangle = 0$$

Coordinates for deformed shapes



$$\dot{x}(t) = \sigma \cdot \eta\big(x(t)\big), \quad x(0) \rightarrow x_\sigma(1)$$

Symbolically $$S(\sigma) = e^{\sigma\eta} S$$

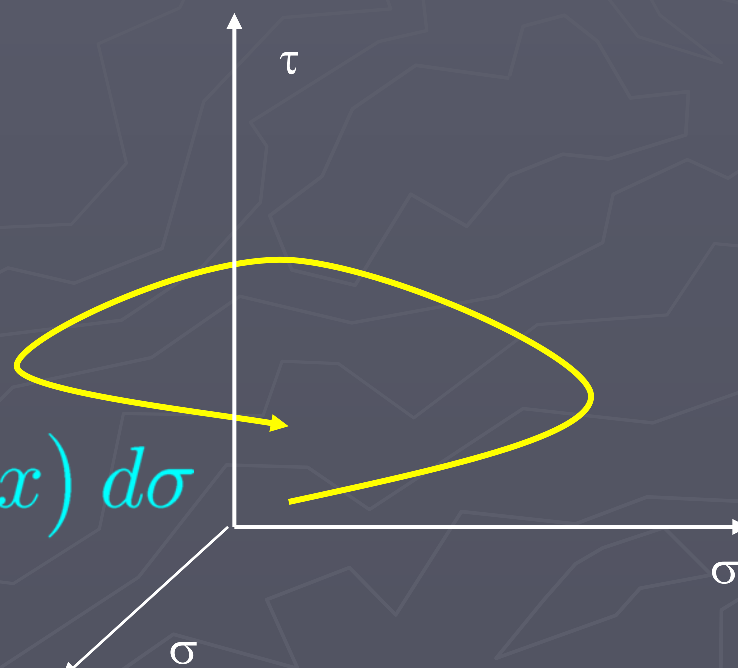
Total space

Coordinates for deformations + Euclidean motions

$$S(\tau,\sigma) = e^{\tau\xi}e^{\sigma\eta}S$$

$$x_0 \longrightarrow x(\tau,\sigma;x_0)$$

$$dx(\tau,\sigma;x_0) = (\partial_\tau x)\, d\tau + (\partial_\sigma x)\, d\sigma$$



Motions is legit if it consistent with zero total momentum

Eq. of Motion

To leading order with σ and τ small

$$dx = \xi d\tau + \eta d\sigma, \quad \xi(x(\sigma,\tau)), \quad \eta(x(\sigma,\tau)),$$

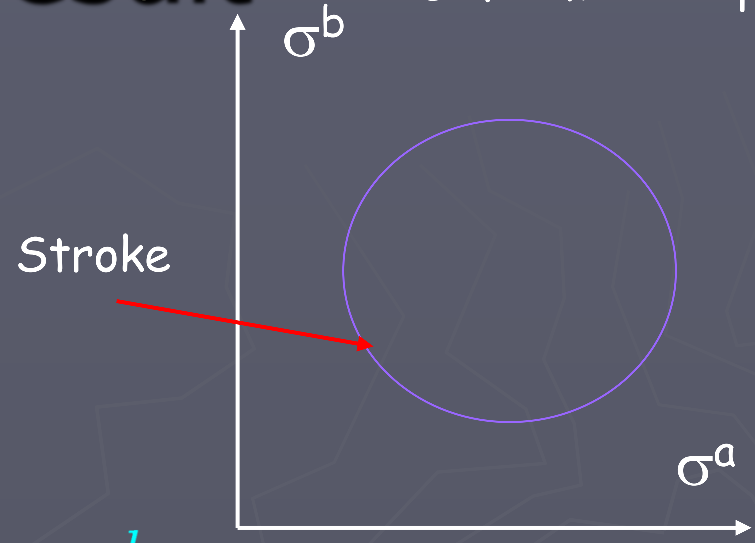
Substitute in conservation law

$$\sum m_n \xi(x_n) \cdot dx_n = 0$$

Gives a linear (system) of equations for dτ

Main result

Deformation space



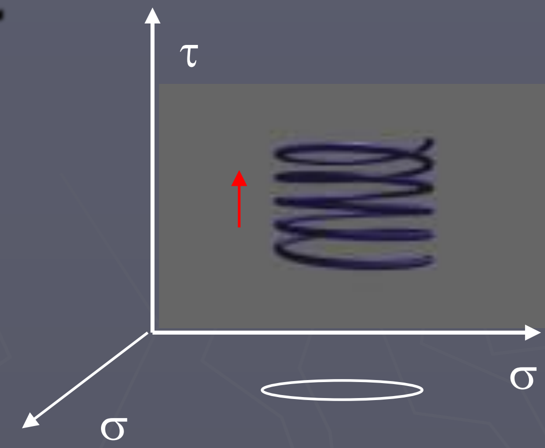
Swimming distance

$$\bar{d}\tau = \langle d\xi | \eta_a, \eta_b \rangle \; d\sigma^a \wedge d\sigma^b$$

Killing fields

Deformation fields

Main result



Swimming distance

$$đ\tau = \langle d\xi | \eta_a, \eta_b \rangle \, d\sigma^a \wedge d\sigma^b$$

Killing fields

Deformation fields

Shape space

$$đ\tau^\alpha = \langle \partial_{[k} \xi^\alpha_{j]} | \eta^k_a \eta^j_b \rangle \, d\sigma^a \wedge d\sigma^b$$

Riemann

Using

$$\nabla_i \nabla_j \xi_\ell = -R_{\ell jik} \xi^k$$

One can write equations for transport for a small swimmer in terms of Riemann

$$M \delta x^k = R_{j\ell ik} \left(\sum m_n x_n^i \eta_a^j (x_n) \eta_b^\ell (x_n) \right) d\sigma^a \wedge d\sigma^b$$

The hero: Oded Kenneth

Thanks to Amos Ori