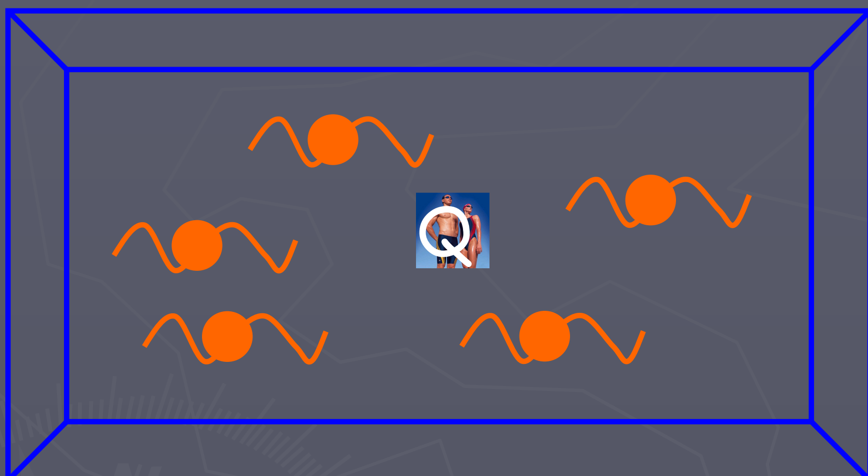
Quantum swimming

Boris Gutkin, David Oaknin

Q-Swimming & scattering



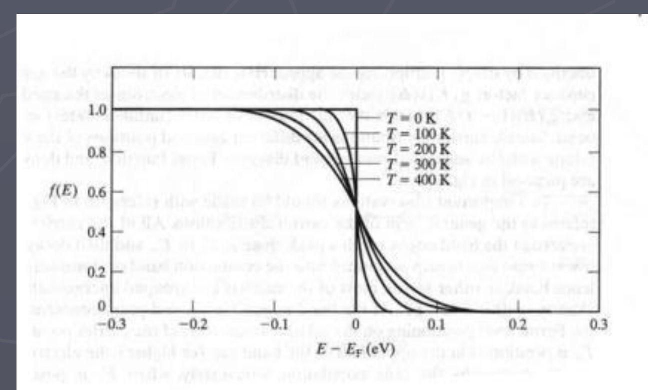
$$S(H, H + V_t)$$

$$\rho(H) = \frac{1}{e^H \pm 1}$$

Swimmer in a quantum sea:

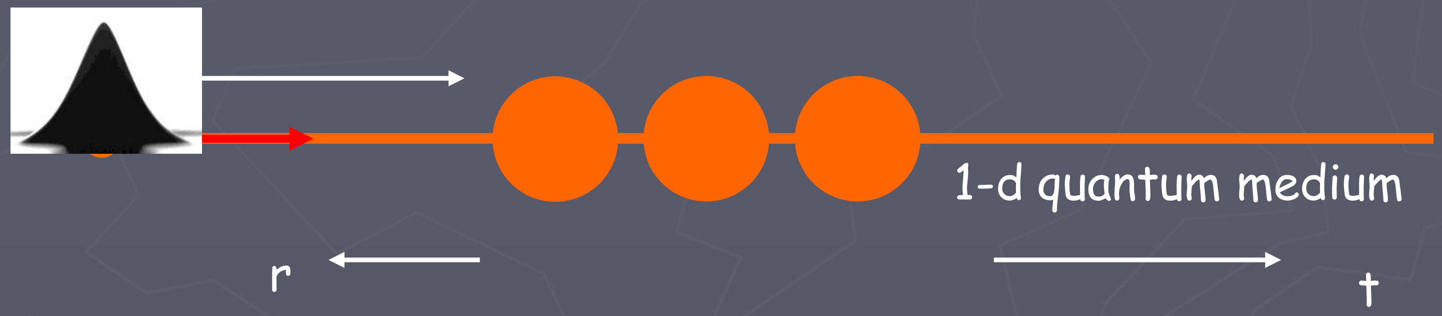
Photon bath

Fermi sea



H

Swimming in one dimension



$$S = \begin{pmatrix} t & r' \\ r & t' \end{pmatrix}$$

On-shell scattering matrix

The swimmer controls the scattering but not its location

Swimming equation

In adiabatic limit

Swimming velocity=F(S,S',ρ)

$$\eta \dot{X} = \frac{i}{2\pi\hbar}\int dE\,\rho'(E) Tr(\dot{S}S^* P)$$

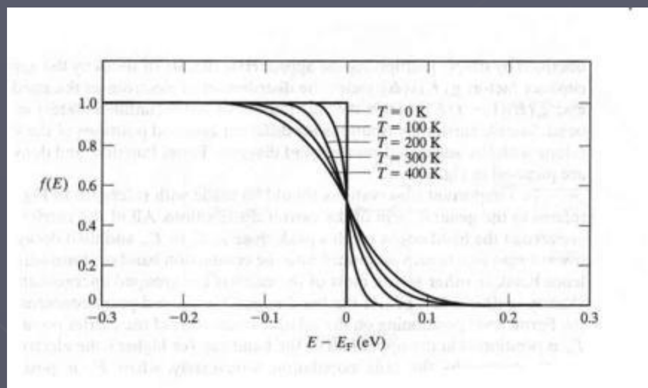
$$P = \begin{pmatrix} p & 0 \\ 0 & -p \end{pmatrix}$$

$$\eta = -\frac{1}{4\pi\hbar}\int dE\,\rho'(E) Tr(|[S,P]|^2)$$

Quantized swimming

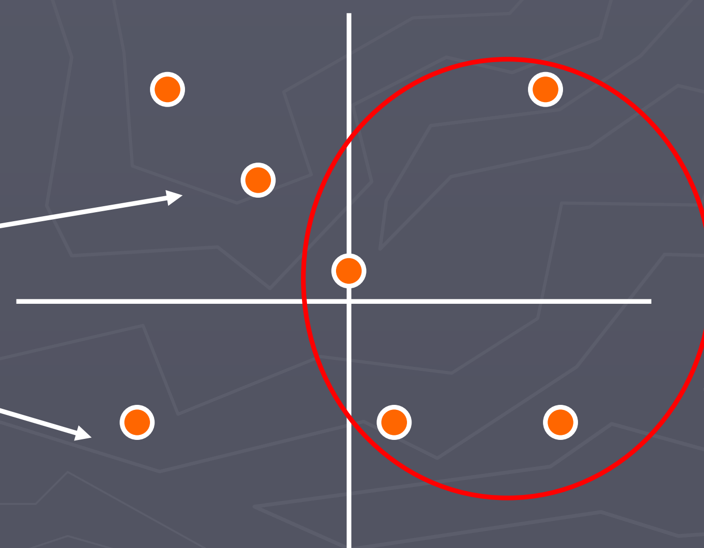
1-d Fermi sea, T=0

ρ



λ

$$\Delta X = \pm \frac{\lambda}{2}$$

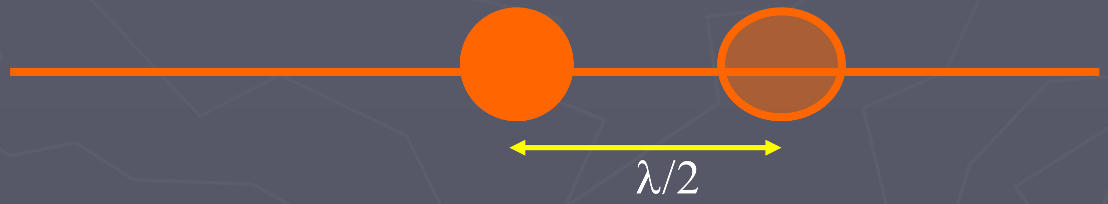


Control space

Points of transparency

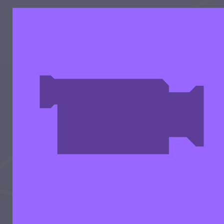
The invisibility principle

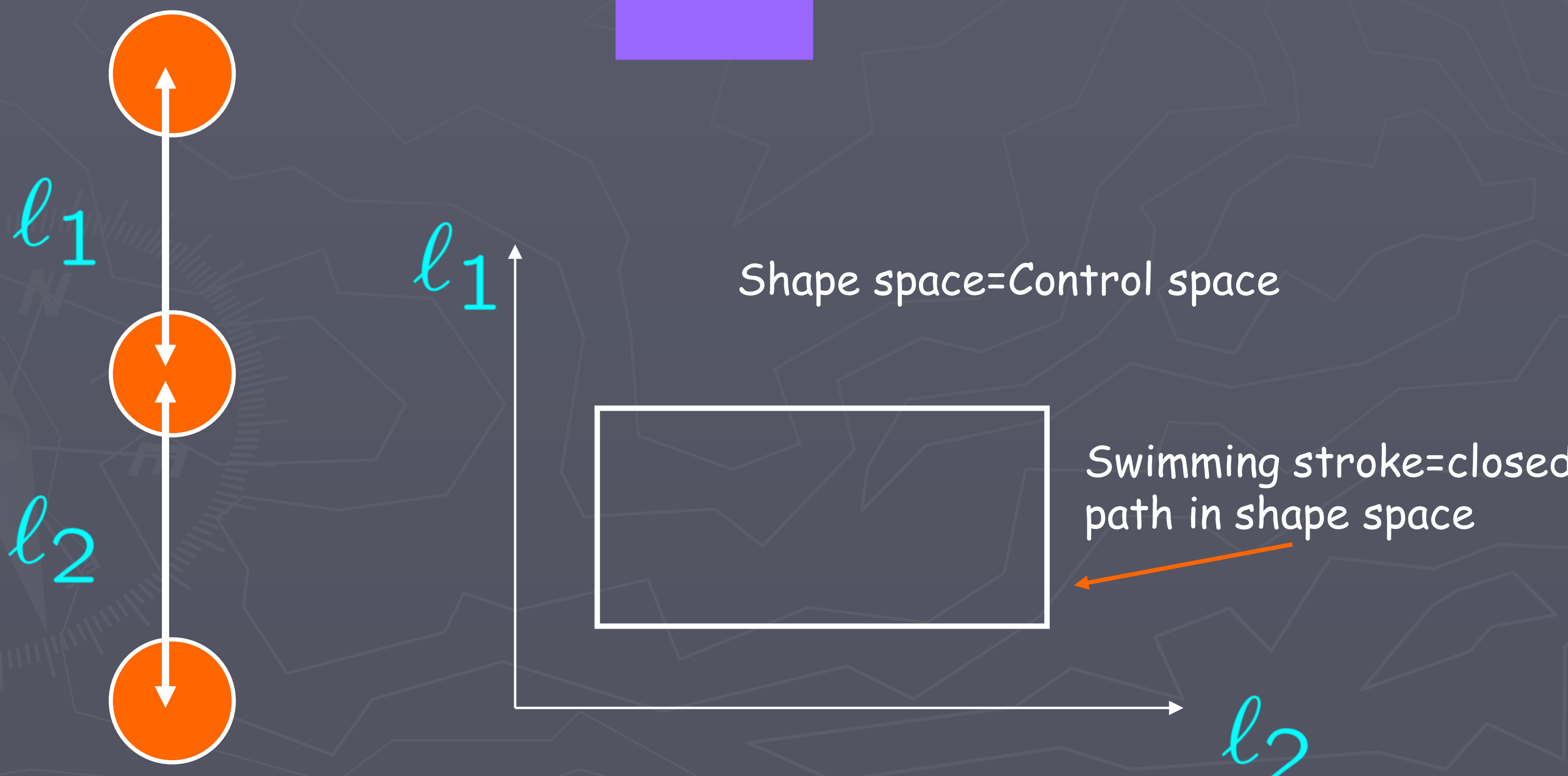
1-d Fermi sea, T=0



The position adjusts so that the medium will think nothing has happened

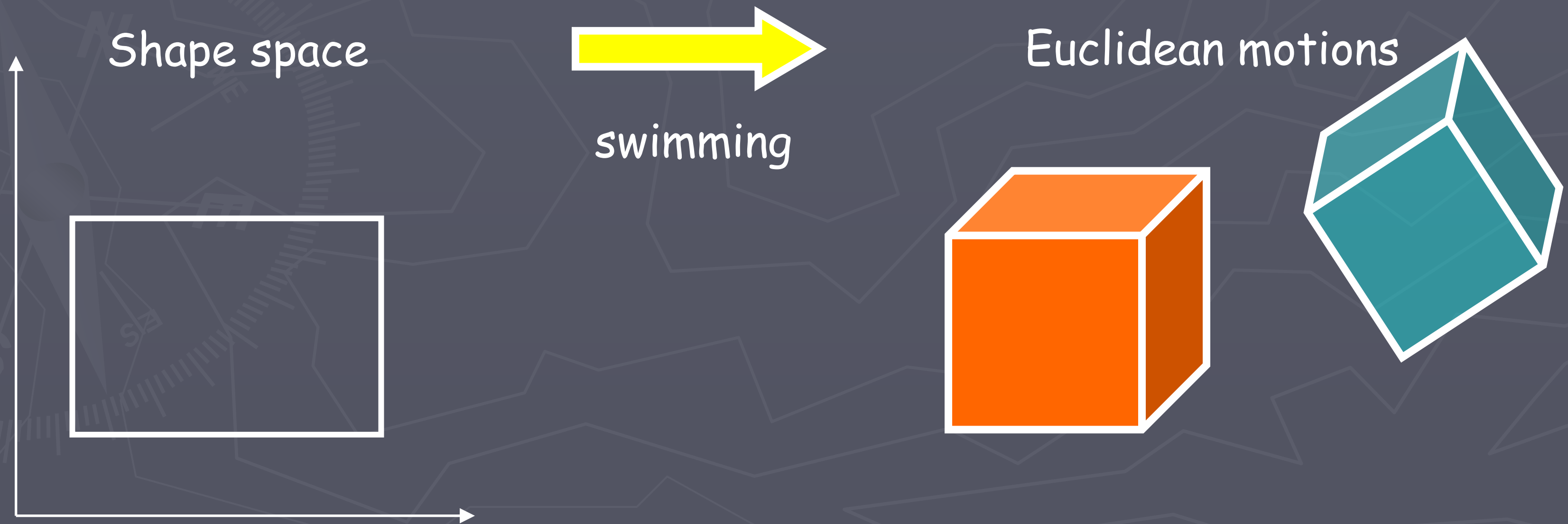
C-swimming competition





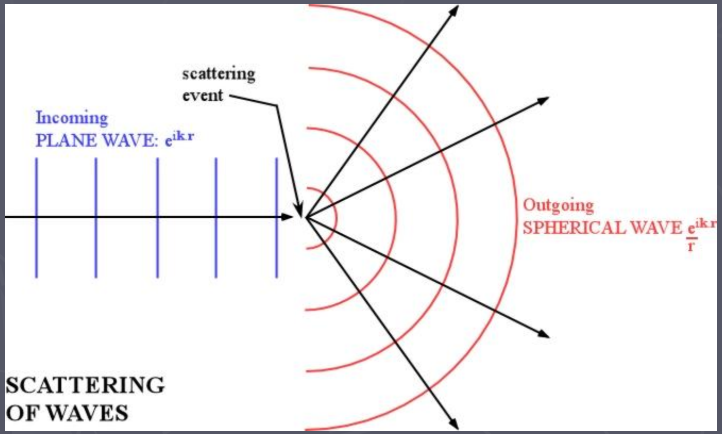
Swimming: definition

Swimming: a map



Untethered scatterers

$$-\Delta + V \longrightarrow -\Delta + V_{x,t}$$



Balanced scatterer

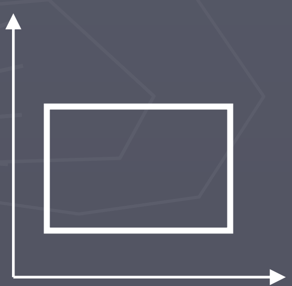
Q-Swimming

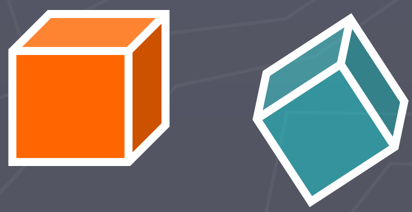
Qswimmer: a unthethered balanced periodic scatterer

$$-\Delta + V \longrightarrow -\Delta + V_{x,t}$$

qswimming

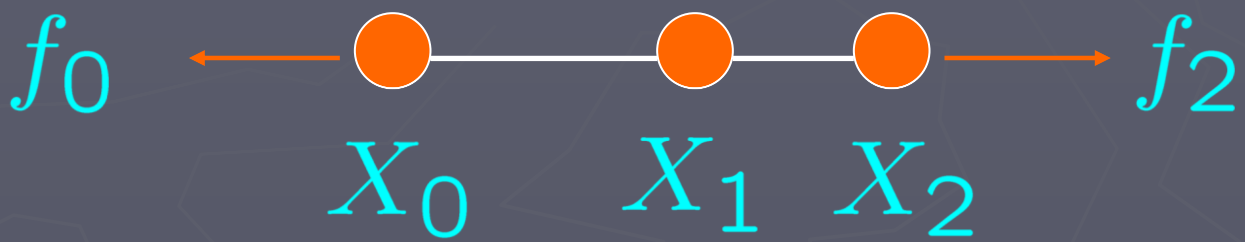
Scattering matrices → Euclidean motions





Need: a principle to fix the location and orientation of balanced scatterer

C-Swimming: The role of friction



Velocity of ball

$$f_j = \sum_k \eta_{jk} \dot{X}_k + \sum_k F_{jk}$$

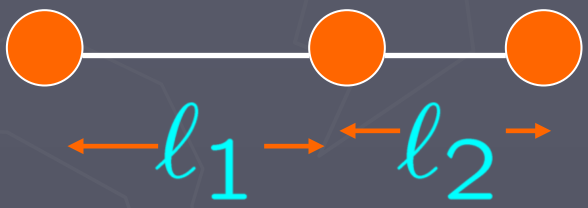
Force on ball

Friction coeff

mutual forces

Control and response

$$\{X_j\} \longleftrightarrow \{\underbrace{X_0}_{rspns}, \underbrace{\ell_1, \ell_2}_{control}\}$$



One unknown, the position X. Equation of motion: equilibrium

$$0 = \sum f_j = \sum \eta_j \dot{X}_j$$

Swimming in geometric

$$\not{d}X = \eta_1 d\ell_1 + \eta_2 d\ell_2$$

Swimming: when dX does not
Integrate to a function

$$\nabla_\ell \times \eta \neq 0$$

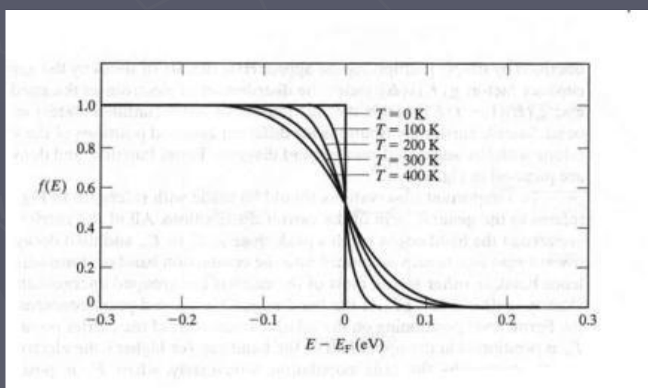
Toolbox

Avron, Buttiker, Elgart, Gutkin, Graf, Oaknin, Pretre, Sadun, Thomas

Key formula

$$S(t) = e^{iHt} S e^{-iHt}, \qquad \dot{S} = i[H, S]$$

ρ



H

$$SHS^* = H + \mathcal{E}$$

$$\mathcal{E} = i\dot{S}S^*$$

$$\rho_{out} = \rho_{in}(H - \mathcal{E}) \approx \rho_{in}(H) - \rho_{in}'(H)\mathcal{E}$$

Avron, Elgart, Graf, Sadun

Conceptual issues

Avron, Buttiker, Elgart, Gutkin, Graf, Oaknin, Pretre, Sadun, Thomas

(E,t) as a canonical pair

$$S(E,t)$$

A function of non-commuting variables

$$E = \frac{p^2}{2}, \qquad t = x/p$$

A canonical transformation of half-line

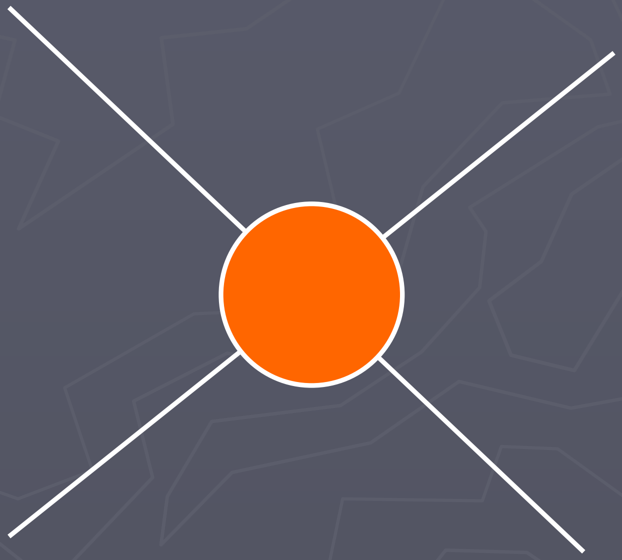
$$\{E,t\} = \{p,x\} = 1$$

Avron, elgart, graf, sadun

Phase space

$S(E, t)$

A function of non-commuting variables



Adiabatic as semi-classical limit