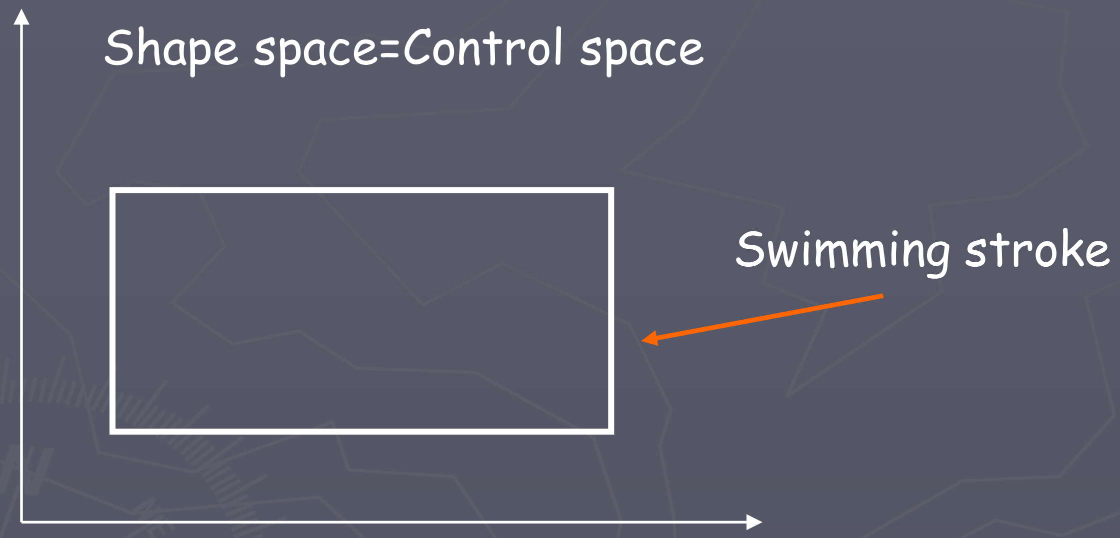
Quantum swimming

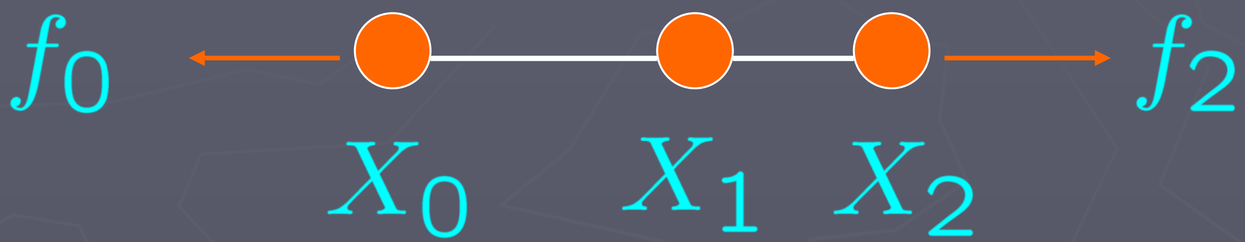
Boris Gutkin, David Oaknin

Swimmers and pumps

Shape and control



Swimming and linear response



Velocity of ball

$$f_j = \sum_k \eta_{jk} \dot{X}_k + \sum_k F_{jk}$$

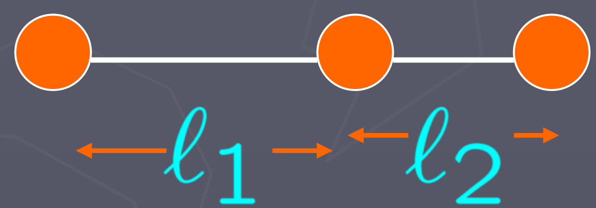
Force on ball

Friction coeff

mutual forces

Control and response

$$\{X_j\} \longleftrightarrow \{\underbrace{X_0}_{rspns}, \underbrace{\ell_1, \ell_2}_{control}\}$$



One unknown, the position X, one equation of motion

$$0 = \sum f_j = \sum \eta_j \dot{X}_j$$

Swimming in geometric

$$\not{d}X = \eta_1 d\ell_1 + \eta_2 d\ell_2$$

Swimming: when dX does not
Integrate to a function

$$\nabla_\ell \times \eta \neq 0$$

Landauer formula for friction

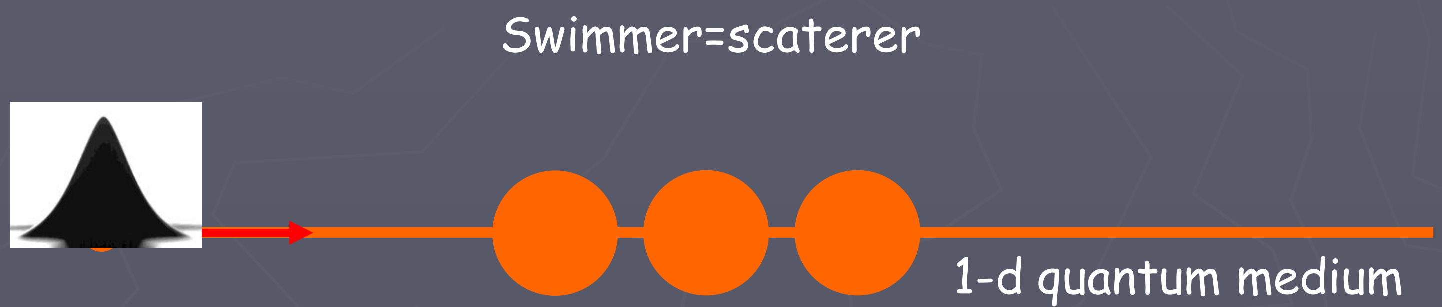
$$\langle f \rangle = \eta \dot{X}$$

$$\eta = \frac{2}{\pi \hbar} p^2 \, |r|^2$$

Reflection

$$[\eta] = \frac{[f]}{[\dot{X}]} = \frac{[p\dot{X}/X]}{[\dot{X}]} = \frac{[p]}{[X]}$$

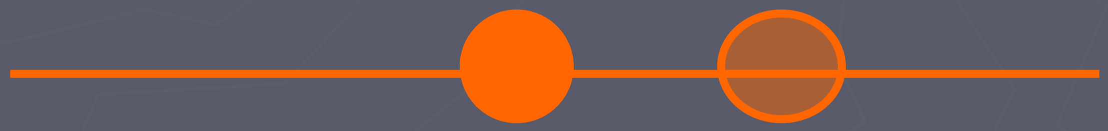
Scattering



$$S = \begin{pmatrix} t & r' \\ r & t' \end{pmatrix} = e^{i\gamma} \begin{pmatrix} i \sin\theta & e^{-i\alpha}\cos\theta \\ e^{i\alpha}\cos\theta & i\sin\theta \end{pmatrix}$$

The swimmer controls the scattering: θ, α, γ

Swimming at T=0

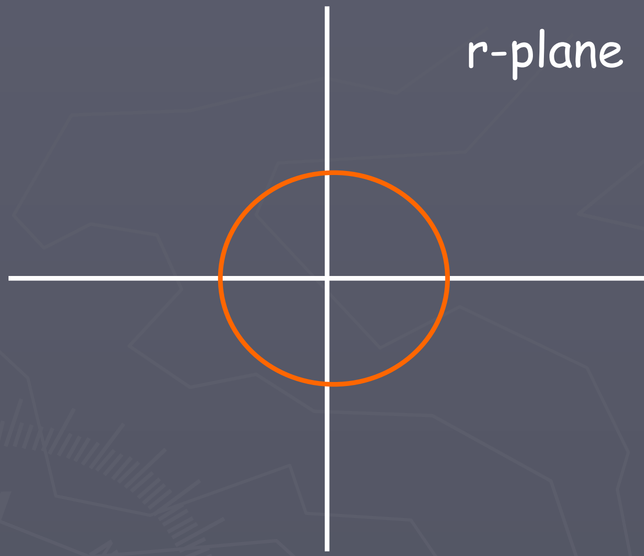


$$r \to e^{4\pi i\,\delta X/\lambda} r, \qquad \alpha \to \alpha - 4\pi\delta X/\lambda$$

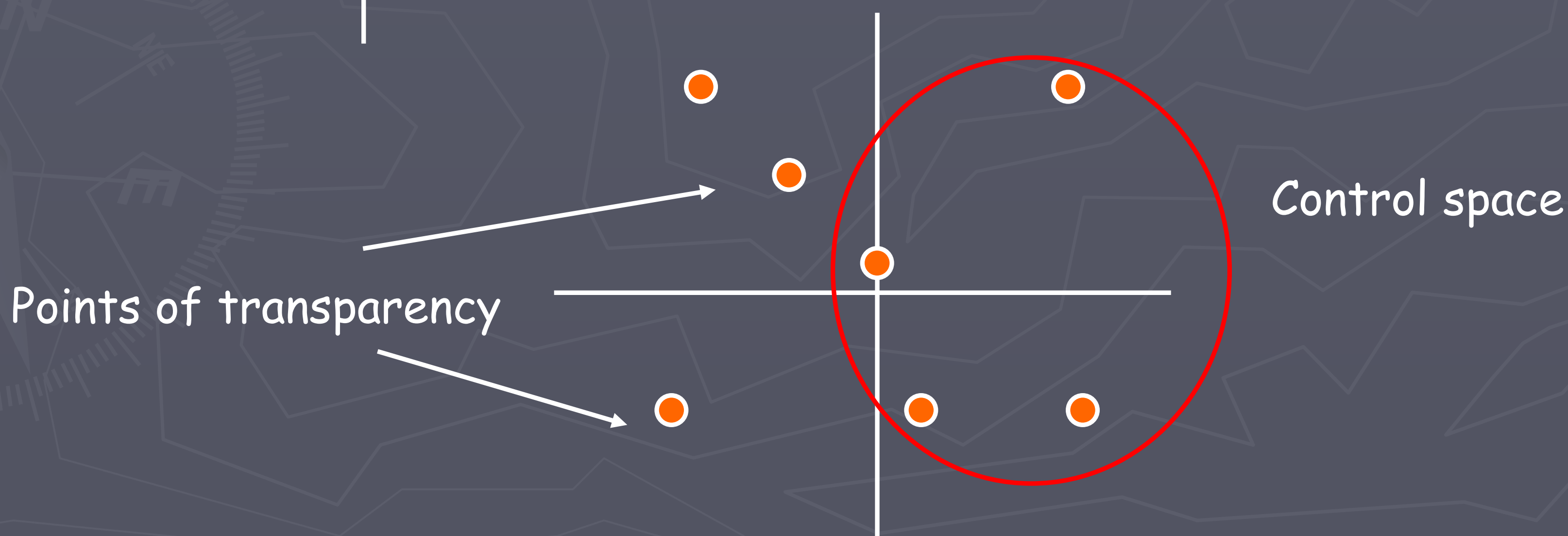
Principle: If the swimmer can control the scattering matrix,
Its position will adjust so that the medium will think nothing has happened

$$\text{đ}X = \frac{\lambda}{4\pi} d\alpha$$

Quantized swimming



$$\Delta X = \frac{\lambda}{2}$$

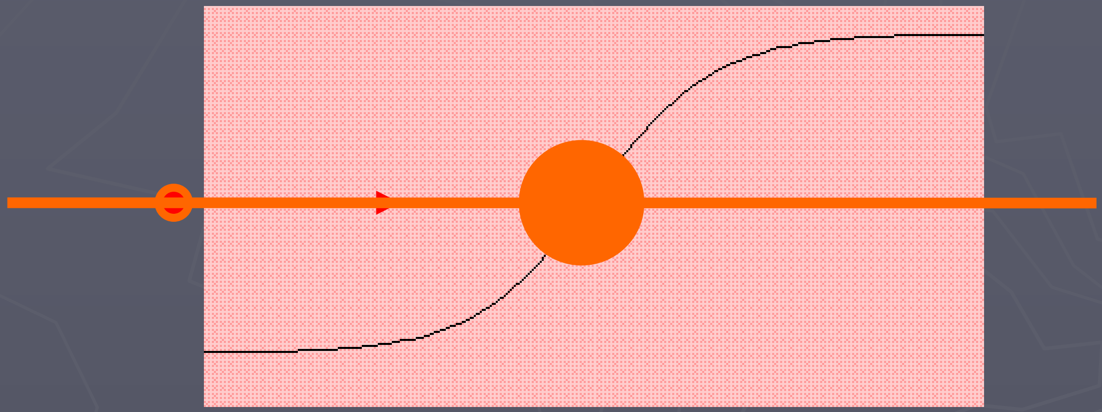


Toolbox

Avron, Buttiker, Elgart, Gutkin, Graf, Oaknin, Pretre, Sadun, Thomas

Energy shift

$$\mathcal{E} = i(\partial_t S)S^\dagger$$



Dual to Wigner time delay

$$\mathcal{T} = i(\partial_E S)S^\dagger$$

Ex: in 2 channel case:

$$\mathcal{E} = \dot{\alpha}\cos^2\theta - \dot{\phi}\sin^2\theta - \dot{\gamma}$$

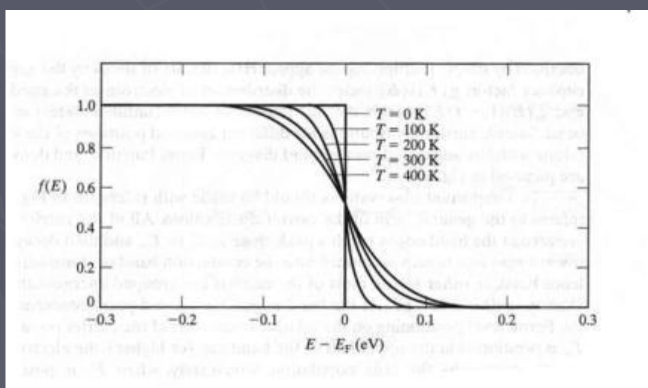
snowplough battery sink

Avron, Elgart, Graf, Sadun

Key formula

$$\rho_{out} = \rho_{in}(H - \mathcal{E}) \approx \rho_{in}(H) - \rho'_{in}(H)\mathcal{E}$$

ρ



H

$$\langle \delta Q \rangle = Tr(\delta\rho\, Q)$$

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BPT

$$\left\langle \dot{Q} \right\rangle (t) = -\frac{1}{2\pi} \int dE \, \rho'(E) \mathcal{E}(E,t)$$

$$\rightarrow \frac{1}{2\pi} \mathcal{E}(E_f, t)$$

$$= \frac{1}{2\pi} (\dot{\alpha} \cos^2 \theta - \dot{\phi} \sin^2 \theta - \dot{\gamma})$$

Standard pumping formula

Landauer formula for friction

$$\left\langle \dot{\mathcal{P}} \right\rangle (t) = -\frac{1}{2\pi}\int dE\, \rho'(E) Tr(\mathcal{E} P)$$

Momentum transfer to
Electron gas

$$P = \left(\begin{array}{cc} p & 0 \\ 0 & -p \end{array} \right)$$

Swimming equation

$$\eta \dot{X} = \frac{1}{2\pi\hbar} \int dE\, \rho'(E) Tr(\mathcal{E} P)$$

Conceptual issues

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(E,t) as a canonical pair

$$S(E,t)$$

A function of non-commuting variables

$$E = \frac{p^2}{2}, \qquad t = x/p$$

A canonical transformation of half-line

$$\{E,t\} = \{p,x\} = 1$$

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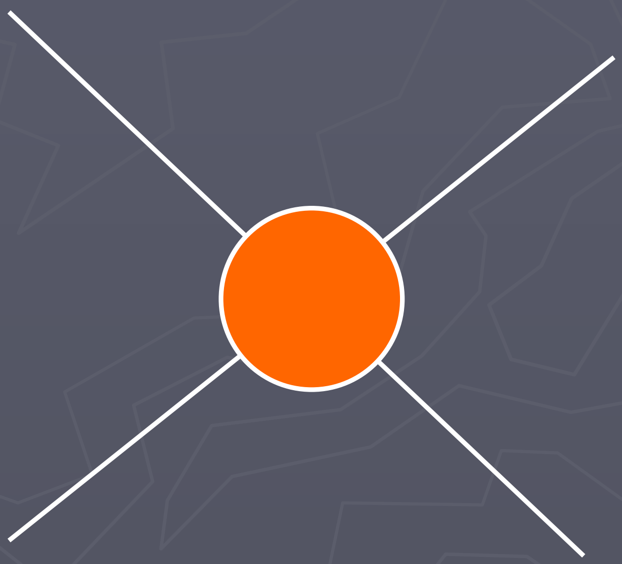
Phase space

$S(E, t)$

A function of non-commuting variables



Weyl calculus

$$Tr(A) = \frac{1}{2\pi\hbar}\int dx\, dp\, A(x,p)$$

operator

symbol

S(E,t) is the classical symbol of the scattering matrix

Avron, elgart, graf, sadun