Micro-swimming, micro-pumping and micro-gliding

With Oren Raz

Outline

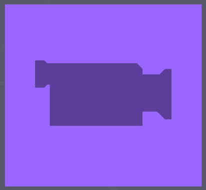
- Showtime
- Helices
- Cox for pedestrians
- Gauge and anchoring
- Triality: Pumps, swimmers & gliders
- Optimal anchoring
- Efficiency: swimmers and pumps

Organisms vs robots

- Interface with macroscopia
- $U_{microbot} \sim 100 \times U_{bacteria}$
- $P_{microbot} \sim 10^4 \times P_{bacteria}$
- Small batteries

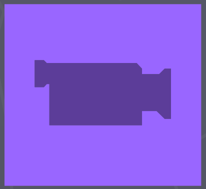
$$P = 6\pi \mu U^2 L$$

Showtime: Robot race



Robot race

Najafi and Golestanian, 2004

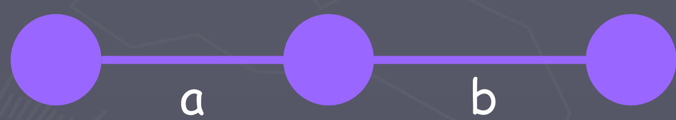


Euglena

http://lifesci.rutgers.edu/~triemer/movies.htm#Metaboly

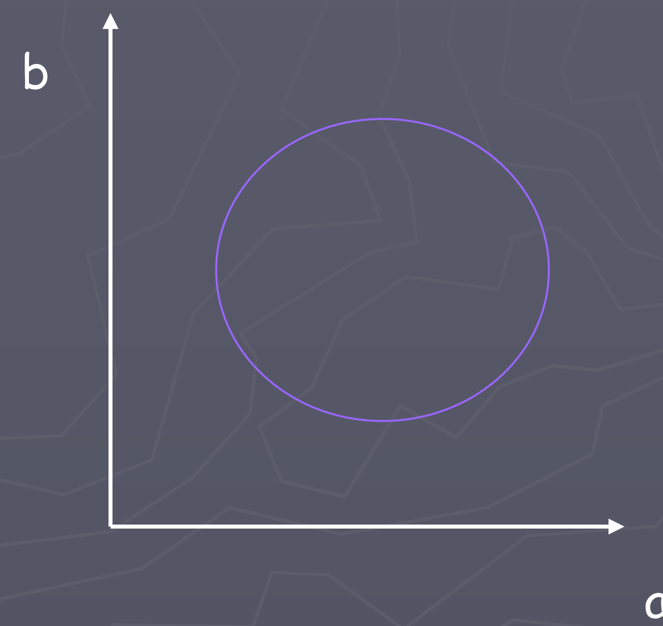
Generic strokes swim

Purcell



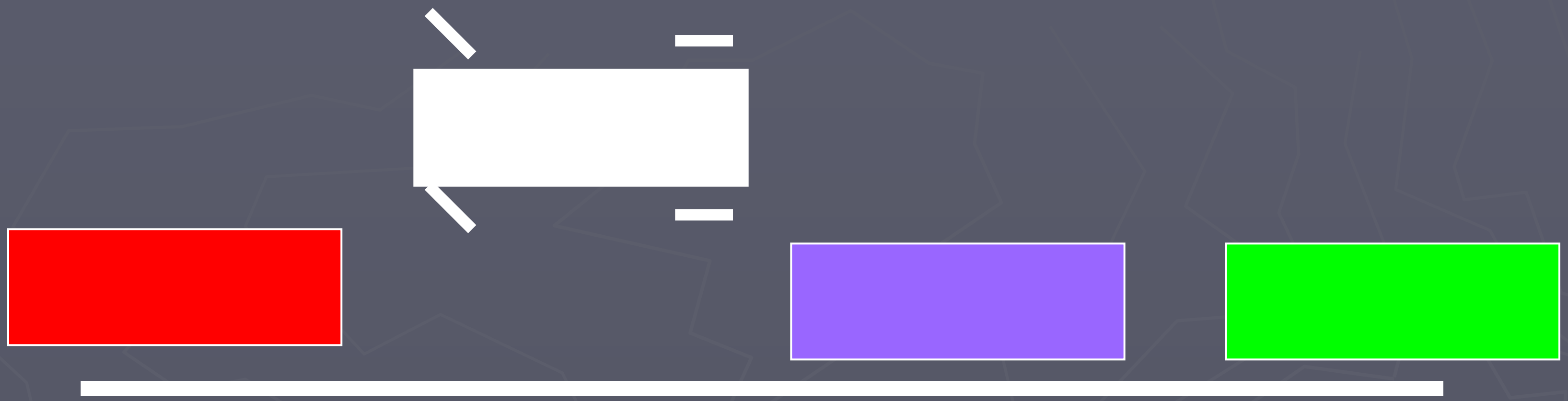
NG 3 linked spheres

pushmepullyou



Distance ~ Area in control space

Swimming and parking

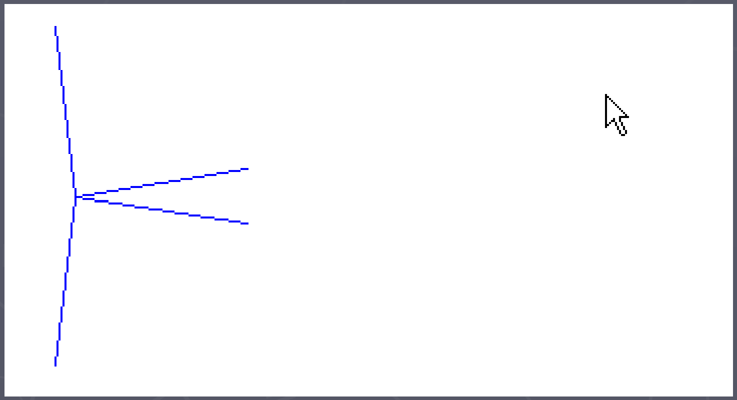


$$E(\theta, drive, park) = \begin{pmatrix} \cos\theta & \sin\theta & drive \\ -\sin\theta & \cos\theta & park \\ 0 & 0 & 1 \end{pmatrix}$$

Park=[steer,drive]

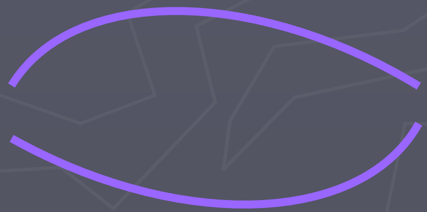
Nelson, J.P. Eckmann

Showtime: Sysiphus



Swimming in quiescent fluid

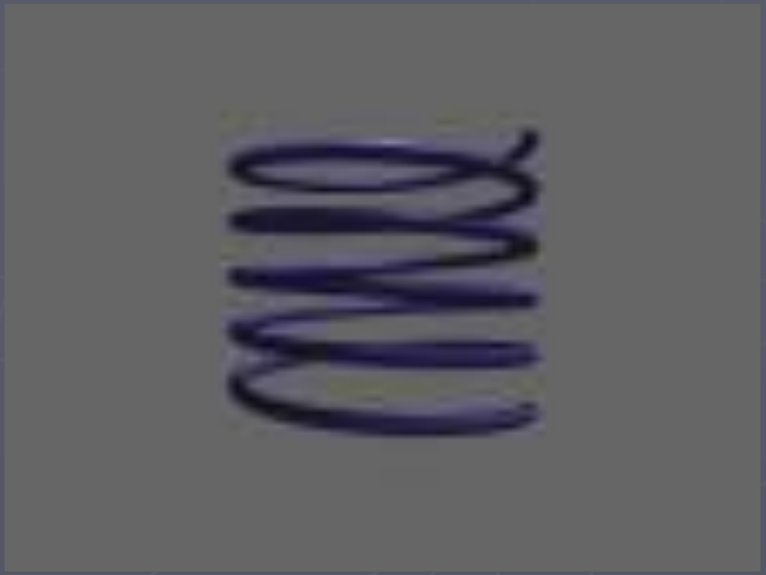
- Generic strokes swim
- Generic swimming is sysiphian
- Effective swimming is quiescent
- Anchor as an engine



pushmepullyou Treadmiller

Helices vs corkscrew

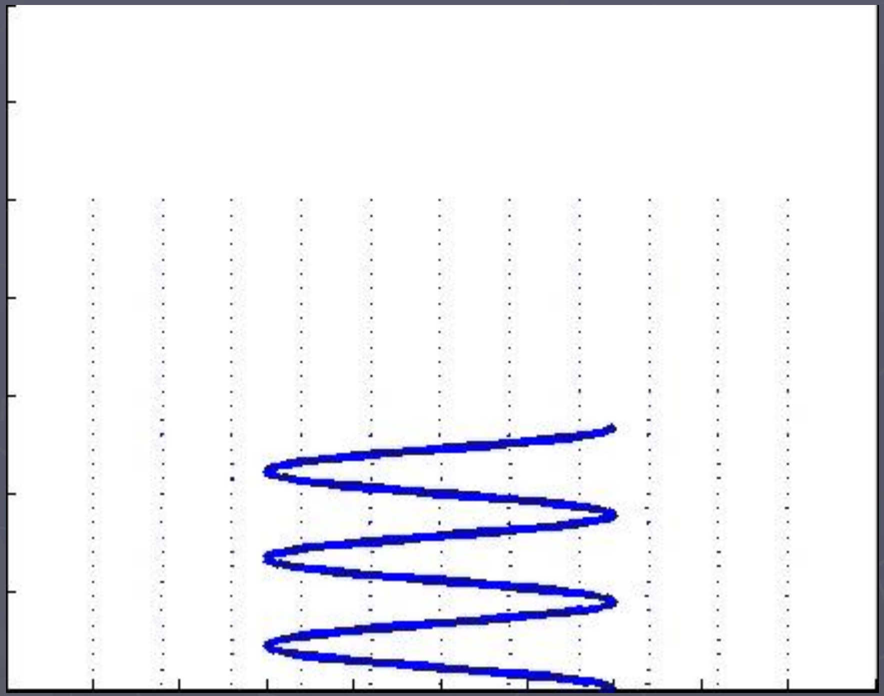
Pitch in one turn



Pitch in many turns

$$\frac{distance}{pitch} = \frac{\cos\theta}{1+\cos^2\theta} \leq \frac{1}{2}$$

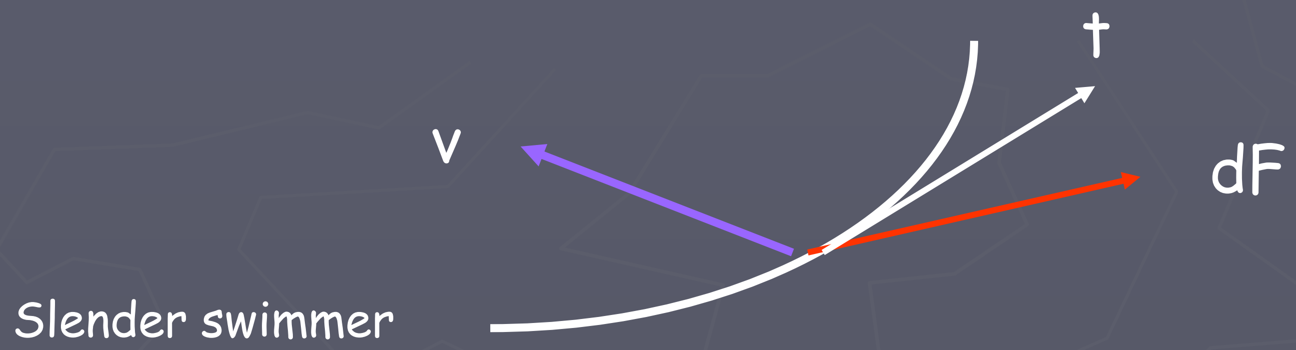
Helices are Sysiphian



Pitch to suit purpose

- Optimal swimming speed $\theta=54.7$
- Optimal pumping force $\theta=45$
- Optimal swimming power $\theta=49.9$
- Optimal pumping power $\theta=42.9$

Cox for pedestrians



$$dF(x) = k(\mathbf{t}(\mathbf{t}\cdot\mathbf{v}) - 2\mathbf{v})d\ell, \quad k = \frac{2\pi\mu}{\ln\kappa}$$

Ratio of drag on a needle in 2:1

Swimming and pumping

- Swimming equation: F=0
- Pumping equation: X=0

Gliding ants

/http://www.canopyants.com

Optimal gliding

$$\min_{\theta} \hat{F} \cdot \hat{V}$$

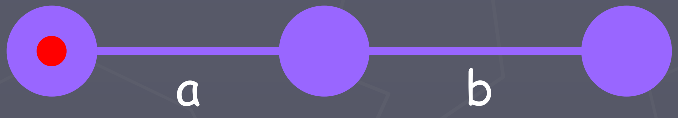
$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \quad n = 2$$

$$\sin^2 \theta = \frac{1}{n+1}, \quad \cos \phi = \frac{2\sqrt{n}}{1+n}$$

$$\phi(n=2) = 19^\circ$$

Swimming, pumping & gliding

$$\mathbf{v}_s = \mathbf{v}_p + \mathbf{v}_g, \quad \mathbf{v}_g = V_s + \omega_s \times \mathbf{x}$$



$$\mathbf{F}_s = \mathbf{F}_p + \mathbf{F}_g$$

$$\mathbf{F}_g = M_g \mathbf{V}_g = -\mathbf{F}_p, \quad M_g = \begin{pmatrix} K & C \\ C^t & \Omega \end{pmatrix}$$

Pumping, like swimming, is geometric

Triality

Lorentz reciprocity

$$\int_{\partial\Sigma} v'_i \, \pi_{ij} \, dS_j = \int_{\partial\Sigma} v_i \, \pi'_{ij} \, dS_j$$

$$P_p = P_s + P_g \geq 0$$

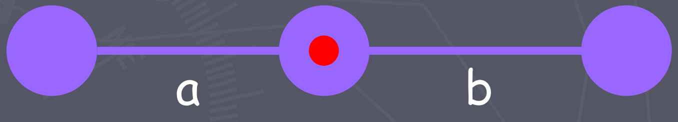
Ehud Yariv, Howard Stone, S Samuel

You can swim for free but you can't pump for free

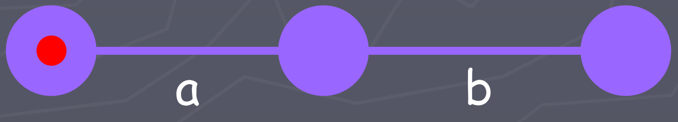
Gauge and anchoring

Euclidean=translation + rotation

Pumps one way



Pumps the other way



Thm: For linear swimmers optimal anchoring is at the joints

Efficiency: swimmers

$$\max\{V_s \mid P_s = const\}$$

With normalized stroke period: $$\delta\left(\frac{v^2}{p}\right) = 0,$$

$$V = \frac{v}{\tau}, \quad P = \frac{p}{\tau^2},$$

$$\underbrace{0 = \frac{\delta V}{V}}_{optimus} = \frac{\delta v}{v} - \frac{\delta \tau}{\tau}, \quad \underbrace{0 = \frac{\delta P}{P}}_{constraint} = \frac{\delta p}{p} - 2\frac{\delta \tau}{\tau}$$

Efficiency: pumps

$$\max\{F_p \mid P_p = const\}$$

With normalized stroke period: $$\delta\left(\frac{f^2}{p}\right) = 0,$$

$$F = \frac{f}{\tau}, \quad P = \frac{p}{\tau^2},$$

- Thanks: A. Leshanski, O. Kenneth, D. Oaknin, O. Gat, H. Stone,