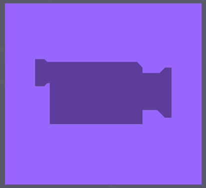
Swimming as a gauge theory

With Oren Raz

Outline

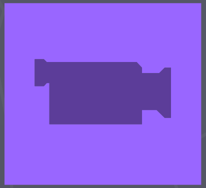
- Adiabatic swimming
- Cox for pedestrians
- Simple swimmer
- Purcell swimmer
- Gauge theory

Showtime: Robot race



Robot race

Najafi and Golestanian, 2004



Euglena

http://lifesci.rutgers.edu/~triemer/movies.htm#Metaboly

Adiabatic swimming is F=0

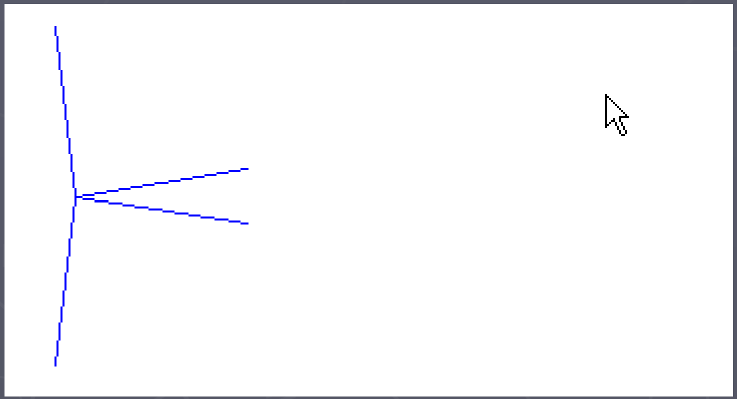
Stokes: $$F = 6\pi\eta RV$$

Newton $$O\left(\frac{R}{T}\right) = F = m\dot{V} = O\left(\frac{R^3}{T^2}\right)$$

In adiabatic limit, for small objects, equation of motion is

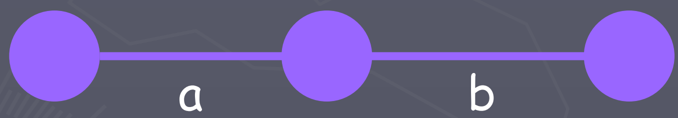
$$F_{total} \approx 0$$

Simple Sysiphian swimmer



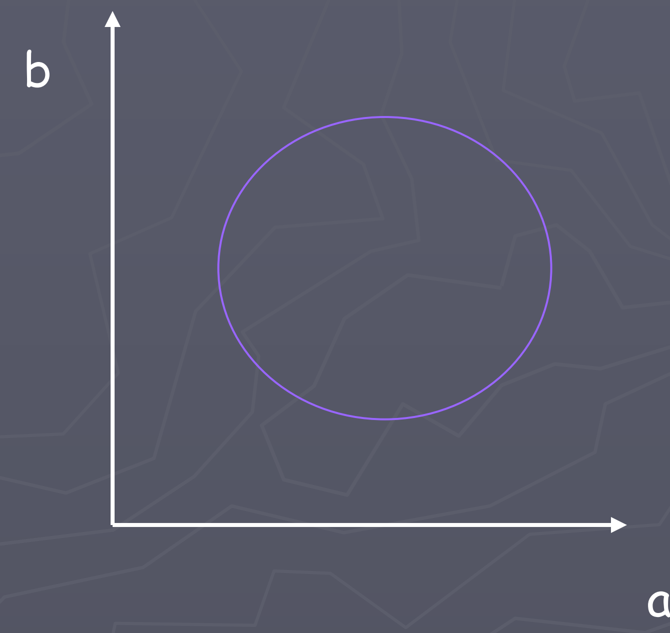
Generic strokes swim

Purcell



NG 3 linked spheres

pushmepullyou

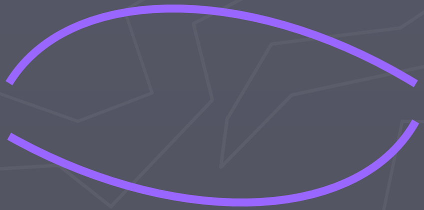


Distance ~ Area in control space

Swimming without moving

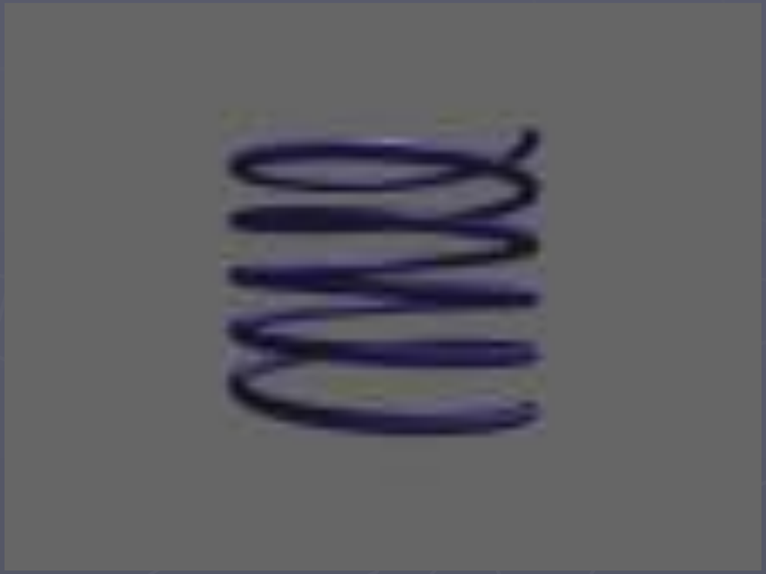
- Generic strokes swim
- Generic swimming is sysiphian
- Effective swimming is quiescent
- Anchor as an engine

pushmepullyou



Treadmiller

Helices vs corkscrew

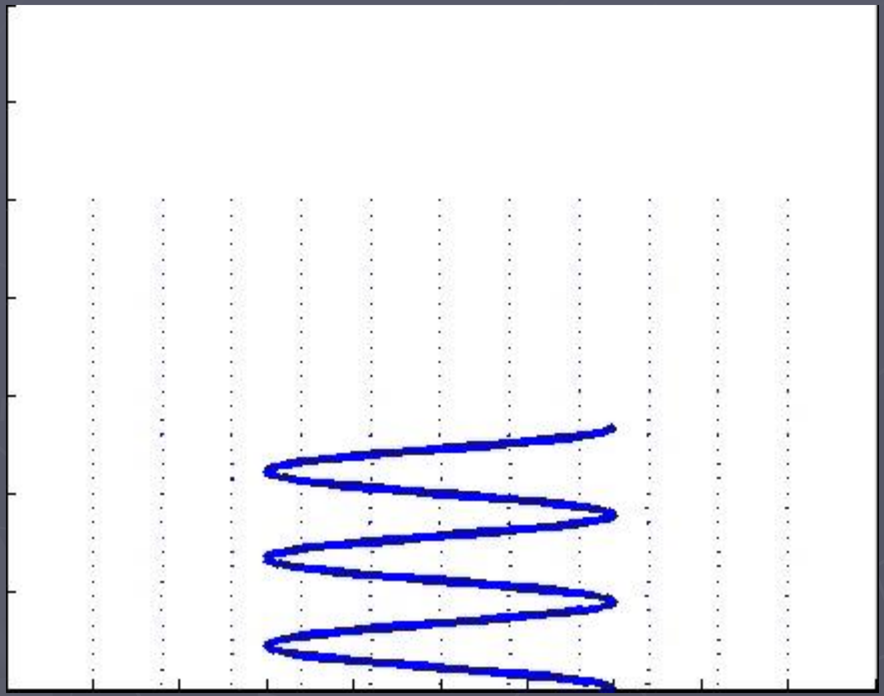


Pitch in one turn

Pitch in many turns

$$\frac{distance}{pitch} = \frac{\cos\theta}{1+\cos^2\theta} \leq \frac{1}{2}$$

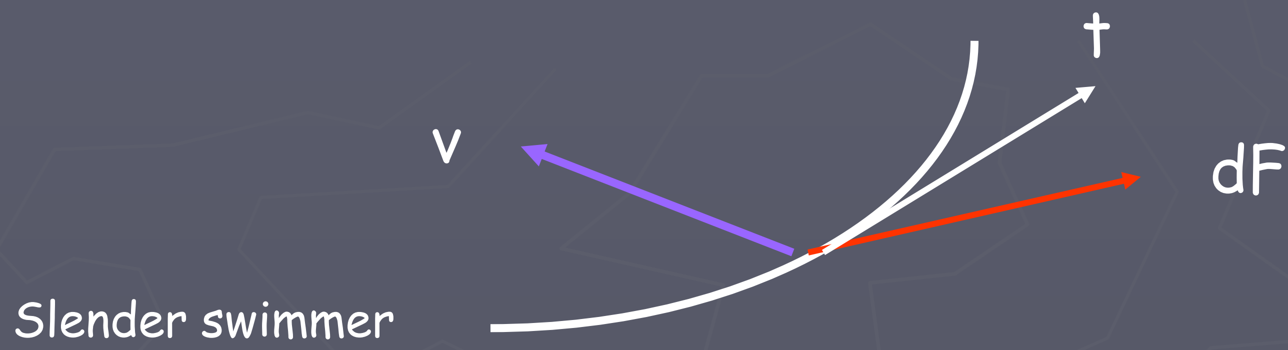
Helices are Sysiphian



Pitch to suit purpose

•Optimal swimming speed	θ=54.7
•Optimal pumping force	θ=45
•Optimal swimming power	θ=49.9
•Optimal pumping power	θ=42.9

Cox for pedestrians



$$dF(x) = k(\mathbf{t}(\mathbf{t}\cdot\mathbf{v}) - 2\mathbf{v})d\ell, \qquad k = \frac{2\pi\mu}{\ln\kappa}$$

Ratio of drag on a needle in 2:1

Swimming and pumping

- Swimming equation: F=0
- Pumping equation: X=0

Gliding ants

/http://www.canopyants.com

Optimal gliding

$$\min_{\theta} \hat{F} \cdot \hat{V}$$

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \quad n = 2$$

$$\sin^2 \theta = \frac{1}{n+1}, \quad \cos \phi = \frac{2\sqrt{n}}{1+n}$$

$$\phi(n=2) = 19^\circ$$

Triality

$$\mathbf{F}_p = -M_g\,\mathbf{V}_s, \quad P_p = P_s + P_g \geq 0$$

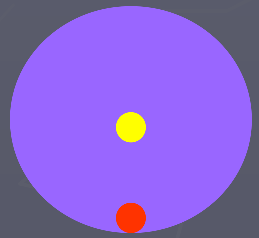
Ehud Yariv, Howard Stone, S Samuel

$$\mathbf{F}_p = (F_p, N_p), \quad \mathbf{V}_s = (V_s, \omega_s)$$

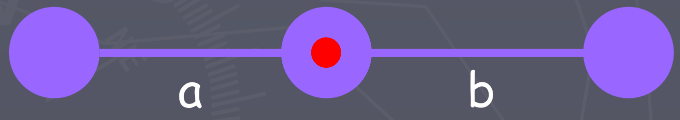
$$\mathbf{F}_g = M\,\mathbf{V}_g, \quad M = \begin{pmatrix} K & C \\ C^t & \Omega \end{pmatrix}$$

Pumping, like swimming, is geometric

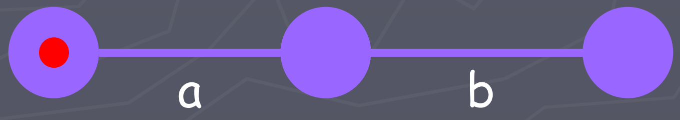
Gauge and anchoring



Pumps one way



Pumps the other way



Thm: For linear swimmers optimal anchoring is at the joints

Efficiency

$$\max\{V_s \mid P_s = const\}$$

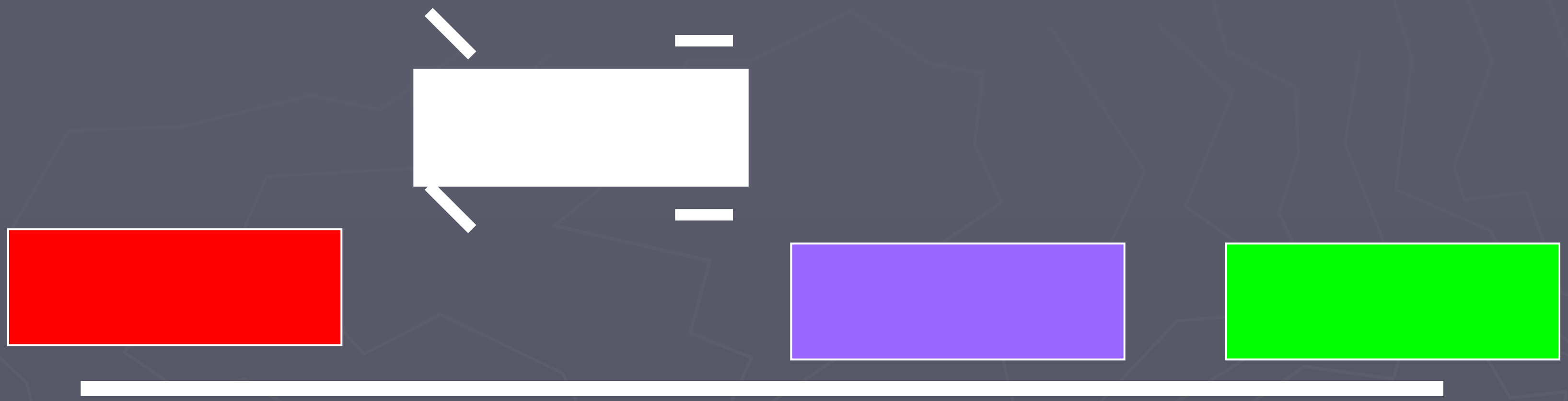
With normalized stroke period:

$$\delta\left(\frac{v^2}{p}\right) = 0,$$

$$V = \frac{v}{\tau}, \quad P = \frac{p}{\tau^2},$$

$$\underbrace{0 = \frac{\delta V}{V}}_{optimus} = \frac{\delta v}{v} - \frac{\delta \tau}{\tau}, \qquad \underbrace{0 = \frac{\delta P}{P}}_{constraint} = \frac{\delta p}{p} - 2\frac{\delta \tau}{\tau}$$

Swimming and parking



$$E(\theta, drive, park) = \begin{pmatrix} \cos\theta & \sin\theta & drive \\ -\sin\theta & \cos\theta & park \\ 0 & 0 & 1 \end{pmatrix}$$

Park=[steer,drive]

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