Duality walls in 5d gauge theories

Hee-Cheol Kim

Perimeter Institute

Based on
arXiv:1506.03871 with Davide Gaiotto

Introduction

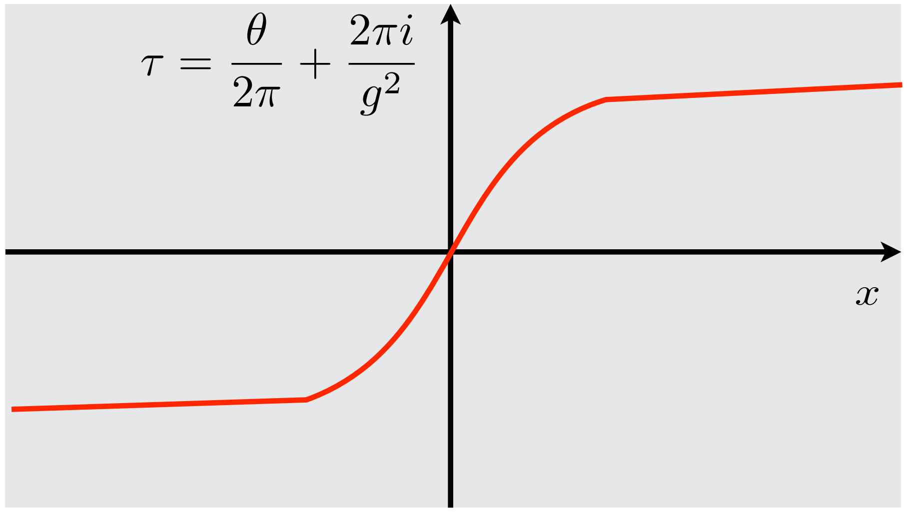
A large class of BPS domain walls in 4d SUSY gauge theories.

[Clack, Karch 05], [D'Hoker, Estes, Gutperle 07],] [Gaiotto, Witten 08], …

Janus domain wall :

Gauge coupling or mass parameters vary along one direction.

[Bak, Gutperle, Hirano 03], [Clack, Freedman, Karch, Schnabl 04], [Gaiotto, witten 08], …



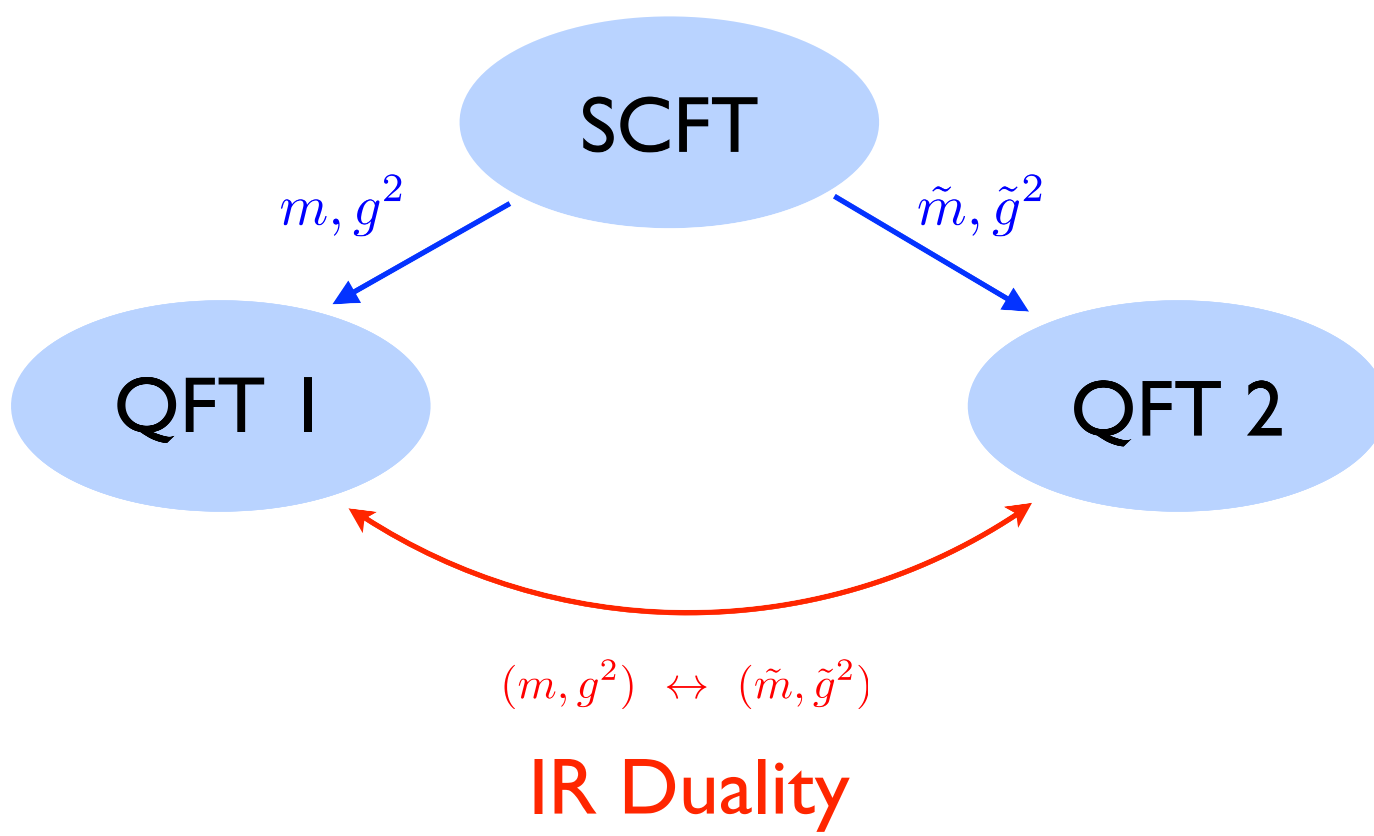
- Construct Janus-like duality domain walls in 5d gauge theories.

Introduction

We focus on 5d gauge theories obtained by mass deformations of the special UV SCFTs having global symmetry enhancement.

[Seiberg 96], [Intriligator-Morrison-Seiberg 97]

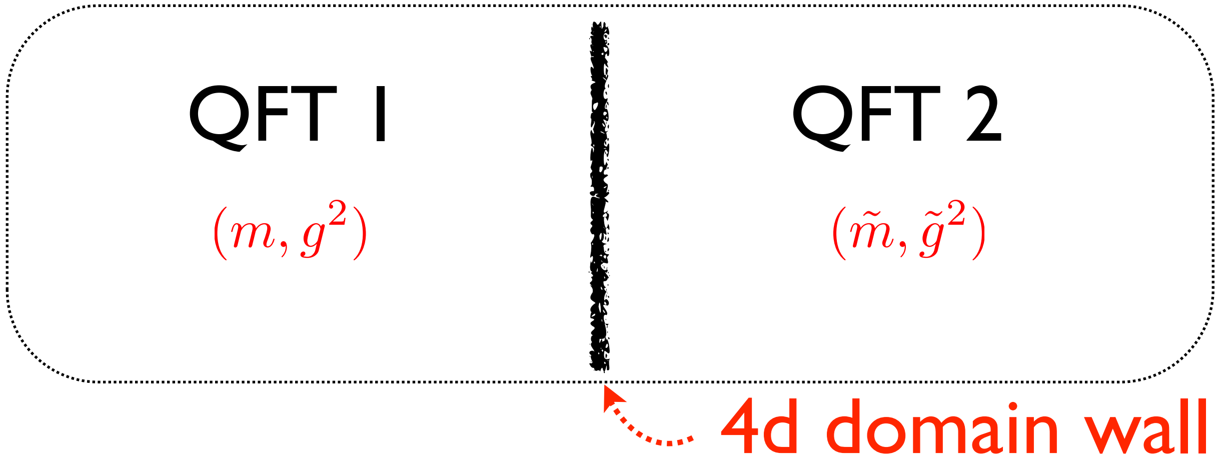
Different mass deformations in general lead to different IR gauge theories.



Symmetry in UV CFT $\longrightarrow$ Duality in IR gauge theories.

Introduction

Duality domain walls



What can we learn?

- UV symmetry enhancement from IR physics.

- Close relation between 5d duality and 4d duality.

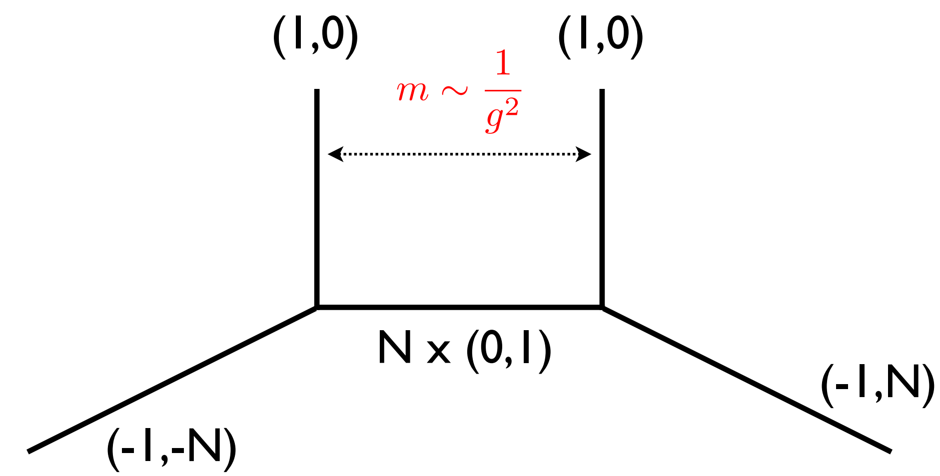
Contents

Duality walls in SU(N) theories

Pure $\mathcal{N}=1$ $SU(N)_N$ gauge theory

Five-brane web construction

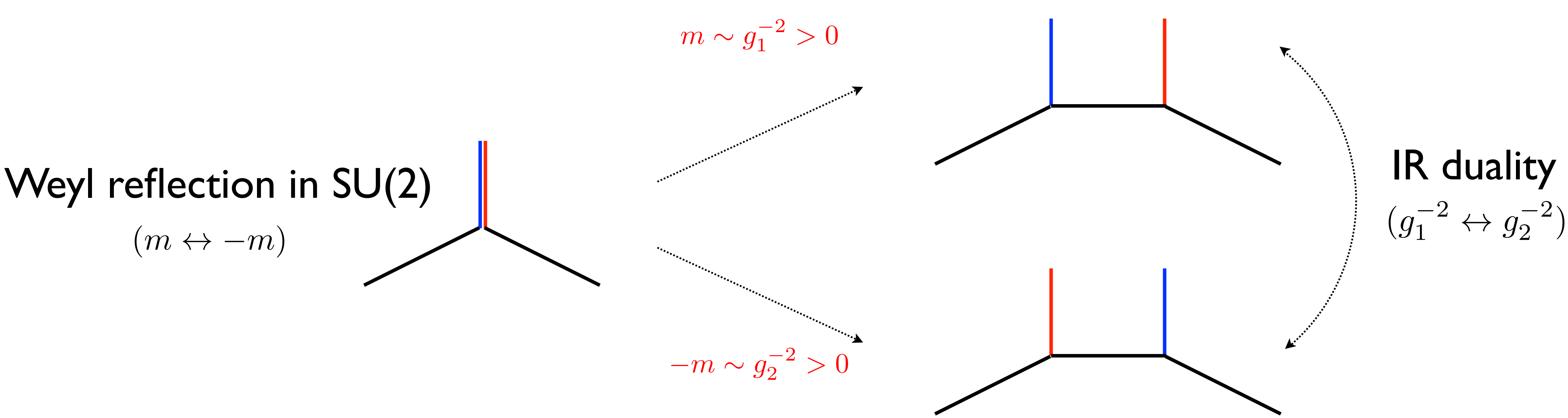
(at the origin of Coulomb branch)



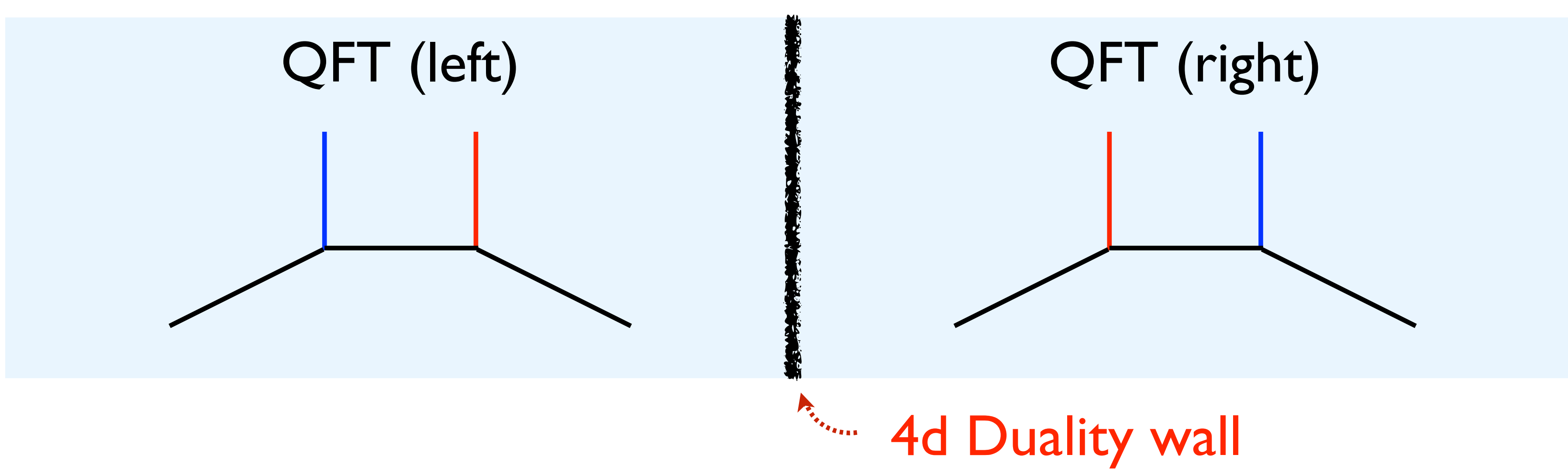
- SU(2) symmetry from parallel NS5 branes when $\kappa = N$.
- Mass deformation m breaks this SU(2) to U(1).
- IR description is $SU(N)_N$ gauge theory with gauge coupling $m \sim \frac{1}{g^2}$

5d Duality in IR gauge theories

- Symmetry in UV CFT reduces to a duality between IR QFTs.



- We propose a 1/2-BPS domain wall connecting two IR dual QFTs.



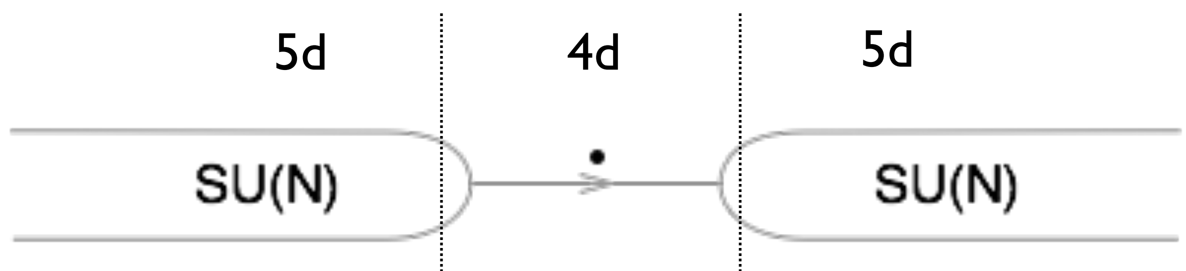
Duality wall

- We couple a 4d theory to Neumann b.c. of the gauge theories on two sides of the wall. (1/2-BPS) [Gaiotto, H.-C Kim 15]

- 4d $\mathcal{N}=1$ theory :

	$SU(N)_l$	$SU(N)_r$	$U(1)_R$	$U(1)_B$
q	N	$\bar{N}$	0	$1/N$
b	1	1	2	$-N$

& $W = b\ \text{det}q$



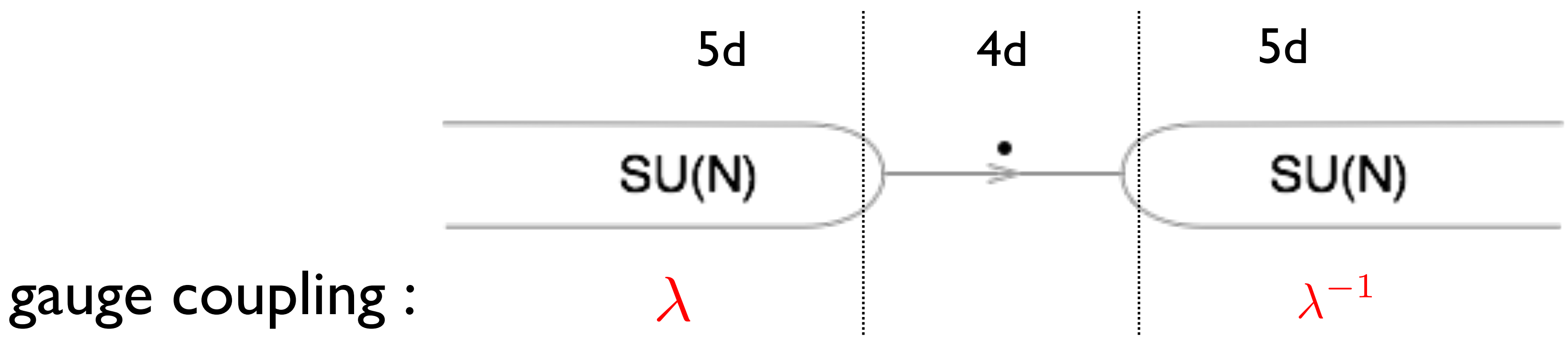
- Strong constraints by anomaly cancellation
  1. 4d theory has cubic anomaly of unit N for each bulk gauge group, and it is cancelled by bulk CS-terms with $\kappa = N$.
  2. Boundary $U(1)_R \subset SU(2)_R$ is fixed by mixed 't Hooft anomaly.
  3. Anomaly-free $U(1)_\lambda$ glues together instanton symmetries $U(1)_{I_l}$ and $U(1)_{I_r}$ with opposite signs.

	$U(1)_\lambda$	$U(1)_B$	$U(1)_{I_l}$	$U(1)_{I_r}$
q	$1/N$	$1/N$	0	0
I_l	1	0	1	0
I_r	-1	0	0	1

Duality wall

Anomaly-free $U(1)_\lambda$ glues together two instanton symmetries on two sides of the wall with opposite signs.

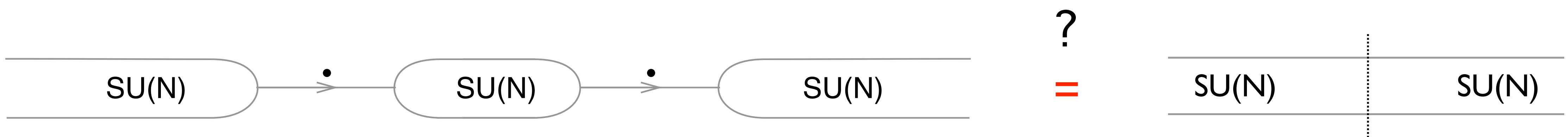
Therefore, duality wall inverts gauge coupling $\lambda \sim e^{-\frac{4\pi^2}{g^2}} \leftrightarrow \lambda^{-1}$.



* Duality wall implements $\mathbb{Z}_2$ action in SU(2) global symmetry of UV CFT.

Composition of duality walls

- Consistency check :



- 4d theory is now $SU(N)$ gauge theory with $q, \tilde{q}, b, \tilde{b}$ & superpotential:

$$W = b\,\mathrm{det}q + \tilde{b}\,\mathrm{det}\tilde{q}$$

- 4d "Seiberg dual" theory in IR consists of a meson $M = \tilde{q}q$ and baryons $B = \mathrm{det}q,\ \tilde{B} = \mathrm{det}\tilde{q}$ subject to a constraint $\mathrm{det}M - B\tilde{B} = \Lambda^{2N}$ with superpotential : $W = b\,B + \tilde{b}\,\tilde{B}$.

➡ Trivial interface!

SUSY indices with Duality walls

SUSY index with duality wall

We now see a more non-trivial check with supersymmetric indices in the presence of the duality wall.

- Superconformal index (SCI)

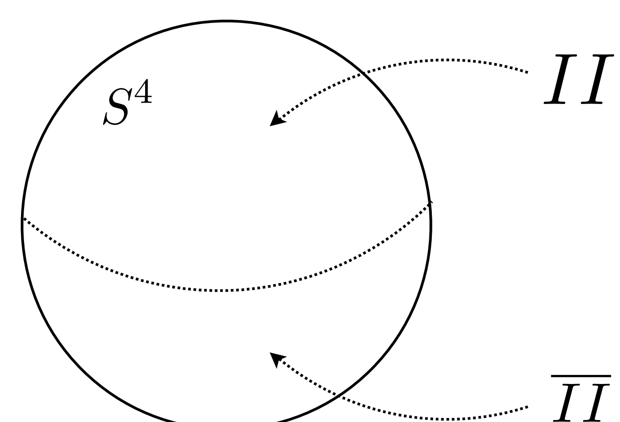
$$I(w_a, \mathfrak{q}; p, q) = \mathrm{Tr}(-1)^F p^{j_1+R} q^{j_2+R} \prod_a w_a^{F_a}\, \mathfrak{q}^k$$

  - j_1, j_2, R are Cartan generators of $SO(2,5) \times SU(2)_R$.
  - F_a are Cartans of flavour symm. and k is instanton number.
  - SCI is equivalent to twisted partition function on $S^1 \times S^4$.

[S.-S Kim, H.-C Kim, K. Lee 12], [Terashima 12]

- SCI factorizes into two "hemisphere" indices by localization.

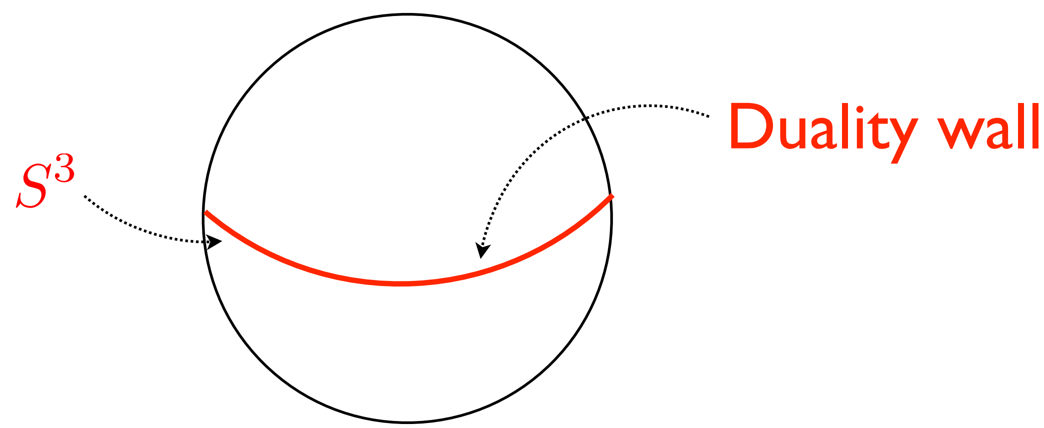
$$I(w_a, \mathfrak{q}; p, q) = \langle II | II \rangle = \oint d\mu_z \overline{II(z, w_a, \mathfrak{q}; p, q)} II(z, w_a, \mathfrak{q}; p, q)$$



$II = Z_{\mathrm{pert}} \cdot Z_{\mathrm{inst}}$: Hemisphere index
= Partition function on $S^1 \times \mathbb{R}^4$ with Omega deformation

SUSY index with duality wall

We can insert a duality wall at the equator (with 1/2-SUSY)



The superconformal index of the full system simply becomes

$$I = \langle II(\mathfrak{q}^{-1})|I^{4d}(\mathfrak{q})|II(\mathfrak{q})\rangle = \oint d\mu_z d\mu_{z'} \overline{II(z,\mathfrak{q}^{-1};p,q)} I^{4d}(z,z',\mathfrak{q};p,q) II(z',\mathfrak{q};p,q)$$

where $I^{4d}(z,z',\mathfrak{q};p,q)$ is the superconformal index of the 4d d.o.f at the interface (which also depends on the boundary condition).

Duality wall action on hemisphere index

- 4d superconformal index at the interface

	$SU(N)_l$	$SU(N)_r$	$U(1)_R$	$U(1)_\lambda$
q	N	$\bar{N}$	0	$1/N$
b	1	1	2	$-N$

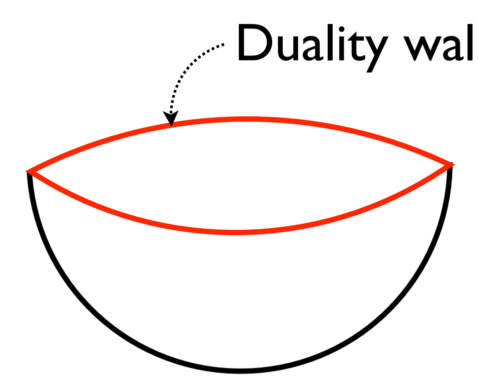
$$\Rightarrow \quad I^{4d} = \frac{\prod_{i,j=1}^{N} \Gamma(\lambda^{1/N} z_i / z'_j)}{\Gamma(\lambda)}\,, \qquad (\mathfrak{q} = \lambda)$$

$\Gamma(x)$: Elliptic gamma function

- We can couple this 4d SCI to hemisphere index :

$$\hat{D} II^N(z,\lambda) \equiv \oint \prod_{i=1}^{N-1} \frac{dz'_i}{2\pi i z'_i} \frac{I^{4d}(z,z',\lambda)}{\prod_{i,j}^{N} \Gamma(z'_i/z'_j)} II^N(z'_i,\lambda)$$

4d SU(N) vectormultiplet



- Duality wall is conjectured to flip the sign of the gauge coupling, therefore, we claim that

$$\hat{D} II^N(z_i,\lambda) = II^N(z_i,\lambda^{-1})$$

Duality wall action on hemisphere index

- Duality wall : $\hat{D} II^N(z_i, \lambda) = II^N(z_i, \lambda^{-1})$ [Gaiotto, H.-C Kim 15]

- The hemisphere index is actually given by a series expansion in instanton number. Thus, this is a very surprising claim since the index $II^N(z,\lambda)$ is expanded by $\lambda^{k\geq 0}$, while the dual index $\hat{D}II^N(z,\lambda)$ is expanded by $(\lambda^{-1})^{k\geq 0}$.

- Can be checked in $x \equiv (pq)^{1/2}$ expansion.
  - Numerical checks for $N = 2, 3, 4$ at least up to x^4 order.

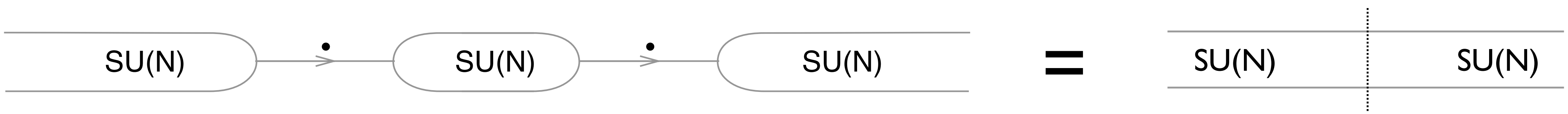
- More surprisingly, assuming $II = Z_{\text{pert}} \cdot Z_{\text{inst}} = Z_{\text{pert}} \cdot (1 + \mathcal{O}(x))$, the integral equation

$$\oint \prod_{i=1}^{N-1} \frac{dz'_i}{2\pi i z'_i} \frac{I^{4d}(z, z', \lambda)}{\prod_{i,j}^{N} \Gamma(z'_i/z'_j)} II^N(z'_i, \lambda) = II^N(z, \lambda^{-1})$$

uniquely determines the instanton partition function Z_{inst} in x expansion!!

Duality wall action on hemisphere index

- Analytic proof of $\hat{D}^2 = I$



- There is an integral formula (elliptic Fourier transform)

[Spiridonov, Warnaar 04]

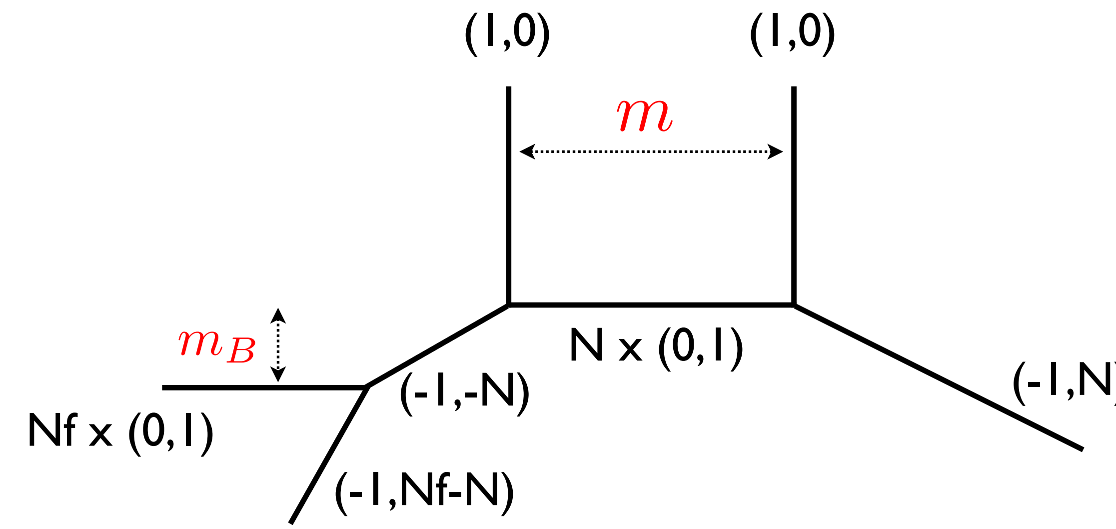
$$\oint d\mu_{z'} \frac{\prod_{i,j}^N \Gamma(\lambda^{1/N} z_i/z'_j)}{\Gamma(\lambda)\prod_{i,j}^N \Gamma(z'_i/z'_j)} \oint d\mu_{z''} \frac{\prod_{i,j}^N \Gamma(\lambda^{-1/N} z'_i/z''_j)}{\Gamma(\lambda^{-1})\prod_{i,j}^N \Gamma(z''_i/z''_j)} f(z'') \sim f(z)$$

- This proves $\hat{D}\hat{D}II(z,\lambda) = II(z,\lambda)$.

SU(N) theories with flavours

SU(N) gauge theory with flavours

Five-brane web construction



- Parallel external NS5 branes when $\kappa = N - N_f/2$.
- SU(2) global symmetry at UV fixed point will be broken to U(1) by mass deformation with m.
- IR gauge coupling is identified as $g^{-2} \sim m + \frac{N_f}{2} m_B$ where m_B is the mass parameter for the overall $U(1)_f \subset U(N_f)$ flavor symmetry.
- We propose a duality domain wall which inverts the sign of the mass parameter $m \leftrightarrow -m$.

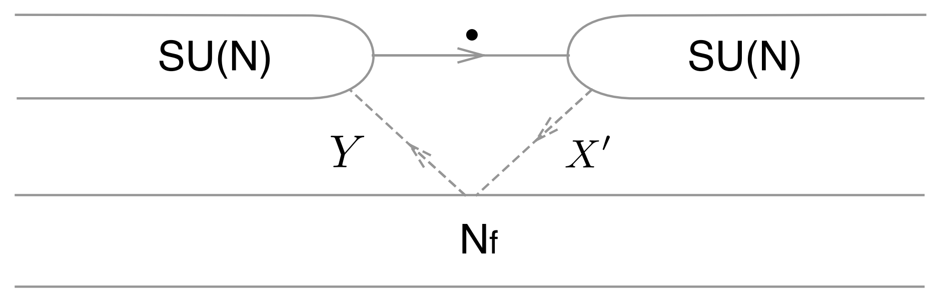
Duality wall with flavours

- Boundary condition for a hypermultiplet $\Phi = (X, Y)$ at the interface is chosen such that $X = 0$ while Y couples to the interface.

- 4d theory at the interface is again the theory of q, b with superpotential

[Gaiotto, H.-C Kim 15]

$$W = b\,\mathrm{det} q + Y q X'$$



- Cubic anomaly $N - N_f/2$ at the interface is cancelled by the bulk CS-term.

- Anomaly-free U(1)'s on the two sides of the wall are identified as (in terms of fugacities)

$$w = \lambda^{1/N} w' \quad \& $$

	q	I_l	I_r	X	X'
fugacity	$\lambda^{1/N}$	$\lambda w^{-N_f/2}$	$\lambda^{-1}(w')^{-N_f/2}$	w	w'

$$e^{-\frac{4\pi^2}{g^2}} = \lambda w^{-N_f/2} ,$$
$$e^{-\frac{4\pi^2}{(g')^2}} = \lambda^{-1}(w')^{-N_f/2} ,$$
$$e^{m_B} = w , \; e^{m'_B} = w'$$

4d Seiberg duality and 5d symmetry enhancement

- Example : $SU(2)$ gauge theory with $N_f = 2$ flavours which has symmetry enhancement $SO(4) \times U(1)_I \rightarrow SU(2) \times SU(3)$ at the UV fixed point.

  [Seiberg 96]

- Since gauge group is pseudo real, we can define two different duality walls for two different choices of b.c. preserving the flavour $SU(2)$

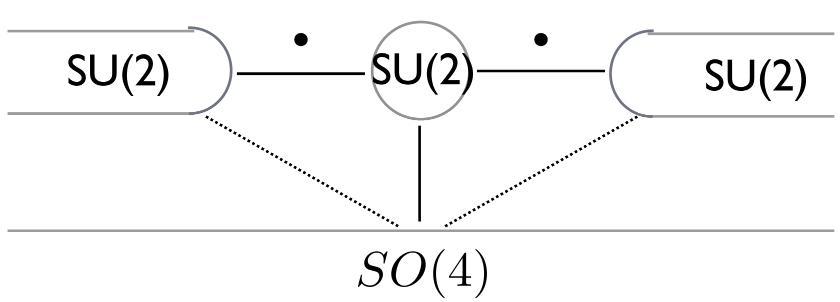
$$\hat{D}_1 \;:\; (X, Y) = (\tilde{Q}_a, Q_a) \;\&\; (X', Y') = (Q'_a, \tilde{Q}'_a)$$

$$\hat{D}_2 \;:\; (X, Y) = (Q_a, \tilde{Q}_a) \;\&\; (X', Y') = (\tilde{Q}'_a, Q'_a)$$

  where $X = Y' = 0$ and Y, X' couple to the interface degrees.

- $\hat{D}_1 \;,\; \hat{D}_2 \;,\; \hat{D}_3 \subset S_3$ in $SU(3)$ where $\hat{D}_3 \;:\; (Q_a, \tilde{Q}_a) \leftrightarrow (\tilde{Q}_a, Q_a)$

- Concatenation of the interfaces



= ?

$\hat{D}_1 \hat{D}_2$

4d Seiberg duality and 5d symmetry enhancement

- Example : $SU(2)$ gauge theory with $N_f = 2$ flavours which has symmetry enhancement $SO(4) \times U(1)_I \rightarrow SU(2) \times SU(3)$ at the UV fixed point.
  [Seiberg 96]

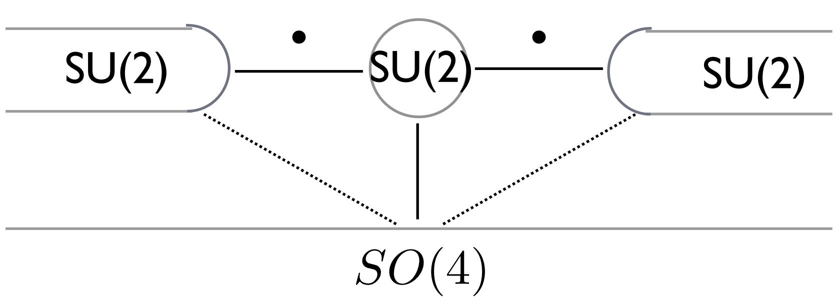
- Since gauge group is pseudo real, we can define two different duality walls for two different choices of b.c. preserving the flavour $SU(2)$

$$\hat{D}_1 \ : \ (X, Y) = (\tilde{Q}_a, Q_a) \ \& \ (X', Y') = (Q'_a, \tilde{Q}'_a)$$
$$\hat{D}_2 \ : \ (X, Y) = (Q_a, \tilde{Q}_a) \ \& \ (X', Y') = (\tilde{Q}'_a, Q'_a)$$

where $X = Y' = 0$ and Y, X' couple to the interface degrees.

- $\hat{D}_1 \ , \ \hat{D}_2 \ , \ \hat{D}_3 \subset S_3$ in $SU(3)$ where $\hat{D}_3 \ : \ (Q_a, \tilde{Q}_a) \leftrightarrow (\tilde{Q}_a, Q_a)$

- Concatenation of the interfaces

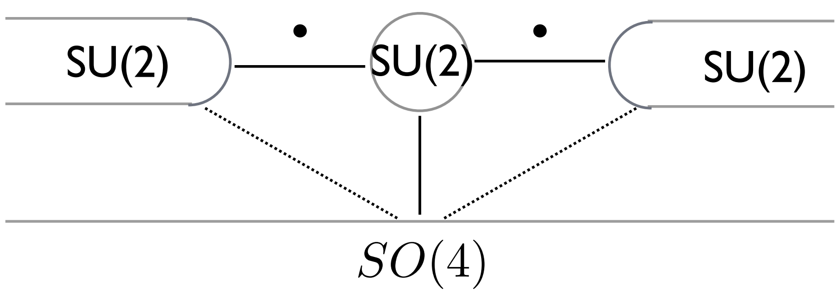


= ?

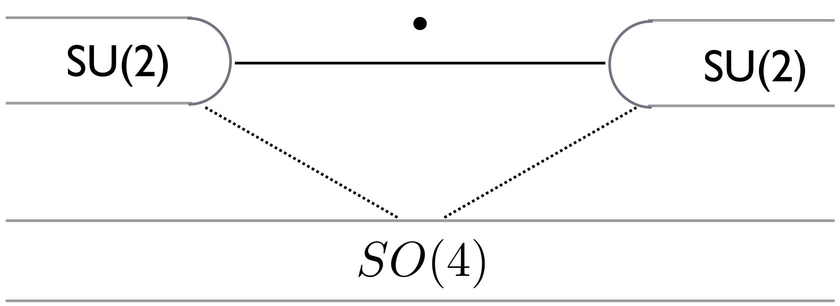
$$\hat{D}_1 \hat{D}_2 \quad = \quad \hat{D}_2 \hat{D}_3 \ (\text{or } \hat{D}_3 \hat{D}_1)$$

Symmetry enhancement and 4d Seiberg duality

- 4d Seiberg duality shows



=



$\hat{D}_1\hat{D}_2$ = $\hat{D}_2\hat{D}_3$ (or $\hat{D}_3\hat{D}_1$)

Seiberg dual

- Therefore, duality wall actions (with help of 4d Seiberg duality) implement Weyl permutations of $SU(3)$ global symmetry at the UV fixed point.

Duality wall action on hemisphere index

- Hemisphere index of the boundary condition $X=0$

$$II^{N,N_f}(z_i,w_a,\mathfrak{q};p,q)=\frac{(pq;p,q)_\infty^{N-1}\prod_{i\neq j}^{N}(pqz_i/z_j;p,q)_\infty}{\prod_{i=1}^{N}\prod_{a=1}^{N_f}(\sqrt{pq}z_i/w_a;p,q)_\infty}Z_{\rm inst}^{N,N_f}(z_i,w_a,\mathfrak{q};p,q)$$

Y contribution

$(x;p,q)_\infty$: q-Pochhammer symbol

- Duality wall action on the hemisphere index

$$\hat{D}II^{N,N_f}(z,w,\lambda)\equiv\oint\prod_{i=1}^{N-1}\frac{dz_i'}{2\pi i z_i'}\frac{I^{4d}(z,z',\lambda)}{\prod_{i,j}^{N}\Gamma(z_i'/z_j')}II^{N,N_f}(z_i',w,\lambda)$$

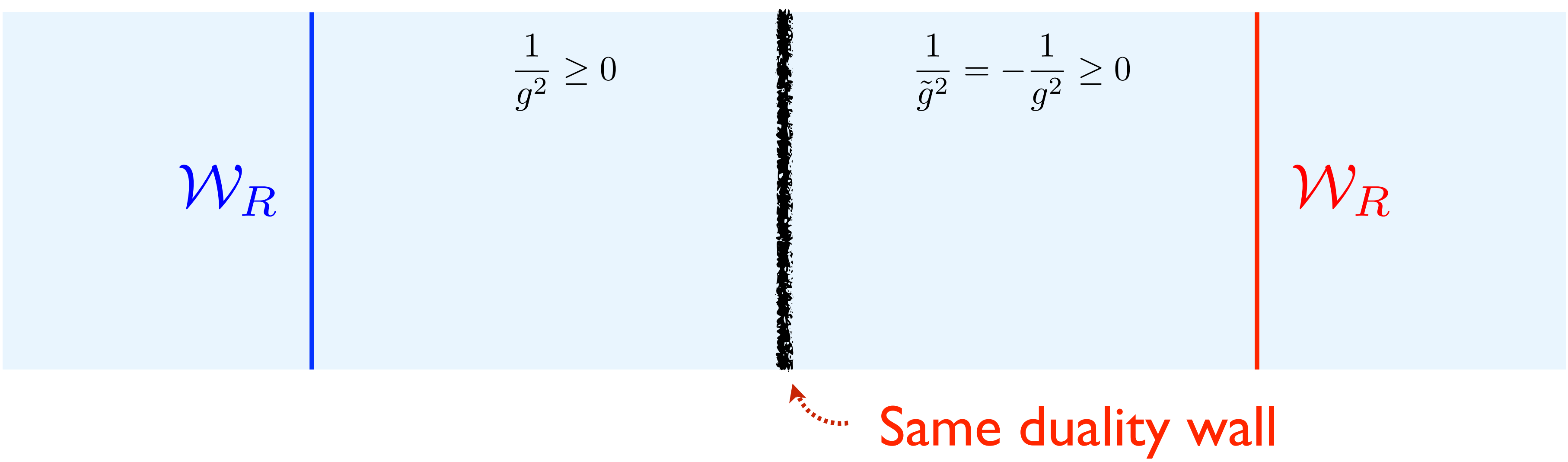
- We claim that $\hat{D}II^{N,N_f}(z_i,w,\lambda)=II^{N,N_f}(z_i,w',\lambda^{-1})$ (with $w=\lambda^{1/N}w'$)

  - Numerical checks for several small N,N_f

- Again, this integral relation of duality wall uniquely determines the full instanton partition function with fund. hypers in $x=(pq)^{1/2}$ expansion.

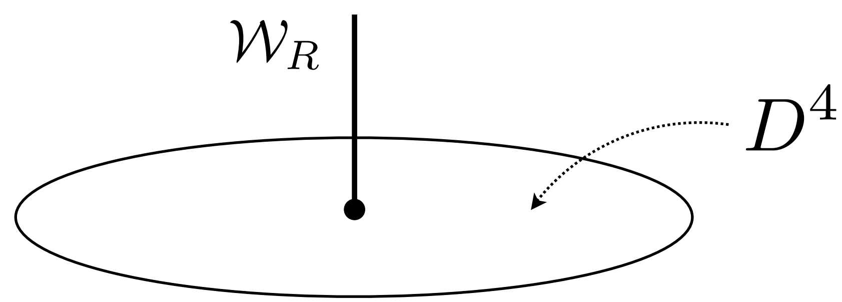
Wilson loops

Duality wall with Wilson loops



We insert Wilson loops in representations R of the gauge group into two dual theories connected by the duality wall.

Hemisphere index with Wilson loops



- Hemisphere index in the presence of a Wilson loop could be computed by inserting the corresponding equivariant Chern character Ch_R into the partition function. [Nekrasov, Okounkov 03], [Gaiotto, H.-C Kim 14]

$$\mathcal{W}_R(z,\mathfrak{q};p,q) = Z_{1-\text{loop}}(z;p,q)\sum_{k=0}^{\infty}\mathfrak{q}^k\frac{1}{|W_k|}\oint[d\rho]\, Ch_R(z,\rho;p,q)\cdot Z_k(z,\rho;p,q)$$

$$= Z_{1-\text{loop}}(z;p,q)\times\left(\text{Tr}_R(z)+\mathcal{O}(\mathfrak{q})\right)$$

instanton correction

- However, there could be additional corrections when the Wilson loop changes asymptotic behaviour of the instanton partition function.
- So far, no systematic way to obtain the "correct" Wilson loop partition function in general representation is known.

Duality wall action

- Duality wall action on the Wilson loop index

$$\hat{D}\mathcal{W}_R \equiv \oint \prod_{i=1}^{N-1} \frac{dz'_i}{2\pi i z'_i} \frac{I^{4d}(z,z',\lambda)}{\prod_{i,j}^{N}\Gamma(z'_i/z'_j)} \mathcal{W}_R(z'_i,w,\lambda)$$

- We claim that the "correct" Wilson loop index satisfies

$$\hat{D}\mathcal{W}_R(z_i,w_a,\lambda) = \underbrace{\lambda^{k(R)/N}}_{\text{Dressing factor}} \mathcal{W}_R(z_i,w'_a,\lambda^{-1})$$

where $k(\text{fund})=1\ ,\ k(\text{symm})=2\ ,\ k(\text{asym})=2\ ,\ k(\text{adj})=3\ ,\cdots$

- A fundamental Wilson loop ends on a local operator q and continues as a fundamental Wilson loop on the other side of the wall. Since q has $U(1)_\lambda$ charge $1/N$, the resulting Wilson loop is dressed by a factor $\lambda^{1/N}$.

- Similarly, Wilson loops in other rep. also acquire dressing factors.

Duality wall action

- Example : SU(2) Wilson loop in symmetric rep. of dimension $L+1$.

$$\hat{D}W^{2,N_f}_{L+1}(a,\lambda^{1/2}w,\lambda) = \lambda^{L/2}W^{2,N_f}_{L+1}(a,w,\lambda^{-1})$$

We have numerically checked that there is no additional correction.

- Example : SU(3) Wilson loop in adjoint rep.

$$\hat{D}\tilde{W}^3_{\text{adj}}(z,\lambda) = \lambda\tilde{W}^3_{\text{adj}}(z,\lambda^{-1}) \quad \text{where} \quad \tilde{W}^3_{\text{adj}}(z,\lambda) = W^3_{\text{adj}}(z,\lambda) - \frac{\lambda}{2}II^3(z,\lambda)$$

correction factor!

- In fact, by plugging perturbative result $W_R(z,w,\lambda) = Z_{1-\text{loop}} \times \left(\text{Tr}_R(z) + \mathcal{O}(x)\right)$, as an input, into the integral equation,

$$\oint \prod_{i=1}^{N-1} \frac{dz'_i}{2\pi i z'_i} \frac{I^{4d}(z,z',\lambda)}{\prod_{i,j}^{N}\Gamma(z'_i/z'_j)} W_R(z'_i,w,\lambda) = \lambda^{k(R)/N} W_R(z_i,w',\lambda^{-1})$$

we can uniquely determine the "correct" Wilson loop index in $x=(pq)^{1/2}$ expansion.

Sp(N) and SU(N+1) duality

(A,C)-type Elliptic integral formula

- We've seen the (A,A)-type inversion formula which is, roughly, a square of the 4d SCI at the duality interface connecting two SU(N) gauge theories.

$$\oint d\mu_{z'} \frac{\prod_{i,j}^{N} \Gamma(\lambda^{1/N} z_i/z'_j)}{\Gamma(\lambda) \prod_{i,j}^{N} \Gamma(z'_i/z'_j)} \oint d\mu_{z''} \frac{\prod_{i,j}^{N} \Gamma(\lambda^{-1/N} z'_i/z''_j)}{\Gamma(\lambda^{-1}) \prod_{i,j}^{N} \Gamma(z''_i/z''_j)} f(z'') \sim f(z)$$

- (A,C) and (C,A)-type inversion formulas [Spiridonov, Warnaar 04]

$$\oint d\mu_{z'} \Delta^{(A)}(z', x, \lambda) \oint d\mu_{z} \Delta^{(C)}(z, z', \lambda) f(z) = f(x) \,,$$

$$\oint d\mu_{z} \Delta^{(C)}(z, x, \lambda) \oint d\mu_{z'} \Delta^{(A)}(z', z, \lambda) f(z') = f(x)$$

$$\Delta^{(A)}(z', z, \lambda) \sim \frac{\prod_{i=1}^{N} \prod_{j=1}^{N+1} \Gamma(\sqrt{\lambda}^{-1} z_i^{\pm}/z'_j) \prod_{i>j}^{N+1} \Gamma(pq\lambda z'_i z'_j)}{\prod_{i\neq j}^{N+1} \Gamma(z'_i/z'_j)} \,,$$

$$\Delta^{(C)}(z, z', \lambda) \sim \frac{\prod_{i=1}^{N+1} \prod_{j=1}^{N} \Gamma(\sqrt{\lambda} z'_i z_j^{\pm}) \prod_{i>j}^{N+1} \Gamma(pq\lambda^{-1}(z'_i z'_j)^{-1})}{\prod_{i>j}^{N} \Gamma(z_i^{\pm} z_j^{\pm}) \prod_{i=1}^{N} \Gamma(z_i^{\pm 2})}$$

- Duality and duality wall between Sp(N) and SU(N+1) gauge theory ?

A proposal for duality domain wall

- We can regard $\Delta^{(C)}(z, z', \lambda)$ as 4d SCI at the interface involving the contribution from the 4d vectormultiplet for Sp(N) gauge group.

$$\Delta^{(C)}(z, z', \lambda) \sim \frac{\prod_{i=1}^{N+1} \prod_{j=1}^{N} \Gamma(\sqrt{\lambda} z'_i z_j^{\pm}) \prod_{i>j}^{N+1} \Gamma(pq\lambda^{-1}(z'_i z'_j)^{-1})}{\prod_{i>j}^{N} \Gamma(z_i^{\pm} z_j^{\pm}) \prod_{i=1}^{N} \Gamma(z_i^{\pm 2})}$$

- We propose that a domain wall connecting Sp(N) and SU(N+1) gauge theories on two sides of the wall is given by the 4d d.o.f at the interface

	$Sp(N)$	$SU(N+1)$	$U(1)_R$	$U(1)_\lambda$
q	N	$N+1$	0	$1/2$
M	1	$N(N+1)/2$	2	-1

[Gaiotto, H.-C Kim 15]

with superpotential $W = \mathrm{Tr} q\, M\, q^T\, w + X\, q\, X'$.

w : symplectic form of $Sp(N)$
X : chiral half of hypermultiplet in $SU(N+1)$
X' : chiral half of hypermultiplet in $Sp(N)$

- We find

$Sp(N)\ ,\ N_f$ | $SU(N+1)_{N+3-N_f/2}\ ,\ N_f$

is Anomaly free!

(Note that $\kappa = N + 3 - N_f/2$ saturates $|\kappa| \leq N + 3 - N_f/2$)

[Bergman, Zafrir 14]

Duality between Sp(N) and SU(N+1) theories

- Recently, it was proposed that $SU(N+1)_{N+3-N_f/2}$ gauge theory with N_f fund. hypers has enhanced $SO(2N_f)$ global symmetry in the UV CFT.

Enhanced global symmetry

N_f	$SU(N)_{\pm(N+1-N_f/2)}$	N_f	$SU(N)_{\pm(N+2-N_f/2)}$
$\leq 2N$	$SU(N_f+1)\times U(1)$	$\leq 2N+1$	$SO(2N_f)\times U(1)$
$2N+1$	$SU(N_f+1)\times SU(2)$	$2N+2$	$SO(2N_f)\times SU(2)$
$2N+2$	$SU(N_f+2)$	$2N+3$	$SO(2N_f+2)$

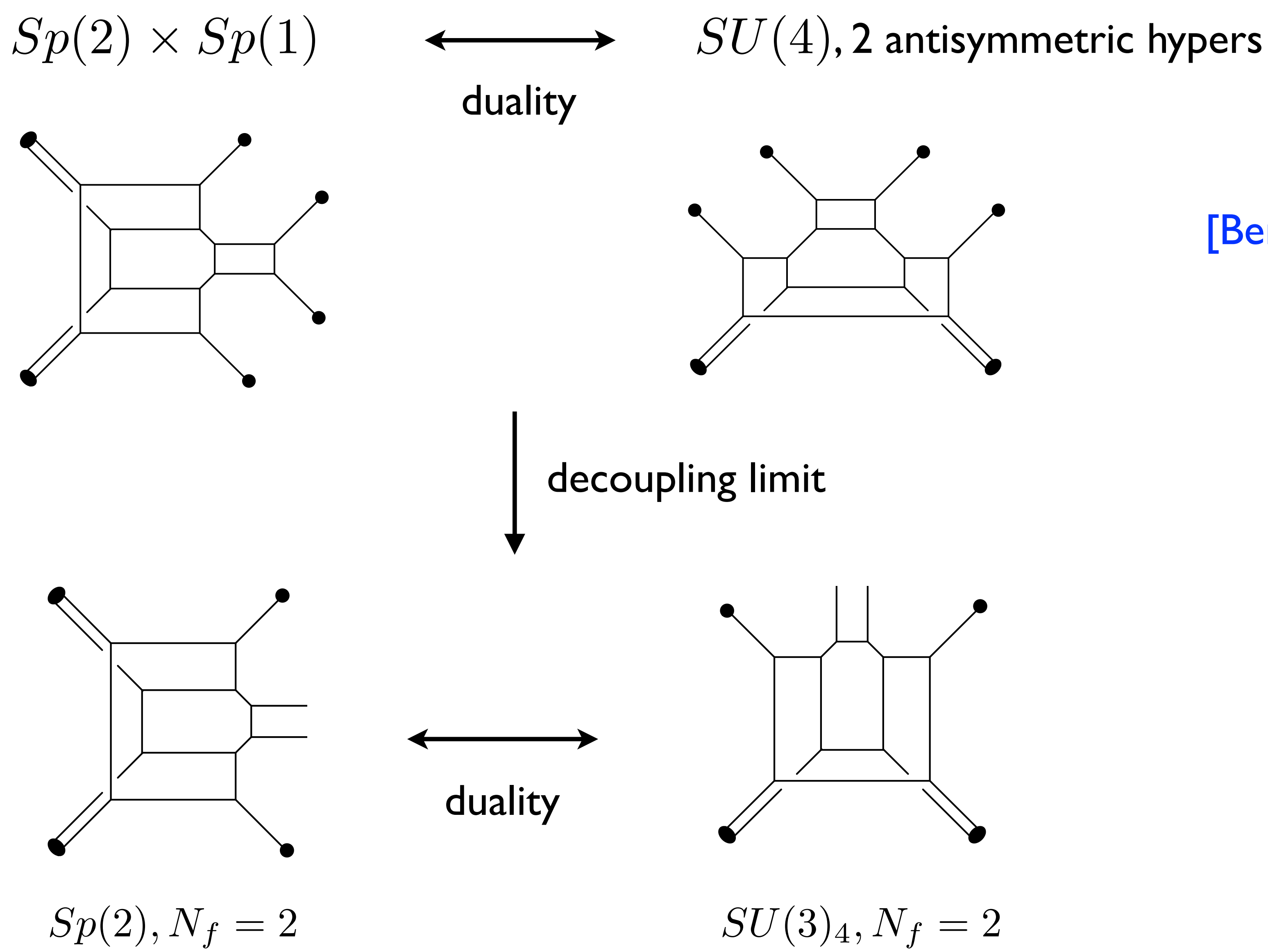
[Bergman, Zafrir 14], [Yonekura 15], [Hayashi, S.-S Kim, K. Lee, Taki, Yagi 15], [Gaiotto, H.-C Kim 15]

- So $Sp(N)$ and $SU(N+1)_{N+3-N_f/2}$ have the same dim of Coulomb branch and the same global symmetry at the UV fixed point.

- We therefore conjecture a new duality between Sp(N) and SU(N+1) gauge theories.
  - Note that $Sp(N)$ theory has UV fixed point when $N_f \leq 2N+4$. [Intriligator, Seiberg, Morrison 97]
  - Whereas, $SU(N+1)_{N+3-N_f/2}$ has UV fixed point when $N_f \leq 2N+6$. [Bergman, Zafrir 14]

Examples for N=2

- Some examples from five-brane web



[Bergman, Zafrir 14]

- Indeed, we have numerically checked that $Sp(2)$ and $SU(3)_{5-N_f/2}$ theories have the same superconformal index.

[Gaiotto, H.-C Kim 15]

Duality wall action on hemisphere index

- Duality action on the hemisphere index of $Sp(N)$ gauge theory.

$$\hat{D}II^{N,N_f}_{Sp} = \oint d\mu_{z_i} \Delta^{(C)}(z,z',\lambda) II^{N,N_f}_{Sp}(z_i, \mathfrak{q}_{Sp}, w_a)$$

- We claim that $\hat{D}II^{N,N_f}_{Sp}(z_i, w_a, \mathfrak{q}_{Sp}; p, q) = II^{N+1,N_f}_{SU}(z'_i, w'_a, \mathfrak{q}_{SU}; p, q)$ (if we identify the parameters as $w_a = \lambda^{1/2} w'_a$, $\mathfrak{q}_{Sp} = \lambda^{(N+1)/2} \prod_{a=1}^{N_f} (w_a)^{-1/2}$, $\mathfrak{q}_{SU} = \lambda^{-1} \prod_{a=1}^{N_f} (w'_a)^{-1/2}$).

- Checked this relation for $N=2$ at least up to x^5 order. (but not for $N>2$)

- Remark : under the constraint $w_a = \lambda^{1/2} w'_a$, we can always rescale $w_a,\ w'_a$ such that the left hand side and right hand side of this duality wall relation are expanded only by positive power and only by negative power of λ, respectively.
  This has been numerically checked for $N=3$.

- This relation can also be used to generate instanton partition functions in x expansion.

Conclusion

Conclusion

- We have proposed duality domain wall connecting two dual SU(N) gauge theories and carried out various tests.

- Remarkabley, enhanced global symmetry in the UV CFT can be seen even in IR gauge theory through the duality wall action and 4d Seiberg duality at the interface.

- New duality between Sp(N) and SU(N+1) gauge theories and the duality wall between them have been proposed.