5d and 6d
Superconformal Field Theories

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Challenges to Quantum Field Theories in Higher Dimensions
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Outline

- 6d SCFTs
- 5d SCFTs
- $SU(3)_\kappa$, N_f=0,1,..,10
- Conclusion

  - Vafa: 6d (1,0) supersymmetric theories and their compactifications
  - Intriligator: Anomalies, RG Flows, and the a-theorem in 6d (1,0) theories
  - Heckman: Geometries of 6d (1,0) SCFTs
  - Jaemo Park: Self-dual strings in 6d SCFT and instanton counting in 5d SCFT

References

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- Danielsson,Ferretti, Kalkkinen, Stjernberg: Notes on … 9703098
- Hanany, Zaffaroni, 9712145
- Kim,Kim,Lee: 5-dim superconformal…1206.6781
- Bergman, Rodrigues-Gomez, Zafriir: 1210.0589,1310.2150,1311.4199
- Heckman,Morrison,Vafa: On the classification… 1312.5746
- Bergman, Zafrir: 1410.2806
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6d SCFTs

- N=(2,0) SCFTs: ADE (nonabelian tensor multiplet)
  - A_N on N M5 branes, D_N on N M5+OM5 branes
  - type IIB on ADE singularity
- N=(1,0) SCFTs: quiver with multiple nonabelian YMs + tensor multiplets + hypermultiplets
  - M5 branes on Γ_{ADE} singularity
  - M5 branes near E_8 M9 wall

Anomaly Free Condition on (1,0) SCFTs

gaugino (1,0)
tensor (0,1)
hyper (0,1)
ε-spinor (1,0)

$$\mathrm{Tr}_R F^4 = \alpha_R \mathrm{tr} F^4 + c_R (\mathrm{tr} F^2)^2$$

$$\alpha_R = 0 \ for \ SU(2), SU(3), G_2, SO(8), F_4, E_6, E_7, E_8$$

$$c_{\mathrm{tot}} = \left[c_{Ad} - \sum_{R \ \mathrm{matter}} c_R \right] \geq 0$$

Green-Schwarz Mechanism

$$H^2 + \sqrt{c_{\mathrm{tot}}} B \wedge \mathrm{tr} F \wedge F + \sqrt{c_{\mathrm{tot}}} \Phi \mathrm{tr} F^2$$

$$dH = \sqrt{c_{\mathrm{tot}}} \mathrm{tr} F \wedge F$$

Global Anomaly free condition

- Bershadsky and Vafa, 9703167
- Global Anomaly: SU(2),SU(3),G_2,

$$\pi_6(SU(2)) = \mathbf{Z}_{12},\ \pi_6(SU(3)) = \mathbf{Z}_6,\ \pi_6(G_2) = \mathbf{Z}_3$$

$$n_2 - 4 = 0 \text{ mod } 2 \text{ for } SU(2)$$
$$n_3 - n_6 = 0 \text{ mod } 6 \text{ for } SU(3)$$
$$n_7 - 1 = 0 \text{ mod } 3 \text{ for } G_2$$

Type	Name	dimension	α_R	c_R
		$SU(n \geq 4)$		
Complex	**fund**	n	1	0
Complex	**asym**	$\frac{n(n-1)}{2}$	$n-8$	3
Complex	**sym**	$\frac{n(n+1)}{2}$	$n+8$	3
Strictly real	**adj**	n^2-1	$2n$	6
		$SO(n \geq 7)$		
Strictly real	**vect**	n	1	0
Strictly real	**adj**	$n(n-1)/2$	$n-8$	3
		$USp(n \geq 4)$		
Pseudo-real	**vect**	n	1	0
Strictly real	**asym**	$\frac{(n+1)(n-2)}{2}$	$n-8$	3
Strictly real	**adj**	$\frac{n(n+1)}{2}$	$n+8$	3

Group	Type	Name	Dimension	α_R	c_R
SU(2)	Pseudo-real	**fund**	2	0	1/2
SU(2)	Strictly real	**adj**	3	0	8
SU(3)	Complex	**fund**	3	0	1/2
SU(3)	Complex	**sym**	6	0	17/2
SU(3)	Strictly real	**adj**	8	0	9
SU(6)	Pseudo-real	**asym3**	20	−6	6
SO(7)	Strictly real	**S**	8	−1/2	3/8
SO(8)	Strictly real	**S**	8	−1/2	3/8
SO(9)	Strictly real	**S**	16	−1	3/4
SO(10)	Complex	**S**	16	−1	3/4
SO(11)	Pseudo-real	**S**	32	−2	3/2
SO(12)	Pseudo-real	**S**	32	−2	3/2
SO(13)	Pseudo-real	**S**	64	−4	3
SO(14)	Complex	**S**	64	−4	3
E_6	Complex	**fund**	27	0	1/12
E_6	Strictly real	**adj**	78	0	1/2
E_7	Pseudo-real	**fund**	28	0	1/24
E_7	Strictly real	**adj**	133	0	1/6
E_8	Strictly real	**adj**	248	0	1/100
F_4	Strictly real	**fund**	26	0	1/12
F_4	Strictly real	**adj**	52	0	5/12
G_2	Strictly real	**fund**	7	0	1/4
G_2	Strictly real	**adj**	14	0	5/2

Simple Gauge group without no quartic

- SU(2): n_2=4,10,16 (LST)
- SU(3): n_3=0,6,12,18 (LST)
- G_2 : n_7=1,4,7,10 (LST)
- SO(8): n(v,c,s) or n(v,2s)=0,1,2,3,4(LST)
- F_4: n_{26}=0,1,2,3,4,5(LST)
- E_6: n_{27}=0,1,2,3,4,5,6(LST)
- E_7: $n_{56/2}$=0,1,2,3,4,5,6,7,8(LST)

Simple Gauge group: quartic

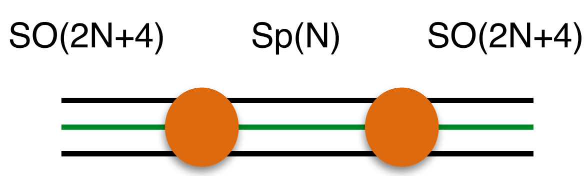
- SU(N) N>3 : $n_F=2N$, or $n_F=n+8$ and $n_A=1$,
- SU(N) N>3: or $n_F=8$ and $n_A=2$ (LST), or $n_S=1$ and $n_A=1$ (LST)
- Sp(N) N>1: $n_F=2N+8$, $n_F=16$ and $n_A=1$ (LST)
- SO(N): nv=N-8, N>7
- SU(6): $n_F= 15$ and $n_{A/2}=1$, $n_F=18$ and $n_{A/2}=2$ (LST)
- SO(7): $n_v=k-1$ and $n_s=2k$ with k=1,2,3,4(LST)
- SO(9): $n_v=k+1$ and $n_s=k$ with k=0,1,2,3,4(LST)
- SO(10): $n_v=k+2$ and $n_s=k$ with k=0,1,2,3,4 (LST)
- SO(11): nv=k+3 and ns=k with k=0,1,2(LST)
- SO(12): nv=k+4 and ns=k with k=0,1,2(LST)
- SO(13):nv=k+5 and ns=0,1(LST)
- SO(14):nv=k+6 and ns=0,1(LST)

product group

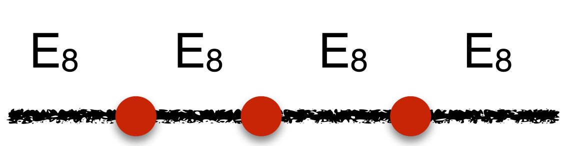
- SU(2) [(2,8)/2] SO(7) [(8,2)/2] SU(2)
- SU(2) [(2,7+1)/2] G_2
- [2N-M] SU(N) [(N,M)] SU(M) [2M-N]
- $[2N+8]_{SO(2N+8)}$ Sp(N) [(2N,2N+8)] SO(2N+8) $[2N]_{Sp(N)}$
- [2M-N] SU(M) [(M,N)] SU(N) [2N-M]

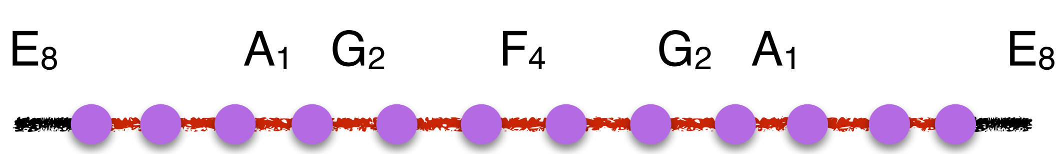
M5s on ADE singularities

- ADE singularity: C^2/Γ_{ADE}
- ADE ALE => ALF : D6, $O6^-$+D6, .., 7d N=1 SYM theories
- 1 NS5 brane gets fractionalized. (G,G) conformal matter
- A_N = N D6 with n NS5 : SU(N)&SU(N)&SU(N)…-&SU(N)
- D_{N+4}= O6-D6 with n NS 5 : SO(2N+4)&Sp(N)&SO(2N+4)&Sp(N) ….Sp(N)&SO(2N+4)



- E_6: 4 pieces, E_6& &SU(3)& &E_6& &SU(3)& … &SU(3)& &E_6
- E_7: 6 pieces, E_7& &SU(2)&SO(7)&SU(2)& &E_7 (2x8_sx1)/2, (1x8_sx2)/2
- E_8: 12 pieces, E_8& & &SU(2)&G_2& &F_4& &G_2&SU(2)& & &E_8 (2x(7+1))/2, ((7+1)x2)/2





- ADE ALE => ALF : D6, $O6^-$+D6, ..
- 1 NS5 brane gets fractionalized.
- A_N : SU(N)-SU(2N)
- D_{N+4}: Sp(N)-SO(4N+16), $O5^-$ plane, SO(2N+8)-Sp(2N), $O5^+$ plane
- E_6: SU(3), E_6
- E_7: SU(2)-SO(7)-SU(2) $(2 \times 8_s \times 1)/2$, $(1 \times 8_s \times 2)/2$, SU(2)-SO(8),$E_7$
- E_8: F4, G_2xSU(2) ((7+1)x2)/2, G_2 $n_7=1$, SU(2) $n_2=4$, E_8

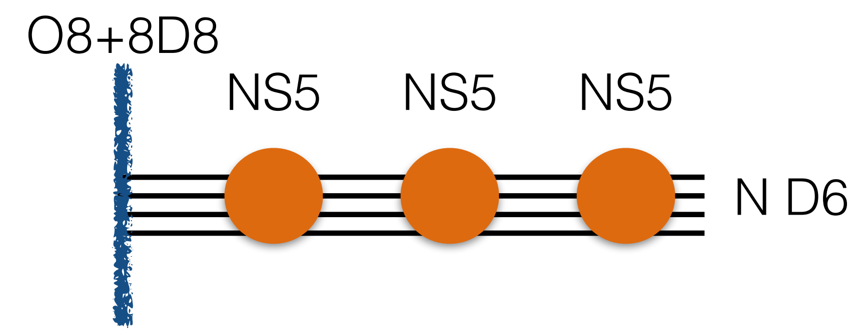
6d Non-Higgsable cluster

D. Morrison, W. Taylor 1412.6112

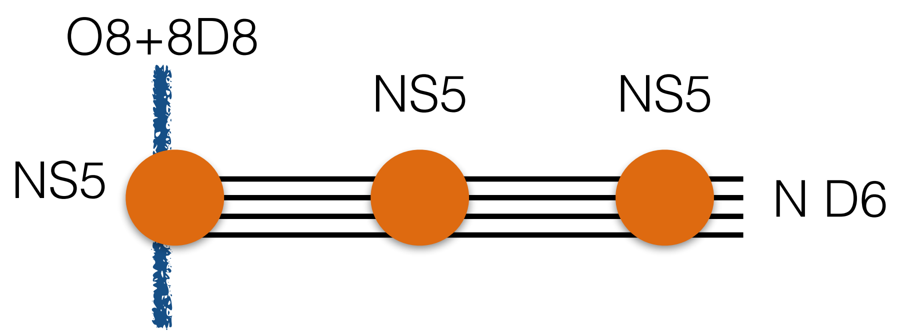
- SU(3), SO(8), F_4, E_6,E_7, E_7 56/2, E_8, G_2xSU(2) (7+1)x2/2, SU(2)xSO(7)xSU(2) (2x8_sx1)/2 (1x8_sx2)/2

Diagram	Algebra	matter	(f, g, Δ)	$\Delta T_{\max}$
-3	$\mathfrak{su}(3)$	0	$(2,2,4)$	$1/3$
-4	$\mathfrak{so}(8)$	0	$(2,3,6)$	1
-5	$\mathfrak{f}_4$	0	$(3,4,8)$	$16/9$
-6	$\mathfrak{e}_6$	0	$(3,4,8)$	$8/3$
-7	$\mathfrak{e}_7$	$\frac{1}{2}\mathbf{56}$	$(3,5,9)$	$57/16$
-8	$\mathfrak{e}_7$	0	$(3,5,9)$	$9/2$
-12	$\mathfrak{e}_8$	0	$(4,5,10)$	$25/3$
$-3, -2$	$\mathfrak{g}_2 \oplus \mathfrak{su}(2)$	$(\mathbf{7}+\mathbf{1}, \frac{1}{2}\mathbf{2})$	$(2,3,6), (1,2,3)$	$3/8$
$-3, -2, -2$	$\mathfrak{g}_2 \oplus \mathfrak{su}(2)$	$(\mathbf{7}+\mathbf{1}, \frac{1}{2}\mathbf{2})$	$(2,3,6), (2,2,4),$ $(1, 1, 2)$	$5/12$
$-2, -3, -2$	$\mathfrak{su}(2) \oplus \mathfrak{so}(7) \oplus \mathfrak{su}(2)$	$(\mathbf{1}, \mathbf{8}, \frac{1}{2}\mathbf{2})$ $+(\frac{1}{2}\mathbf{2}, \mathbf{8}, \mathbf{1})$	$(1,2,3), (2,4,6),$ $(1, 2, 3)$	$1/2$

6d SCFTs on E_8 or D8 $O8^-$Wall



- [8]-Sp(N)-SU(2N)-…SU(2N)—[2N]
- [8]-Sp(N)-SU(2N)-…SU(2N)-Sp[N]-[8] (LST)



- [8,A]-SU(N)-…SU(N)-[N]
- [S]-SU(N)-SU(N-8)-[N-16]
- [8,A]-SU(N)-[8,A] (LST)

5d SCFTs and enhancement of global symmetry

- UV completion, strong coupling limit in the symmetric phase
- Instantons: topological U(1) symmetry
- D,NS5 brane web
- superconformal index: localization or topological vertex

SU(2), n_2=0,1,2,,,,7,8

- theta term: $\theta=0,\pi$ (Seiberg,Intriligator, Morrison….)
- $E_0,\tilde{E}_1,E_1$=SU(2),E_2=SU(2)xU(1),E_3,=SU(3)xSU(2), E_4,=SU(5),E_5,=SO(10), E_6,E_7,E_8
- A (p,q) 5 brane terminates at [p,q] 7 brane (Dewolfe, Hanany,Iqbal,Katz)
- superconformal index: localization, instanton operators
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- topological vertex & branes
  - Bao, Mitev, Pomoni, Taki, Yagi 1310.3841, H. Hayashi, H.C. Kim, Nishinaka 1310.3854, S.S. Kim, M. Taki, F. Yagi 1504.03672

D5,NS5 brane web and 7 branes

- A (p,q) 5 brane terminates at [p,q] 7 brane.

- The [P,Q] 7 brane monodromy is

$$K_{[P,Q]} = \begin{pmatrix} 1+PQ & -P^2 \\ Q^2 & 1-PQ \end{pmatrix}.$$

- Transformation of (p,q) 5 brane and [p,q] 7 brane counterclockwise crossing [P,Q] 7 brane experience the K[P,Q] monodromy,

$$\begin{pmatrix} p' \\ q' \end{pmatrix} = \begin{pmatrix} 1+PQ & -P^2 \\ Q^2 & 1-PQ \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$$

- [1,0], D7 brane

- A=[0,1], B=[1,-1], C=[1,1] 7 branes

D5,NS5 brane web and 7 branes

5D Field Theory	Brane Configuration	Geometry
E_0	$\mathbf{E_0 = X_{[2,-1]} X_{[-1,2]} C}$	$\mathbb{P}^2$
$\tilde{E}_1$	$\mathbf{A X_{[2,-1]} X_{[-1,2]} C}$	$\mathcal{B}_1$
E_1	$\mathbf{BCBC}$	$\mathbb{P}^1 \times \mathbb{P}^1$
$E_n, n \geq 2$	$\mathbf{A}^{n-1}\mathbf{BCBC} = \mathbf{A}^n \mathbf{X_{[2,-1]} C X_{[-1,2]}}$	$\mathcal{B}_n$

5d SU(N)

- SU(N)+ N_f fundamental hyper + κ-Chern-Simons level
- N_f + 2|κ| =2$\mathbb{Z}$: anomaly (the fermion determinant in the U(1) theory.
- $N+2|\kappa| \leqq 2N$ (Intriligator, Morrison, Seiberg)
- $N+2|\kappa| \leqq 2N+4$, Enhanced global symmetry at UV (Bergman, Zafrir)
  - (Sung-Soo Kim, Masato Taki, Futoshi Yagi, Tao-diagram..)
  - (Hirotaka Hayashi, Sung-Soo Kim, KL, Masato Taki,Futoshi Yagi)
  - (Kazuya Yonekura, Gaiotto and Hee-Cheol Kim)
- $SU(N)_0$, N_f =2N+4 with finite coupling: 6d completion

5d SU(3)κ Enhanced Global Symmetry

For total rank of G = N_f +1, abelian part may be added
N+2|κ| ≦ 10

N_f	$G_{\|\kappa\|}$ (κ is the Chern-Simons level)				
10	$SO(20)_0$				
9	$SO(20)_{\frac{1}{2}}$				
8	$SU(10)_0$,	$[SO(16)\times SU(2)]_1$			
7	$[SU(8)\times SU(2)]_{\frac{1}{2}}$,	$SO(14)_{\frac{3}{2}}$			
6	$[SU(6)\times SU(2)\times SU(2)]_0$,	$SU(7)_1$,	$SO(12)_2$		
5	$[SU(5)\times SU(2)]_{\frac{1}{2}}$,	$SU(6)_{\frac{3}{2}}$,	$SO(10)_{\frac{5}{2}}$		
4	$SU(4)_0$,	$[SU(4)\times SU(2)]_1$,	$SU(5)_2$,	$SO(8)_3$	
3	$SU(3)_{\frac{1}{2}}$,	$[SU(3)\times SU(2)]_{\frac{3}{2}}$,	$SU(4)_{\frac{5}{2}}$,	$SO(6)_{\frac{7}{2}}$	
2	$SU(2)_0$,	$SU(2)_1$,	$[SU(2)\times SU(2)]_2$,	$SU(3)_3$,	$SO(4)_4$
1	$SU(2)_{\frac{5}{2}}$,	$SU(2)_{\frac{7}{2}}$			
0	$SU(2)_3$				

5d SU(N)κ Enhanced Global Symmetry

For total rank of G = N_f +1, abelian part may be added
N+2|κ| ≦ 2N+4

N_f	$G_{\|\kappa\|}$
$2n+4$	$SO(4n+8)_0$
$2n+3$	$SO(4n+8)_{\frac{1}{2}}$
$2n+2$	$SU(2n+4)_0$, $\left[SO(4n+4)\times SU(2)\right]_1$
$2n+1$	$\left[SU(2n+4)\times SU(2)\right]_{\frac{1}{2}}$, $SO(4n+2)_{\frac{3}{2}}$
$2n$	$\left[SU(2n)\times SU(2)\times SU(2)\right]_0$, $SU(2n+1)_1$, $SO(4n)_2$

From 6 to 5 dim and to 6d

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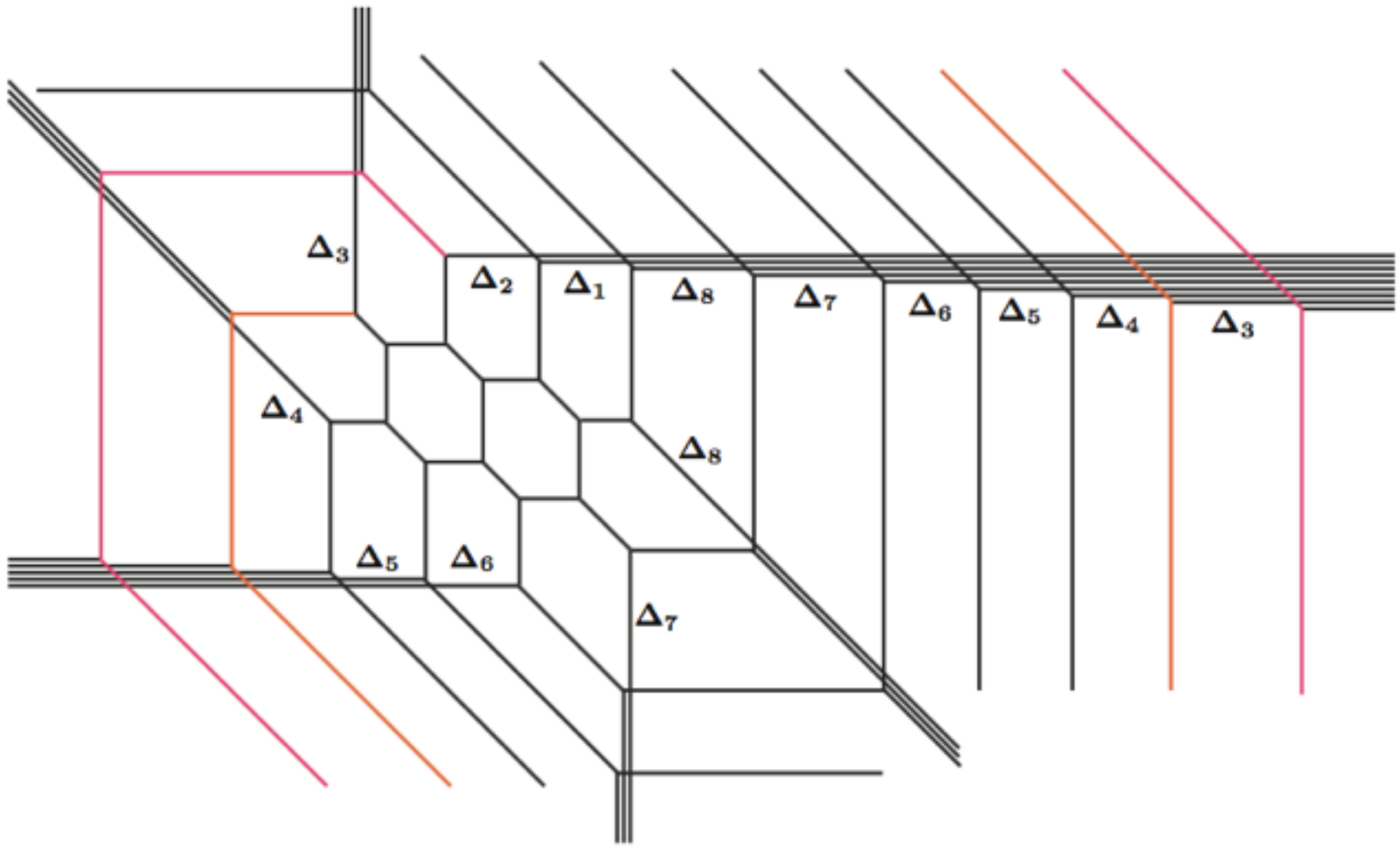

Tao-diagram

Enhanced Global Symmetry

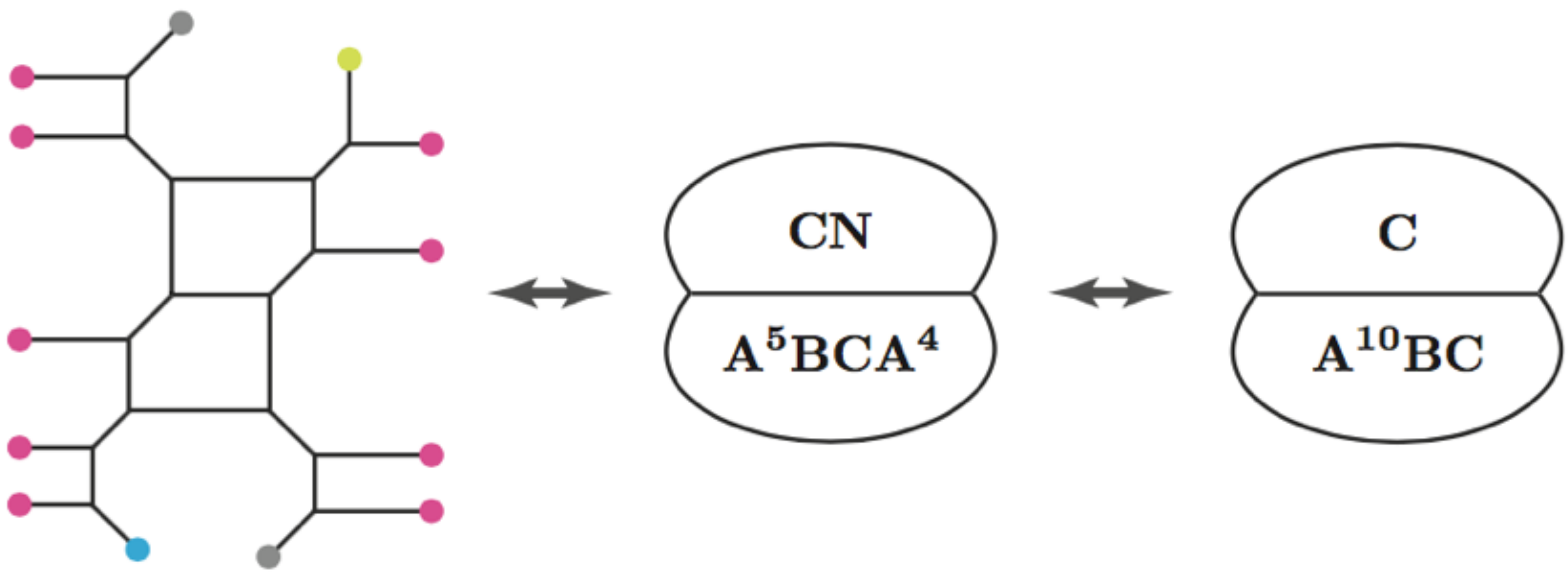

Figure 8: *A web diagram of* $SU(3)$ *theory with* $N_f = 8$ *flavors with CS level 1 (left) and with* $N_f = 9$ *flavors with CS level* $\frac{1}{2}$ *(right). They are obtained by taking mass decoupling limit from Tao diagram.*

Conclusion

- A lot more to be learned about 5d and 6d SCFTs
- Further relation between 5d and 6d SCFTs
- More on 6d SCFTs
- 6d (2,0) and (1,0) Little String Theories