M5-branes and Wilson Surfaces in AdS_7/CFT_6 Correspondence

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PARTICLE THEORY OSAKA

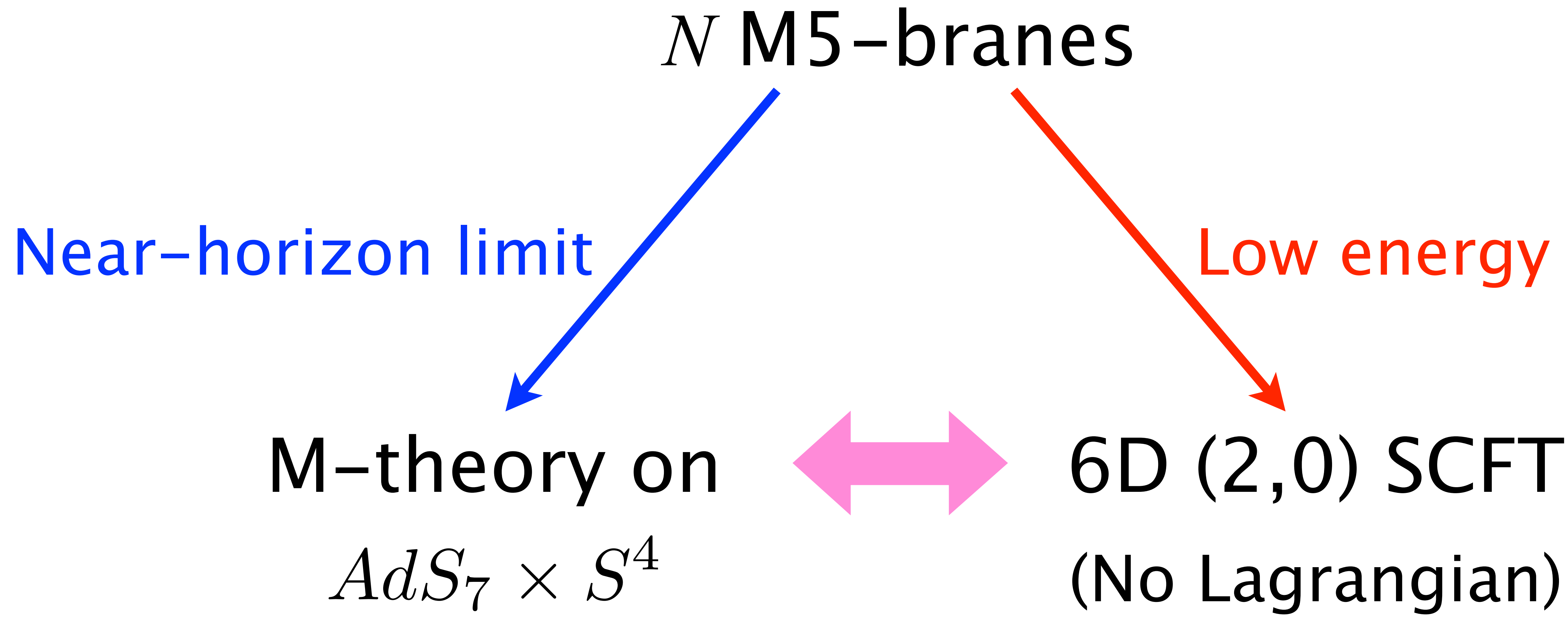
arXiv:1404.0930, work in progress collaborated with
Satoshi Yamaguchi (Osaka Univ.)

2015/06/29, Challenges to Quantum Field Theory in Higher Dimensions @ Technion, Israel

What we did: test of

AdS_7/CFT_6

AdS_7/CFT_6


N M5-branes
Near-horizon limit
Low energy
M-theory on
$AdS_7 \times S^4$
6D (2,0) SCFT
(No Lagrangian)

6D (2,0) SCFT

[Douglas '10]
[Lambert, Papageorgakis, Schmidt-Sommerfeld '10]

equivalent (?) || compactified on S^1

5D maximally SYM

Recent progress: Exact computation

[Källén, Zabzine '12] [Hosomichi, Seong, Terashima '12]
[Källén, Qiu, Zabzine '12] [Kim, Kim '12] [Imamura '12]
[Kim, Lee '12] [Fukuda, Kawano, Matsumiya '12] [Kim, Kim, Kim '12]
[Qiu, Zabzine '13] [Kim, Kim, Kim, Lee '13] [Schmude '14] ....

AdS_7/CFT_6

M-theory on $AdS_7 \times S^4$ ⟷ 6D (2,0) SCFT

Exact results

Comparison

AdS_7/CFT_6

M-theory on $AdS_7 \times S^4$

(Boundary= $S^5 \times S^1$)

⟺

6D (2,0) SCFT on $S^5 \times S^1$

Exact results

M2/M5-brane

Bubbling geometry

⟺

Wilson surface

≀

Wilson loop

in 5D MSYM on S^5

▢Summary

M2/M5-brane		Wilson surface
M2-brane on $\underset{AdS_7}{\underset{\cap}{AdS_3}}$	⇔ [Minahan, Nedelin, Zabzine '13]	Fundamental rep. $\exp\left[\frac{\beta N}{2}\right]$
M5-brane on $\underset{AdS_7}{\underset{\cap}{AdS_3}} \times S^3$	⇔ New!	k-th symmetric rep. $\exp\left[\frac{\beta N}{2}k\left(1-\frac{k}{N}\right)\right]$
M5-brane on $\underset{AdS_7}{\underset{\cap}{AdS_3}} \times \underset{S^4}{\underset{\cap}{\tilde{S}^3}}$	⇔ New!	k-th anti-symmetric rep. $\exp\left[\frac{\beta N}{2}k\left(1+\frac{k}{2N}\right)\right]$

Not the 't Hooft limit ⇒ not stringy, but M-theoretic

Plan

1. Bubbling geometry
2. Matrix model
3. Wilson surface by M5-brane
4. Summary & Outlook

6D (2,0) SCFT

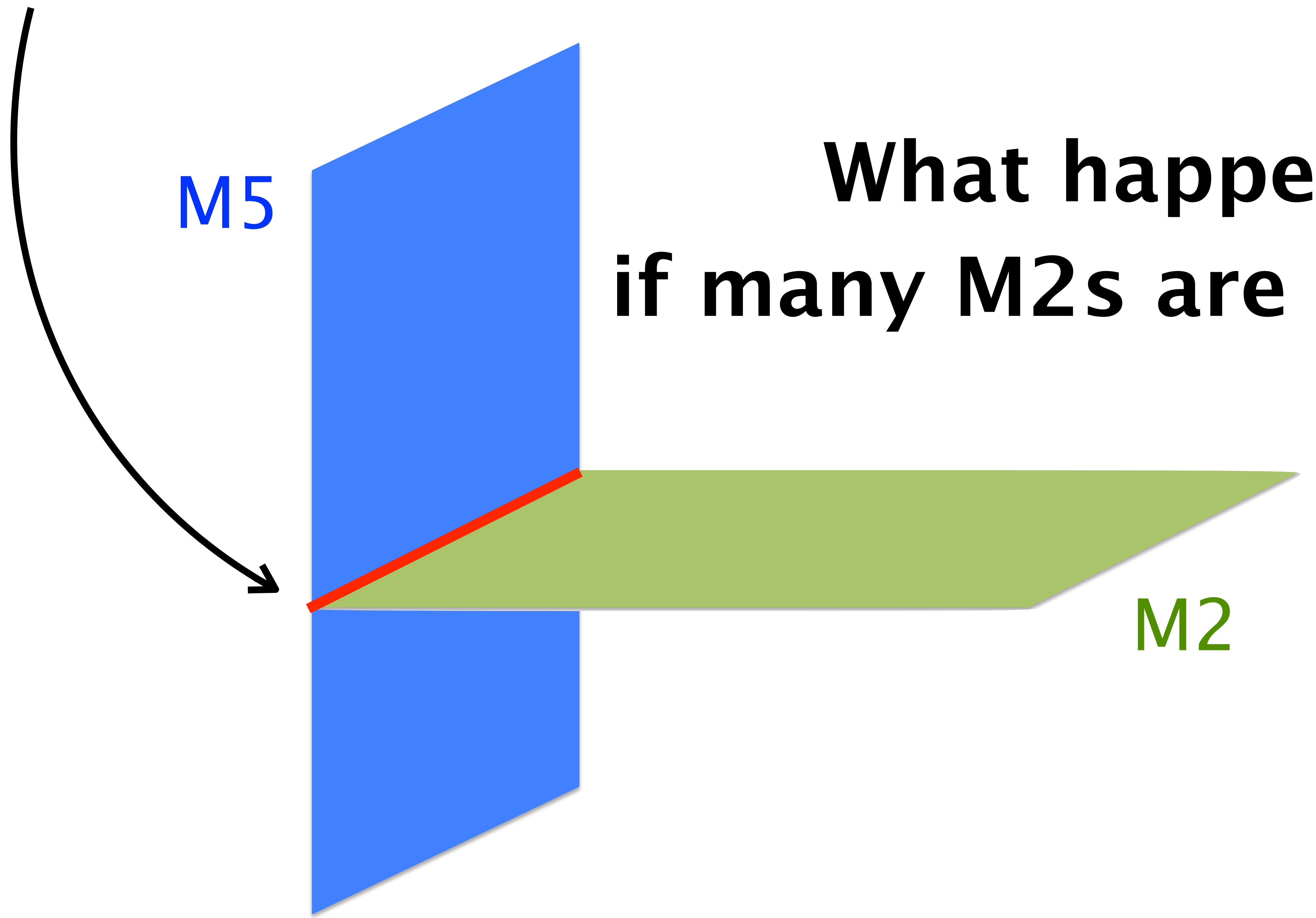
= Low energy theory on N M5–branes

(No known Lagrangian description)

Wilson surface

= Boundary of M2 ending on M5

= Non-local operator extending on 2-dim space



What happens if many M2s are put on?

What happens
if many M2s are put on?

(I) Become an M5–brane

(II) Back–reaction to gravity

$\Rightarrow$ Bubbling geometry

1. Bubbling geometry

Strategy to find SUGRA solution [Lin, Lunin, Maldacena '04]

Symmetry: $\overbrace{\mathrm{SO}(2,2)}^{\text{2D conformal}} \times \underbrace{\mathrm{SO}(4)}_{\text{rotation in transverse direction}} \times \overbrace{\mathrm{SO}(4)}^{\text{remnant of R-sym}}$

⬇

11D geometry $\supset$ $AdS_3 \times S^3 \times S^3$

⬇

Ansatz

⬇

SUSY condition: $\delta(\mathrm{gravitino}) = 0$

1. Bubbling geometry

A class of classical half-BPS solutions of 11D supergravity

[Lin, Lunin, Maldacena '04]
[Yamaguchi '06] [Lunin '06]
[D'Hoker, Estes, Gutperle, Krym '08]

$$ds^2 = e^{2A} d\check{\Omega}_3^2 + d\Sigma_2^2 + e^{2B} d\Omega_3^2 + e^{2C} d\widetilde{\Omega}_3^2$$

$$d\Sigma_2^2 = \frac{1}{-e^{2B+2C} + e^{2A+2B} + e^{2A+2C}} \left(dy^2 + dx^2\right)$$

$$\begin{cases} A, B, C : \text{functions on } \Sigma_2, \\ F, J, K : \text{1-forms on } \Sigma_2. \end{cases}$$

$$G_4 = -6F E^0 E^1 E^2 + 6J E^5 E^6 E^7 + 6K E^8 E^9 E^{10}$$

$$y = e^{A+B+C}$$

$$\begin{cases} 6F = 4\dfrac{df_0}{g_1} - \dfrac{f_0 dg_1}{g_1^2} + \dfrac{2}{g_1^2}(f_2 \tilde{d} f_3 - f_3 \tilde{d} f_2), \\ 6J = 4\dfrac{df_3}{g_1} - \dfrac{f_3 dg_1}{g_1^2} + \dfrac{2}{g_1^2}(-f_0 \tilde{d} f_2 + f_2 \tilde{d} f_0), \\ 6K = -4\dfrac{df_2}{g_1} + \dfrac{f_2 dg_1}{g_1^2} + \dfrac{2}{g_1^2}(-f_0 \tilde{d} f_3 + f_3 \tilde{d} f_0), \end{cases}$$

$$f_0 = e^A, \quad f_3 = pe^B, \quad f_2 = qe^C \quad (p, q\text{: constants}),$$

$$g_1 = \sqrt{f_0^2 - f_2^2 - f_3^2}.$$

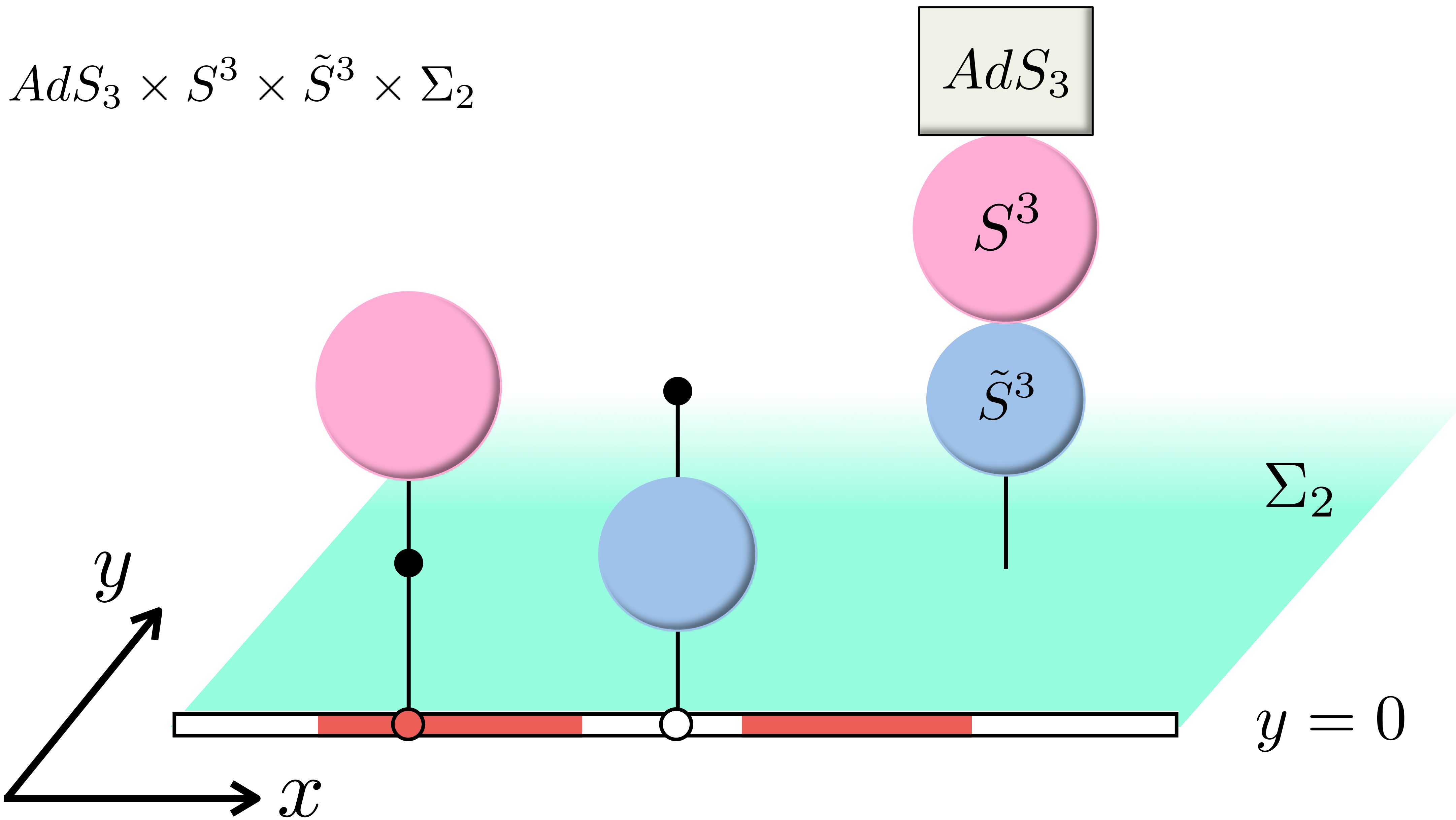
and differential equations.

1. Bubbling geometry

$$\mathrm{d}s^2 = e^{2A}\mathrm{d}\check{\Omega}_3^2 + \mathrm{d}\Sigma_2^2 + e^{2B}\mathrm{d}\Omega_3^2 + e^{2C}\mathrm{d}\widetilde{\Omega}_3^2$$

$$\mathrm{d}\Sigma_2^2 = \frac{1}{-e^{2B+2C} + e^{2A+2B} + e^{2A+2C}}\left(\mathrm{d}y^2 + \mathrm{d}x^2\right)$$

$$\mathcal{M}_{11} = AdS_3 \times S^3 \times \tilde{S}^3 \times \Sigma_2$$



1. Bubbling geometry

Bubbling geometry is labeled by

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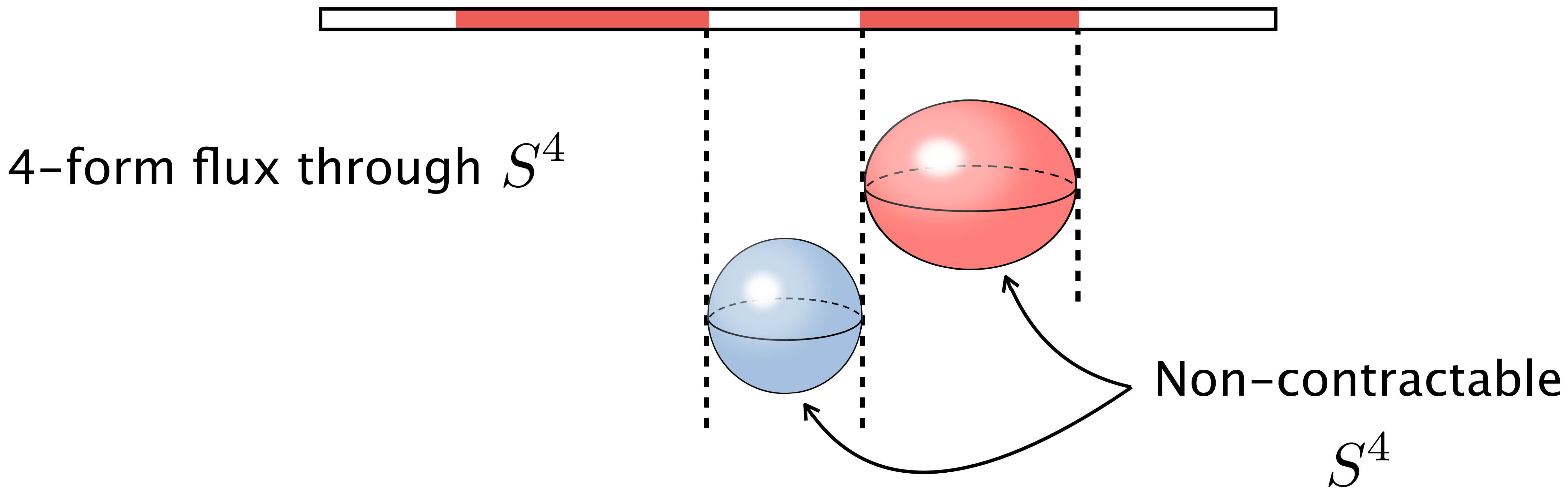
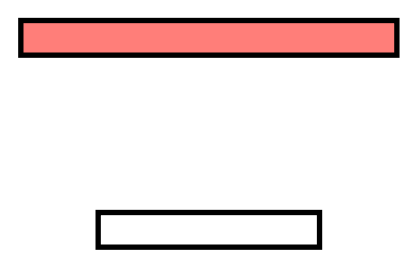


1. Bubbling geometry

Bubbling geometry is labeled by

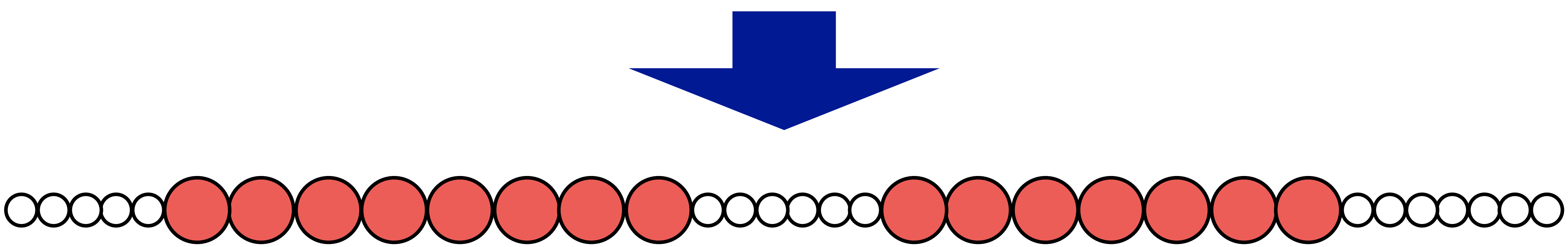
4-form flux through S^4
Quantization condition $\Longrightarrow$ Length of each segment is quantized



$: a\mathbb{Z}$

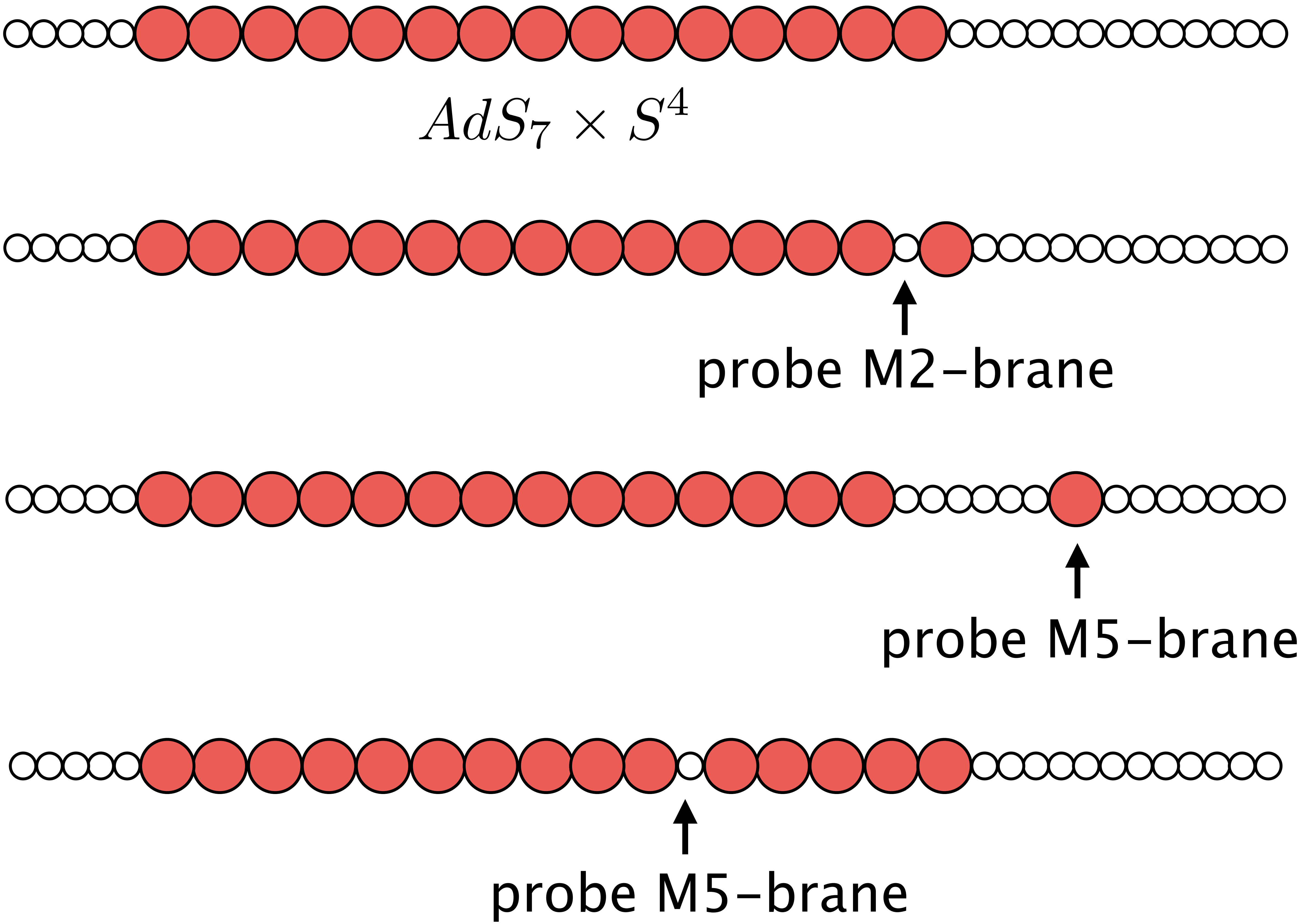
$: \frac{a}{2}\mathbb{Z}$

$a = 2\pi\ell_p$



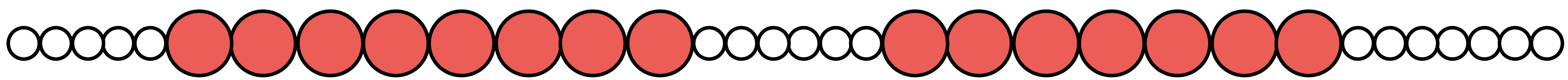
"Maya diagram"

1. Bubbling geometry



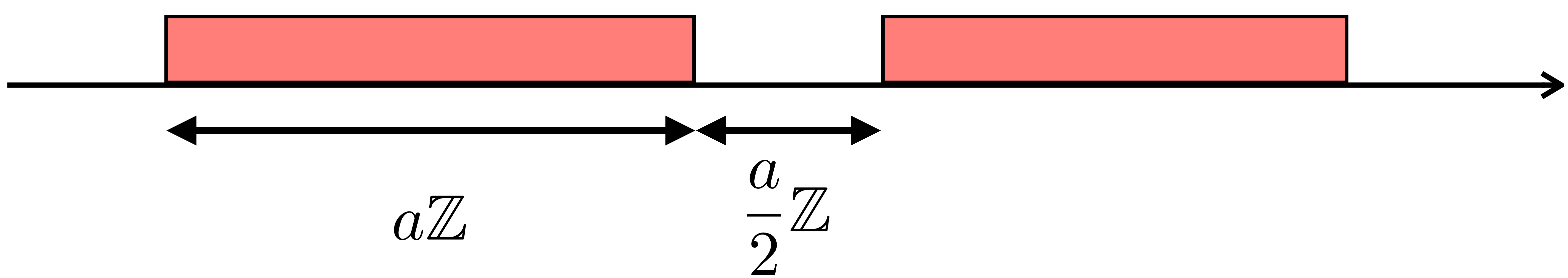
1. Bubbling geometry

Bubbling geometry is labeled by



From experiences of other examples,

Maya diagram ⇒ eigenvalue density



if the matrix model for Wilson surfaces exists.

Plan

1. Bubbling geometry

2. Matrix model

3. Wilson surface by M5-brane

4. Summary & Outlook

2. Matrix model

6D (2,0) on $S^5 \times S^1$

$R_6 = \frac{g_{YM}^2}{8\pi^2}$

$(R_6 : \text{radius of } S^1)$

5D MSYM on S^5

Chern-Simons matrix model [Källén, Qiu, Zabzine '12] [Kim, Kim '12]

$$\mathcal{Z} \sim \int \prod_{i=1}^{N} \mathrm{d}\nu_i \exp\left[-\frac{N^2}{\beta} \sum_{i=1}^{N} \nu_i^2 + \sum_{i \neq j} \ln \left| 2 \sinh \frac{N(\nu_i - \nu_j)}{2} \right| \right]$$

$$\beta = \frac{g_{\mathrm{YM}}^2}{2\pi r} = \frac{4\pi R_6}{r}, \quad r : \text{radius of } S^5$$

2. Matrix model

6D (2,0) on $S^5 \times S^1$ $\supset$ Wilson surface $(S^1 \times S^1)$

|| ||

5D MSYM on S^5 $\supset$ Wilson loop (S^1)

$\langle W_{\mathbf{R}} \rangle$

Chern-Simons matrix model

[Källén, Qiu, Zabzine '12]
[Kim, Kim '12]

$$\mathcal{Z} \sim \int \prod_{i=1}^{N} \mathrm{d}\nu_i \exp\left[-\frac{N^2}{\beta} \sum_{i=1}^{N} \nu_i^2 + \sum_{i \neq j} \ln \left| 2 \sinh \frac{N(\nu_i - \nu_j)}{2} \right| \right]$$

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2. Matrix model

6D (2,0) on $S^5 \times S^1$ $\supset$ Wilson surface $(S^1 \times S^1)$

|| ||

5D MSYM on S^5 $\supset$ Wilson loop (S^1)

⇩

Chern–Simons matrix model

[Källén, Qiu, Zabzine '12]
[Kim, Kim '12]

$$\langle W_{\mathbf{R}} \rangle = \frac{1}{\mathcal{Z}} \int \prod_{i=1}^{N} \mathrm{d}\nu_i \left(\mathrm{Tr}_{\mathbf{R}} e^{N\nu}\right) \exp\left[-\frac{N^2}{\beta} \sum_{i=1}^{N} \nu_i^2 + \sum_{i \neq j} \ln \left| 2 \sinh \frac{N(\nu_i - \nu_j)}{2} \right| \right]$$

$$\beta = \frac{g_{\mathrm{YM}}^2}{2\pi r} = \frac{4\pi R_6}{r}, \quad r : \text{radius of } S^5$$

2. Matrix model

Chern-Simons matrix model

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$$\beta = \frac{g_{\mathrm{YM}}^2}{2\pi r} = \frac{4\pi R_6}{r}, \quad r : \text{radius of } S^5$$

We want to evaluate this in the limit

$$\beta \text{ fixed}, \; N \to \infty$$

NOTE: This is NOT the 't Hooft limit.

2. Matrix model

Chern–Simons matrix model

[Källén, Qiu, Zabzine '12]
[Kim, Kim '12]

$$\mathcal{Z} \sim \int \prod_{i=1}^{N} \mathrm{d}\nu_i \exp\left[-\frac{N^2}{\beta}\sum_{i=1}^{N}\nu_i^2 + \sum_{i\neq j}\ln\left|2\sinh\frac{N(\nu_i-\nu_j)}{2}\right|\right]$$

$O(N^3) \Rightarrow$ Saddle point method

β fixed, $N \to \infty$

$$\mathcal{Z} \sim \int \prod_{i=1}^{N} \mathrm{d}\nu_i \exp\left[-\frac{N^2}{\beta}\sum_{i=1}^{N}\nu_i^2 + \frac{N}{2}\sum_{i\neq j}|\nu_i-\nu_j|\right]$$

2. Matrix model

Chern–Simons matrix model

[Källén, Qiu, Zabzine '12]
[Kim, Kim '12]

$$\mathcal{Z} \sim \int \prod_{i=1}^{N} \mathrm{d}\nu_i \exp\left[-\frac{N^2}{\beta}\sum_{i=1}^{N}\nu_i^2 + \frac{N}{2}\sum_{i\neq j}|\nu_i - \nu_j|\right]$$

Saddle point eqs: $0 = -\frac{2N^2}{\beta}\nu_i + N\sum_{j,i\neq j}\mathrm{sign}\left(\nu_i - \nu_j\right)$

Assume $\nu_1 > \nu_2 > \cdots > \nu_N$

Eigenvalues: $\nu_i = \frac{\beta}{2}\left(1 - \frac{2i}{N}\right)$

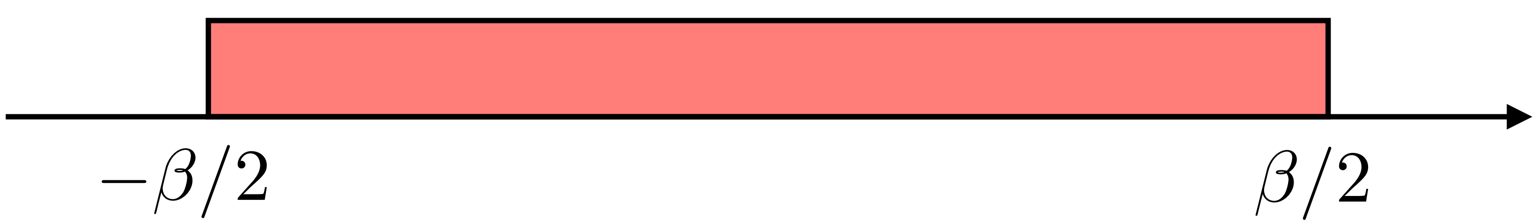
2. Matrix model

Chern-Simons matrix model

[Källén, Qiu, Zabzine '12]
[Kim, Kim '12]

Eigenvalues: $\nu_i = \frac{\beta}{2}\left(1 - \frac{2i}{N}\right)$

Eigenvalue density: $\rho(\nu) = \begin{cases} \frac{1}{\beta} & \text{for } |\nu| \leq \beta/2, \\ 0 & \text{for } |\nu| > \beta/2. \end{cases}$



This is consistent with bubbling geometry!

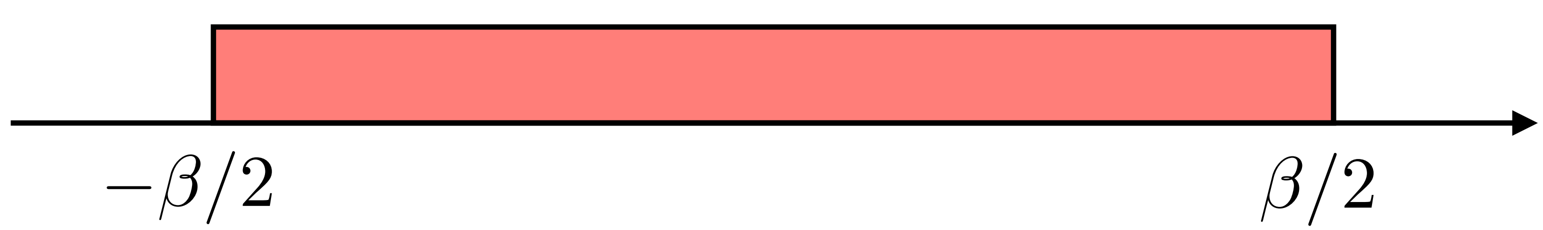
2. Matrix model

Fundamental rep.

$$\mathrm{Tr}_{\Box} e^{N\nu} = \sum_i e^{N\nu_i}$$

$\Longrightarrow$ Largest contribution: $e^{N\nu_1}$

$$\langle W_{\Box} \rangle \sim \exp\left[N\nu_1 \right]\big|_{\text{saddle point}} \sim \exp\left[\frac{N\beta}{2} \right]$$



2. Matrix model

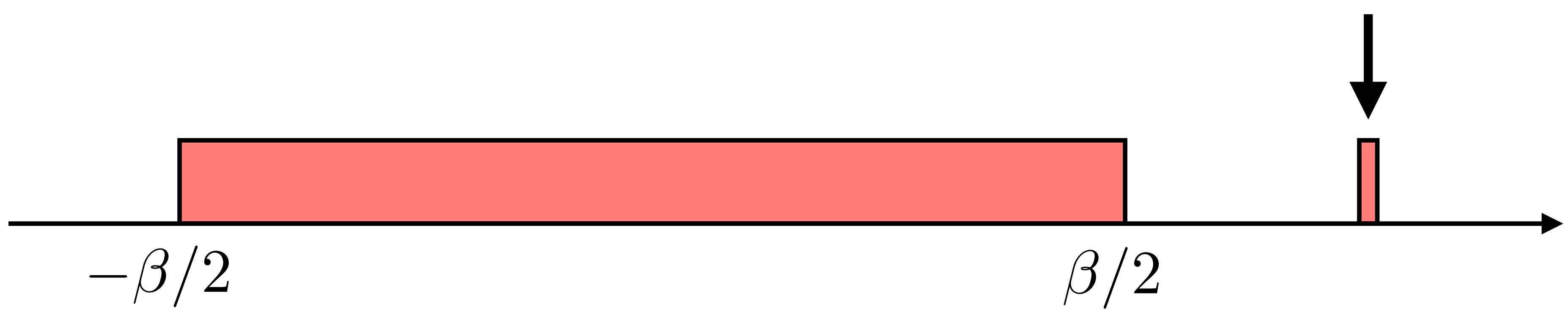
<u>k–th symmetric rep.</u>

$$\mathrm{Tr}_{\underbrace{\square\square\cdots\square}_{k}} e^{N\nu} = \sum_{1\leq i_1 \leq \cdots \leq i_k \leq N} \exp\left[N \sum_{l=1}^{k} \nu_{i_l} \right]$$

$\Longrightarrow$ Largest contribution: $\exp\left[Nk\nu_1 \right]$

$$\langle W_{\underbrace{\square\square\cdots\square}_{k}} \rangle = \frac{1}{\mathcal{Z}} \int \prod_{i=1}^{N} \mathrm{d}\nu_i \exp\left[-\frac{N^2}{\beta} \sum_i \nu_i^2 + \frac{N}{2} \sum_{i\neq j} |\nu_i - \nu_j| + Nk\nu_1 \right]$$

$\Longrightarrow$ Saddle point configuration changes: $\nu_1 = \frac{\beta}{2}\left(1 + \frac{k}{N}\right)$



2. Matrix model

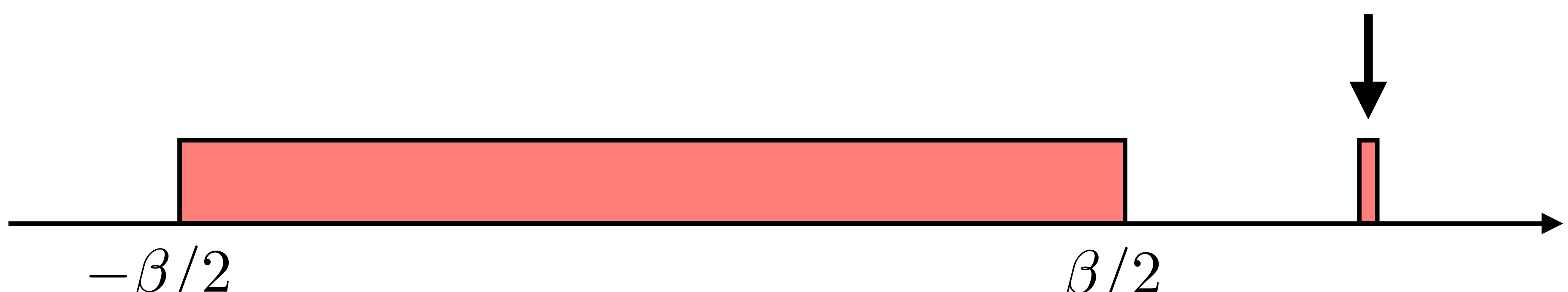
k–th symmetric rep.

$$\langle W_{\underbrace{\square\square\cdots\square}_{k}} \rangle \sim \exp\left[-\frac{N^2}{\beta}\sum_i \nu_i^2 + \frac{N}{2}\sum_{i\neq j}|\nu_i-\nu_j| + Nk\nu_1 \right]\Bigg|_{\text{saddle point}}$$

$$\sim \exp\left[-\frac{N^2}{\beta}\nu_1^2 + N\sum_{j=2}^{N}|\nu_1-\nu_j| + Nk\nu_1 + (\text{terms independent of } k) \right]\Bigg|_{\text{saddle point}}$$

$$\sim \exp\left[\frac{\beta N}{2} k \left(1 + \frac{k}{2N} \right) \right]$$

$$\nu_1 = \frac{\beta}{2}\left(1+\frac{k}{N}\right)$$



2. Matrix model

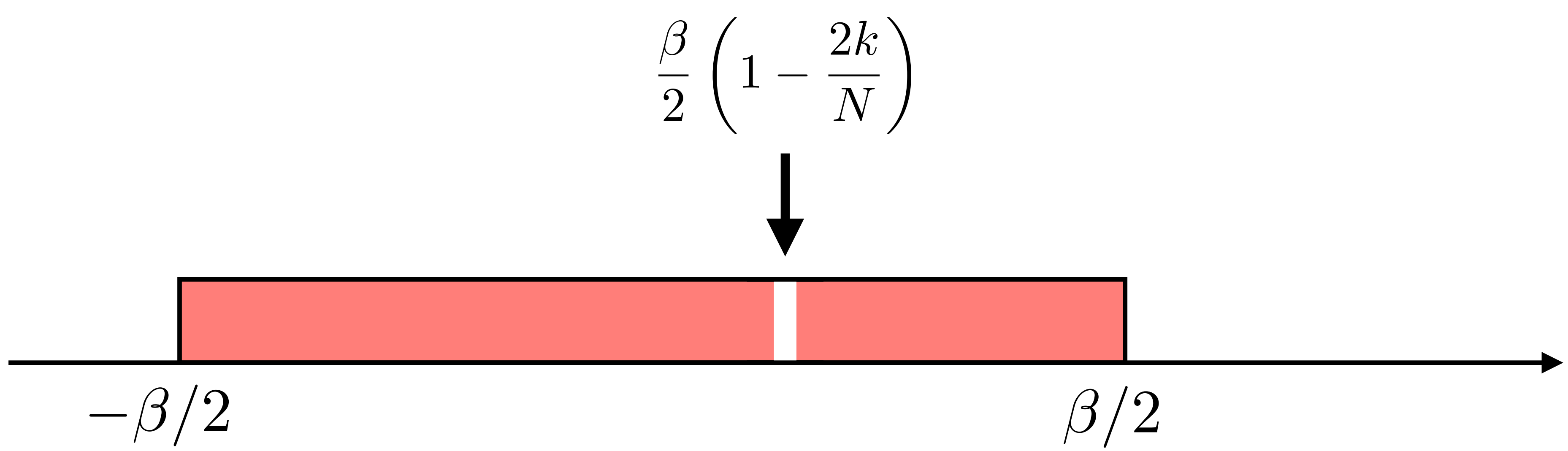
k–th anti–symmetric rep.

$$\mathrm{Tr}_{k\left\{\begin{smallmatrix}\square\\ \square\\ \vdots\\ \square\end{smallmatrix}\right.} e^{N\nu} = \sum_{1\le i_1<\cdots<i_k\le N} \exp\left[N\sum_{l=1}^{k}\nu_{i_l}\right]$$

$\Longrightarrow$ Largest contribution: $\exp\left[N(\nu_1+\cdots+\nu_k)\right]$

$\Longrightarrow$ Saddle point configuration does not change

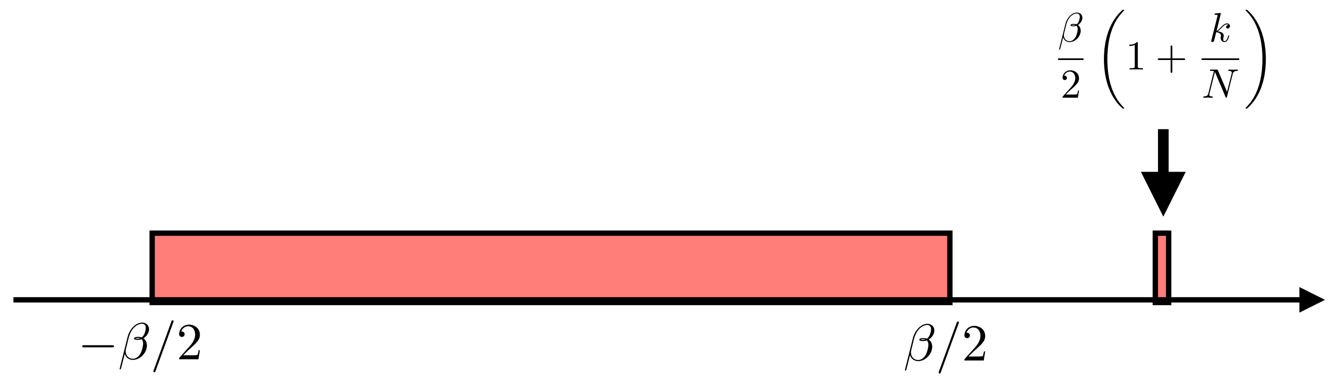
$$\langle W_{k\left\{\begin{smallmatrix}\square\\ \square\\ \vdots\\ \square\end{smallmatrix}\right.}\rangle \sim \exp\left[N(\nu_1+\cdots+\nu_k)\right]\Big|_{\text{saddle point}} \sim \exp\left[\frac{\beta N}{2}k\left(1-\frac{k}{N}\right)\right]$$



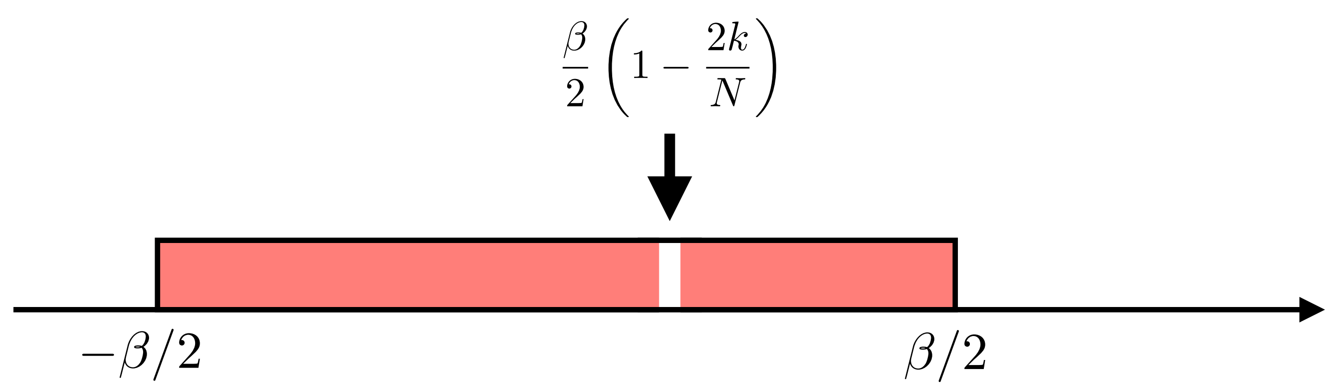
2. Matrix model

$$\beta = \frac{g_{\rm YM}^2}{2\pi r} = \frac{4\pi R_6}{r}, \quad r : \text{radius of } S^5$$

Symmetric : $$\langle W_{\underbrace{\square\square\cdots\square}_{k}} \rangle \sim \exp\left[\frac{\beta N}{2} k \left(1 + \frac{k}{2N}\right)\right]$$



Anti-symmetric : $$\langle W_{k\left\{\begin{smallmatrix}\square\\ \square\\ \vdots \\ \square\end{smallmatrix}\right.} \rangle \sim \exp\left[\frac{\beta N}{2} k \left(1 - \frac{k}{N}\right)\right]$$



Plan

1. Bubbling geometry

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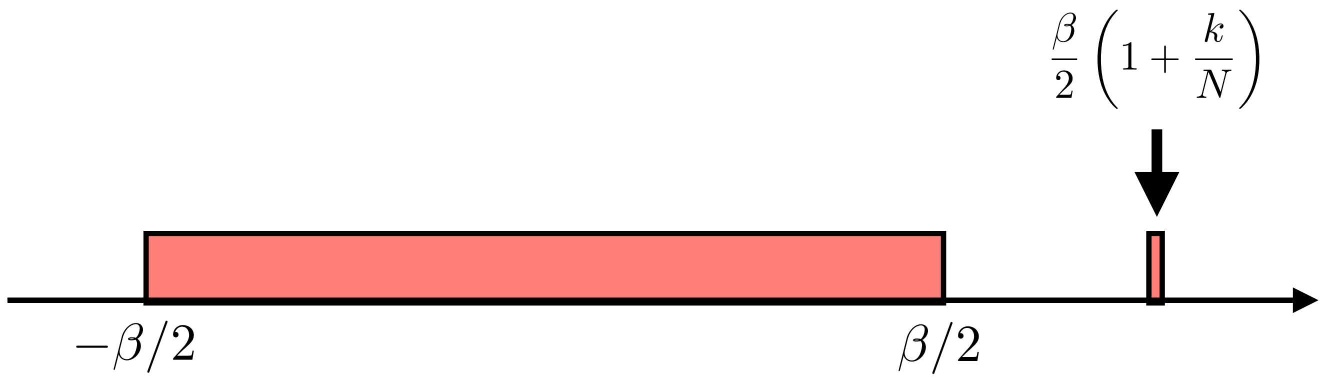
3. Wilson surface by M5-brane

4. Summary & Outlook

3. Wilson surface by M5-brane

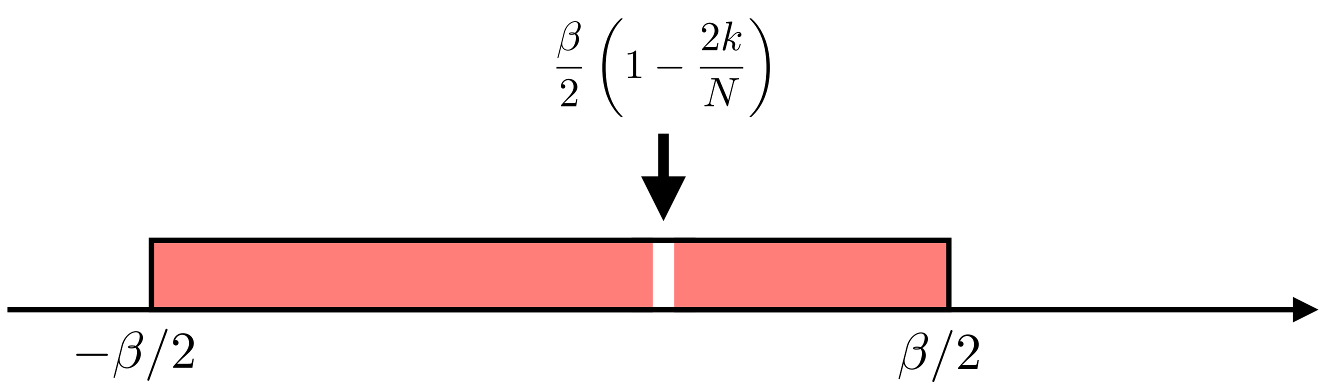
Symmetric :

$$\langle W_{\underbrace{\Box\Box\cdots\Box}_{k}} \rangle \sim \exp\left[\frac{\beta N}{2} k \left(1 + \frac{k}{2N}\right)\right]$$



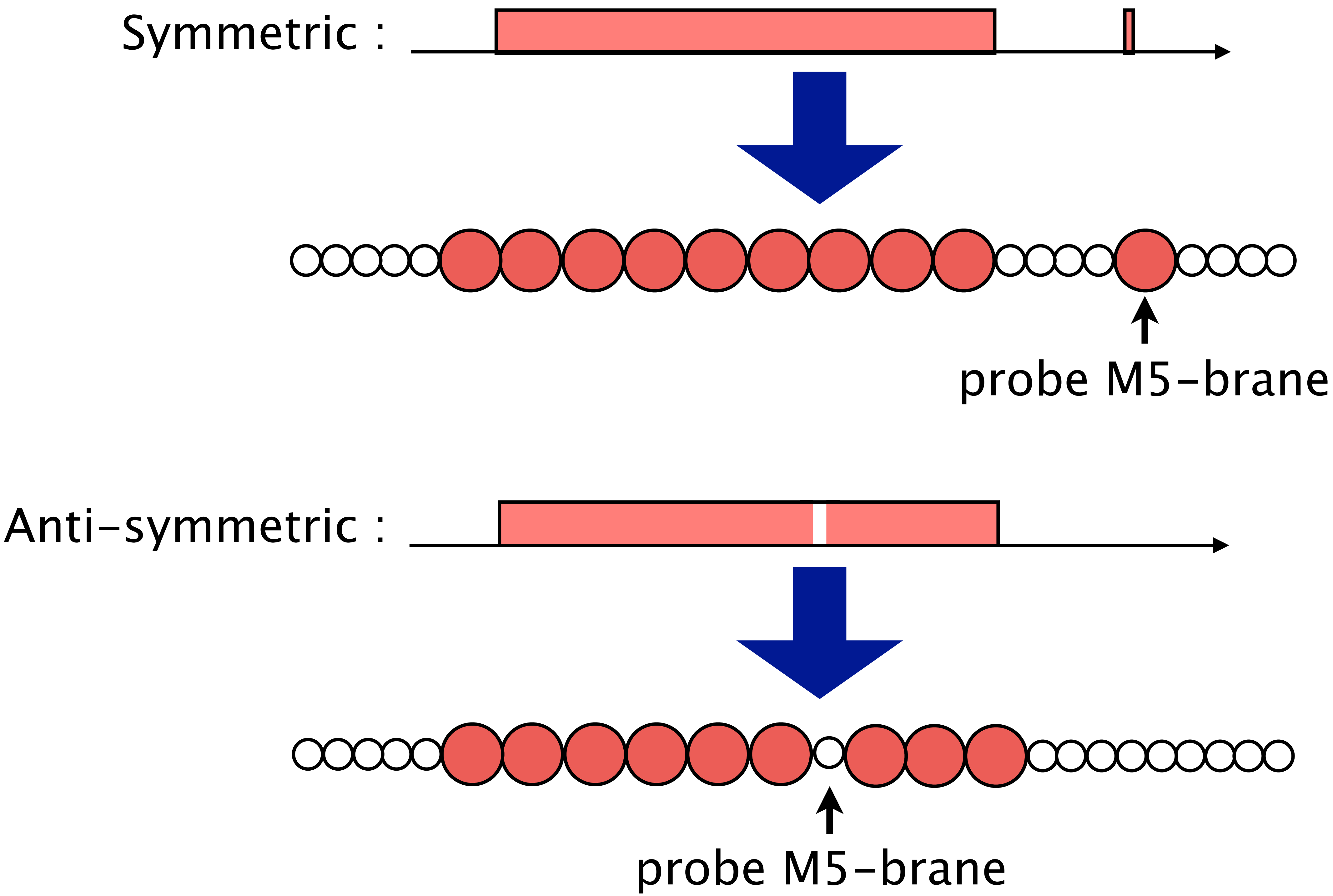
Anti-symmetric :

$$\langle W_{k\left\{\begin{smallmatrix}\Box\\ \Box\\ \vdots \\ \Box\end{smallmatrix}\right.} \rangle \sim \exp\left[\frac{\beta N}{2} k \left(1 - \frac{k}{N}\right)\right]$$

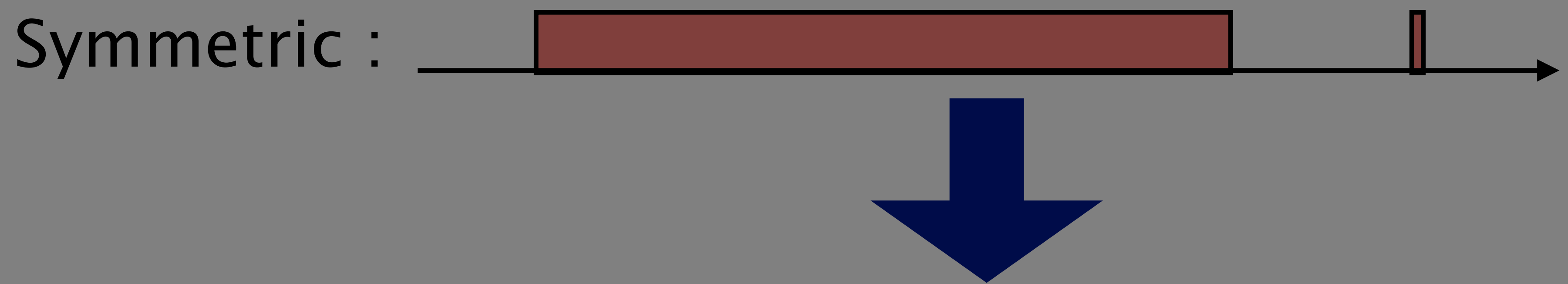


What are gravity duals to those Wilson surfaces?

3. Wilson surface by M5-brane

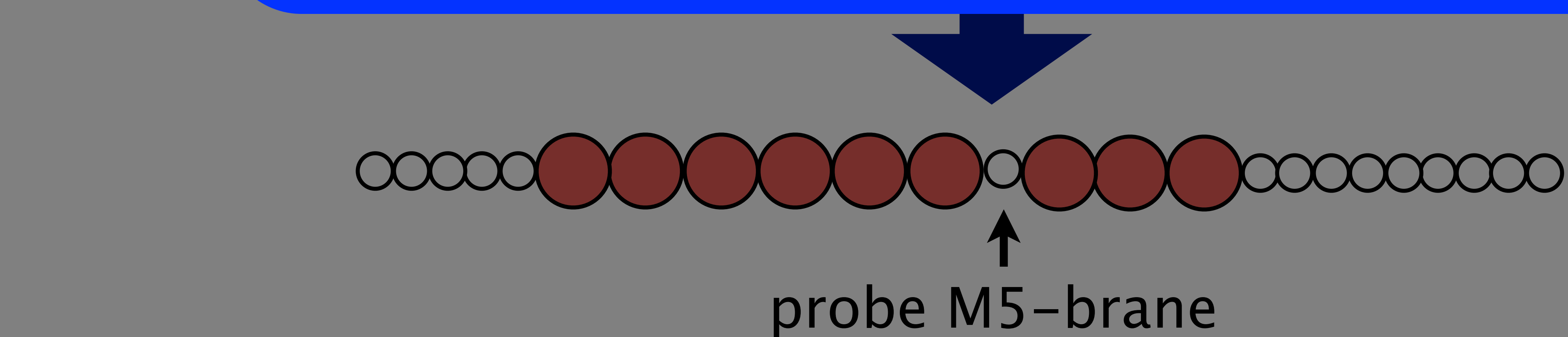


3. Wilson surface by M5-brane



○ AdS/CFT correspondence

$$\langle W_{\mathbf{R}} \rangle \sim \exp\left[-S_{\mathrm{M5}}\right]$$



3. Wilson surface by M5-brane

$$\langle W_{\mathbf{R}} \rangle \sim \exp\left[-S_{\mathrm{M5}}\right]$$

PST action [Pasti, Sorokin, Tonin '97] ← on a single M5-brane

$$S_{\mathrm{M5}} = T_5 \int d^6x \sqrt{-g} \left[\mathcal{L} + \frac{1}{4}\tilde{H}^{mn} H_{mn}\right] + T_5 \int \left(C_6 - \frac{1}{2} C_3 \wedge H_3\right)$$

$$\mathcal{L} = \sqrt{\det\left(\delta_m{}^n + i\tilde{H}_m{}^n\right)}$$

$$v_p = \frac{\partial_p a}{\sqrt{-g^{mn}\partial_m a \partial_n a}}$$

g_{mn} : induced metic

a : auxiliary field

$$T_5 = \frac{1}{(2\pi)^5 \ell_p^6}$$

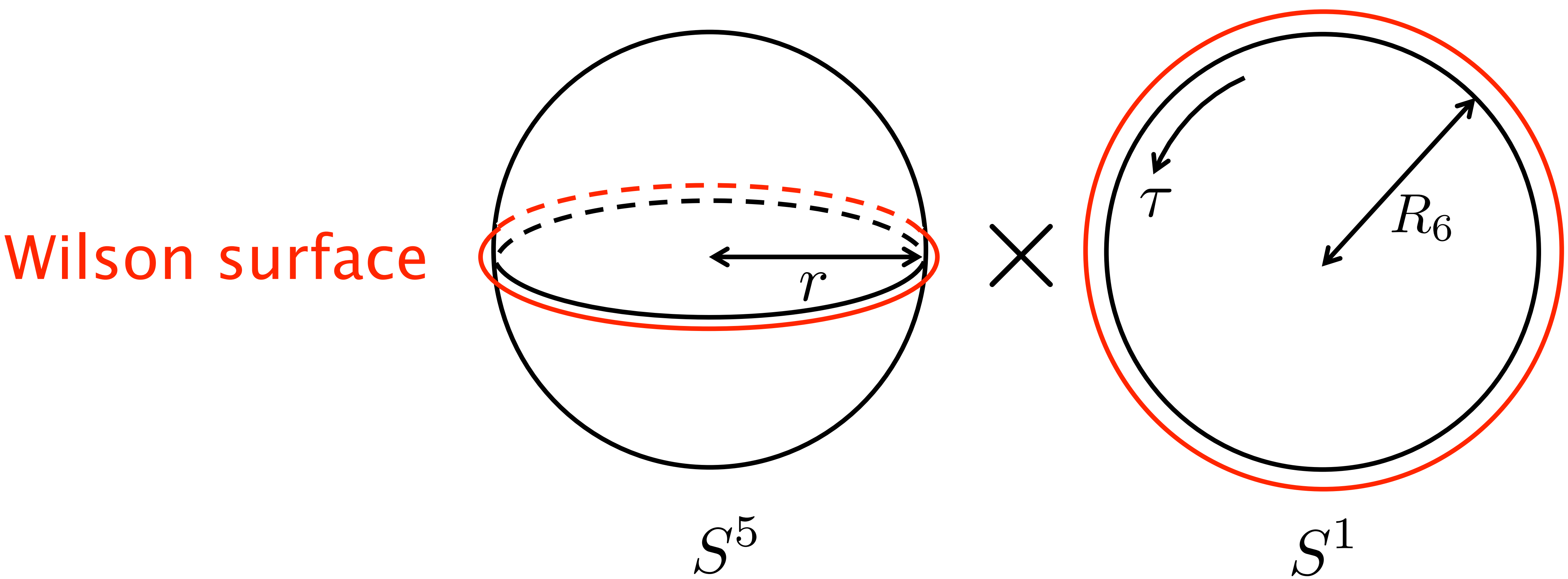
$$H_3 = dA_2 - C_3, \quad H_{mn} = H_{mnp} v^p, \quad \tilde{H}^{mn} = (*_6 H)^{mnp} v_p$$

3. Wilson surface by M5-brane

$\underline{AdS_7 \times S^4}$

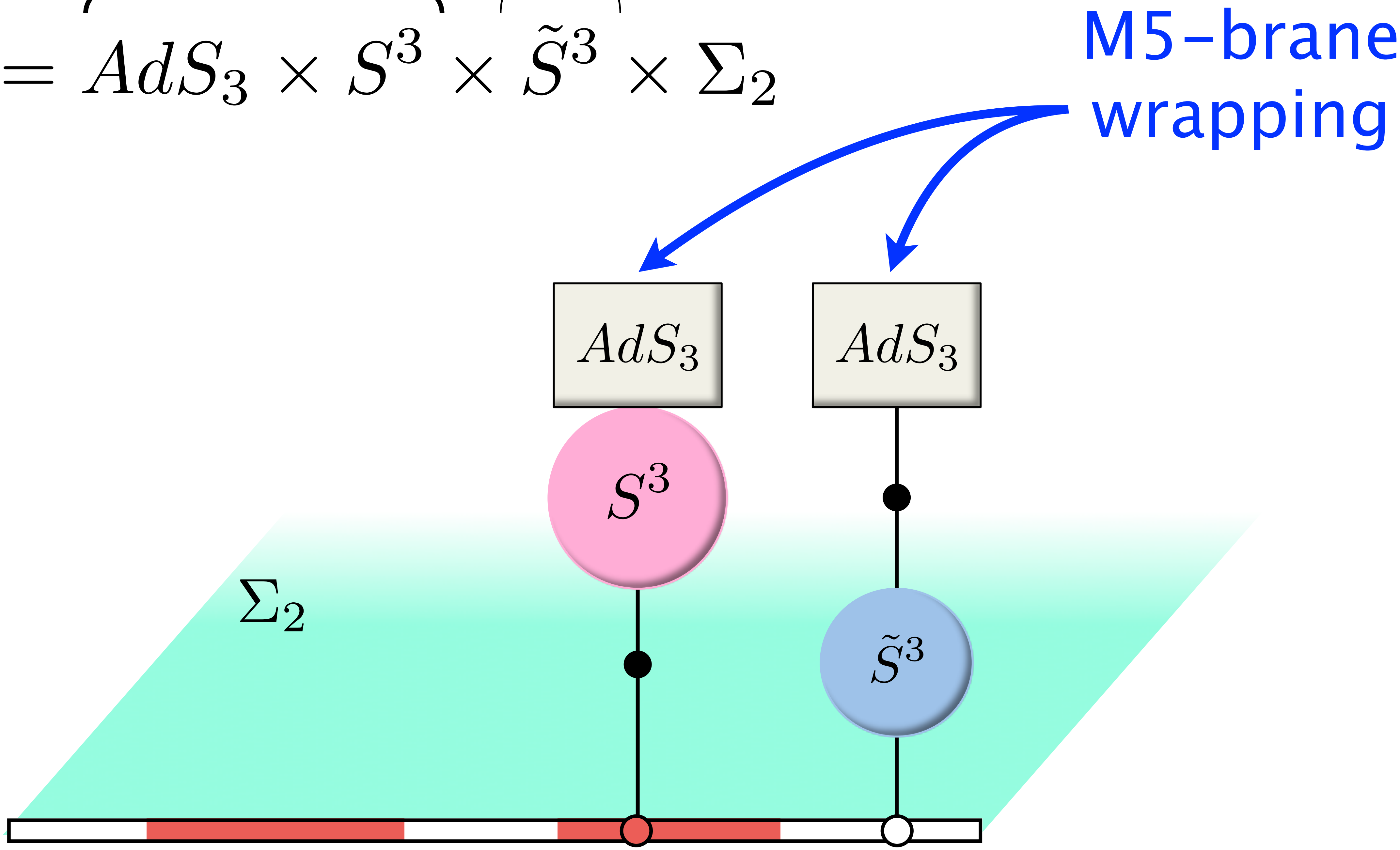
Metric: $$ds^2 = L^2\left\{\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho(d\chi^2 + \cos^2\chi d\phi^2 + \sin^2\chi d\Omega_3^2)\right\} + \frac{L^2}{4}\left(d\theta^2 + \sin^2\theta d\tilde{\Omega}_3^2\right)$$

Boundary = $S^5 \times S^1$ **Identification:** $\tau \sim \tau + 2\pi\frac{R_6}{r}$



3. Wilson surface by M5-brane

$$\mathcal{M}_{11} = \overbrace{AdS_3 \times S^3}^{AdS_7 \, \cup} \times \overbrace{\tilde{S}^3}^{S^4 \, \cup} \times \Sigma_2$$



3. Wilson surface by M5-brane

$$\underset{\displaystyle AdS_3}{\overset{AdS_7}{\cup}} \times S^3$$

We must add the local counter term $S_{\rm bdy}$ to regularize $S_{\rm M5}$.

To determine it, using the fact

Poincare coordinate $\Rightarrow S_{\rm M5} + S_{\rm bdy} = 0$

⬇

on-shell $S_{\rm M5} \propto$ (Volume on the boundary)

||

$S_{\rm bdy}$

3. Wilson surface by M5-brane

$$\overset{\displaystyle AdS_7}{\overset{\cup}{AdS_3 \times S^3}}$$

Global coordinate: $\mathrm{d}s^2_{AdS_3\times S^3} = L^2 \left\{\cosh^2 u(\cosh^2 w \mathrm{d}\tau + \mathrm{d}w^2 + \sinh^2 w \mathrm{d}\phi^2) + \mathrm{d}u^2 + \sinh^2 u \mathrm{d}\Omega_3^2\right\}$

Flux quantization: $\sinh u_k = \sqrt{\dfrac{k}{2N}}$ $\qquad k \in \mathbb{Z}_{\geq 0}$

$$\Rightarrow S_{\mathrm{M5}} = \frac{4\pi R_6}{r} Nk\left(1+\frac{k}{2N}\right)\sinh^2 w_0 \qquad w_0 : \text{cut-off}$$

$$S_{\mathrm{bdy}} \propto \sinh w_0 \cosh w_0 \qquad \beta = \frac{4\pi R_6}{r}$$

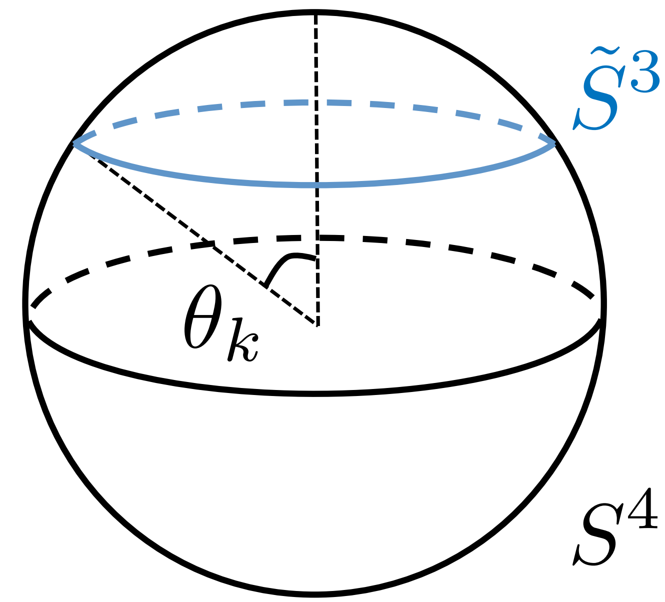
$$\Rightarrow S^{\mathrm{reg}}_{\mathrm{M5}} = -\frac{\beta N}{2} k\left(1+\frac{k}{2N}\right) \quad \Longleftrightarrow \quad \langle W_{\underbrace{\Box\Box\cdots\Box}_{k}}\rangle$$

agree

3. Wilson surface by M5-brane

$$\begin{matrix} AdS_7 & & S^4 \\ \cup & & \cup \\ AdS_3 & \times & \tilde{S}^3 \end{matrix}$$

Global coordinate



Flux quantization: $\cos\theta_k = 1 - \dfrac{2k}{N}$

$$\Rightarrow \; S_{\mathrm{M5}} = \frac{4\pi R_6}{r} N k \left(1 - \frac{k}{N}\right) \sinh^2 \rho_0$$

ρ_0 : cut-off

$$S_{\mathrm{bdy}} \propto \sinh\rho_0 \cosh\rho_0 \qquad \beta = \frac{4\pi R_6}{r}$$

$$\Rightarrow \; S^{\mathrm{reg}}_{\mathrm{M5}} = -\frac{\beta N}{2} k \left(1 - \frac{k}{N}\right)$$

agree ⟷ $\langle W_{k\left\{\begin{smallmatrix}\square\\ \vdots\\ \square\end{smallmatrix}\right.} \rangle$

Plan

1. Bubbling geometry

2. Matrix model

3. Wilson surface by M5-brane

4. Summary & Outlook

4. Summary & Outlook

M2/M5-brane		Wilson surface
M2-brane on $\underset{AdS_7}{\underset{\cap}{AdS_3}}$	$\Longleftrightarrow$	Fundamental rep. $\exp\left[\frac{\beta N}{2}\right]$
[Minahan, Nedelin, Zabzine '13]		
M5-brane on $\underset{AdS_7}{\underset{\cap}{AdS_3}} \times S^3$	$\Longleftrightarrow$ New!	k-th symmetric rep. $\exp\left[\frac{\beta N}{2}k\left(1-\frac{k}{N}\right)\right]$
M5-brane on $\underset{AdS_7}{\underset{\cap}{AdS_3}} \times \underset{S^4}{\underset{\cap}{\tilde{S}^3}}$	$\Longleftrightarrow$ New!	k-th anti-symmetric rep. $\exp\left[\frac{\beta N}{2}k\left(1+\frac{k}{2N}\right)\right]$

Not the 't Hooft limit ⇒ not stringy, but M-theoretic

4. Summary & Outlook (in progress)

1. Calculating by using bubbling geometry

2. Deriving the matrix model from gravity side