Conformal field theories in higher dimensions from string theory

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based on
1506.05462 with A. Passias, A. Rota;
1502.06620 with F. Apruzzi, M. Fazzi, A. Passias; 1502.06622 with A. Rota
1309.2949 with F. Apruzzi, M. Fazzi, D. Rosa; 1404.0711 with D. Gaiotto
1407.6359 with M. del Zotto, J.Heckman, C.Vafa

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Introduction

Several reasons to be interested in CFTs in $d > 4$.

- Mothers of interesting theories in $d \leq 4$

[Gaiotto '09, Alday, Gaiotto, Tachikawa '09...]

- Harder to define.

e.g. $\mathrm{Tr}(F_{\mu\nu})^2$ relevant in $d > 4$. Similar problem to $\sqrt{-g}R$ in $d > 2$

- They might allow us to get a handle on the elusive (2,0) theory living on M5-brane stacks

crucial features:

- number of degrees of freedom $\sim N^3$
- 'chiral tensors': $b_{\mu\nu}$ such that $h_{\mu\nu\rho}$ is self-dual

Plan

- Classification of AdS_7 solutions in type II

  - infinitely many; analytical
  - They generate anal. infinitely many AdS_5 and AdS_4 solutions

$$\begin{tikzcd} & \mathrm{AdS}_7 \arrow[dl, shift left] \arrow[dr, shift left] & \\ \mathrm{AdS}_4 \times \Sigma_3 \arrow[ur, shift left] \arrow[rr, shift left] & & \mathrm{AdS}_5 \times \Sigma_2 \arrow[ul, shift left] \arrow[ll, shift left] \end{tikzcd}$$

- Their CFT_6 duals: NS5-D6-D8 brane constructions
  - 'daughters' of the (2,0) theory
  - linear quiver (away from conformal point)
  - a similar classification from F-theory; "fractional M5's"

AdS7 classification

- AdS$_7 \times M_4$ in 11d sugra:

cone over M_4 should have reduced holonomy $\Rightarrow$ $M_4 = S^4/\Gamma_{\text{ADE}}$

- AdS$_7 \times M_3$ in type II: 'pure spinor' methods [Apruzzi, Fazzi, Rosa, AT '13]

originally applied to AdS$_4 \times M_6$ in type II [Graña, Minasian, Petrini, AT '05]

later extended to any 10d solution in type II [AT '11]

we will later see a similar classification for AdS$_5 \times M_5$ in IIA [Apruzzi, Fazzi, Passias, AT '15]

• IIB: no solutions! [but: see later about F-theory]

• IIA: internal M_3 is locally S^2-fibration over interval

$\boxed{\text{\{no Ansatz necessary\}}}$

$$ds^2 \sim e^{2A(r)} ds^2_{\mathrm{AdS}_7} + dr^2 + v^2(r) ds^2_{S^2}$$

This S^2 realizes the SU(2) R-symmetry of a $(1,0)$ 6d theory.

Fluxes: $F_0, F_2 \sim \mathrm{vol}_{S^2}, H \sim dr \wedge \mathrm{vol}_{S^2}$

$A(r)$, $\phi(r)$, $v(r)$ determined by ODEs

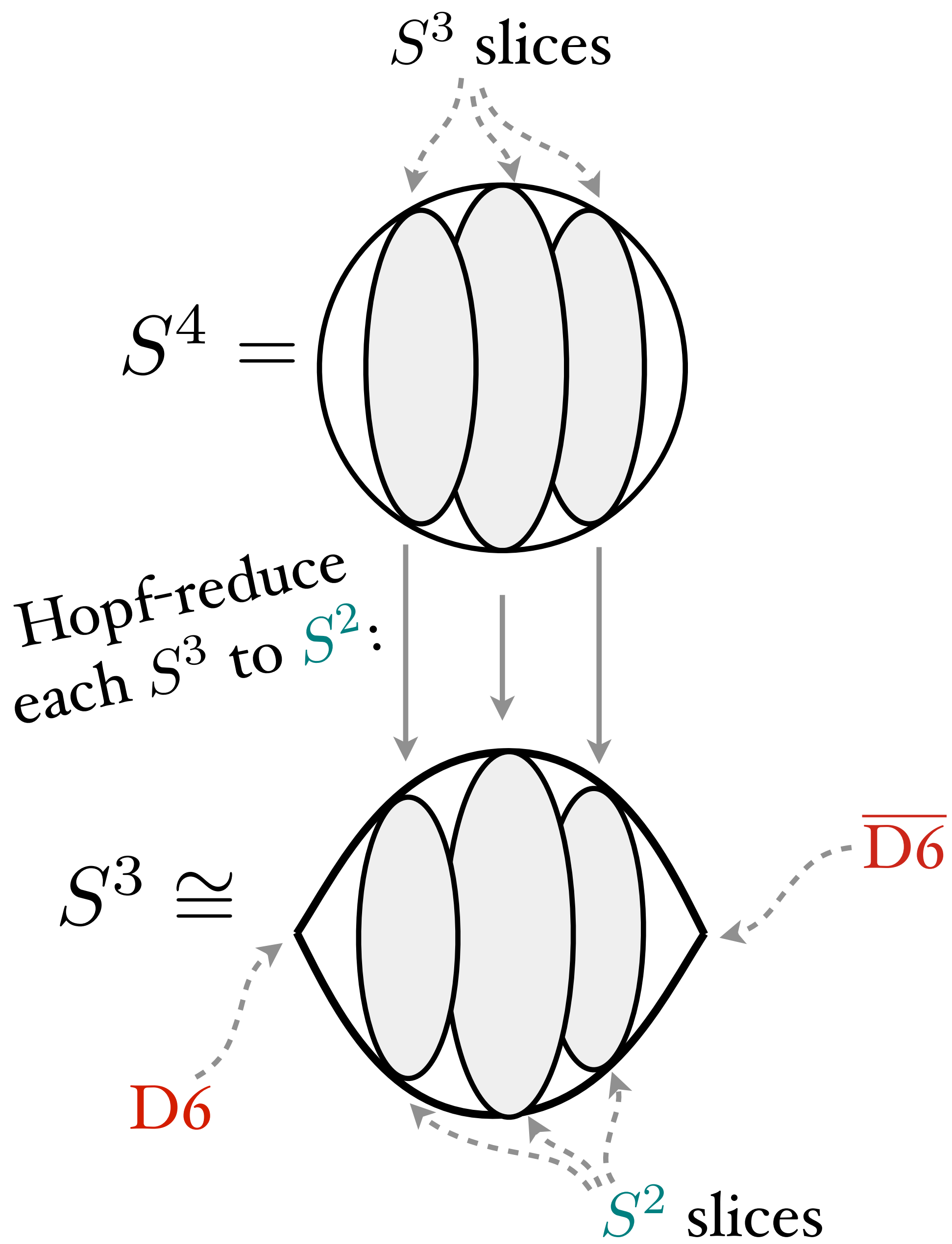
solved at first numerically [Apruzzi, Fazzi, Rosa, AT '13]

then analytically with the help of AdS4 and AdS5

[Rota, AT '15] [Apruzzi, Fazzi, Passias, AT '15]

- Warm-up: $F_0 = 0$

We can reduce
$\mathrm{AdS}_7 \times S^4$ to IIA:



- $F_0 \neq 0$: many new solutions

we can make
one of the poles regular:

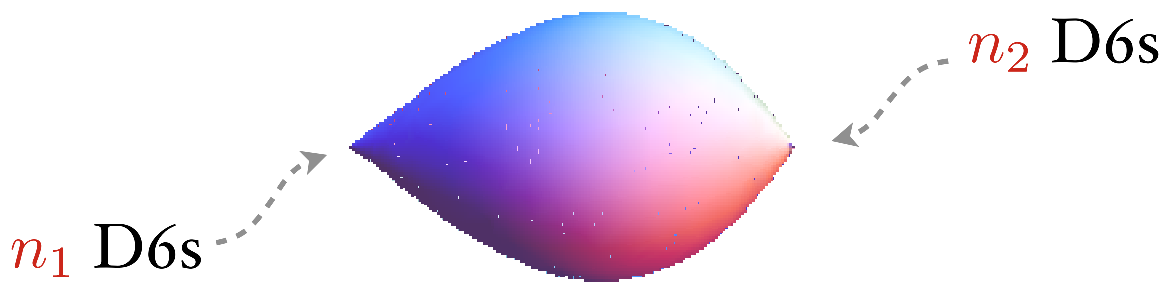
$$ds^2_{M_3} = \frac{n_{\rm D6}}{F_0}\left(\frac{dy^2}{4(1-y)\sqrt{y+2}} + \frac{1}{3}\frac{(1-y)(y+2)^{3/2}}{8-4y-y^2}ds^2_{S^2}\right) .$$

reg. point

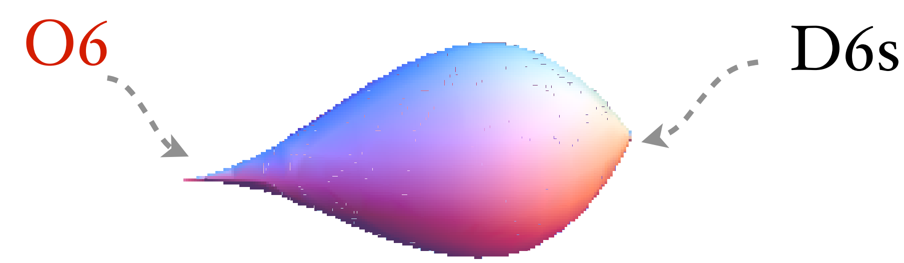
D6

local solutions also in [Blåbäck, Danielsson, Junghans, Van Riet, Wrase, Zagermann '11]
susy-breaking? in [Junghans, Schmidt, Zagermann '14]

more generally we can have
two unequal D6 stacks



or also an O6 and a D6 stack



these solutions are also analytic, but a bit more complicated.

If you're curious about the analytic expressions:

- All is determined by a single function $\beta(y)$ where $\left(\frac{y^2\beta}{\beta'^2}\right)' = \frac{F_0}{72}$

$$ds^2 = \frac{4}{9}\sqrt{-\frac{\beta'}{y}}\left[ds^2_{\mathrm{AdS}_7} - \frac{1}{16}\frac{\beta'}{y\beta}dy^2 + \frac{\beta/4}{4\beta - y\beta'}ds^2_{S^2}\right]$$

[it's easy to solve]

- β has single zero $\Rightarrow$ regular point; double zero $\Rightarrow$ D6 stack

examples:

$F_0 = 0$, two D6 stacks $\beta \propto (y^2 - y_0^2)^2$

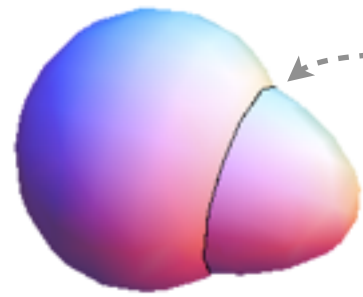
$F_0 \neq 0$, one D6 stack $\beta \propto (y - y_0)(y + 2y_0)^2$

$F_0 \neq 0$, most general: $\beta \propto \left(\sqrt{\hat{y}} - 6\right)^2 \left(\hat{y} + 6\sqrt{\hat{y}} + 6b_2 - 72\right)^2$

$$\hat{y} \equiv 2b_2\left(\frac{y}{y_0} - 1\right) + 36$$

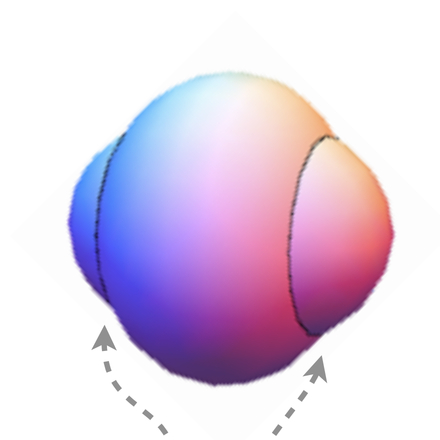
we can also include D8’s:

D8–D6 stack



actually, ‘magnetized’ D8’s
||
D8–D6 bound states

metric: gluing of two pieces of earlier metric



intuitively: D8’s don’t slip off because of electric attraction

stacks with opposite D6 charge

metric: gluing of two pieces of metric in prev. slide
+ central region from two slides ago

Generalization:

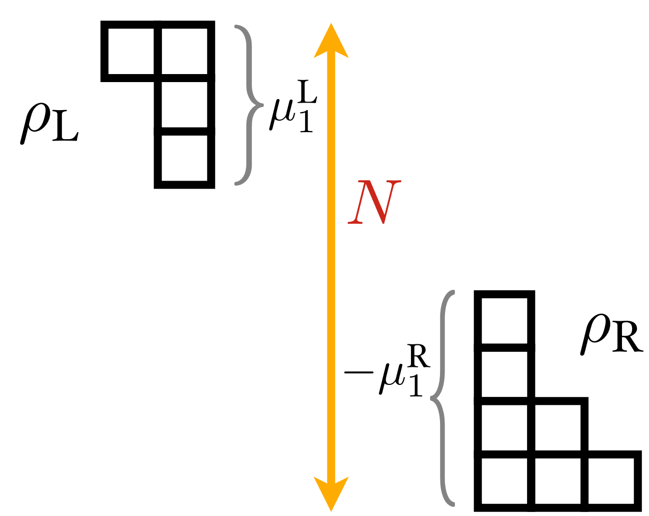
[Apruzzi, Fazzi, Rosa, AT '13;
Gaiotto, AT '14]

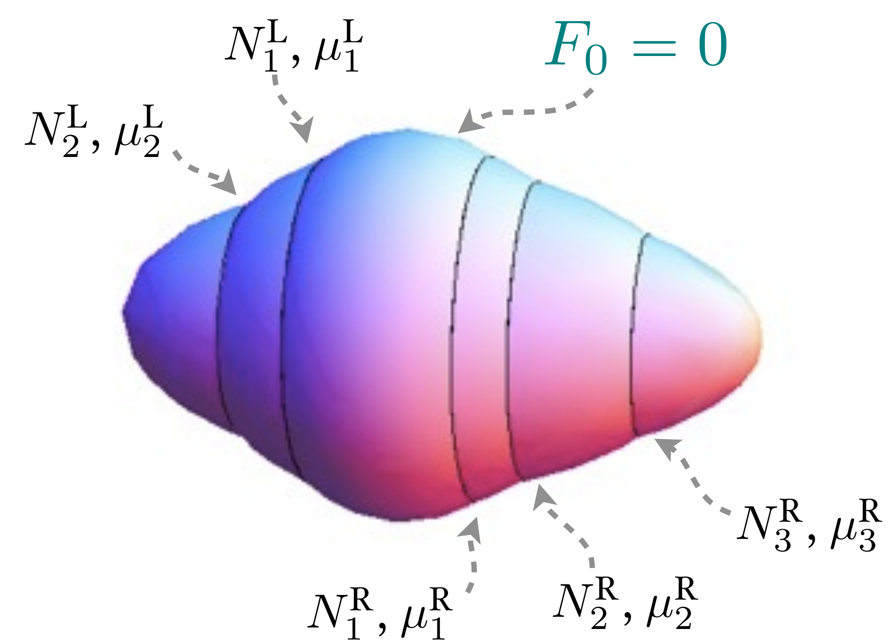
- numbers N_i of D8's, and their D6 charges μ_i
- flux integer $N \equiv \frac{1}{4\pi^2}\int H$

subject to constraints:

- μ_i positive and growing for $F_0 > 0$
  negative and growing for $F_0 < 0$ → Young diagrams ρ_L, ρ_R
- $N \geq |\mu_1^L| + |\mu_1^R|$

bordering
$F_0 = 0$ region.





7d effective description.

To any of our solutions

$$e^{2A}ds^2_{\mathrm{AdS}_7}+dr^2+v^2ds^2_{S^2}$$

[Apruzzi, Fazzi Passias, AT '15]

$$\frac{3}{4}e^{2A}(ds^2_{\mathrm{AdS}_5\times\Sigma_2})+dr^2+\frac{v^2}{1-4v^2}e^{2A}ds^2_{S^2}$$

dual to $\mathrm{CFT}_4\cong\mathrm{CFT}_6/\Sigma_2$

[twisted compactification]

[Rota, AT '15]

$$\frac{5}{8}e^{2A}(ds^2_{\mathrm{AdS}_4\times\Sigma_3})+dr^2+\frac{v^2}{1-6v^2}e^{2A}ds^2_{S^2}$$

dual to $\mathrm{CFT}_3\cong\mathrm{CFT}_6/\Sigma_3$

[twisted compactification]

we came to suspect that there was a more general story:

To any of our solutions

$$e^{2A} ds^2_{\mathrm{AdS}_7} + dr^2 + v^2 ds^2_{S^2}$$

$$e^{2A} ds^2_7 + dr^2 + \frac{v^2}{1+16(X^5-1)v^2} e^{2A} ds^2_{S^2}$$

this is in fact an Ansatz for a consistent truncation!

For any AdS7 solution in IIA there is a consistent truncation to 'minimal gauged 7d sugra'

fields: $g^{(7)}_{\mu\nu}$, A^i_μ, X

[Passias, Rota, AT '15]

One can use it to establish

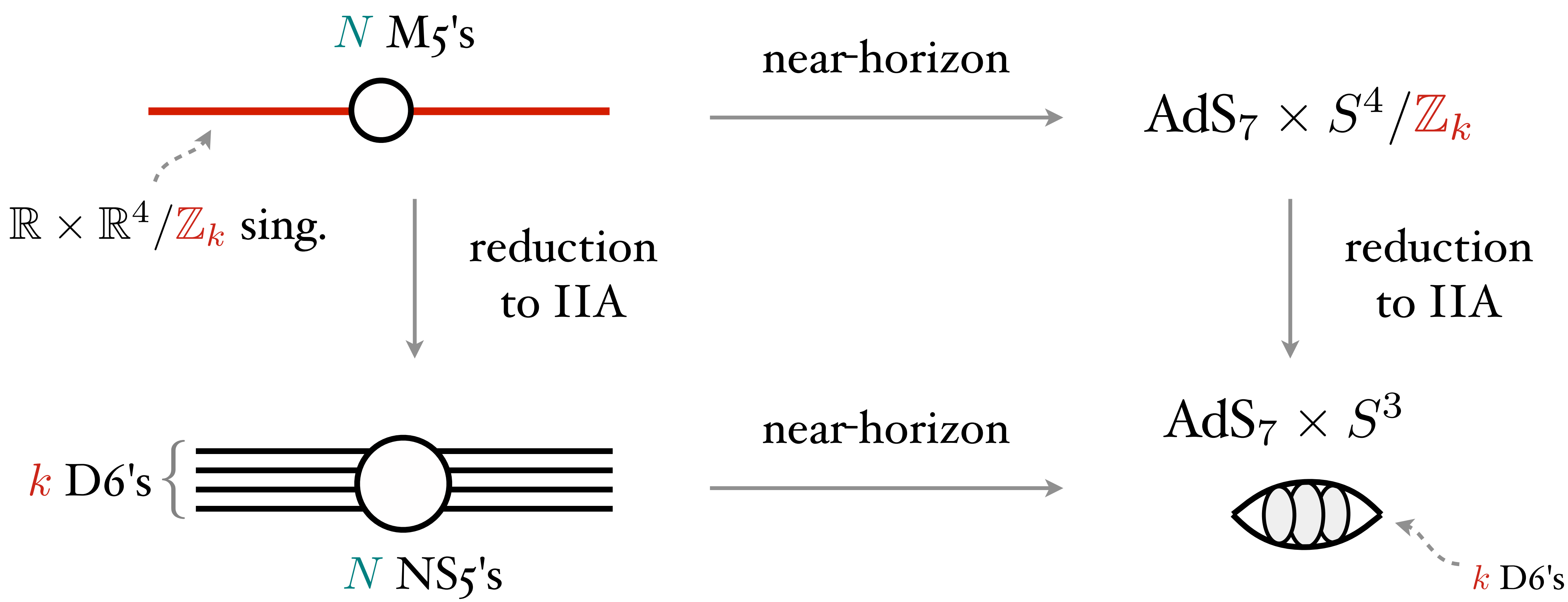
- RG flows from AdS$_7$ to AdS$_5 \times \Sigma_2$ and AdS$_4 \times \Sigma_3$
- AdS$_3$ to AdS$_3 \times \Sigma_4$ solutions
- non-susy AdS$_7$ solution

CFT$_6$ duals

[Gaiotto, AT '14]

Often one finds a CFT dual using a brane configuration.

For the $F_0 = 0$ solution this can be done:

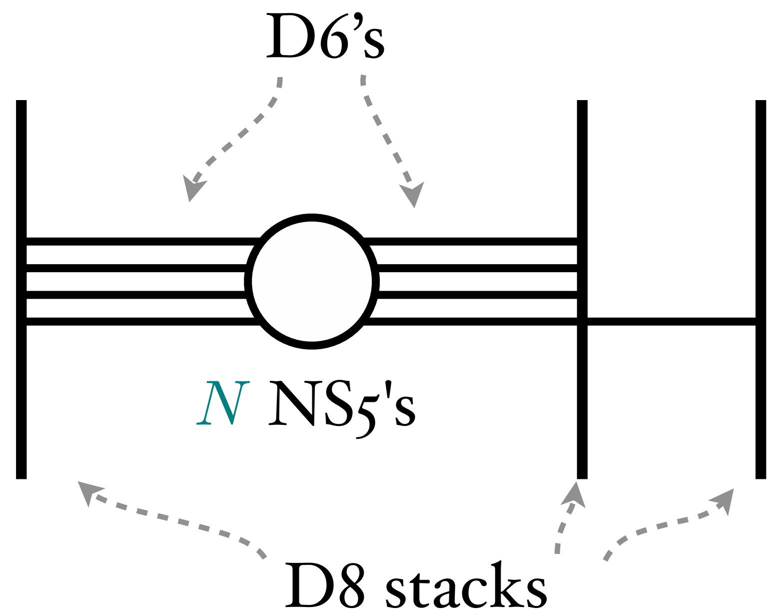


T_k^N: CFT$_6$ with

- $(1,0)$ supersymmetry
- $N^3 k^2$ degrees of freedom
- $\mathrm{SU}(k) \times \mathrm{SU}(k)$ flavor symmetry

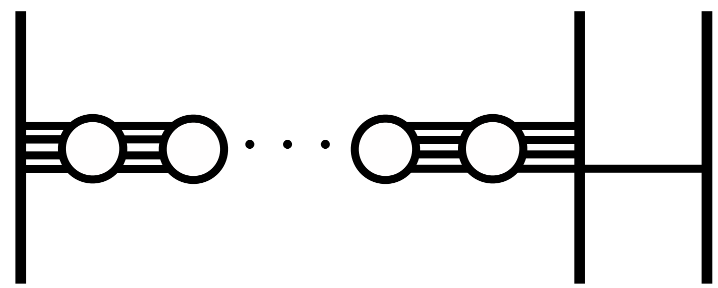
To include $F_0 \neq 0$, we should introduce D8-branes

for example, configurations like



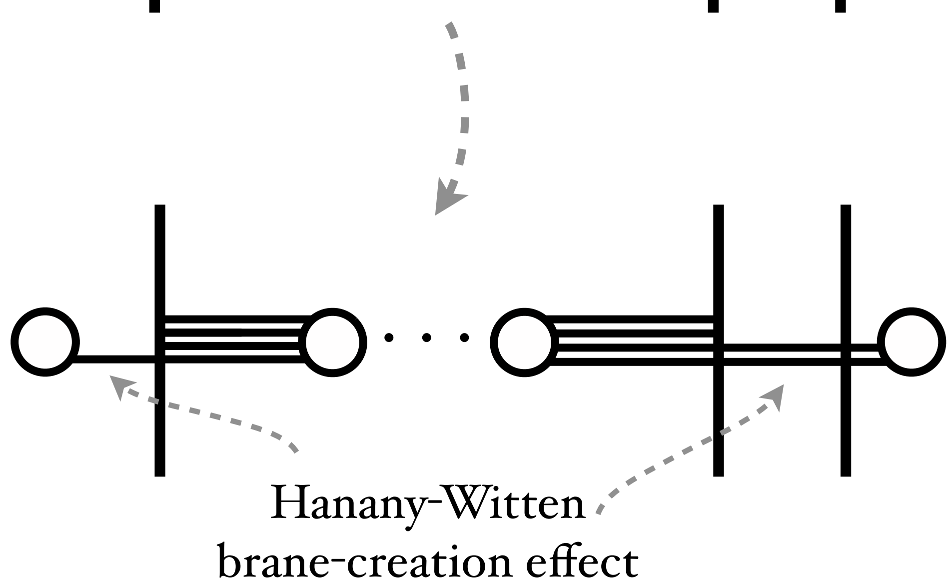
the field theory is still non-Lagrangian

However, we can separate the NS5's



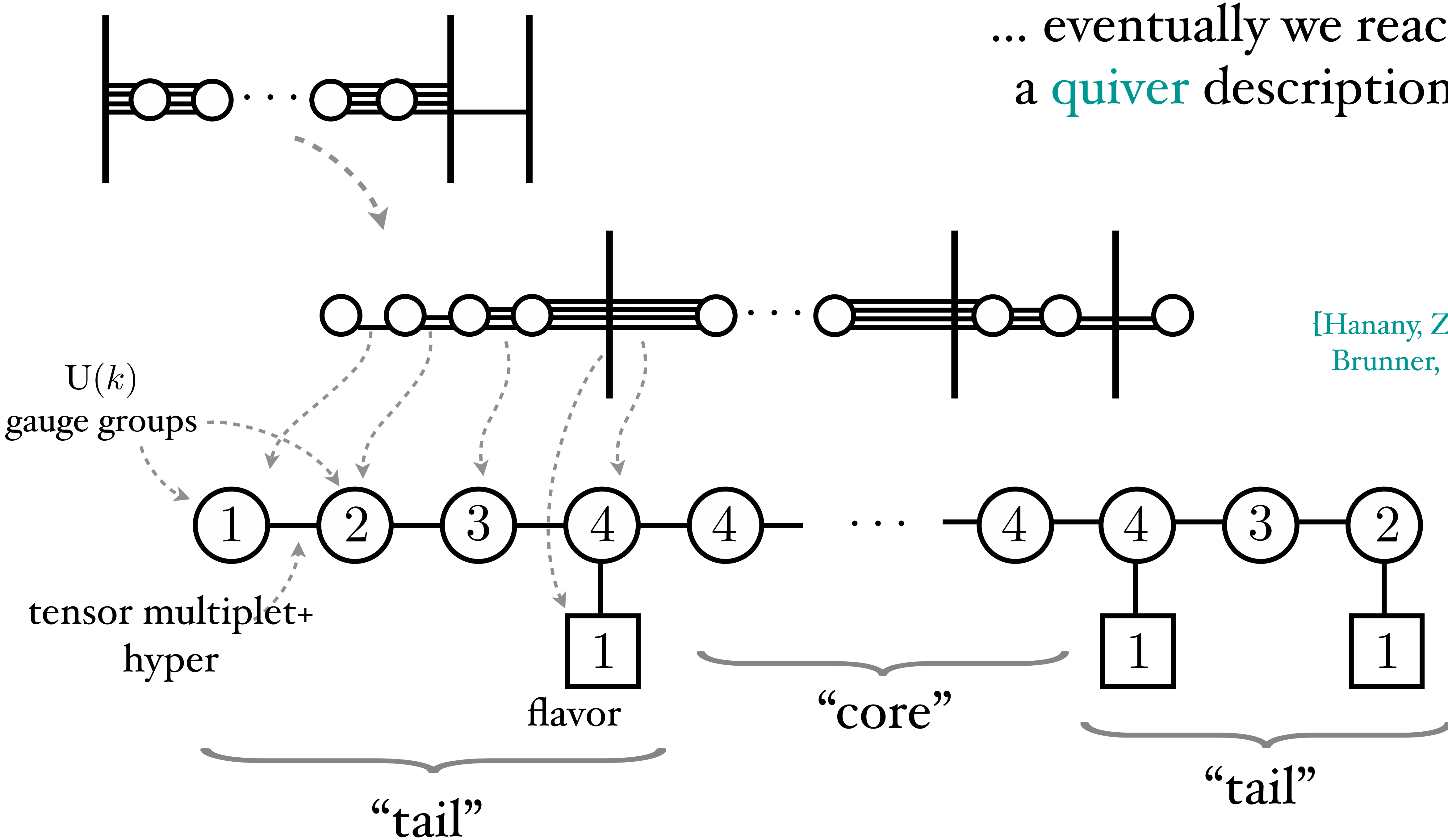
[but the conformal point is when the NS5's coincide!]

if we pull the D8's in...



... eventually we reach
a quiver description

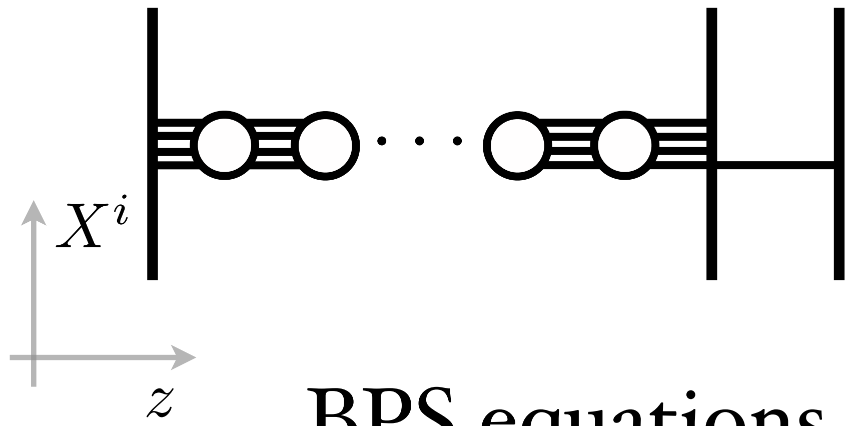
[Hanany, Zaffaroni '97;
Brunner, Karch '97]



scalars in the tensor multiplets
couple to vectors:

$$(\phi_{i+1} - \phi_i)\mathrm{Tr}(F^2_{\mu\nu})_i \sim \frac{1}{g^2_{\mathrm{YM}}}$$

conformal point:
coincident ϕ_i



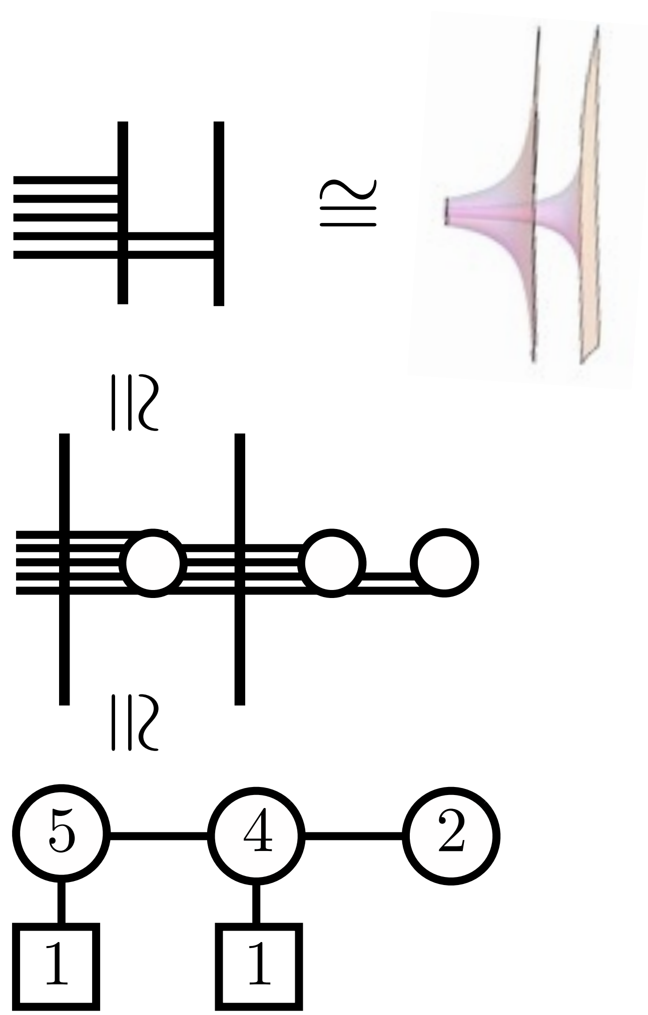
we can also study these theories by looking at their moduli spaces

Adapting methods developed in [Gaiotto, Witten '08] for 3d theories

BPS equations on D6:
Nahm equations

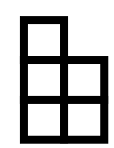
$$\partial_z X^1 = [X^2, X^3] \text{ etc.}$$

D8's can be thought of as a certain boundary condition:
Nahm poles for the D6's.



$$X^i \sim \frac{t_i}{z} \dashrightarrow \text{su}(2) \text{ subalgebra of su}(k)$$

$\cong$

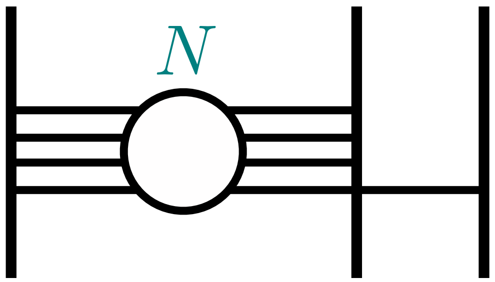


Young diagram

$\cong$

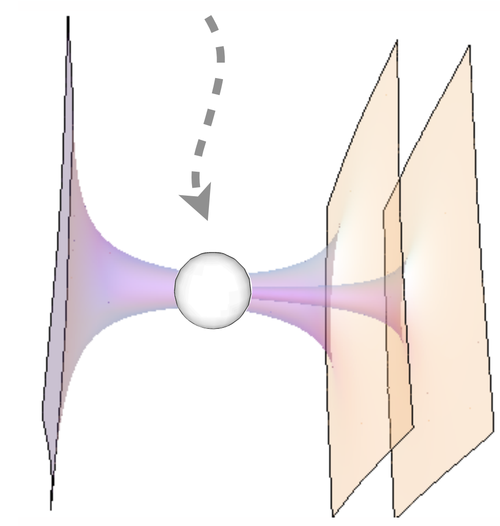
quiver tail

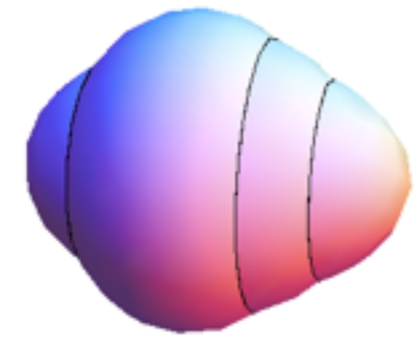
Conjecture:



$\cong$

near-horizon
on the NS5 stack...





...and the funnels become
our D8–D6 bound states!

More precisely:

D6's ending on a D8 $\longleftrightarrow$ $\mu \equiv$ D6 charge of the D8

N = # NS5's $\longleftrightarrow$ flux integer $\int_{M_3} H$

One-to-one correspondence
with AdS7 solutions!

More quantitative checks?

- AdS/CFT: (# deg. freedom) $\cong \mathrm{vol}(M_3)$

we can compare it with the R-symmetry anomaly in field theory

cancel gauge anomalies using GS mechanism;
then compute SU(2) R-symmetry anomaly

[Intriligator '15; Ohmori, Shimizu, Tachikawa, Yonekura '15]

One example:

simple case:
$$\rho_{\mathrm{L}} = \rho_{\mathrm{R}} = \left.\begin{array}{c}\square\\\square\\\vdots\\\square\end{array}\right\}\mu$$

$$\frac{1}{12}\left(N^3 - 4N\mu^2 + \frac{16}{5}\mu^3\right)$$

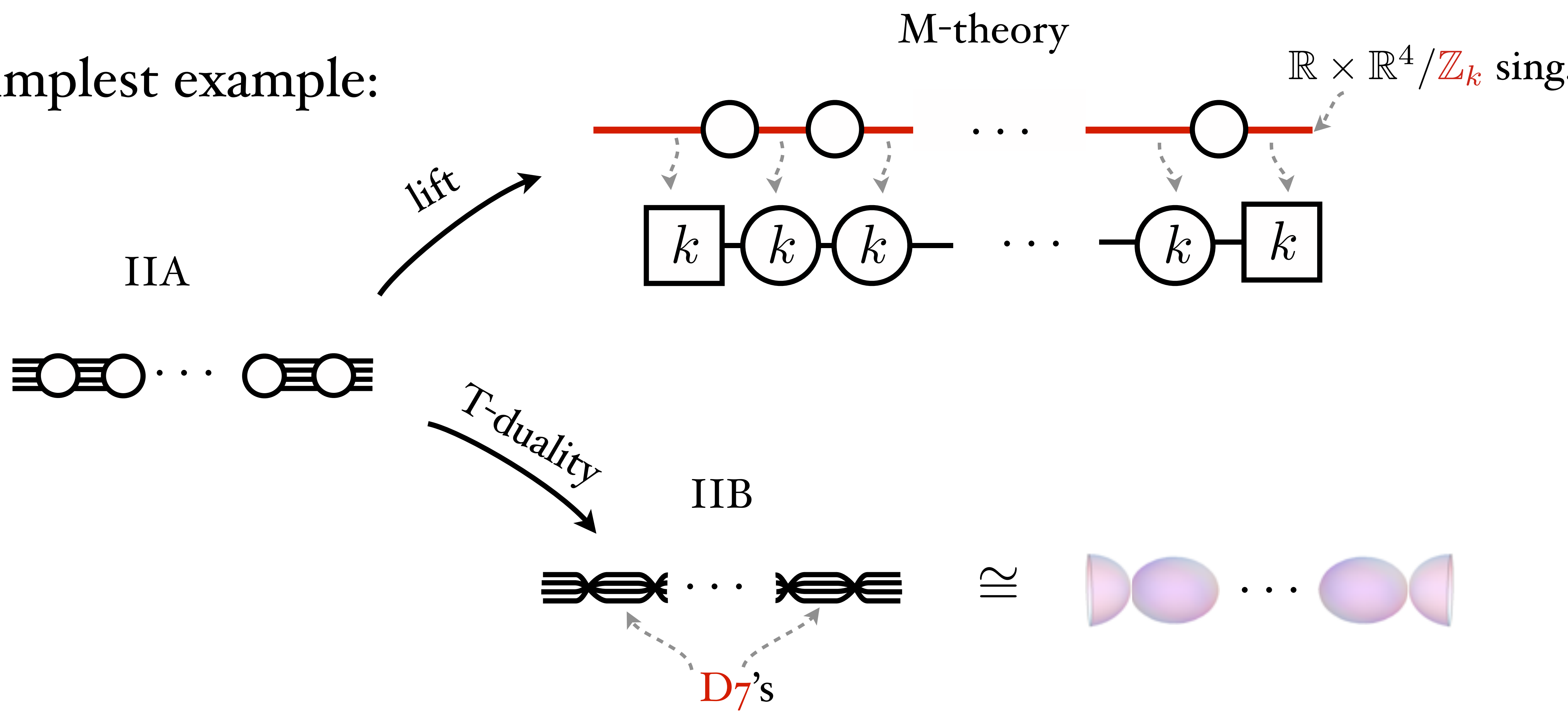
from both computations!

[work in progress with S. Cremonesi]

More general CFT6 from F-theory

So far we have seen chains of $SU(N)$ gauge groups

simplest example:



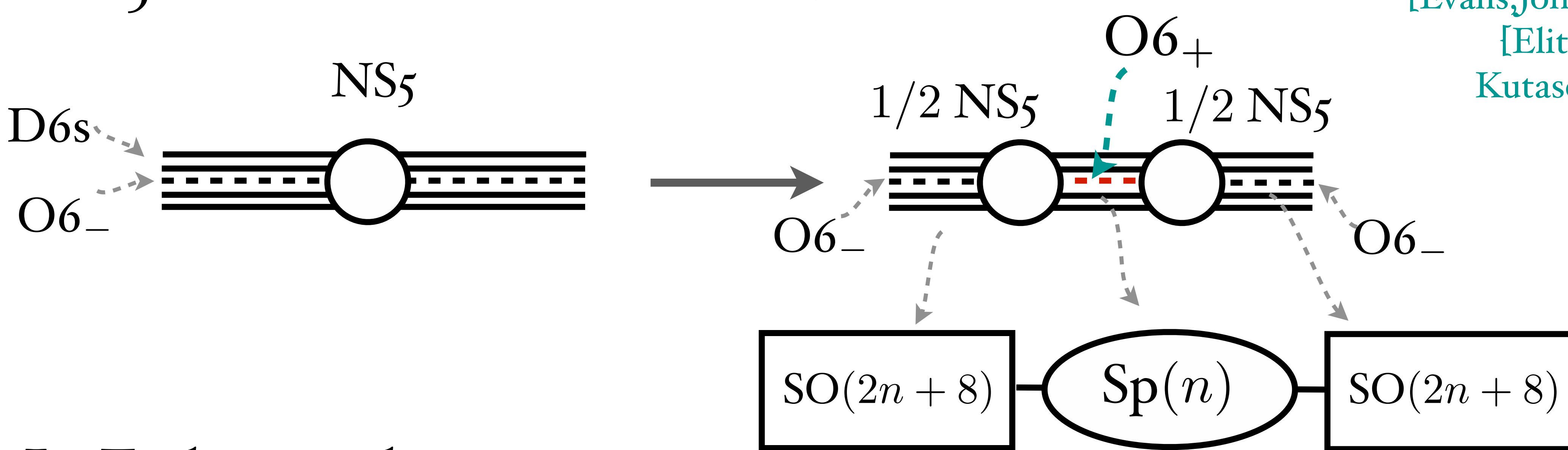
- F-theory allows to include more general gauge groups
- The D8's should be dual in F-theory to an object called "T-brane"

[del Zotto, Heckman, AT, Vafa '14]

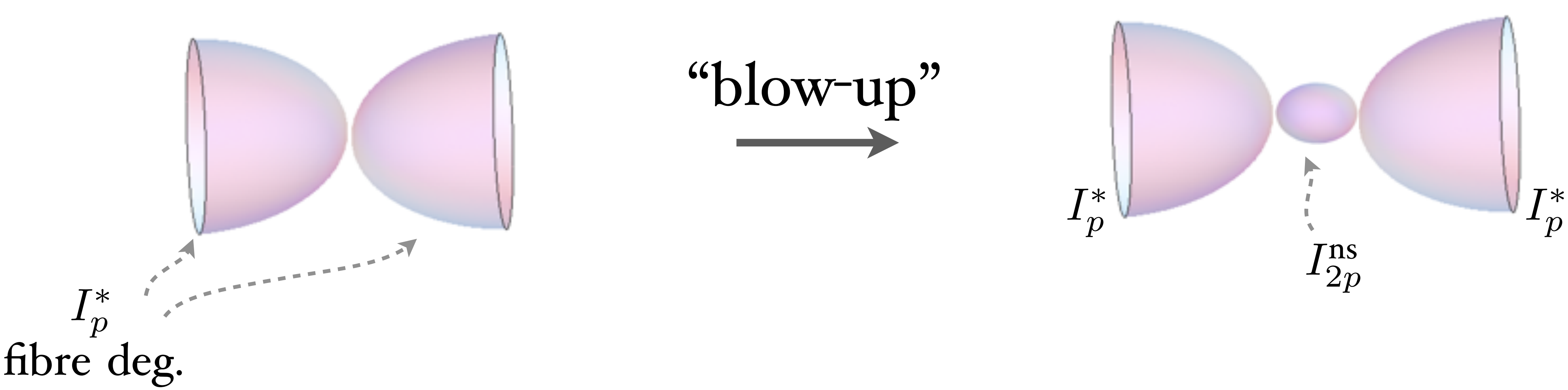
- First generalization: SO/Sp gauge groups

known IIA phenomenon:
an NS5 can 'fractionate' on an O6

[Evans,Johnson,Shapere '97]
[Elitzur,Giveon, Kutasov, Tsabar '98]

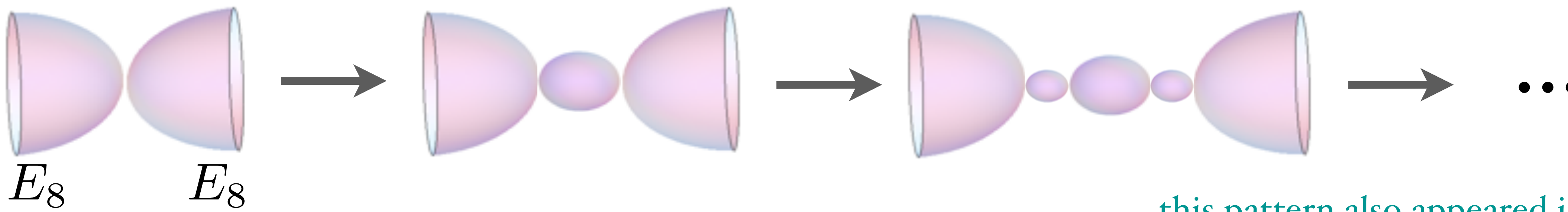


In F-theory this is
reproduced geometrically:



- There is also an analogue for exceptional gauge groups

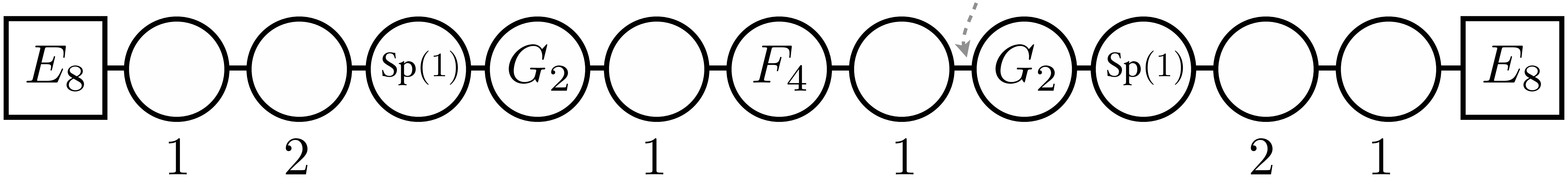
now we need several blowups...

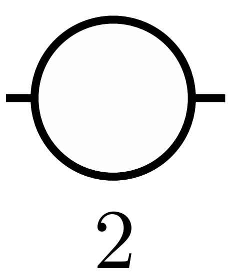


this pattern also appeared in
[Berhadsky, Johansen '96]
[Aspinwall, Morrison '97]
[Intriligator'97]...

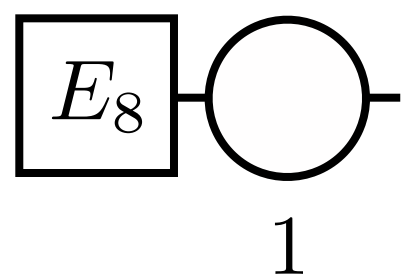
Final result: the (E_8, E_8) theory

tensor multiplets

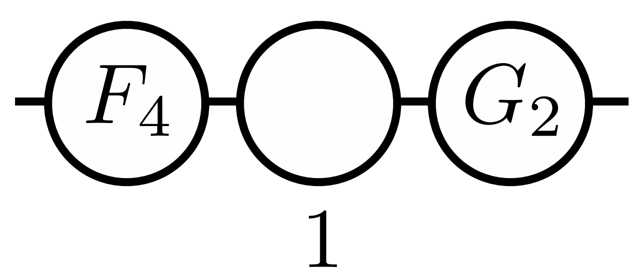




= two tensors (no gauge group)

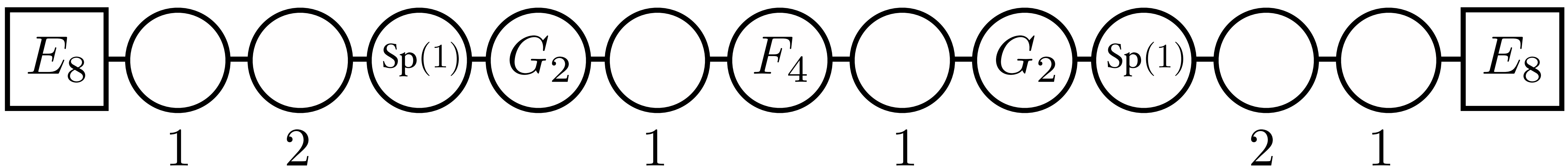


= "E-string" (no gauge group) E_8 flavor symmetry

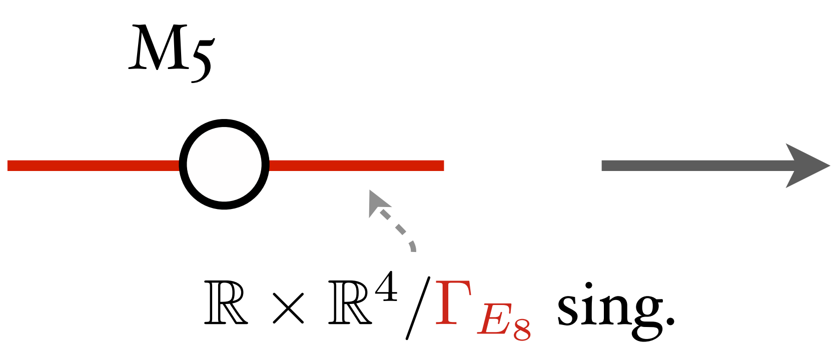


= again "E-string" where $F_4 \times G_2 \subset E_8$ has been gauged

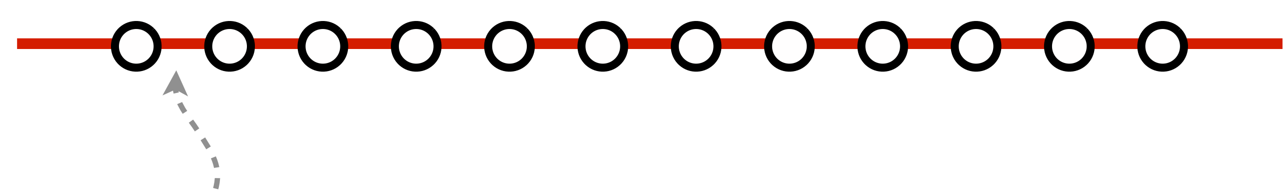
Final result: the (E_8, E_8) theory



In M-theory:



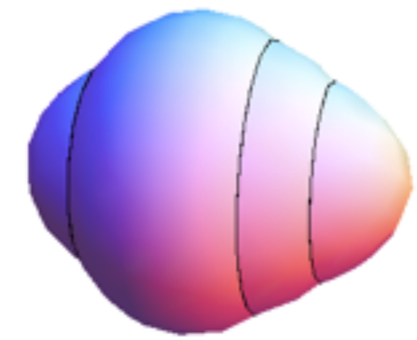
Conjecture: 12 fractional M5's



a 'discrete flux' is created whenever
a fractional M5 is crossed

for a nice alternative explanation
[Ohmori, Shimizu, Tachikawa, Yonekura '15]

Conclusions

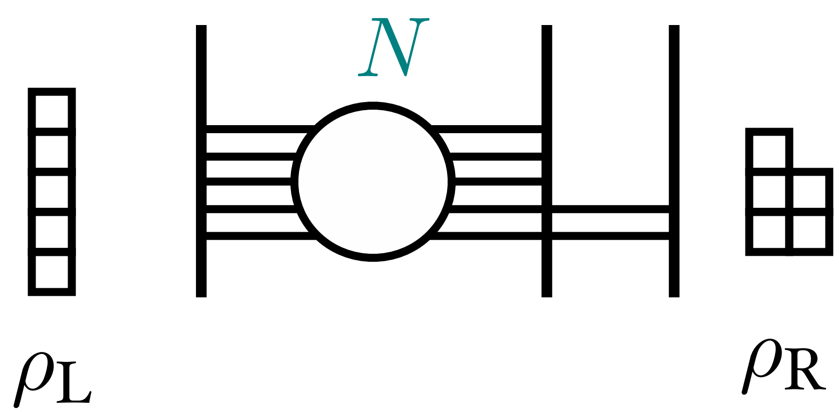


- Classification of type II AdS7 solutions
- Infinitely many analytic AdS7, AdS5, AdS4 solutions
- Related by simple universal analytic map
- Dual field theories: strong coupling points in linear $\mathrm{U}(k)$ quivers

NS5-branes

$$T^{N}{}_{\rho_{\mathrm{L}},\rho_{\mathrm{R}}}$$

pattern of D6's
ending on D8's



- More general quivers from F-theory; fractional M5's?