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Simulated annealing in application to telescope phasing

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ABSTRACT

One interdisciplinary application of Statistical Physics is the mapping of roughening transitions (and their solution by Metropolis Monte Carlo/simulated annealing) to the telescope phasing problem. We describe the topic, some calculations and applications, the connection to Stauffer and recent progress.

1. Introduction

This contribution describes an example of the application of a model of statistical physics to a different area of physics/engineering, videlicit telescope (or microscope) phasing. The route leads via an analogy of the roughening model of crystal growth to the optical flattening of a telescope surface. Simulated annealing or rather Monte Carlo Metropolis simulation is today quite a standard optimization technique outside statistical physics, but in 1990 its application to telescope phasing was rather more esoteric. Note that in crystal models a minimum energy is desirable, for optical systems a maximal brightness. A practical implementation of simulated annealing is described in [1].

For this volume a Stauffer connection is desirable. It is also very natural in our case because of the roughening transition analogy. Both Stauffer [2] and one of us [3] were modeling roughening transitions at that time — JA has an acknowledgment to Stauffer in one of her roughening transition papers.

Two common crystal roughening models — 2d surfaces in 3d Ising models and the SOS (Solid-on-Solid) model were often studied with Metropolis simulation over a range of temperatures, and minimization of the interface energy was trivial in terms of timesteps if approached from the groundstate. However, approach from a high temperature excited state was timestep consuming, especially for the SOS version. A realization of a telescope model is shown in Fig. 1, for the crystal version just replace the disk on the pole with a column of atoms, and the peaks of phasing with the flatness of the surface.

At this time the other author, ER, was studying the then fairly new field of adaptive optics for optical phasing of a telescope. Simulated annealing was a possible evaluation algorithm for this. In this problem multiple mirrors (or a flexible sheet with pistons below) need to be phased to provide an optically flat surface. Here, starting from an optically flat surface means phasing is trivial, but structural vibration or atmospheric interference means that in practice it is very problematic to retain or obtain a flat surface.

We [4] developed an analogy between the SOS and telescope models that enabled the application of simulated annealing to telescope phasing. Specifically we made a mapping analogy to crystal growth with telescope elements corresponding to columns of atoms, optimal reflected wavefront to the groundstate, noise to temperature, achieving optimal image to reaching the groundstate, and telescope adjustment to the movement of an atom. The Hamiltonians

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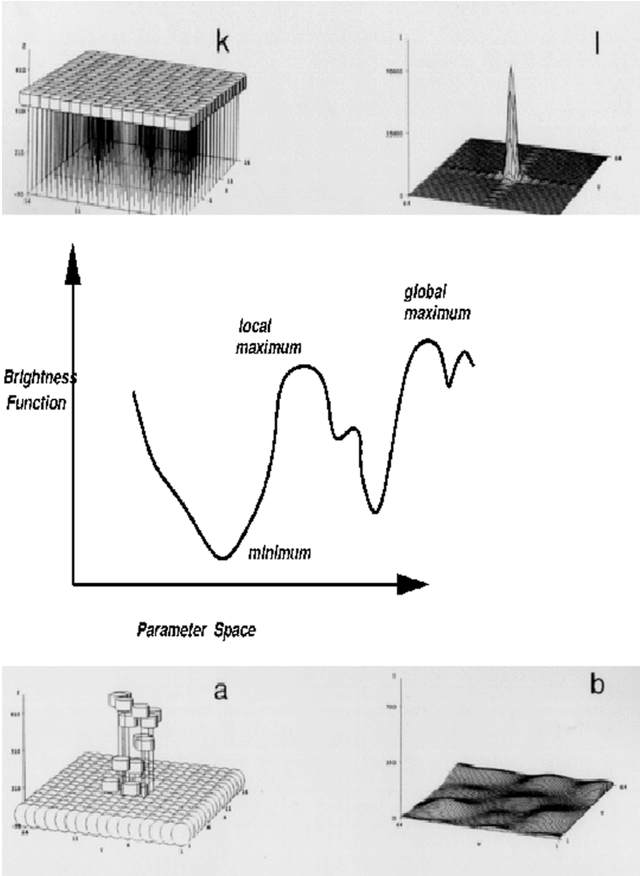

Fig. 1. Parts of images from [4] and [1] showing (upper left)an optically flat telescope surface and its (upper right) intensity, a graph of the brightness function in parameter space — this is the 'upside down' of the crystal energy, (lower left) a "rough" telescope surface and (lower right) the corresponding low intensity.

of an SOS model and of a multiple mirror telescope were shown in [4] to take the same functional form, although the SOS is usually a sum over nearest neighbor pairs and the telescope over all pairs.

A Hamiltonian for the energy of the SOS model on a planar lattice, where there is a column of atoms of height h_k (relative to some arbitrary reference plane) at the position k can be written

$$\mathcal{H} = \sum_{k>l} J_{kl}(h_k - h_l)^2, \tag{1}$$

where J_{kl} is a function of the distance between the sites k and l. The ground state of the system is that choice of h_k which minimizes the value of the total $\mathcal{H}$; this will occur when all pairs of height differences are as small as possible.

By analogy, we define an energy ("Hamiltonian") function for the telescope which will be minimized when the telescope is phased (i.e. the wavefront reflected from its aperture is spherical, flat if the image is at infinity). For this Hamiltonian, we choose

$$\mathcal{H} = \sum_{k>l} \mid \Gamma_{kl} \mid^2 [1 - \cos(\frac{2\pi}{\lambda}(h_k - h_l))]. \tag{2}$$

The wavelength of measurement is λ and Γ_{kl} is the mutual coherence function between points k and l on the telescope aperture. In this case h_k is measured from a spherical reference wavefront focused on the reference plane. When $h_k \sim h_l$, Eq. (2) reverts to Eq. (1). The coherence function $\Gamma_{kl} = 1$ for point sources, which is why we use them, and not extended sources. For extended sources we revert again to Eq. (1), since this Γ_{kl} drops to zero when the distance between patches k, l becomes large. Now the electromagnetic waves at the telescope aperture and at its focal plane are a Fourier pair, in which case Hamaker et al. [5] showed that minimizing the Hamiltonian in Eq. (2) is equivalent to maximizing Muller and Buffington's sharpness function [6]

$$\mathcal{S} = \sum_{m} I_m^2, \tag{3}$$

where I_m is the light intensity measured by a detector at position m in the focal (Fourier) plane of the telescope. According to the van Cittert–Zernike theorem [7], Γ_{kl} is the Fourier transform of the (stellar) object intensity. In a real optical system, it is faster to measure the image sharpness from Eq. (3), while Eq. (2) is faster to calculate in a computer simulation.

A comparison of Eqs. (1) and (2) clearly illustrates the analogy between the two systems. J_{kl} corresponds to $|\ \Gamma_{kl}\ |^2$, both being functions of the distance between two sites. These both multiply functions of the height differences $h_k - h_l$ which are respectively minimized when the solid state system is in its flat ground state and when the telescope is phased, i.e. the outgoing wavefront is flat, modulo λ.

The scheduling aspects of the telescope phasing are rather tricky, possibly due to the long-range interaction. In fact another input from simulated annealing experience provided by the originator of out-of statistical physics applications for simulated annealing, S. Kirkpatrick, to initially block some mirror rings and phase the telescope by successive rings was crucial.

2. Adaptive optics

The optical telescope application has led to many other optical applications, such as ocular ones, [8]. There have also been multiple hardware realizations of these systems, [9]. Optical design deals with finding the best configuration of lenses, their refractive indices and surface curvatures. Even a system with two lenses and a mirror have more that twelve variables that need to be adjusted according to some criteria such as sharp images, reduction of color effects, or large field of view. For larger systems there are many variables and there can be many local maxima of image sharpness.

One example is the fine alignment of a segmented telescope. The primary mirror of such a telescope is segmented for reasons of weight, volume, or difficulty of polishing a non-segmented mirror, especially a large telescope, on the ground or in space. There can be hundreds or thousands of local minima in the image produced by such a telescope. Such a telescope can focus itself by measuring the brightness of a distant star. Many of the new large telescopes use adaptive optics, for example the Keck telescope that is one of the heroes of the recent Physics Nobel Prize, since it was used for some of the black hole measurements.

Another example of adaptive optics is when the surface of one of the elements (usually a mirror) is flexible, and allows to correct for optical aberrations elsewhere in the system. Adaptive optics was first used in astronomy to counteract the effects of atmospheric turbulence. Each time the aberrations change, they are measured in a wave front sensor. This device divides the incoming stellar beam into small patches and measures the local aberrations, to send a correcting signal to a closed-loop servo system. For larger telescopes, this might mean hundreds to thousands of parallel measurements and corrections within each atmospheric time scale (usually milliseconds).

A more recent simulated annealing study of telescope phasing using additional phasing techniques was made in [10]. This study also included a laboratory model that went thru a hardware annealing process. In this project extensive use was made of additional image processing software and image reconstruction.

Adaptive optics was later adopted for ocular imaging, to improve images of the retina through variable aberrations in the cornea and ocular motions. Instead of using a star, one measures instead the quality of a light beam, reflecting off a spot in the retina. The wave front sensor uses this reflection in the same manner as in astronomical set ups, but with few to tens of measurements and corrections, with similar time scales. Because the system is simpler and more stable (the major aberrations of the eye, such as defocus and astigmatism, do not vary much), it turns out the simulated annealing can be used instead to sharpen the image of that spot. This method was adopted by many, and dubbed "sensorless adaptive optics", although the sensor, namely the imaging camera view of the retina, is the main feed-back tool. With fewer local extrema in the sharpness (or energy) map, many have adopted a derivative of simulated annealing, stochastic parallel gradient descent (SPGD) which is somewhat faster, but tends to reach local extrema.

3. Recent developments

The simulated annealing technique has become a standard in optical phasing problems. (The question of its independent (or not) application will not be broached here, but our initial application derived from the roughening transition analogy is as far as we know original — as evidenced by initial acceptance problems and then by citations and publication dates.) A recent study of note is [9] with many further references within. Throughout Refs. [8–10] several laboratory realizations are described, we choose to show one in Fig. 2.

We return briefly to the Ref. [1] we chose to use for simulated annealing. (Although its publication date is after [4] the project preceded [4] by quite a few years.) In the atomistic context we have experimented with variations on simulated annealing, notably with a master–slave parallel genetic algorithm [11], motivated by the need to find the groundstate quickly. Unfortunately this is not practical for the telescope, since it would require multiple hardware realizations, but it has a double Stauffer link. Firstly, Stauffer visited and helped us use the parallel machine, and when he found out about our algorithm, passed it on to his associate Gropengeisser [12] for a successful implementation to find a spin glass groundstate. Today there are many other techniques used as well as or in competition to simulated annealing one or more of which might be useful for adaptive optics models.

The simple telescope described in [4] has up/down movement at different heights (motivated by varied atomic heights) but the more recent ones may also have tilt, or split mirrors and thus the computational complexity is much greater.

There is perhaps less current interest in nearest neighbor roughening transitions in a theoretical sense today, but applications in crystal modeling continue — for example [13]. Roughening transitions with longer range interactions have also been studied, for example in [14]. A dominant effect is a substantial change in the roughening temperature.

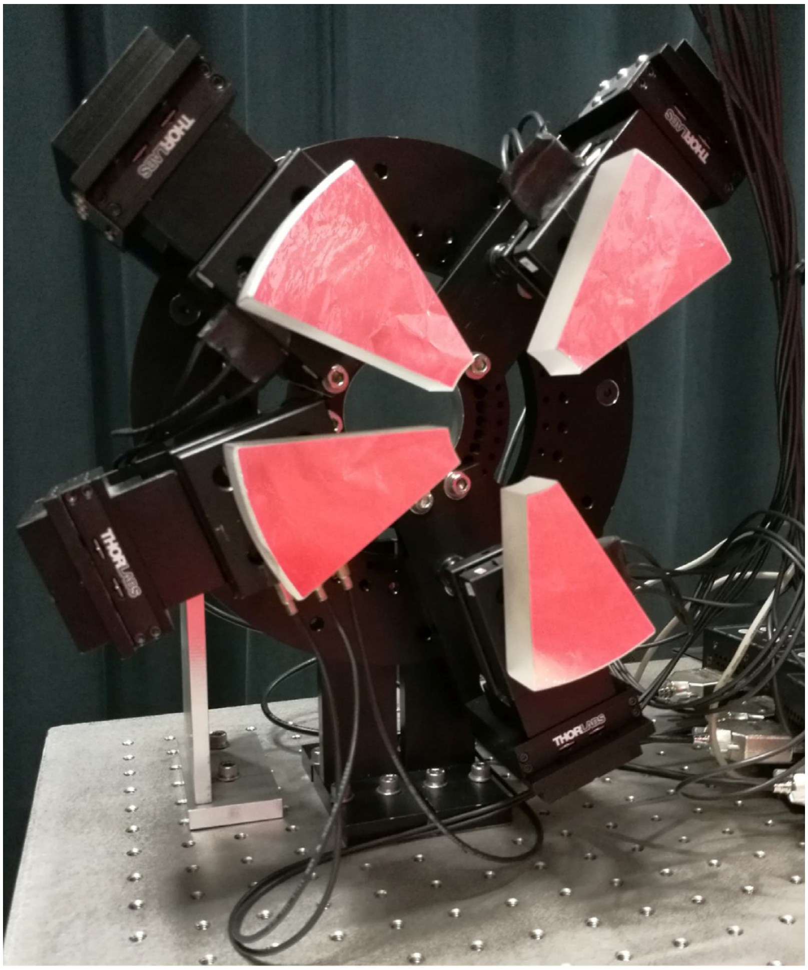

Fig. 2. An image from [9] of a laboratory telescope.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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