STATISTICAL PROPERTIES OF THE SPECTRUM OF MANY BODY SYSTEMS

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Statistical properties of the energy spectrum of many body interacting systems are investigated. We calculate the spectral form factor using a semiclassical description for the partition function (or the evolution operator) of the system. The coupling constant describing the interaction plays a role analogous to $\hbar$ in the usual semiclassical approximation of quantum mechanics. It is shown that the equivalent of the Van Vleck determinants are given by Fredholm determinants which for the case of electrons interacting through a Coulomb potential are related to the dielectric function calculated within the RPA . The low energy part of the many body spectrum is then shown to obey a Poisson statistics in accordance with recent numerical results. The validity of this description is finally discussed.

I. INTRODUCTION

The aim of this work is to present a method for obtaining some analytical results about the statistics of the energy spectrum of many body systems. The interest for such statistical properties is at least twofold. First, recent numerical investigations of many body hamiltonians [1] did show a connection between the nature of the statistics of their energy levels and their integrability .This was somehow unexpected since unlike one body systems (e.g. billiards), most of these many body hamiltonians do not have a classical limit. Therefore, it is rather difficult to associate to the observed statistics a possible chaotic behaviour in the classical limit.

More surprising is the result [2], [3] showing that for some models, there is a crossover between Poisson and Wigner statistics when going higher in the energy spectrum. The nature of this crossover is certainly a central issue in the existence of a Fermi liquid behaviour.

A second reason for the raise of interest in the statistical properties of energy spectra of many body systems is the connection recently worked out between the behaviour of non interacting, disordered mesoscopic systems and the semi-classical analysis of their energy spectra. More precisely, under certain conditions, it is possible to relate some thermodynamic and transport quantities to the behaviour of the energy levels under external perturbations like a magnetic Aharonov-Bohm flux (or a change of boundary conditions). The generalization of these results to describe transport properties of many body systems would certainly be of great interest.

Unfortunately, we do not have at the moment in order to describe the statistics of the many body spectra powerful tools like the semi-classical approach to the single particle quantum mechanics or the periodic orbit description. The standard perturbation methods usually describe systems in the thermodynamic limit and therefore are not adapted to the description of discrete spectra. There is nevertheless a possible way to obtain a "semi-classical" description of many

body systems. The basic methodology was first developped in the context of field theory [5] and nuclear physics [6]. It is the aim of this paper to extend them to study the statistics of energy spectra of many body systems.

II. STATISTICAL PROPERTIES OF THE ENERGY SPECTRUM

A successful characterization of the nature of a quantum system is given by the distribution function $P(s)$ of the distance s between neighbouring energy levels measured in units of the mean level spacing Δ. Two kinds of behaviour are usually observed. $P(s)$ is either of the Poisson type ($P(s) \propto e^{-s}$) or of the Wigner type ($P(s) \propto s^{\beta} e^{-c_\beta s^2}$) where β depends on the symetry of the hamiltonian. The Poisson distribution in the context of single particle quantum mechanics is believed to describe systems which are integrable in the classical limit. In contrast, the Wigner distribution deals mostly with a non integrable (or chaotic) classical limit. These distributions are universal and depend only on very general symetry considerations.

If $P(s)$ is easily studied numerically, it is rather difficult to obtain analytical results to describe the statistics even for the one particle problem. This is certainly one of the reasons for the success of phenomenological approaches like the Random Matrix Theory (RMT), which was indeed designed to describe the excitation spectrum of interacting systems, namely heavy nuclei [7]. In the framework of the semi-classical approach to quantum mechanics, there is nevertheless a quantity which conveys a lot of information about the nature of the system. This is the spectral form factor $K(t)$ defined as the Fourier transform of the two point correlation function of the density of states $K(\epsilon_1 - \epsilon_2) = < \rho(\epsilon_1)\rho(\epsilon_2) >_c$, where $\rho(\epsilon) = -\frac{1}{\pi} Im \mathrm{Tr}\, G^R(\epsilon)$ and $G^R(\epsilon)$ is the resolvent operator. Some statistical quantities describing the energy spectrum can be simply derived from the knowledge of $K(t)$. For instance the number variance measuring the fluctuation of the number of energy levels $N(E)$ in a band of width E is very often considered. It is defined by $\Sigma_2(E) \equiv < \delta N^2(E) > = < N^2(E) > - < N(E) >^2$ and can be written as a function of $K(t)$ as:

$$\Sigma_2(E) = 2\int_0^\infty dt \frac{K(t)}{t^2} \sin^2(\frac{Et}{2}) \tag{1}$$

Within the framework of the RMT, $K(t)$ is found to be proportional to t. This gives for Σ_2 the well-known Dyson relation $\Sigma_2(E) = \frac{1}{2\pi^2\beta} \ln(\frac{E}{\gamma})$ which describes the rigidity of the levels distributed according to the Wigner law. A great success of the semi-classical approach to weakly disordered mesoscopic metals was to relate, in the diffusive limit ($k_f l \gg 1$), the spectral form factor $K(t)$ to the return probability $P(t)$ of a classically diffusive particle [8]. $P(t)$ being a central quantity for the calculation of either thermodynamic or transport quantities [9], this relation did provide a connection with the statistical properties of the energy spectrum of disordered metals without interactions. It is the extension of the calculation of $K(t)$ to many body systems which is discussed in the rest of this paper.

III. "SEMI-CLASSICAL" DESCRIPTION OF A MANY BODY SYSTEM

In order to evaluate $K(t)$, we need to express the density of states of the many body spectrum. It is given by the inverse Laplace transform of the partition function:

$$\rho(E) = \frac{1}{2i\pi} \int_{-i\infty}^{+i\infty} d\beta e^{\beta E} Z(\beta) \tag{2}$$

where $Z(\beta) = \mathrm{Tr}\, e^{-\beta H}$. The hamiltonian H of the system is given by:

$$H = \sum_i \frac{p_i^2}{2m} + \frac{\lambda}{2} \sum_{i \neq j} v\,(x_i - x_j) \tag{3}$$

It describes particles interacting through the binary potential v of strength λ. We shall concentrate on the case of electrons. Introducing fermion field operators which obey the usual anticommutation relations, we can rewrite H as:

$$H = H_0 + V = \int dx \psi^\dagger(x) h(x) \psi(x) + \frac{\lambda}{2\Omega} \int\int dx dy \psi^\dagger(x) \psi^\dagger(y) v(x-y) \psi(y) \psi(x) \tag{4}$$

where $h(x)$ is the kinetic energy (to which could be added as well a part describing either disorder or periodic potential).

It is possible to rearrange the interaction term V by using the commutation relations. There are three different ways to do this which do correspond respectively to the Hartree, Fock and pairing rearrangements. Since in this paper we consider only normal Fermi systems, only the first two possibilities are relevant, the third dealing with the BCS case. Moreover, as will appear clearer later, the Hartree and Fock arrangements are equivalent in the semi-classical limit considered here.Therefore, we shall consider the Hartree case V_H which leads to simpler and more transparent calculations since it involves only two density operators $\rho(x) = \psi^\dagger(x)\psi(x)$. It is given by:

$$V_H = \frac{\lambda}{2\Omega} \int\int dx dy \psi^\dagger(x) \psi(x) v(x-y) \psi^\dagger(y) \psi(y) \tag{5}$$

plus a constant term incorporated in a redefinition of the energies. To go further and calculate the partition function we need to decouple the quartic term. This is done using the usual Hubbard Stratanovich transformation [6]. To that purpose, we introduce an auxiliary field $\sigma(x,s)$ over which we integrate. Then,

$$Z(\beta) = \frac{1}{\mathcal{N}} \int \mathcal{D}[\sigma(x,s)] e^{\frac{-1}{2\lambda} \int_0^\beta ds \int dx dx' \sigma(x,s) v^{-1}(x-x') \sigma(x',s)} Z[\sigma;\beta] \tag{6}$$

where the partition function $Z[\sigma;s]$ describes a one body hamiltonian but in the unknown field $\sigma(x,s)$ which depends both on the imaginary time and on the position :

$$Z[\sigma;\beta] = \mathrm{Tr}\, \mathcal{P} \exp\{- \int_0^\beta ds \int dx \psi^\dagger(x) [h(x) + \sigma(x,s)] \psi(x)\} \tag{7}$$

where $\mathcal{P}$ is the time ordering. The normalization constant $\mathcal{N}$ is given by:

$$\mathcal{N} = \int \mathcal{D}[\sigma(x,s)] e^{\frac{-1}{2\lambda} \int_0^\beta ds \int dx dx' \sigma(x,s) v^{-1}(x-x') \sigma(x',s)} \tag{8}$$

Our initial interacting problem is now replaced by a non interacting one. The price is that now the one body partition function $Z[\sigma;s]$ describes a non trivial, β-dependent Hamiltonian. A strategy to study such a problem was proposed by Dashen, Hasslacher and Neveu [5] in the framework of quasi-classical approximation to find particle states in quantum field theory. Eq. (6) for the partition function suggests looking for a stationnary phase approximation for the functional integral. This is justified for a large value of the inverse coupling strength $\frac{1}{\lambda}$ which for our problem plays a role analogous to the inverse Planck constant for the usual semi-classical approximation. Rewriting the partition function as:

$$Z(\beta) = \frac{1}{\mathcal{N}} \int \mathcal{D}[\sigma] e^{\frac{1}{\lambda} S[\sigma_0, \beta]} \tag{9}$$

we define the action:

$$S[\sigma_0,\beta]=-\frac{\beta}{2}\left[\int dxdx'\sigma_0(x)v^{-1}(x-x')\sigma_0(x')+\frac{\lambda}{2}\log \mathrm{Tr}\ \exp\left\{\int dx\psi^{\dagger}(x)(h(x)+\sigma_0(x))\psi(x)\right\}\right] \tag{10}$$

For the sake of simplicity, we are looking for a β-independent σ, which is justified precisely for the Hartree arrangement here considered. We obtain the saddle point field σ_0 through the stationarity condition of the action $\delta S[\sigma_0,\beta]=0$. It gives:

$$\sigma_0(x)=\int dx'v(x-x')\sum_l n(\epsilon_l,\beta)|\phi_l(x)|^2 \tag{11}$$

where $n(\epsilon_l,\beta)$ is the Fermi Dirac distribution function with a chemical potential given at zero temperature by the Fermi energy ϵ_f. Since we are dealing with a one particle Hamiltonian, we then have to solve the Shrodinger equation:

$$\left[-\frac{\hbar^2}{2m}\nabla^2+\sigma_0(x)\right]\phi_l(x)=\epsilon_l\phi_l(x) \tag{12}$$

for an electron in the potential $\sigma_0(x)$. The basis $\{\phi_l(x),\epsilon_l\}$ describes the one particle spectrum out of which the many particle states are built as Slater determinants. Eq. (12) turns out to be the usual self consistent Hartree equation. The thermodynamic potential which corresponds to the classical action is obtained from the eigenenergies of (12) in the usual way:

$$\Omega_0=\frac{1}{2}\sum_{l,l'}n(\epsilon_l,\beta)n(\epsilon_{l'},\beta)\int dxdy\ |\phi_l(x)|^2v(x-y)|\phi_{l'}(x)|^2-\frac{1}{\beta}\sum_l\log\left(1+e^{-\beta\epsilon_l}\right) \tag{13}$$

For the usual electron gas model in the translationally invariant system the Hartree field is zero, so the last expression simplifies to

$$\Omega_0=-\frac{1}{\beta}\sum_k\log\left(1+e^{-\beta\epsilon_k}\right) \tag{14}$$

where k denotes the momentum states of one-particle. The partition function at this stage can be written as

$$Z_0=e^{-\beta\Omega_0}=\prod_k\left(1+e^{-\beta\epsilon_k}\right) \tag{15}$$

or using the many-body states with energy as a functional of the occupation numbers:

$$Z_0=\sum_{\{n_k\}}e^{-\beta E\{n_k\}},\quad E\{n_k\}=\sum_k n_k\epsilon_k \tag{16}$$

It is worth emphasizing at that point that had we started with the Fock arrangement for V, we would have obtained the self consistent Hartree Fock equation as a saddle point . In our case, the Fock term will be recovered as part of the quadratic fluctuations around the stationary solution that we shall study hereafter.

IV. QUADRATIC FLUCTUATIONS AROUND THE STATIONARY SOLUTION

Again in the spirit of the standard semi-classical description of path integrals, we consider now the quadratic corrections around the stationary phase approximation. To that purpose, we consider small variations of the field $\sigma(x,t)$ around σ_0 of the form $\sigma(x,t) = \sigma_0 + \eta(x,t)$. Then, the action is expanded up to second order :

$$S[\sigma] \simeq S[\sigma_0] + \frac{1}{2}\int_0^\beta \int_0^\beta dtdt'\, \eta(t) \frac{\delta^2 S[\sigma_0]}{\delta\sigma(t)\delta\sigma(t')}\bigg|_{\sigma=\sigma_0} \eta(t') \tag{17}$$

The dependence of η on the coordinate x in the rhs of Eq. (17) is understood as well as periodic boudary conditions in the time integrals. The second derivative which appears in Eq. 17 can be written:

$$\frac{\delta^2 S[\sigma_0]}{\delta\sigma(t)\delta\sigma(t')}|_{\sigma=\sigma_0} = -v^{-1}(x-x')\delta(t-t') - \Gamma(x,x';t,t') \tag{18}$$

where $\Gamma(x,x';t,t')$ can be expressed in terms of the density-density response function χ_0 given by a product of one particle propagators G_0 calculated within the Hartree approximation:

$$\Gamma(x,x';t,t') = iG_0(x-x',t-t')G_0(x'-x,t'-t) = \chi_0(x-x',t-t') \tag{19}$$

The correlation function which appears depends on the kind of interaction we do consider. For a Coulomb interaction, it is the density-density correlation function. For other models we have to consider the appropriate one (for instance, the spin correlation function for a Heisenberg model). Then, we can perform the integral over the auxiliary field and obtain for the partition function the expression:

$$Z = \frac{1}{\sqrt{D(\beta)}} e^{-\beta\Omega_0} \tag{20}$$

where Ω_0 is the Hartree thermodynamic potential and $D(\beta) = \mathrm{Det}\ \epsilon(r-r',t-t')$ where ϵ is:

$$\epsilon(r-r',t-t') = \delta(r-r')\delta(t-t') - \int dx v(r-x)G_0(x-r',t-t')G_0(r'-x,t'-t) \tag{21}$$

The determinant $D(\beta)$ is defined in the space of periodic functions with imaginary period β in addition to periodic boundary conditions in the real space. Within this subspace, we can write its Fourier transform

$$\epsilon(q,i\omega_n) = 1 - V_q \sum_k \frac{n(\epsilon_{k+q},\beta) + n(\epsilon_k,\beta)}{i\omega_n - \epsilon_{k+q} + \epsilon_k} \tag{22}$$

where $\omega_n = \frac{2\pi n}{\beta}$ are the usual Matsubara frequencies and $n(\epsilon_k)$ the Fermi Dirac factor. For the Coulomb interaction, ϵ appears to be the dielectric function calculated within the RPA.

The nature of the approximation might be more transparent writing $D(\beta)$ in a formal way as the ratio of the determinants of two operators:

$$D(\beta) = \frac{\mathrm{Det}\ (\hat{G_0}^{-1} + \hat{\Sigma})}{\mathrm{Det}\ \hat{G_0}^{-1}} \tag{23}$$

where the diagonal matrix element of the operator in the denominator

$$G_0{}^{-1}(i\omega_n, k, q) = i\omega_n - \epsilon_{k+q} + \epsilon_k \tag{24}$$

is the Green's function describing the propagation of an electron-hole pair in the non-interacting case and

$$\hat{G}^{-1} = \hat{G_0}{}^{-1} + \hat{\Sigma} \tag{25}$$

is the equivalent quantity for the interacting case written with the help of the Dyson equation. The matrix elements of the self-energy operator $\hat{\Sigma}$ are given in our approximation by

$$\Sigma(k, q, \beta) = V_q[n(\epsilon_{k+q}, \beta) - n(\epsilon_k, \beta)] \tag{26}$$

Det $\hat{G_0}{}^{-1}$ has zeroes at $\omega^0_{k,q} = \epsilon_{k+q} - \epsilon_k$ while Det $\hat{G}^{-1}$ has zeroes $\omega_{k,q}$ corresponding both to particle-hole excitations, which go to $\omega^0_{k,q}$ in thermodynamic limit together with zeroes ω_q emerging from the continuum and describing bound states. Thus one obtains for $D(\beta)$ the structure of a Fredholm determinant. Its poles describe the excitation energies of the non interacting system while its zeros are those of the interacting one at this approximation which is nothing but the well known RPA. In this description, the non perturbative character of the method appears clearly. It is well known otherwise that it corresponds to the resummation of a infinite number of diagrams [10] implied by the Dyson equation (25). The ground state energy and the density of states are easily obtained within this formalism [11] as an extension of the Krein-Friedel formula derived in context of scattering theory. This determinant structure which appears here for a many body problem is the equivalent of the Van Vleck determinant of the semi-classical description associated with the amplitude of the corresponding periodic orbits. At that stage, this analogy is mostly formal since we do not know how to identify an analogous periodic orbit structure.

V. THE SPECTRAL FORM FACTOR AND THE NUMBER VARIANCE OF THE MANY BODY SPECTRUM AT LOW ENERGIES

We are now in a position to calculate the two point correlation function of the density of states and the spectral form factor $K(t)$. The partition function is given by $Z(\beta) = \frac{1}{\sqrt{D(\beta)}} e^{-\beta\Omega_0}$ where $D(\beta)$ is the Fredholm determinant defined above and Ω_0 is the thermodynamic potential of the many body system obtained within the stationary phase approximation given by (13).

We then write the spectral form factor as the Fourier transform of the two point correlation function of the density of states:

$$K(\epsilon) = < \rho(E + \frac{\epsilon}{2})\rho(E - \frac{\epsilon}{2}) >_c \tag{27}$$

where the average is taken over a interval L in the energy spectrum large compared to the mean level spacing centered at some energy E_c. This supposes that we are in a homogeneous part of the spectrum where the mean level spacing keeps its meaning, for instance low enough in energy. This average over the spectrum is equivalent to the so called diagonal approximation used in the semi-classical description [4], namely

$$K(\epsilon) = \frac{1}{L}\int_{E_c - L/2}^{E_c + L/2} dE\ \rho(E + \frac{\epsilon}{2})\rho(E - \frac{\epsilon}{2}) \tag{28}$$

The density of states is given by the Laplace transform (2) of the partition function. By virtue of this expression we are able to rewrite $K(\epsilon)$ as

$$K(\epsilon) = \frac{1}{L}\int_{E_c-L/2}^{E_c+L/2} dE \left(\frac{1}{2\pi i}\right)^2 \int_{-i\infty}^{+i\infty}\int_{-i\infty}^{+i\infty} d\beta_1 d\beta_2\, D(\beta_1)^{-1/2} D(\beta_2)^{-1/2} \sum_{\{n_k\},\{n'_k\}} e^{-\beta_1 E\{n_k\} - \beta_2 E\{n'_k\}} \tag{29}$$

We perform the integration over the energy first. Using the fact that $\frac{1}{x}\sin\frac{Lx}{2} \to \pi\delta(x)$ when $L \to \infty$ we obtain

$$K(\epsilon) = \frac{1}{4\pi i L}\int_{-i\infty}^{+i\infty} d\beta\, D(\beta)^{-1/2} D(-\beta)^{-1/2} \sum_{\{n_k\},\{n'_k\}} e^{-\beta\left(E\{n_k\} - E\{n'_k\}\right)} \tag{30}$$

Along with our diagonal approximation, we keep only the diagonal terms $\{n_k\} = \{n'_k\}$ giving the main contribution to $K(\epsilon)$. We normalize the correlation function dividing it by a constant L/C, C being equal to the remaining sum of constant terms. Then, at low enough temperature, where our approximation of a homogeneous spectrum is meaningful we can write for the Fourier transform of $K(\epsilon)$:

$$K(t) = 1/|D(it)| \tag{31}$$

where $D(\beta)$ is the Fredholm determinant defined above within the RPA.

The number variance is then given by Eq.(1), i.e.:

$$\Sigma_2(E) = 2\int_0^\infty dt \frac{|D(it)|^{-1}}{t^2} \sin^2(\frac{Et}{2}) \tag{32}$$

This relation is the central result of this paper. It relates a statistical property of the many body spectrum to the dielectric function through $D(it)$, i.e a quantity which in principle can be calculated from the usual perturbation methods developped for interacting systems in the thermodynamic limit.

VI. APPLICATIONS AND DISCUSSION

In this last section, we would like to discuss more in details the meaning of the various approximations considered in this description of the statistical properties of the many body spectrum. This might be of some importance especially if we are interested comparing it with the Fermi liquid description. It is first of all worth mentionning that the spectrum calculated through this functional integral approach gives well defined energy levels. The only width of the level to this approximation is proportional to $e^{-\frac{a}{\lambda^2}}$ (a being a constant) i.e. independent of the system size. It describes the probability of tunneling towards another minimum σ_0 of the functional integral. This would correspond to the spontaneous decay of a metastable state. It is very small within the weak coupling approximation here considered.

To obtain now the statistics of these levels we have to study the behaviour of $K(t)$. To that purpose, we rewrite the Fredholm determinant $D(\beta)$ under the form:

$$D(\beta) = \prod_n \prod_q \epsilon(q, \frac{2\pi n}{\beta}) \tag{33}$$

or, by definition of the Fredholm determinant as the ratio of the product of the eigenvalues of the interacting system over those of the non interacting one, i.e.:

$$D(\beta) = \frac{\prod_\lambda \sinh(\frac{\beta\omega_\lambda}{2})}{\prod_{\lambda_0} \sinh(\frac{\beta\omega_{\lambda_0}}{2})} \tag{34}$$

we can express both the numerator and the denominator as a distribution of harmonic oscillators:

$$\frac{1}{\prod_{\nu_\lambda \neq 0} 2\sin\frac{1}{2}\nu_\lambda t} = \sum_{\{n_\lambda\}} \exp\{-i \sum_{\nu_\lambda \neq 0} [(n_\lambda + \frac{1}{2})\nu_\lambda t - \frac{\pi}{2}]\} \tag{35}$$

where $\{n_\lambda\}$ means summation over all sets of the boson occupation numbers $n_\lambda = 0, 1, 2....$ Similarly, we can decompose the denominator of Eq.(34) and retain only the first term for the Hartree Fock expansion. We finally obtain:

$$D(it)^{-1} = \exp[\frac{1}{2}it\sum_{\lambda_0}\omega_{\lambda_0}] \sum_{\{n_\lambda\}} \exp\{-i \sum_{\omega_\lambda \neq 0} (n_\lambda + \frac{1}{2})\omega_\lambda t\} \tag{36}$$

We then have a spectrum obtained as a superposition of an infinite number of harmonic oscillators. For one particle quantum mechanics, Berry and Tabor [12] did show that a system whose spectrum is described by a finite number of harmonic oscillators does not exhibit level clustering and therefore does not have a Poisson statistics as expected for such an integrable case. Our description goes beyond this limit since we have an infinite number of such oscillators in order to describe our many body spectrum. Since these harmonic oscillators do have incommensurate frequencies, we can approximate $D(it)^{-1}$ given by Eq.(36) by a superposition of incoherent waves such that $K(t)$ given by Eq.(31) is a constant. This leads for the number variance to $\Sigma_2(E) \propto N^2(E)$ i.e. to a spectrum of the Poisson type.

Then, the question arises about the range of validity of our approximation or more precisely about the number of levels that can be safely described within it. To estimate this number or at least a lower bound, we may establish a connection between our approach and the Fermi liquid theory. There, a central role is played by the width Γ of the quasiparticle states where $\Gamma = k(\epsilon - \epsilon_f)^2$. The energy ϵ is measured from the Fermi energy ϵ_f and k is the effective coupling constant obtained by considering higher order corrections in λ, $k(\lambda) = \lambda(1 + a_1(\lambda) + ...)$. Within our description, Γ can be obtained from the partition function $Z(\beta)$ by adding sources and calculating the one particle Green's function as a second derivative of $Z(\beta)$ as usual in field theory. Then, well defined quasiparticles have the propagator (in the proper eigenbasis) :

$$G_{\alpha,\alpha'}(t - t') = \delta_{\alpha,\alpha'} Z_\alpha e^{-i(\epsilon_\alpha - i\Gamma_\alpha)(t-t')} \tag{37}$$

with $\Gamma_\alpha \ll \epsilon_\alpha$. For the Coulomb potential, the imaginary part Γ is given within the RPA [13] by:

$$\Gamma = \frac{\pi^2\sqrt{3}}{128}\omega_p(\frac{\epsilon - \epsilon_f}{\epsilon_f})^2 \tag{38}$$

where ω_p is the plasma frequency and then $k \propto \frac{\omega_p}{\epsilon_f^2}$. In our approximation, the spectrum is defined by a characteristic energy scale ω_0 obtained as the level spacing between the zeroes of the Fredholm determinant D. We can now evaluate a lower bound for the number of levels that can be described within our approximation. It is obtained for $\Gamma \simeq \omega_0$. At the energy ϵ the number n of levels is $\epsilon - \epsilon_f = n\omega_0$. Then our description is valid for energies $\epsilon - \epsilon_f \leq \sqrt{\frac{\omega_0}{k}}$ i.e. at weak coupling. This is not surprising, since our description has the validity of the RPA which consists in a bosonization of the low energy excitations [11] which is a good approximation at weak coupling.

Higher in energy, the statistics of the spectrum appears to be described by a distribution of the Wigner type [2], [3]. The nature of this crossover which might be viewed as a phase transition in the Hilbert space of the system is beyond the scope of this article. Nevertheless,

we believe that the present method may provide the good tool in order to investigate this question.

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Propriétés statistiques du spectre d' énergie de systèmes à N corps

Cet article présente une étude des propriétés statistiques du spectre d' énergie de systèmes à N corps en interaction. Nous calculons le facteur de forme spectral en utilisant une description semiclassique de la fonction de partition (ou de la trace de l'opérateur d' évolution) du système. La constante de couplage qui décrit les interactions joue un role analogue à celui de $\hbar$ dans la description semiclassique habituelle de la mécanique quantique. Nous montrons de plus que l' équivalent des determinants de Van Vleck est donné par des determinants de Fredholm qui, pour le cas d'une interaction de type Coulomb, sont reliés à la fonction diélectrique calculée dans l'approximation de la RPA. La partie basse énergie du spectre du problème à N corps obeit à une statistique du type Poisson ce qui est en accord avec des résultats numériques récents. La validité de notre description est finalement discutée.