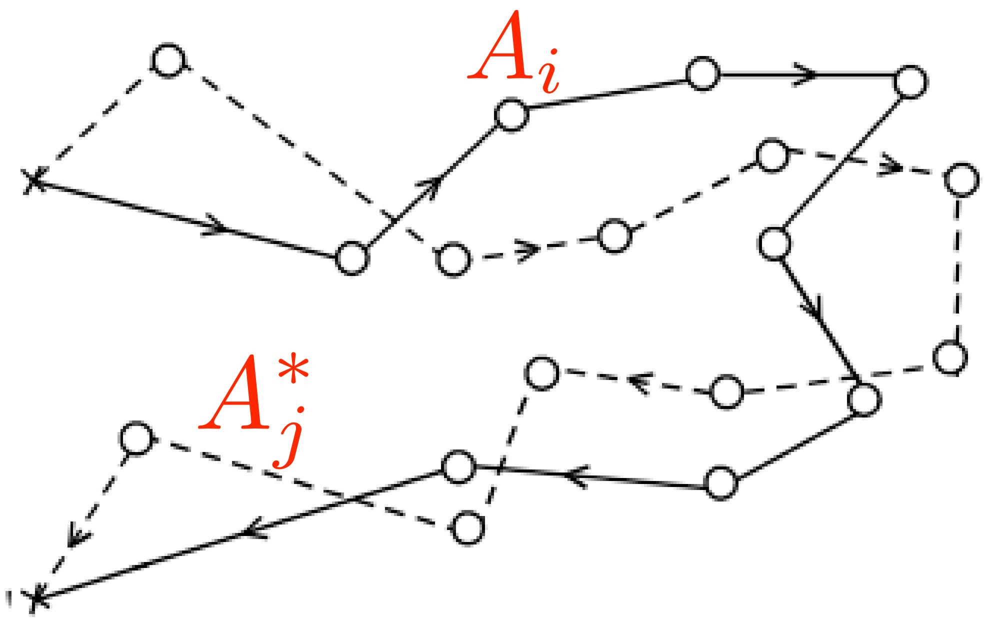
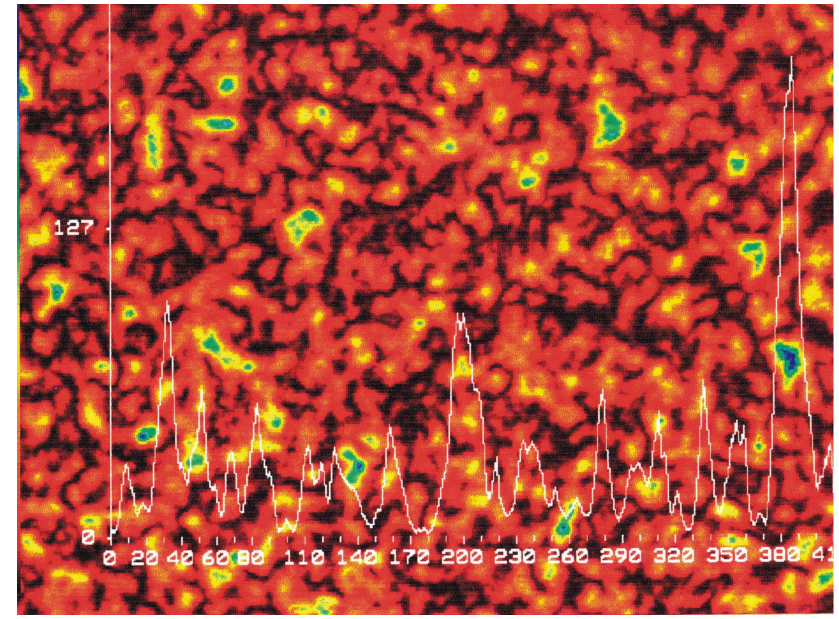
Lecture 3

- Correlation of speckle pattern: optics and atomic physics
- Diffusons, Quantum crossings and weak disorder
- Angular correlations of speckle patterns
- Dephasing-decoherence and dynamics of scatterers
- Time correlations of speckle patterns
- Photons correlations induced by disorder in a quantum mesoscopic gas

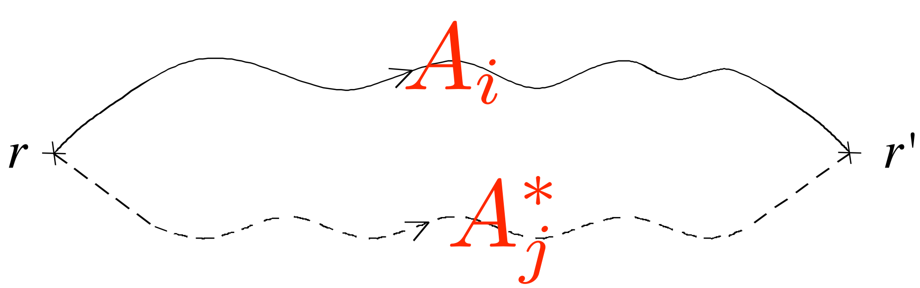




Before averaging : speckle pattern (full coherence)

Configuration average: most of the contributions vanish because of large phase differences.

A new design !



Vanishes upon averaging...up to quantum crossings

$$P_{cl}(\mathbf{r},\mathbf{r}') = \overline{\sum_j |A_j(\mathbf{r},\mathbf{r}')|^2}$$

Diffuson

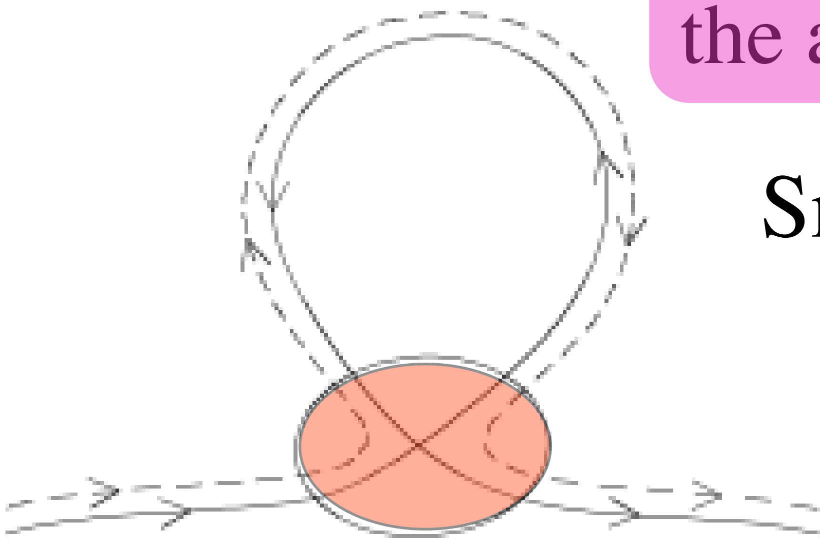
Solution of a diffusion equation

$$\left[\frac{\partial}{\partial t} - D\Delta\right] P_{cl}(\mathbf{r},\mathbf{r}',t) = \delta(\mathbf{r}-\mathbf{r}')\delta(t)$$

Quantum crossings

A diffuson is the product of 2 complex amplitudes: it can be viewed as a" diffusive trajectory with a phase".

Smallest phase defect: crossing which interchanges the amplitudes and pair them differently : phase shift.



Small phase shift $\leq 2\pi \Rightarrow$ localized crossing

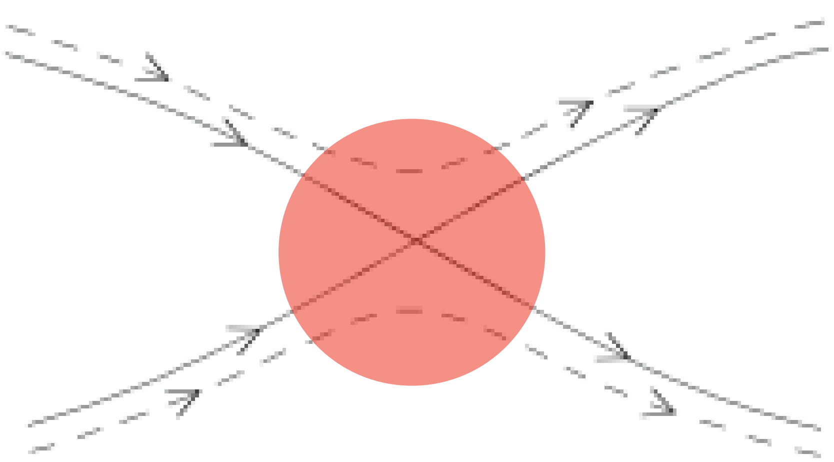
Crossing probability of 2 diffusons:

$$p_\times(\tau_D) = \frac{1}{g}$$

$$\tau_D = L^2 / D$$

volume of a crossing

$$\lambda^{d-1} l_e$$

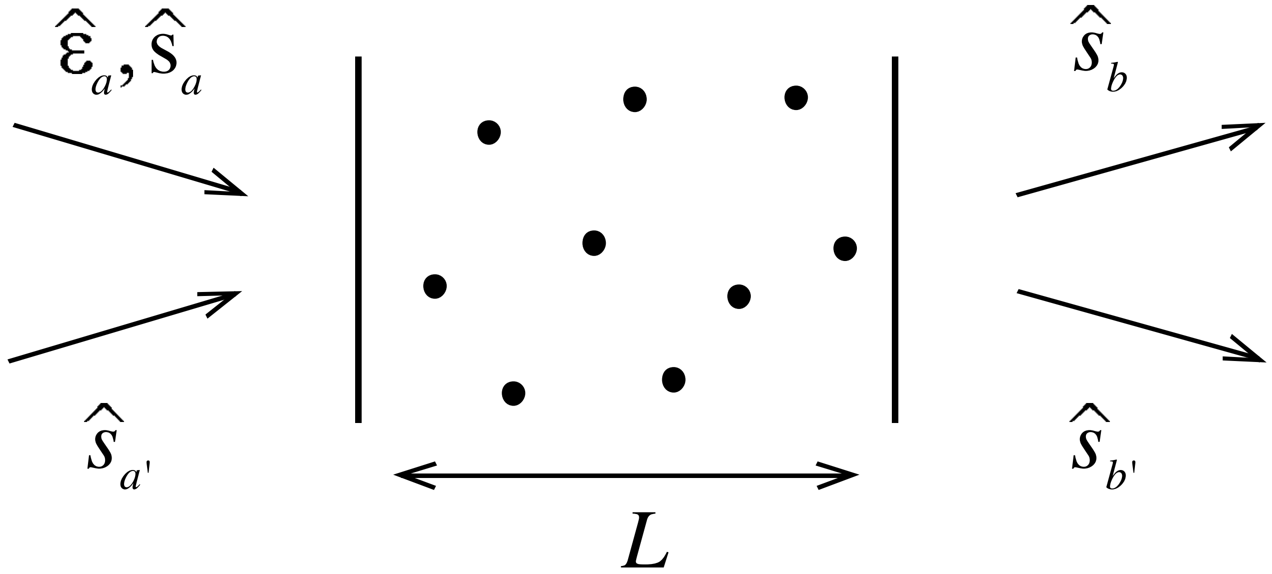


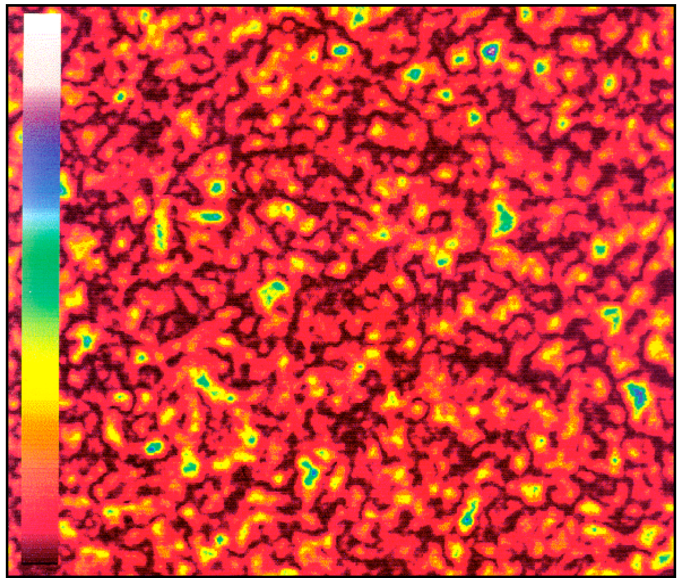
$$g = \frac{l_e}{3\lambda^{d-1}} L^{d-2} \gg 1$$

Weak disorder physics

- Weak disorder limit: $g \gg 1$
- The probability of a quantum crossing is small : phase-coherent corrections to the classical limit are small.
- Quantum crossings are independently distributed : generate higher order corrections to the Diffuson as an expansion in powers of $1/g$

Consider the transmission coefficient $\mathcal{T}_{ab}$





Outgoing photons build a speckle pattern
i.e. an interference picture

How to characterize a speckle pattern ?

Calculate the *angular correlation function*

$$C_{aba'b'} = \frac{\overline{\delta\mathcal{T}_{ab}\delta\mathcal{T}_{a'b'}}}{\overline{\mathcal{T}}_{ab}\,\overline{\mathcal{T}}_{a'b'}}$$

with $\delta\mathcal{T}_{ab} = \mathcal{T}_{ab} - \overline{\mathcal{T}}_{ab}$

The transmission coefficient $\mathcal{T}_{ab}$ is a random variable.

For scattering of a classical wave by classical scatterers, a speckle is well characterized by the Rayleigh law:

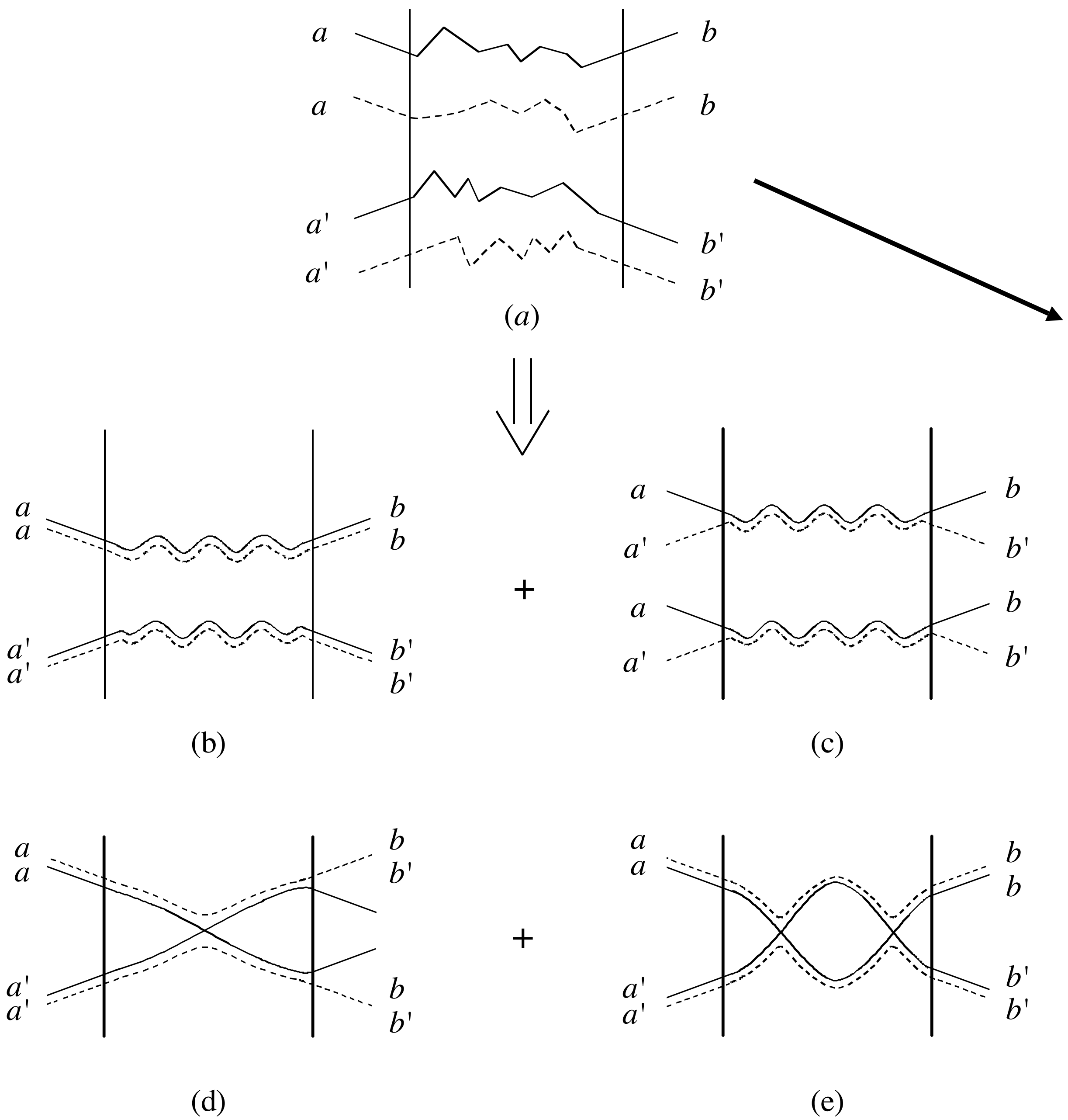
$$\overline{\delta \mathcal{T}^2} \equiv \overline{\mathcal{T}^2} - \overline{\mathcal{T}}^2 = \overline{\mathcal{T}}^2$$

which accounts for the granular aspect of a speckle : relative fluctuations are of order unity.

The Rayleigh law simply expresses that a speckle pattern results from the coherent superposition of a large number of uncorrelated random, complex valued amplitudes.

The Rayleigh law characterizes the interference pattern of a complex wave system.

Fluctuations and correlations



Slab geometry

transmission coefficient

$$T_{ab} = |t_{ab}|^2$$

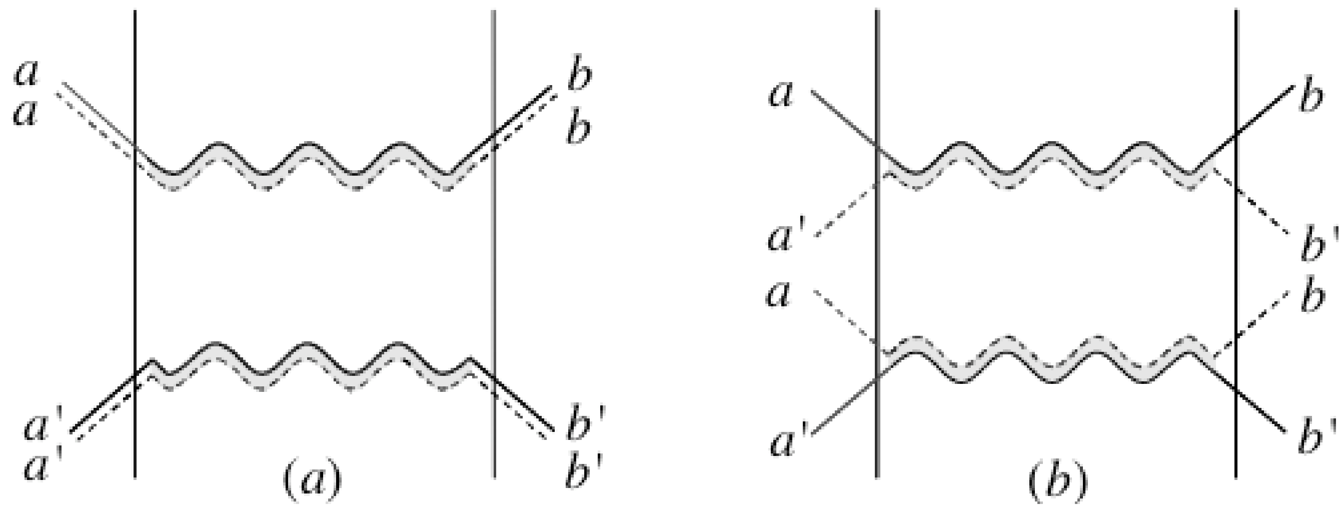
correlations involve the product of 4 complex amplitudes with or without quantum crossings

Correlation function of the transmission coefficient :

$$C_{aba'b'} = \frac{\overline{\delta T_{ab} \delta T_{a'b'}}}{\overline{T}_{ab} \overline{T}_{a'b'}}$$

Angular correlations of speckle patterns

0-quantum crossing correlations:



$$C^{(1)}_{aba'b'} = \delta_{\Delta\hat{s}_a,\Delta\hat{s}_b}\left(\frac{q_a L}{\sinh q_a L}\right)^2$$

with $q_{a,b} = k\left|\Delta\hat{s}_{a,b}\right|$

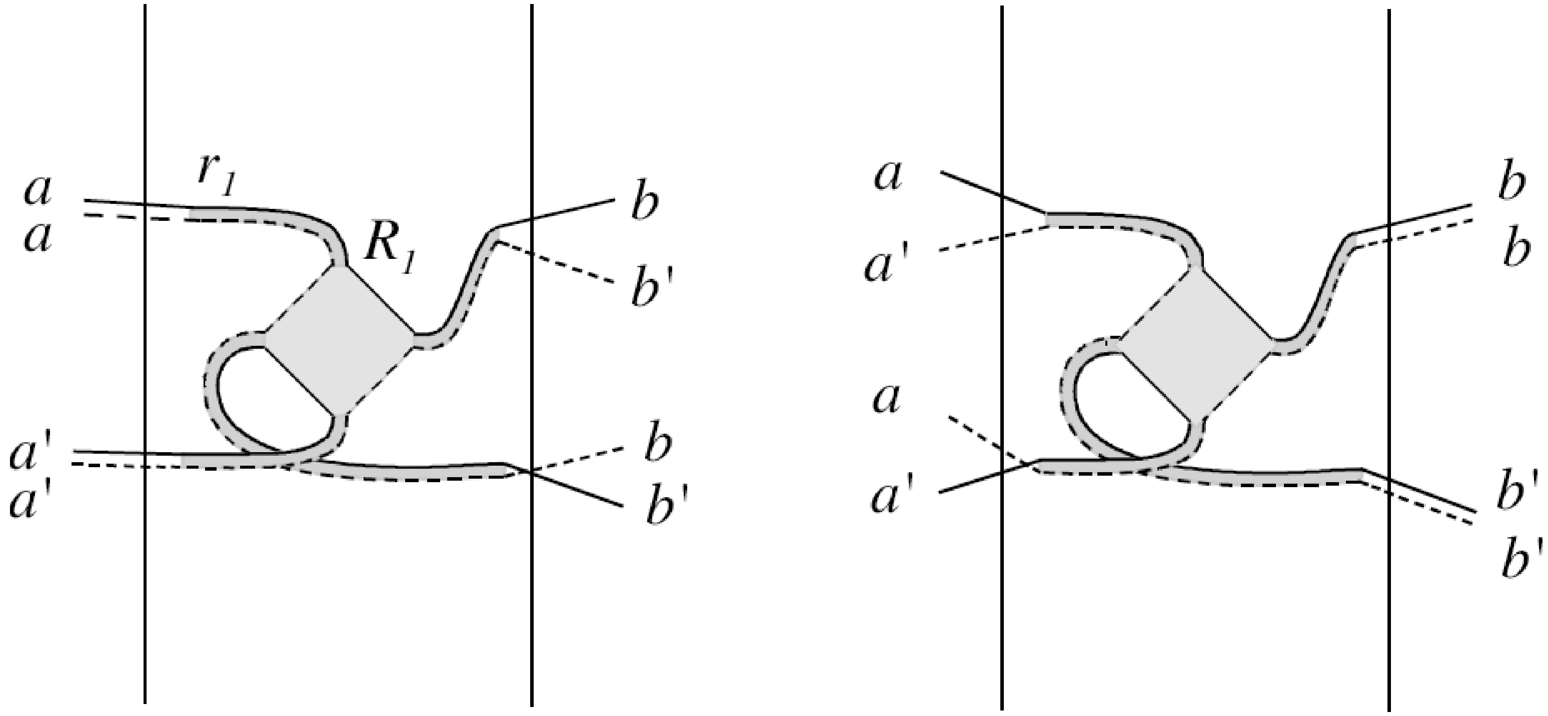
$$\Delta\hat{s}_a = \hat{s}'_a - \hat{s}_a$$

$$C^{(1)}: \qquad (aa')(aa') \longrightarrow (bb')(bb')$$

Rayleigh law:

$$C^{(1)}_{abab} = 1 \quad \Longrightarrow \quad \overline{\delta \mathcal{T}^2_{ab}} = \overline{\mathcal{T}}_{ab}^{\,2}$$

1-quantum crossing correlations



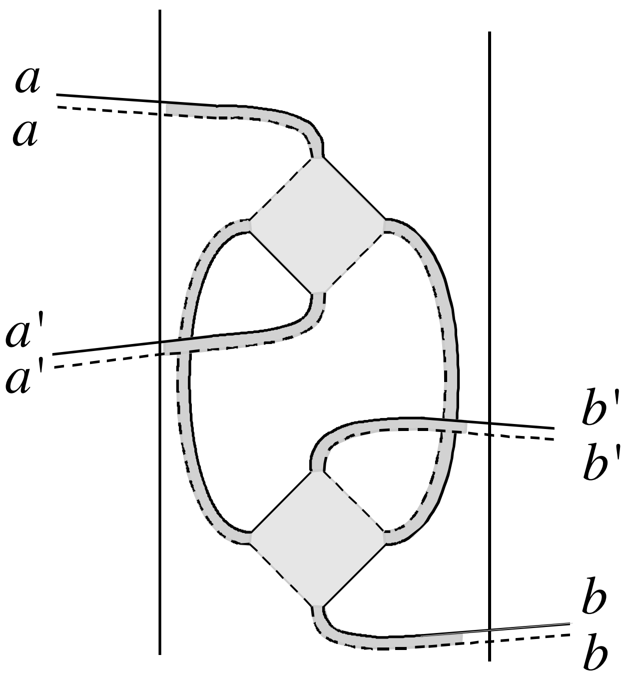
Long range correlation : $C^{(2)}_{aba'b'} = \frac{1}{g}\left[F_2(q_a L) + F_2(q_b L)\right]$

$$F_2(x) = \frac{1}{\sinh^2 x}\left(\frac{\sinh 2x}{2x} - 1\right) \Longrightarrow C^{(2)}_{abab'} \xrightarrow{b \neq b'} \frac{2}{3g}$$

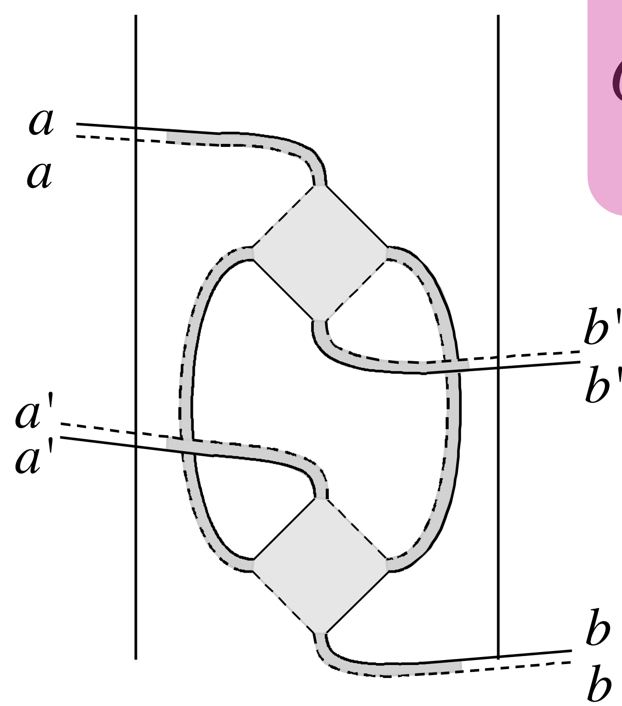
$$C^{(2)} : \begin{array}{l} (aa)(a'a') \longrightarrow (bb')(bb') \\ (aa')(aa') \longrightarrow (bb)(b'b') \end{array}$$

$C^{(2)}$ does not contribute to universal conductance fluctuations

2-quantum crossings correlations



(K_1^d)

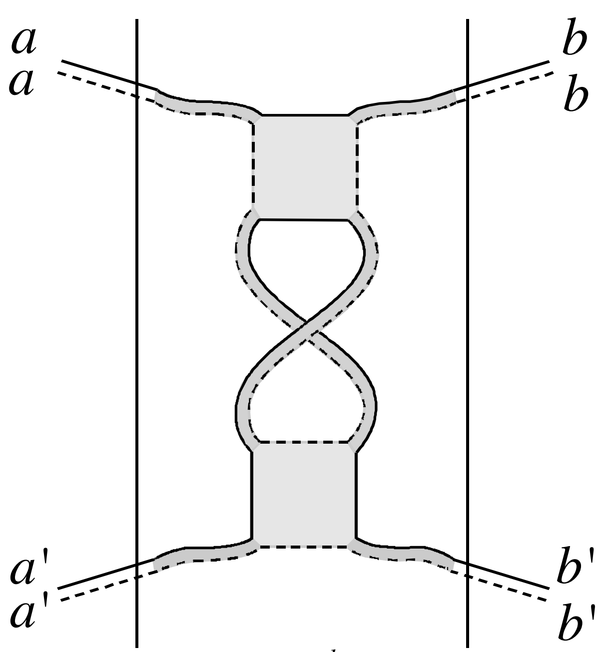


(K_2^c)

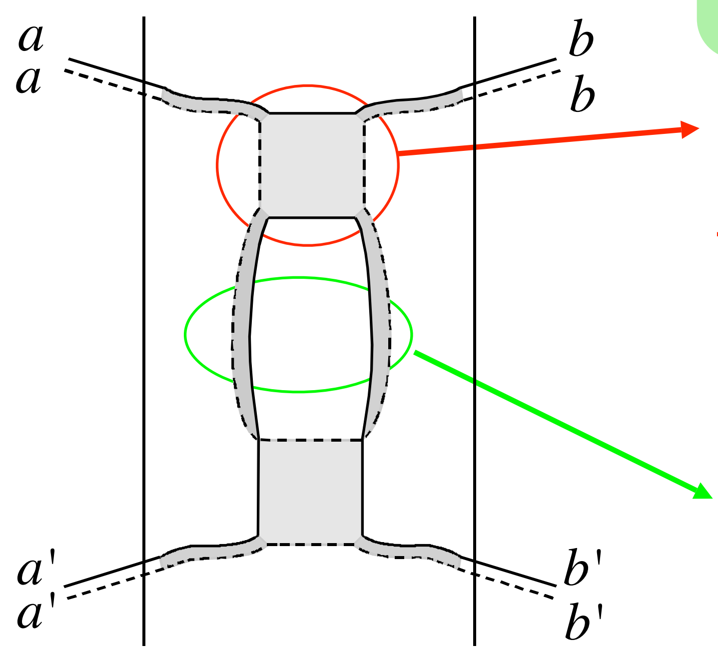
$$C^{(3)}_{aba'b'} = \frac{12}{g^2}\frac{D^2}{L^4}\int_0^L \int_0^L dz dz' P_{int}(z,z')^2$$

$$= \frac{2}{15}\frac{1}{g^2}$$

$C^{(3)}$ is independent of angular channels: uniform background that extends over the entire system.



(K_3^d)



(K_3^c)

Crossings: coherent effects, spatially localized in a volume $\lambda^{d-1}l$

Long range diffusons (classical)

Time dependent speckle patterns

Uneasy experimentally to separate the contributions to the angular correlations: long range parts are small compared to $C^{(1)}$ and the universal $C^{(3)}$ contribution affects equally all the speckle spots

To separate the different contributions we study their dependence as a function of an additional parameter : time correlations

Dephasing-decoherence

Quantum mesoscopics effects result from interferences and as such are very sensitive to dephasing.

Sources of dephasing:

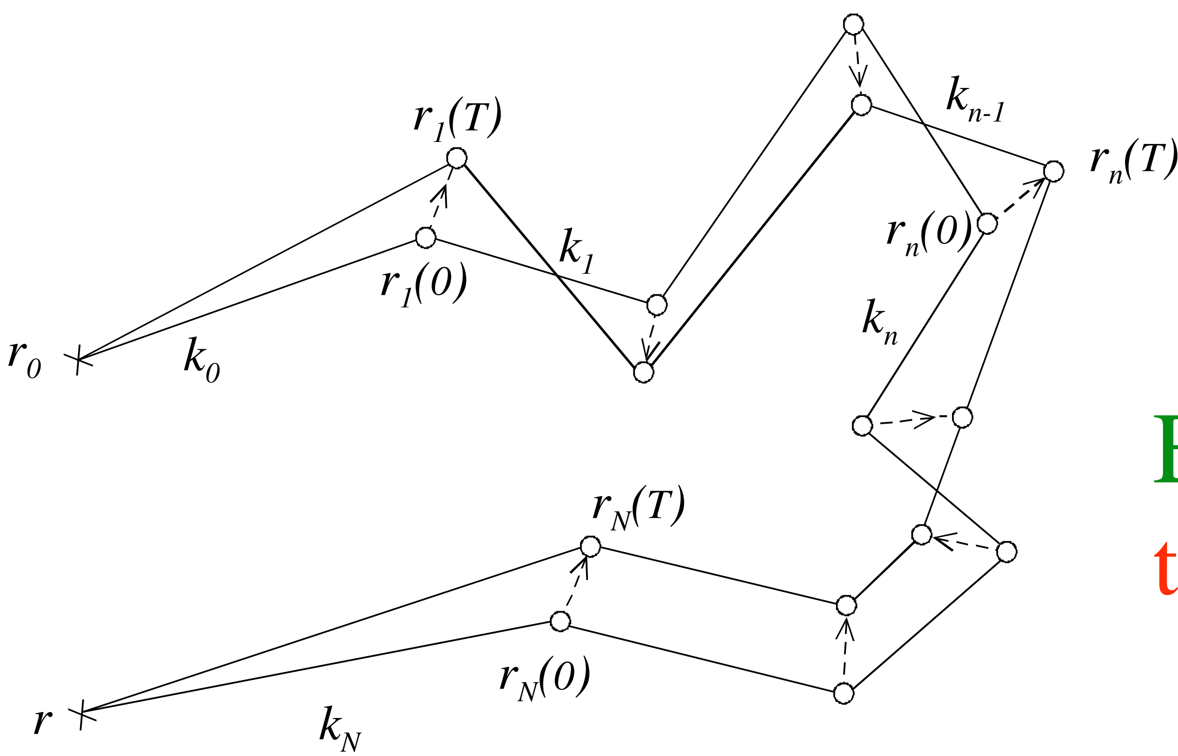
- External magnetic field, Aharonov-Bohm flux,
- Degrees of freedom of the waves (electron spin, photon polarization)
- Degrees of freedom of the scatterers: dynamics, Zeeman sub-levels

The probability $P(r,r',t)$ is affected by a global dephasing

$$\left\langle e^{i\Delta\phi(t)} \right\rangle \approx e^{-t/\tau_\varphi}$$

$L_\varphi = D\tau_\varphi$ is the phase coherence length

Dynamics of the scatterers



Complex amplitude

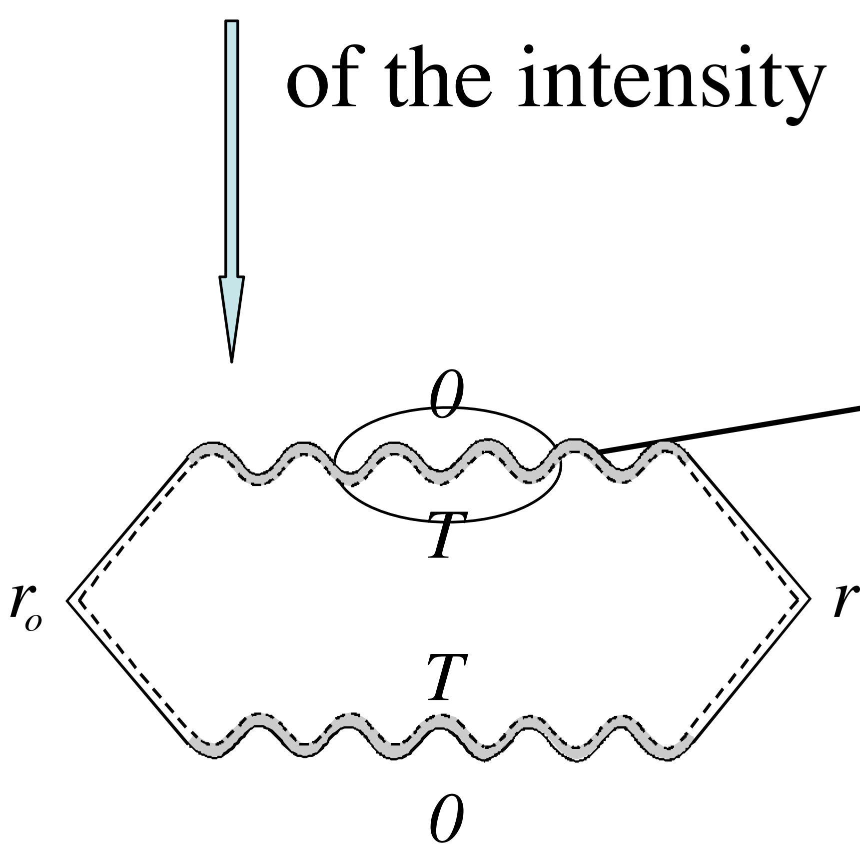
$$E(r_0,r,t) = \sum_{C_N} A(r_0,r,C_N(t))\, e^{i\phi_N(t)}$$

Ergodic assumption: $C_N(t) = C_N(0)$

the ensemble of multiple scattering sequences is independent of time

Time correlation function: of the intensity

$$\langle E(r,T)E^*(r,0)\rangle = \frac{c}{4\pi}\int_0^\infty P_{cl}(r_0,r,t)e^{-t/\tau_\phi(T)}dt$$

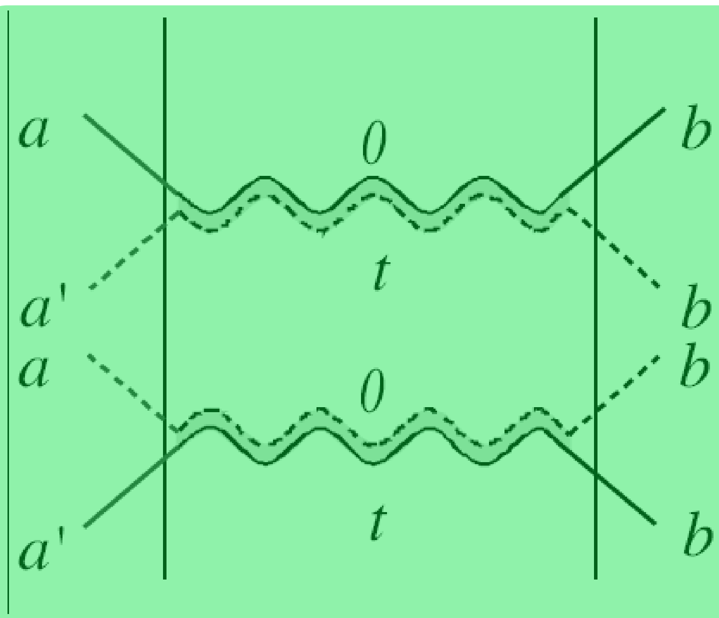


Dephasing effects result from Diffusons built from the pairing of 2 amplitudes which belong to distinct and out of phase realisations.

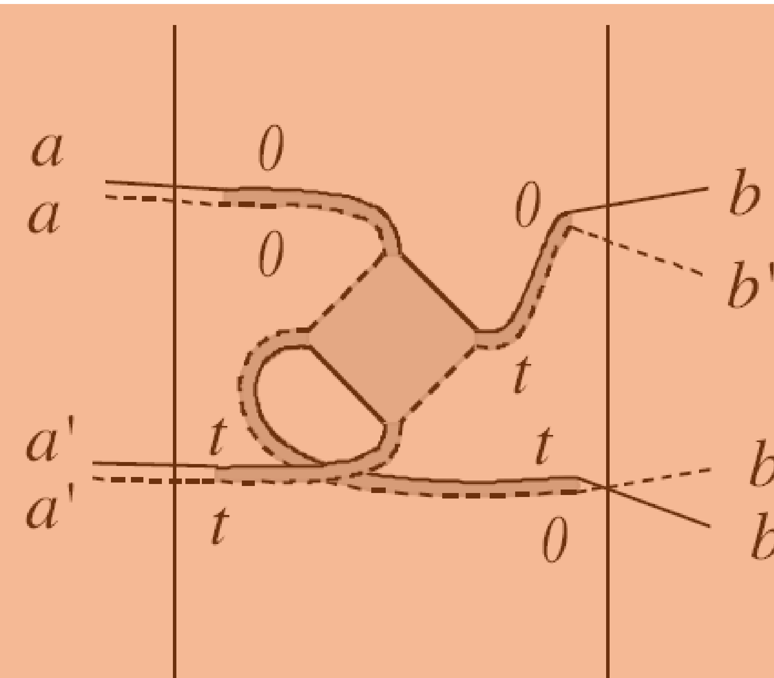
Time correlations of speckle patterns

Uneasy experimentally to separate the contributions to the angular correlations: long range parts are small compared to $C^{(1)}$ and the universal contribution affects equally all the speckle spots $C^{(3)}$

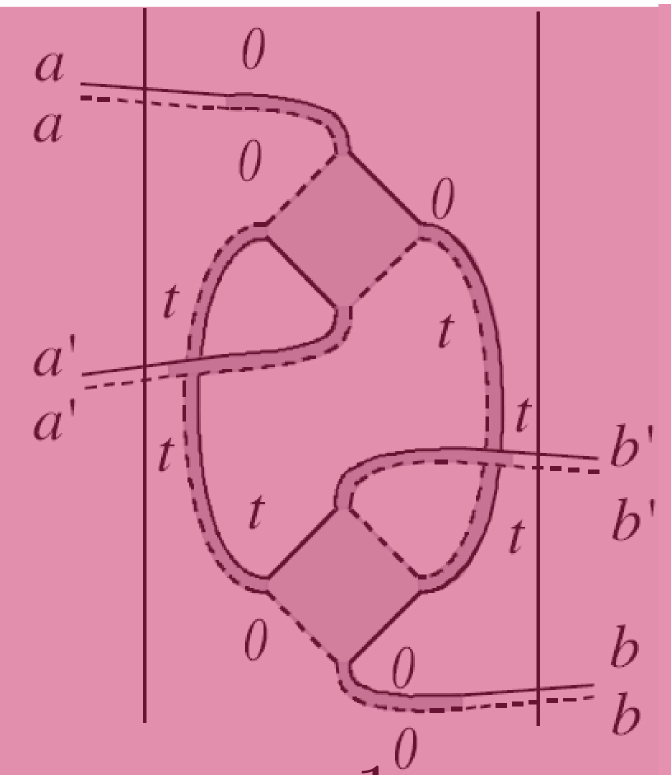
To separate the different contributions we study their dependence as a function of an additional parameter : time correlations



$$C^{(1)}(t) = \left(\frac{L/L_\phi}{\sinh L/L_\phi}\right)^2$$



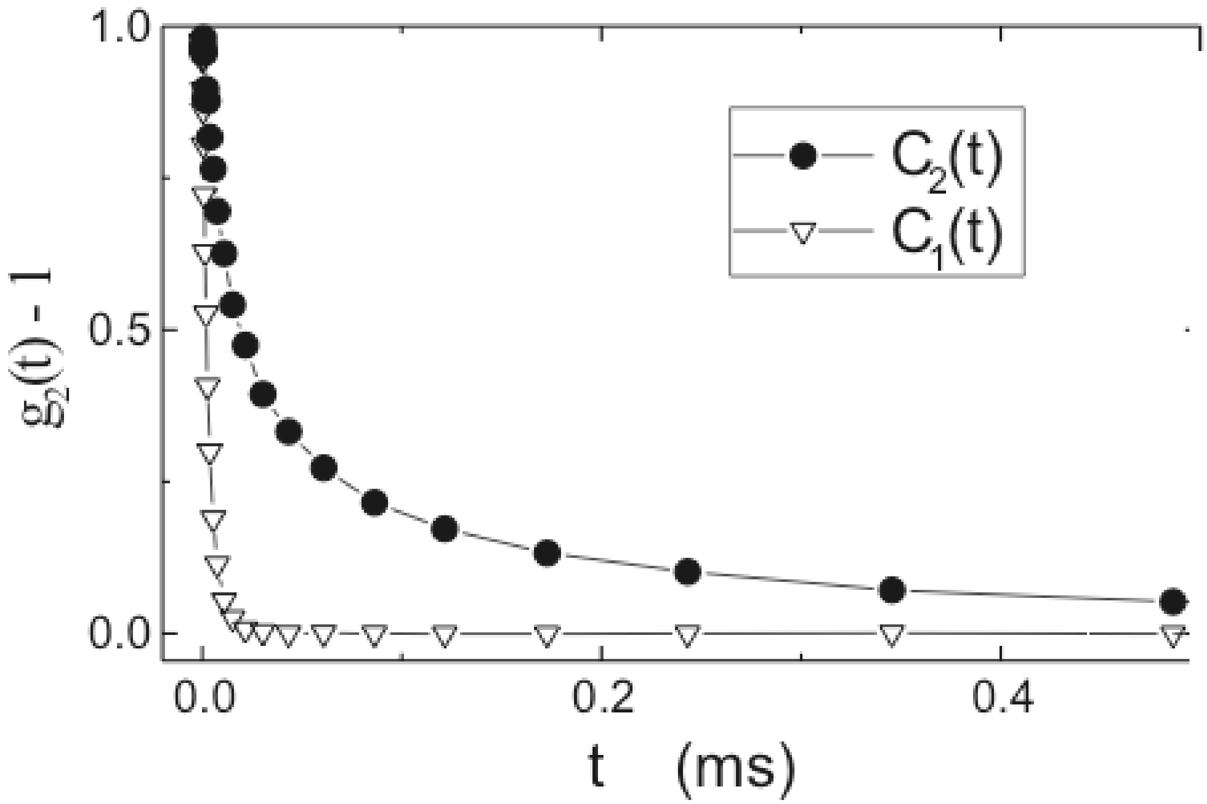
$$C^{(2)}(t) = \frac{2}{g} F_2(L/L_\phi)$$

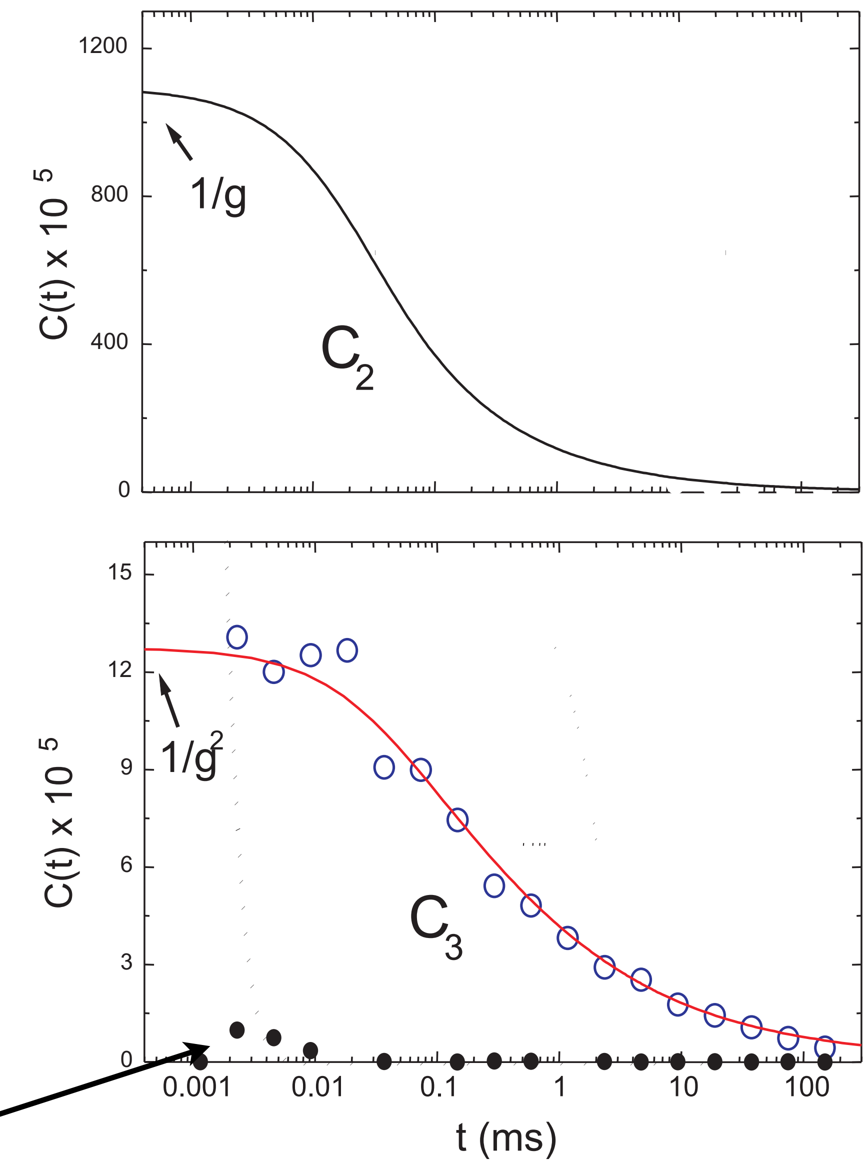


$$C^{(3)}(t) = \frac{1}{g^2} F_3(L/L_\phi)$$

Experiments

F. Scheffold and G. Maret (1998)





$C^{(1)}(t)$

Frequency dependence: A.Lagendijk et al. (1992)

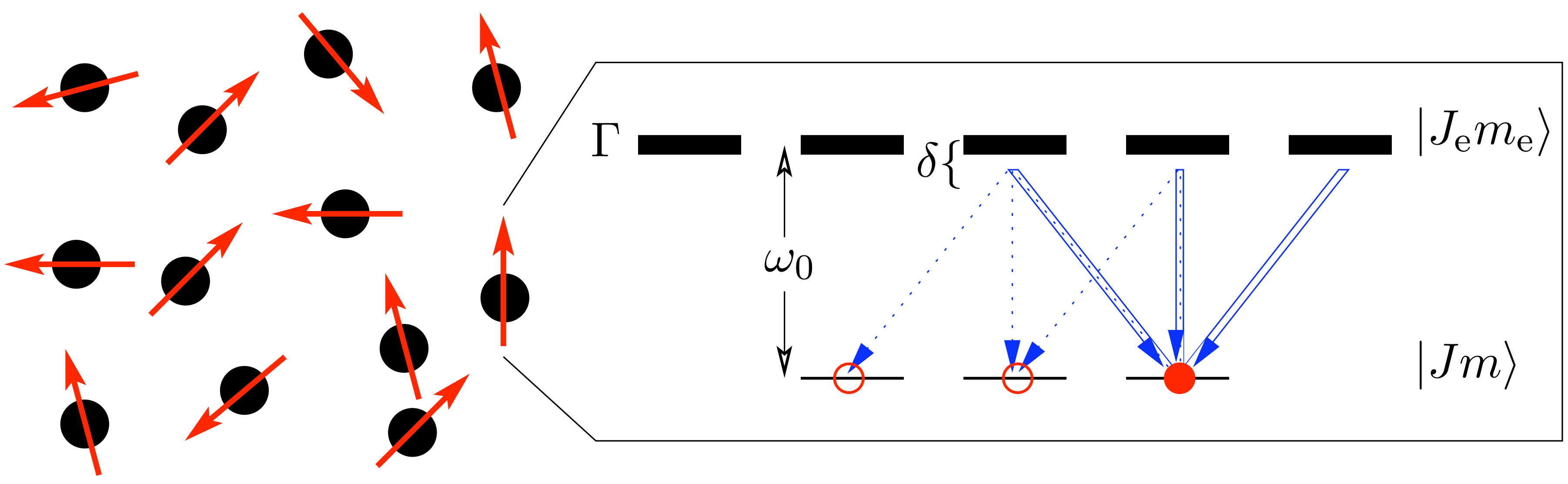
Photon correlations induced by disorder in a quantum mesoscopic gas

Include atomic degrees of freedom

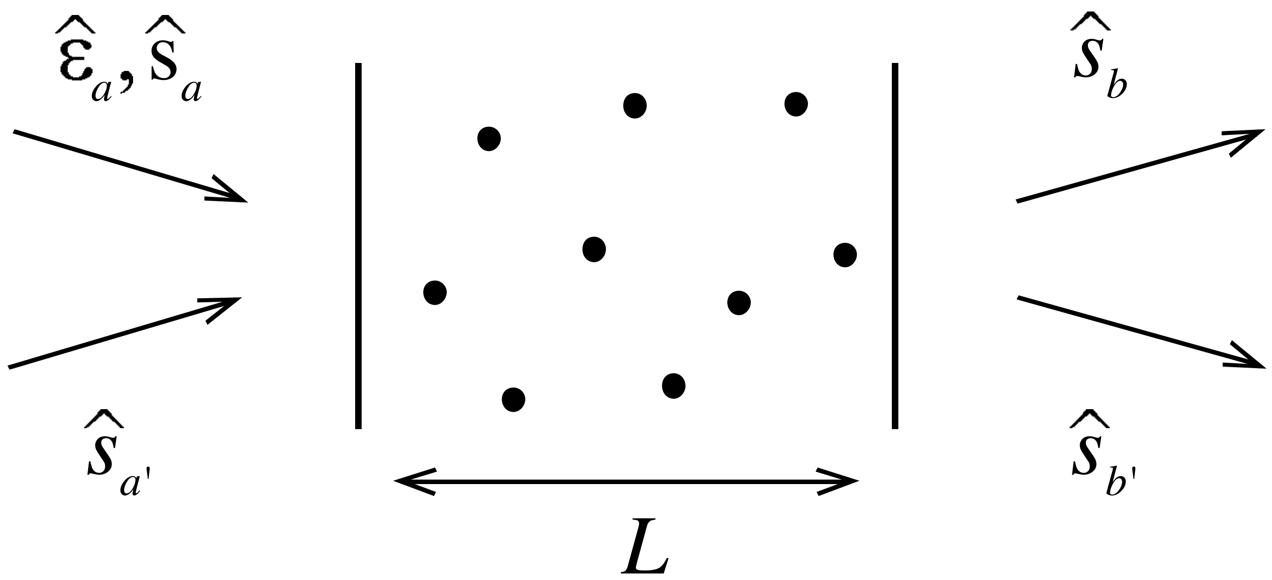
Ohad Assaf, E.A.

Framework:

Multiple scattering of photons by a cold atomic gas.



Consider the transmission coefficient $\mathcal{T}_{ab}$



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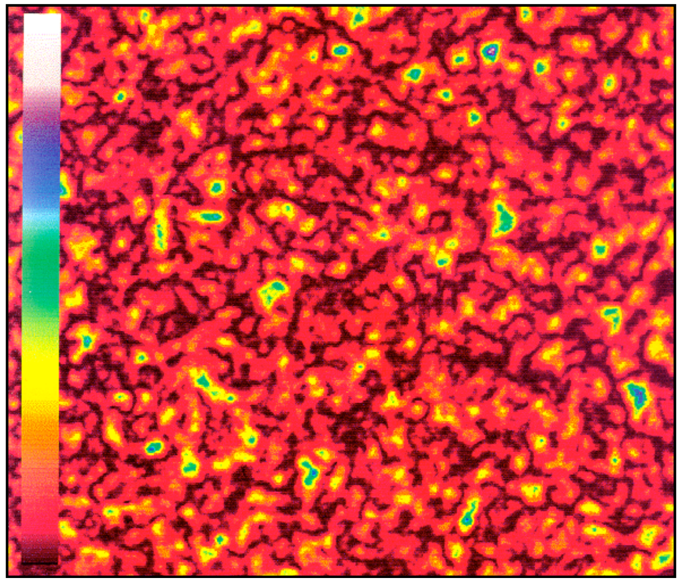

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The Rayleigh law characterizes the interference pattern of a complex wave system.

Two new features

- For a gas of atoms with degenerate Zeeman sublevels, the intensity correlations are enhanced well above the Rayleigh limit:

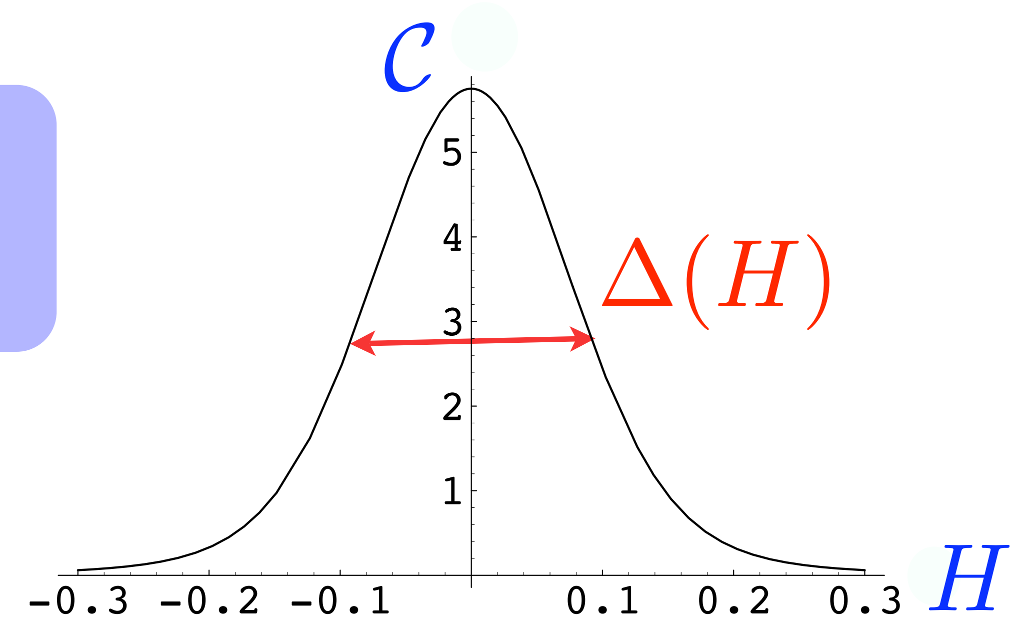
$$\overline{\delta\mathcal{T}^2} > \overline{\mathcal{T}}^2$$

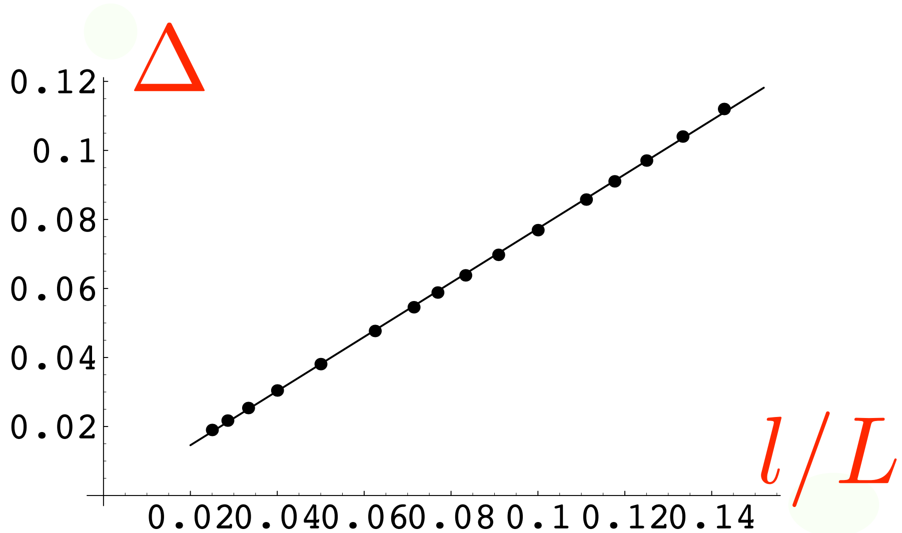
Contrast between bright and dark spots is enhanced.

This enhanced variance results from interference based on the effect of spatial disorder and atomic quantum internal degrees of freedom.

- This large contrast speckle interference pattern is very sensitive to dephasing such as Zeeman splitting induced by a magnetic field H which removes the ground state degeneracy of the atoms. Resonant-like shape.

$$\mathcal{C}^2 = \frac{\overline{\delta\mathcal{T}^2}}{\overline{\mathcal{T}}^2} - 1$$

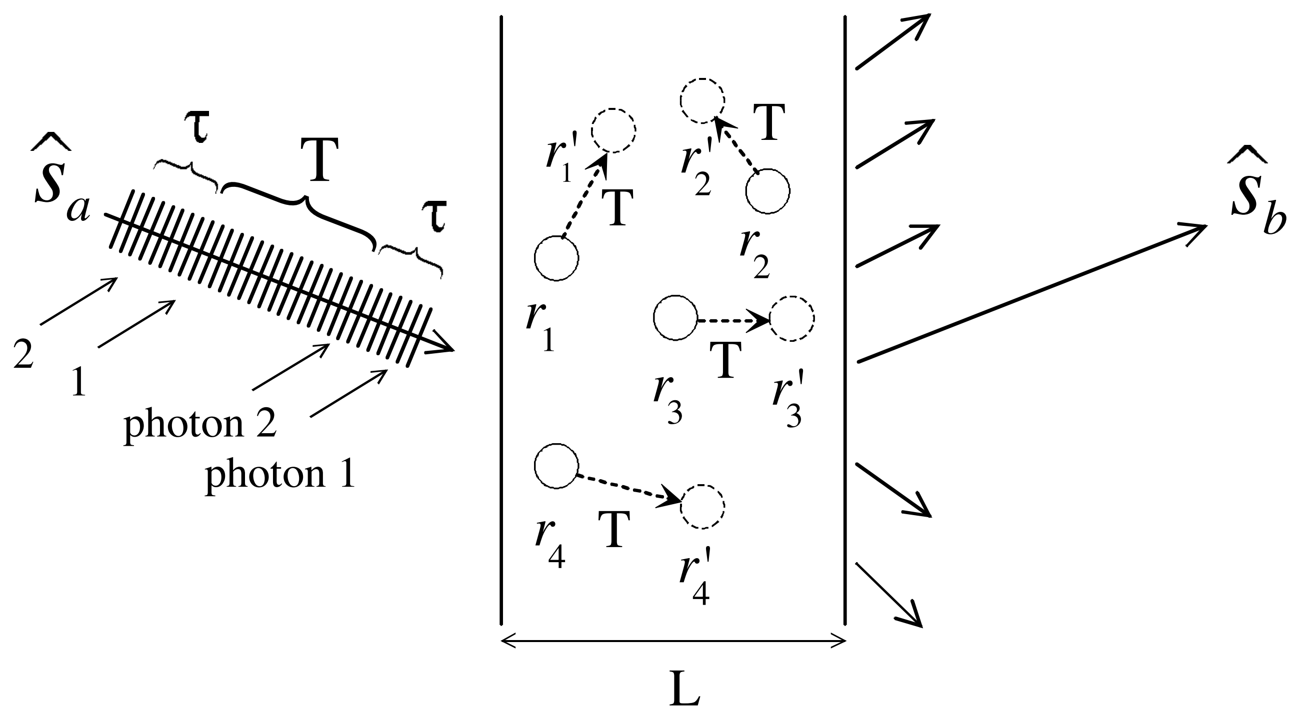




$$\Delta(H) \simeq \frac{\hbar\Gamma}{g\mu_0}\frac{l}{L}$$

The linewidth $\Delta(H)$ is reduced by a large factor L/l as compared to other spectroscopic techniques routinely used (Hanle or Franken effects)

Experimental setup



During the time $\tau \gg \Gamma^{-1}$ the atoms stay at rest. However the two photons 1 and 2 experience different atomic internal configurations due to all photons between them.

The measurement is repeated after a time $T \gg \tau$ during which atoms move.

The configuration average results from this motion.

Correlation enhancement

1) Single atom: detected photon intensity

$$I_s = \frac{1}{2j_g+1}\sum_{\{m\}}\left|A^{\{m,R\}}\right|^2 \qquad \{m\} = (m_1, m_2)$$

Summation over m_1 is a statistical average over the atomic ground state with a weight $1/(2j_g+1)$

Summation over m_2 results from non-detected final states.

$A^{\{m,R\}}$ is a sum over quantum amplitudes:

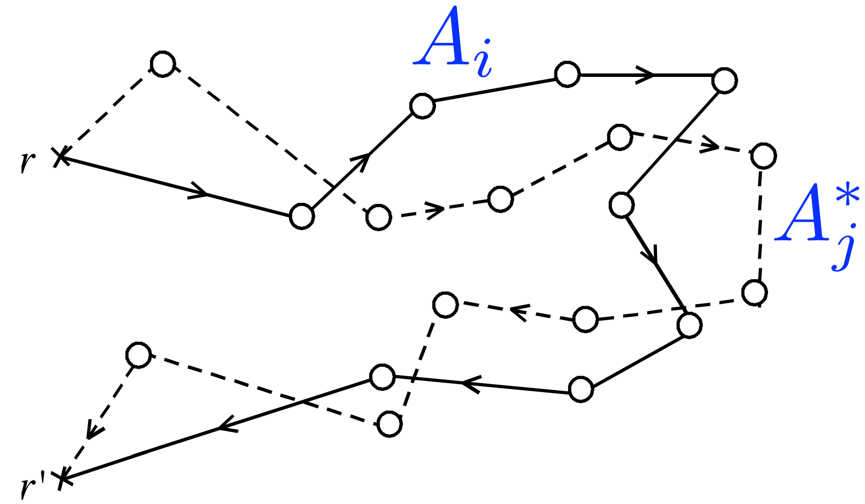
$$A^{\{m,R\}} = e^{i\phi(R)}\sum_{m_e}\frac{\langle m_2|V|m_e\rangle\langle m_e|V|m_1\rangle}{\omega-\omega_{m_1 m_e}+i\frac{\Gamma}{2}}$$

that corresponds to a given and fixed position $\{R\}$ of the atom.

The operator $V=-\mathbf{d}.\mathbf{E}$ describes the dipolar interaction between atoms and photons.

classical phase: $\phi = ik(|\mathbf{R}-\mathbf{R}_s|+|\mathbf{R}_d-\mathbf{R}|)$

2) Multiple scattering by more than one atom:



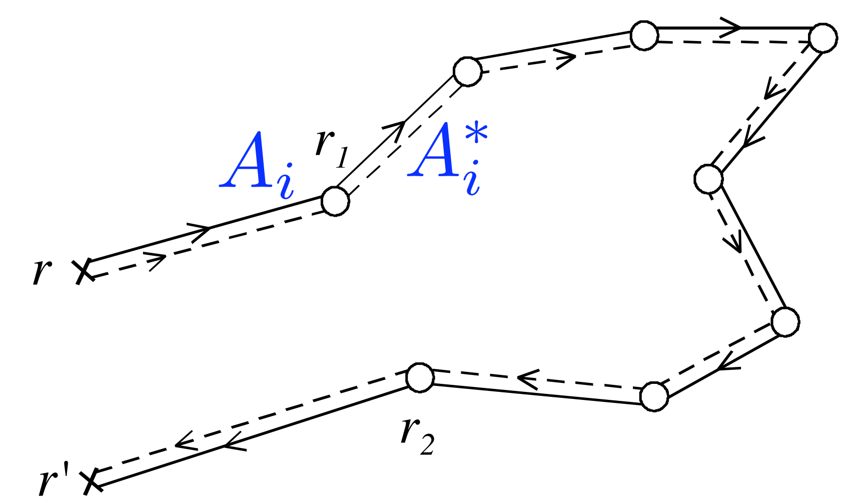
$$I(R) = \frac{1}{(2j_g+1)^2} \sum_{\{m\}} \left| \sum_i A_i^{\{m,R\}} \right|^2$$

$\{R\}$ spatial configuration of the atoms

$\{m\}$ configuration of the ground state sublevels

Internal atomic degrees of freedom are independent of spatial positions : $A_i^{\{m,R\}} = A_i^{\{m\}} e^{i\phi_i(\{R\})}$

Configuration average : $\langle e^{i(\phi_i - \phi_j)} \rangle = \delta_{ij}$

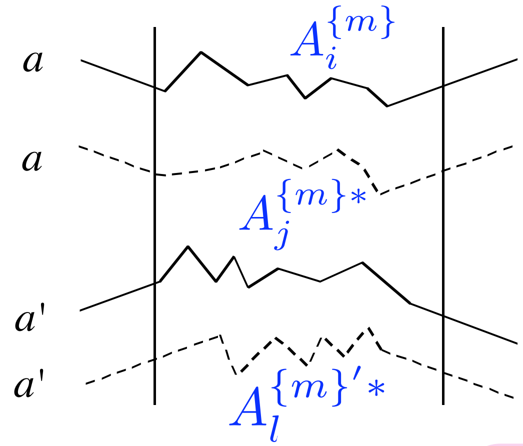


$$\langle I \rangle = \frac{1}{(2j_g+1)^2} \sum_{\{m\}} \sum_i |A_i^{\{m\}}|^2$$

The average intensity is a sum of intensities i.e. without interferences between scattering trajectories. But each $A_i^{\{m\}}$ remains a sum of quantum amplitudes thus leading to quantum interference effects.

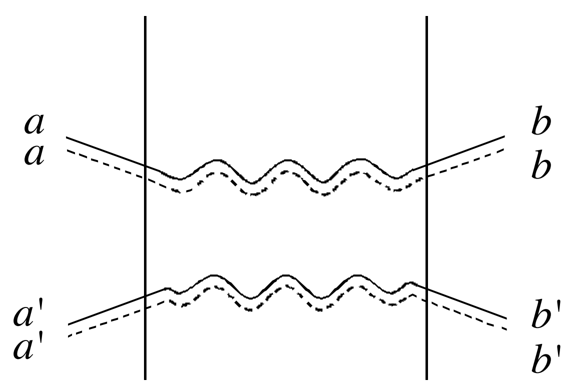
Correlations

$$\langle I(R)I'(R)\rangle = \frac{1}{(2j_g+1)^4}\sum_{\{m\}\{m'\}}\sum_{ijkl} A_i^{\{m\}}A_j^{\{m\}*}A_k^{\{m'\}}A_l^{\{m'\}*}\langle e^{i(\phi_i-\phi_j+\phi_k-\phi_l)}\rangle$$



$$\langle e^{i(\phi_i-\phi_j+\phi_k-\phi_l)}\rangle = \delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk}$$

$\Downarrow$



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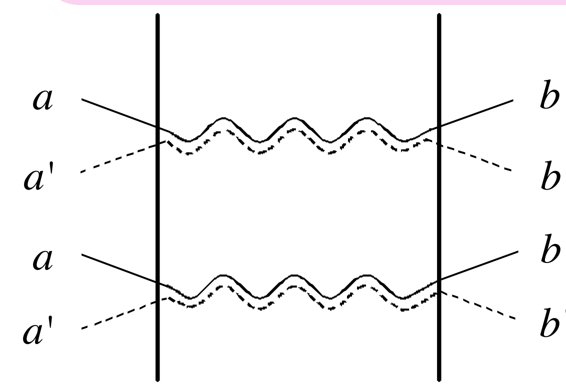


$\delta_{ij}\delta_{kl}$

$\langle I\rangle\langle I'\rangle$

$\delta_{il}\delta_{jk}$

Interference terms between distinct atomic configurations

$$\mathcal{C}(I,I') = \langle I(R)I'(R)\rangle - \langle I\rangle\langle I'\rangle$$

$$= \frac{1}{(2j_g+1)^4}\sum_{\{m\}\{m'\}}\sum_{i\neq j}\left(A_i^{\{m\}}A_i^{\{m'\}*}\right)\left(A_j^{\{m'\}}A_j^{\{m\}*}\right)$$

For a non degenerate atomic ground state : $j_g = 0$ and $\{m\} = \{m'\}$

and for $N \gg 1$ we recover the Rayleigh law:

$$\mathcal{C}(I, I') \equiv \langle I(R) I'(R) \rangle - \langle I \rangle \langle I' \rangle = \langle I \rangle \langle I' \rangle$$

But for a degenerate ground state, we always have:

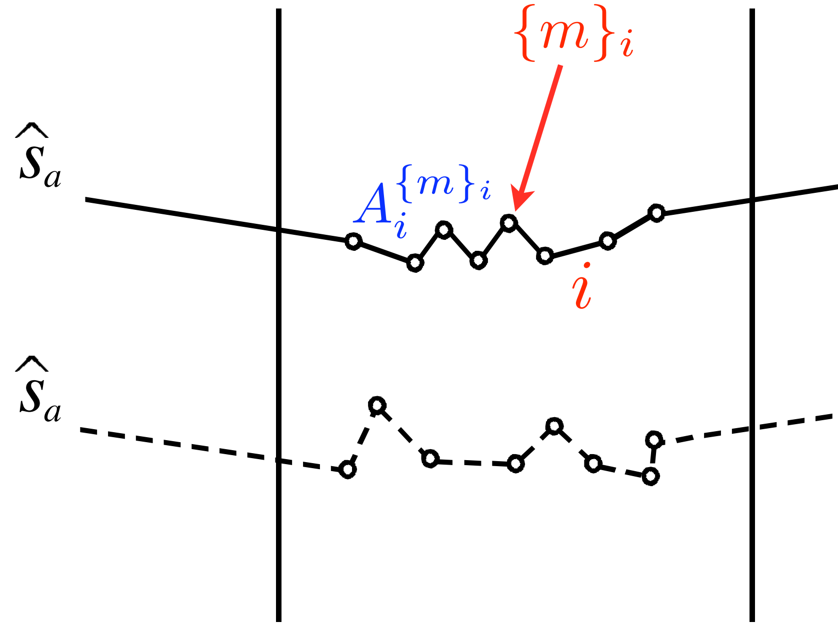
$$\mathcal{C}(I, I') > \langle I \rangle \langle I' \rangle$$

namely correlations enhanced above the Rayleigh limit.

Proof in the limit of a large number of atoms N>>1.

For a fixed spatial configuration $\{R\}$ of atoms, among all possible couples of scattering trajectories of a photon, the ratio of those sharing at least one common scattering event to those without any common scatterer becomes negligible for large N.

$$\langle I \rangle \propto \sum_{\{m\}} \sum_i \left| A_i^{\{m\}} \right|^2 = (2j_g+1)^N \sum_i (2j_g+1)^{-N_i} \sum_{\{m\}_i} \left| A_i^{\{m\}_i} \right|^2$$



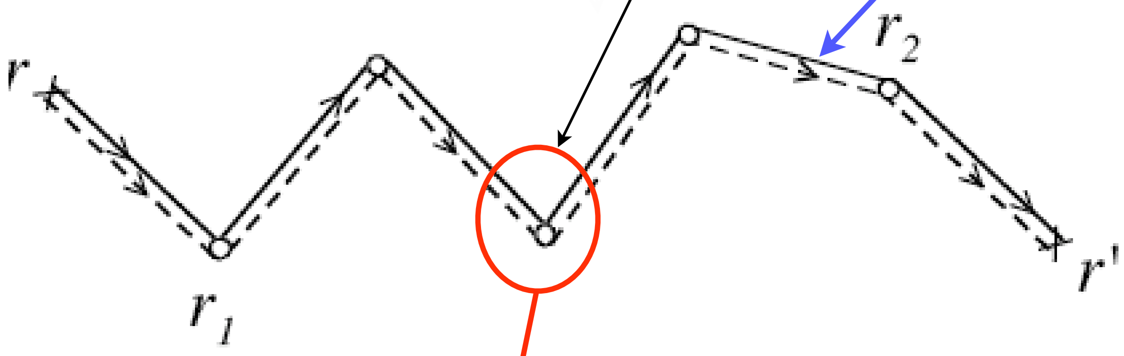
A typical scattering sequence is a succession of N_i independent scattering events, each being composed of paired single scattering amplitudes weighted by the statistical factor $1/(2j_g+1)$

$$\sum_{\{m\}\{m'\}} \sum_i A_i^{\{m\}} A_i^{\{m'\}*} \simeq (2j_g+1)^N \sum_i (2j_g+1)^{-2N_i} \sum_{\{m\}_i \{m'\}_i} A_i^{\{m\}_i} A_i^{\{m'\}_i *}$$

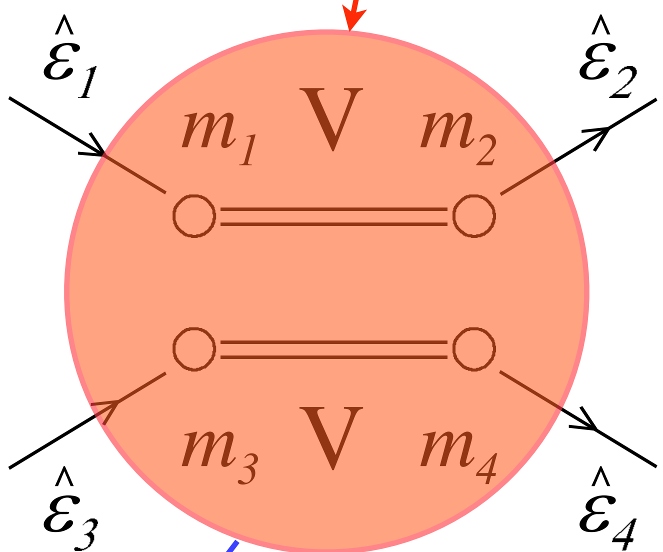
so that $\mathcal{C}(I, I') \geq \langle I \rangle \langle I' \rangle$

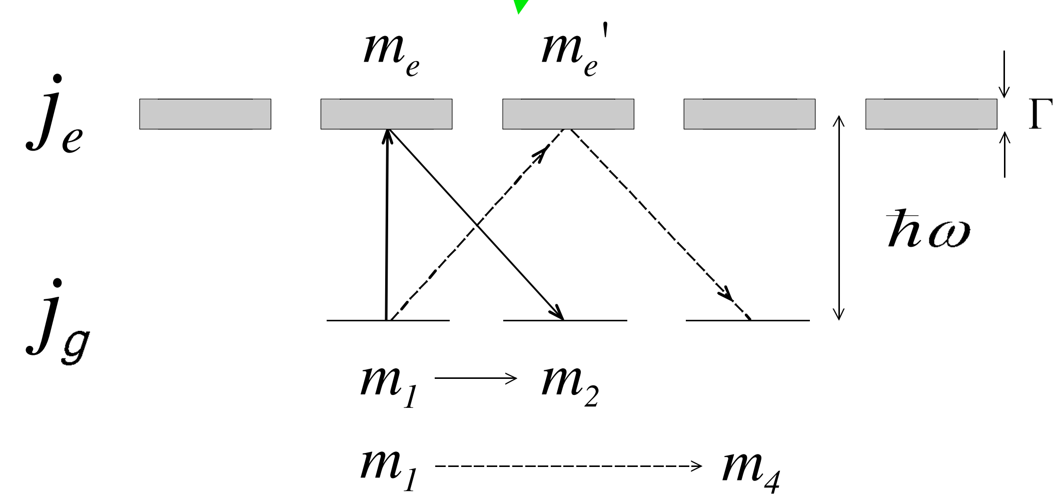
Iteration of an elementary vertex

$$\mathcal{D} = \mathcal{V} + \mathcal{V}\mathcal{W}\mathcal{V} + \cdots = \mathcal{V} + \mathcal{D}\mathcal{W}\mathcal{V}$$



process that contributes to the correlation but not to the average intensity



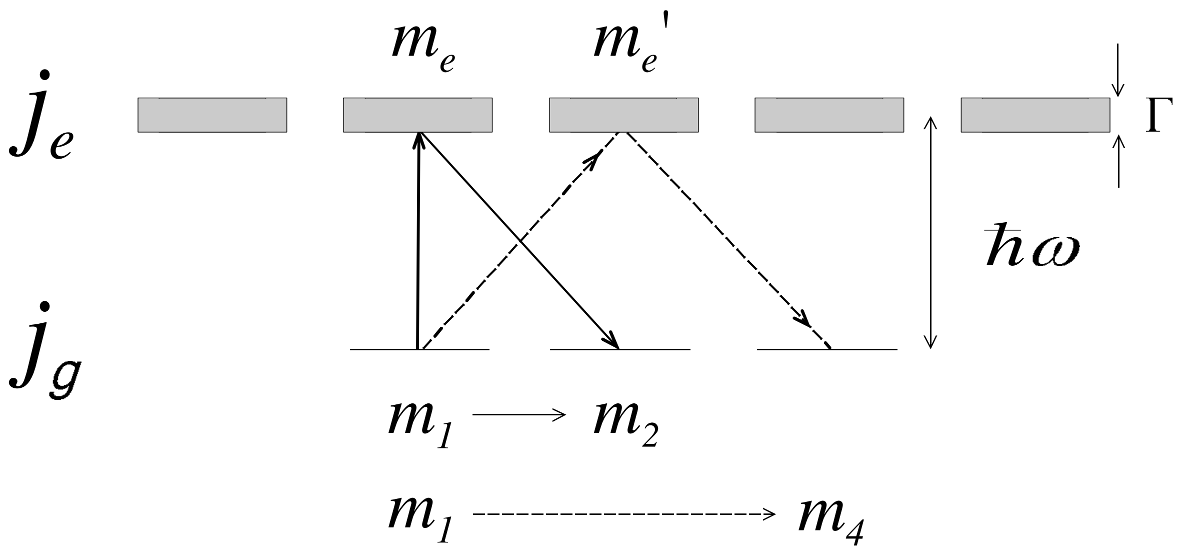


$$\mathcal{V}(\{m\},\{m'\}) = \frac{1}{(2j_g+1)^2} \sum_{m_e m_e'} \frac{\langle m_2|V|m_e\rangle\langle m_e|V|m_1\rangle\langle m_4|V|m_e'\rangle^*\langle m_e'|V|m_3\rangle^*}{(\omega - \omega_{m_1 m_e} + i\frac{\Gamma}{2})(\omega - \omega_{m_3 m_e'} - i\frac{\Gamma}{2})}$$

Dephasing by a magnetic field

The enhanced correlation is an interference mesoscopic effect : it is sensitive to a dephasing process.

For instance an applied magnetic field *H* removes the ground state degeneracy so that the enhanced correlation reduces back to the Rayleigh law.



this contribution vanishes for a large enough magnetic field since Zeeman splitting takes it far from resonance

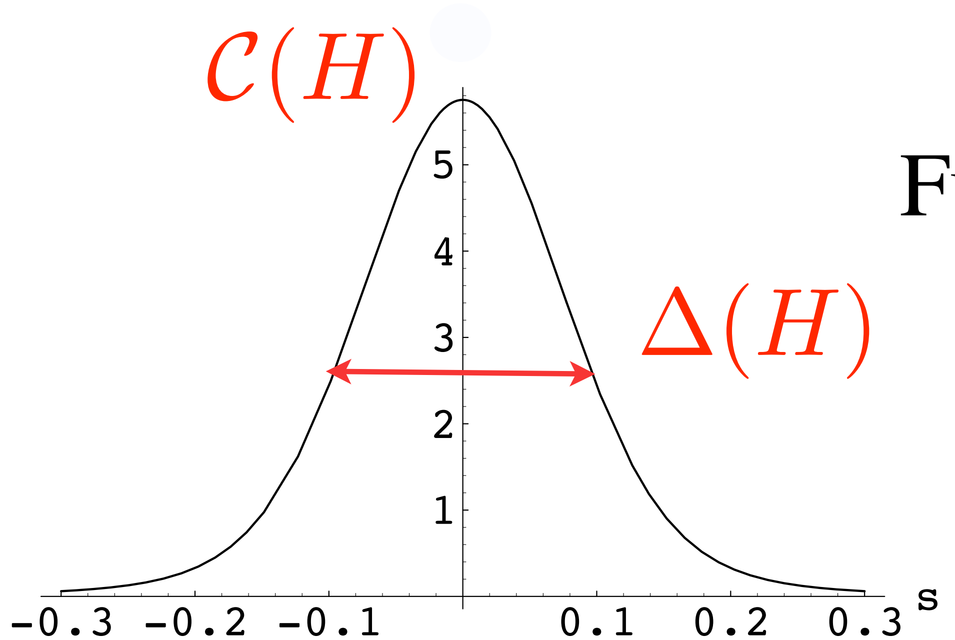
Calculate $\mathcal{C}(H)$ using the diffusion approximation for the multiple scattering of photons leads to

$$\mathcal{C} = Y_0^{(c)} \left(\frac{\sin^2(\frac{X}{b})}{X \sin X} - 2 \sin^2(\frac{\pi}{b}) \frac{e^{-\pi^2+X^2}}{\pi^2 - X^2} \right)$$ (not a lorentzian !)

$b = L/l$: optical depth

$X = b\sqrt{|f_0 - f_2 s^2|}$

$(f_0, f_2, Y_0^{(c)})$ depend on the features of the atomic transition



Full width at half max.
(FWHM)

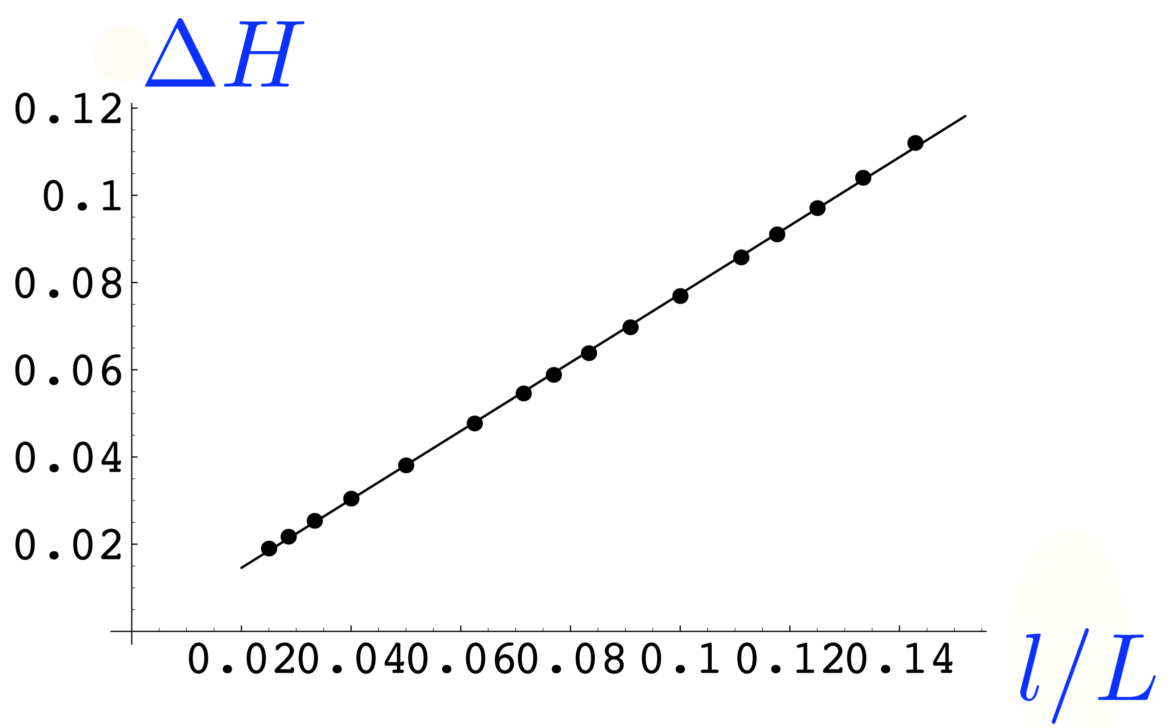
Full width at half maximum (FWHM) $\Delta(H)$ a simple derivation.

- Single scattering : the vertex $\mathcal{V}$ integrated over the frequency ω has a lorentzian shape : $1/(\delta^2+\Gamma^2)$ of FWHM Γ
- Two independent scatterings : the corresponding width is a product of 2 lorentzians so that the FWHM becomes $\Gamma(\sqrt{2}-1)^{1/2}$
- For $n \gg 1$ independent scatterings, the FWHM becomes $\simeq \Gamma/\sqrt{n}$
- Assuming a diffusion process for the photons, leads to $n \simeq (L/l)^2$ so that the FWHM becomes

$$\Delta(H) \simeq \Gamma \frac{l}{L}$$

the exact expression is $\Delta H \simeq a \frac{\hbar \Gamma}{g \mu_0} \frac{l}{L}$

with $a = 2\sqrt{\ln 2 / f_2}$



The FMHW of the "resonance" of $\mathcal{C}(H)$ may be, in principle, much smaller than the atomic linewidth Γ.

Typically, the optical depth $b = L/l \simeq 10^2$ so we can gain about two orders of magnitude compared to other spectroscopic methods (Hanle or Franken effects).