Lecture 5

Non exponential relaxations in mesoscopic conductors

- Quasiparticles and Fermi liquid theory
- Weakly disordered conductors
- Coherent effects
- Measurements
- The Fermi golden rule: non exponential decay
- Probing the non exponential decay

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Quasiparticles and Fermi liquid theory

Metals : electrons interacting through a strongly screened Coulomb interaction.

Quasiparticles : electrons dressed by the screening cloud of the other electrons

→ gas of "independent" particles.

Landau theory of Fermi liquids

Properties of the gas of QP's are those of a noninteracting electron gas with renormalized physical parameters.

QP's have a finite lifetime due to their residual interaction

Probability that a QP remains in its initial state is $P(t) = e^{-t/\tau_{ee}(\epsilon,T)}$

$$\frac{1}{\tau_{ee}(\epsilon, T)} = \max\left(\frac{\epsilon^2}{\epsilon_F}, \frac{T^2}{\epsilon_F}\right)$$

Energy ϵ is measured from the Fermi energy

At the Fermi level, QP's are well defined since the width $\hbar/\tau_{ee}$ of a state vanishes more rapidly than its energy .

Weakly disordered conductors (Altshuler- Aronov)

For a weak disorder ($k_F l \gg 1$), QPs have a diffusive motion.

Then, the Coulomb interaction between the QPs is **enhanced** as compared to a ballistic motion

$\longrightarrow$ Decrease of the QP lifetime.

$$\frac{1}{\tau_{ee}(\epsilon)} = \Delta \left(\frac{\epsilon}{E_c} \right)^{d/2}$$

$\Delta = 1/L^d \rho_0 =$ Mean level spacing

$E_c = \hbar D/L^2$ energy to diffuse in a volume L^d

Is the temperature dependence of $\tau_{ee}(\epsilon = 0, T)$ obtained by replacing $\epsilon \rightarrow T$ like for ballistic systems ?

Correct for $d = 3$ but not for $d \leq 2$

More details:

$$\frac{1}{\tau_{ee}} = 4\pi v_0^3 \int_0^{\epsilon} \omega W^2(\omega) d\omega$$

the function $W^2(\omega)$ accounts for effects due to disorder and to Coulomb interactions.

It is given by $$W^2(\omega) \propto \frac{1}{\omega^2} \left(\frac{\omega}{E_c} \right)^{d/2}$$

so that $$\boxed{\frac{1}{\tau_{ee}} \propto \int_0^{\infty} \frac{Z(t)}{t} \sin^2\left(\frac{\epsilon t}{2}\right)}$$

where $Z(t)$ is the probability to have a close diffusive trajectory (return to the origin) within a finite volume Ω

$$Z(t) = \Omega/(4\pi D t)^{d/2}$$

$$\frac{1}{\tau_{ee}} \propto \int_0^{\infty} \frac{Z(t)}{t} \sin^2\left(\frac{\epsilon t}{2}\right)$$

$$Z(t) = \Omega/(4\pi D t)^{d/2}$$

τ_{ee} decreases with the dimensionality d

large return probability corresponds to a large number of returning diffusive trajectories: in the excitation spectrum there is a large number of low energy ω excitations. (Polya theorem)

Finite temperature : $T \neq 0$

Convolution with the Fermi Dirac statistical factors:

$$\frac{1}{\tau_{ee}(\epsilon, T)} = 4\pi v_0^3 \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} F(\epsilon, \epsilon', \omega) W^2(\omega) d\epsilon'$$

Decay time at the Fermi level: $\tau_{ee}(\epsilon = 0, T)$

$$\boxed{\frac{1}{\tau_{ee}(T)} \propto T \int_0^T \frac{d\omega}{\omega^2} \left(\frac{\omega}{E_c}\right)^{d/2}}$$

Additional factor T/ω is responsible for the low energy divergence.

Large number of low energy ω excitations !

Self-consistent description:

(Altshuler, Aronov, Khmelnitskii, 1985)

The states are defined with an energy larger than $\hbar/\tau_{in}(T)$

Therefore
$$\frac{1}{\tau_{in}(T)} \propto T \int_{\hbar/\tau_{in}}^{T} \omega^{\frac{d}{2}-2} d\omega$$

namely for $d = 1$ quasi-1d wire of section S)

$$\boxed{\tau_{in}(T) = \left(\frac{Te^2\sqrt{D}}{S\sigma\hbar^2}\right)^{-2/3}}$$

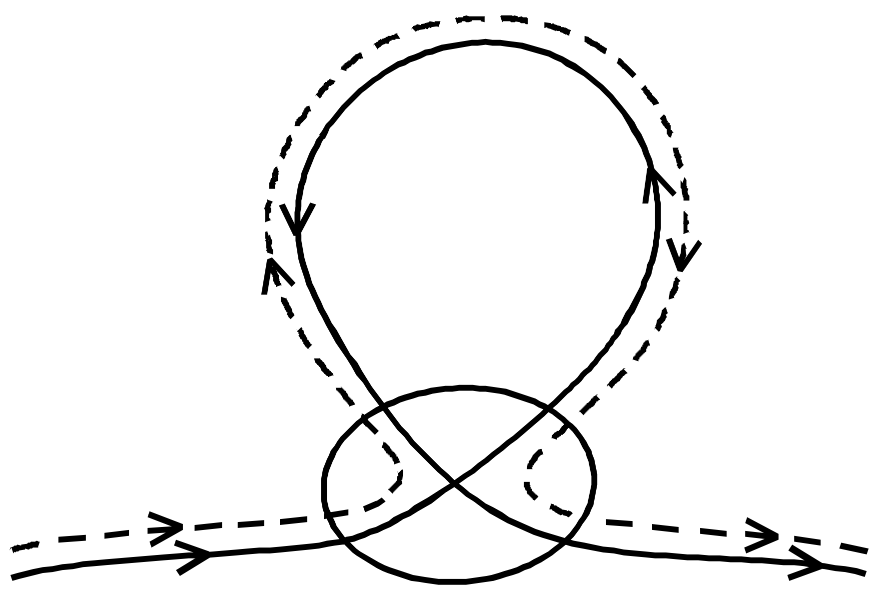
The QP relaxation is still exponential.

Remark: This does not affect the validity of the Fermi liquid description: $\hbar/\tau_{in} \ll T$

How to see this unusual dependence of τ_{in} in $d = 1$?

Coherent effects in multiple scattering:

Weak disorder $\longrightarrow$ corrections to the conductivity that arise from the **coherent pairing between time reversed trajectories.**



Interference effects are sensitive to dephasing !

<u>How is it related to Coulomb interactions ?</u>

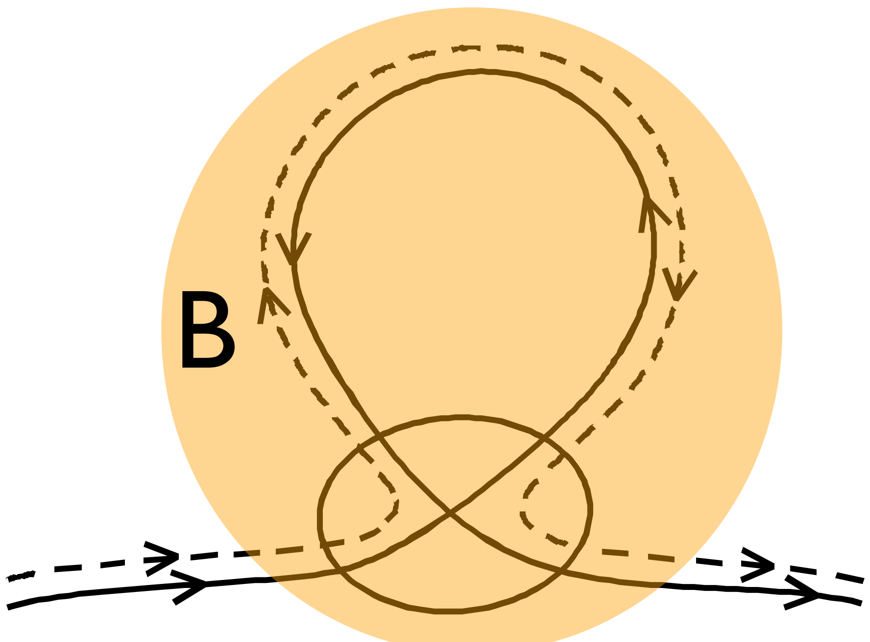
The two paired multiple scattering trajectories correspond to a given QP state. For $t > \tau_{in}(T)$ this state decays and interferences are washed out.

The relaxation of the cooperon that results from the decoherence is characterized by a **phase coherence time** $\tau_\phi(T)$

We thus expect $\boxed{\tau_\phi(T) \simeq \tau_{in}(T)}$

Cooperon $P_c(r, r', t) =$ probability for a wavepacket to move from r to r' in a time t by means of a coherent process.

In the presence of dephasing (*e.g.* interactions):



$$P_c(r, r', t) \rightarrow P_c(r, r', t)\langle e^{i\Delta\phi}\rangle$$

$\Delta\phi$ is the phase shift induced between the 2 coherent trajectories by the Coulomb interaction, *i.e.* due to the QP decay.

Apply a homogeneous magnetic field B and measure transport properties

Assuming that $\langle e^{i\Delta\phi} \rangle = e^{-t/\tau_{\phi(T)}}$, gives for the **magnetoresistance**

$$\Delta R = R(B) - R(0)$$

$$\boxed{\frac{\Delta R}{R} = \frac{e^2 R}{\pi \hbar L}\left[\frac{3}{2}\left(\frac{1}{L_\phi^2} + \frac{4}{3L_{so}^2} + \frac{S}{3L_H^4}\right)^{-1/2} - \frac{1}{2}\left(\frac{1}{L_\phi^2} + \frac{S}{3L_H^4}\right)^{-1/2}\right]}$$

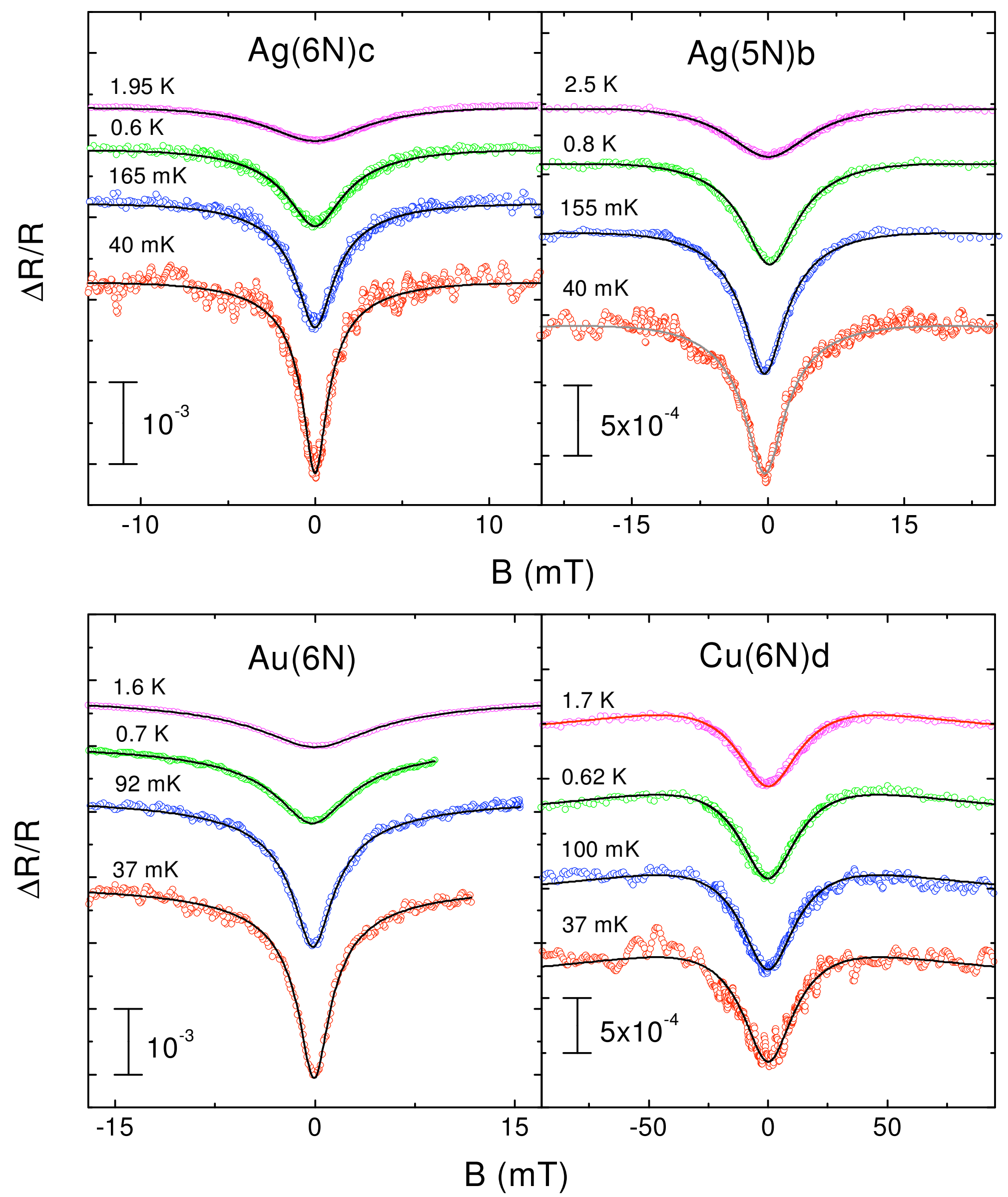
$L_\phi^2 = D\tau_\phi(T)$ and $L_H^2 = \hbar/eB$ is the magnetic length, B the applied magnetic field, L the length of the metallic wire, S its section.

L_{so} is the spin-orbit length.

Measurements: (D.Esteve et al. , Saclay group, 2003)

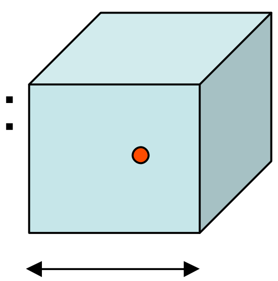
Magnetoresistance of quasi-1d metallic wires (Au, Ag, Cu) of different nominal purity

Measuring $\tau_\phi(T)$: raw data



5N = 99.999 % source purity
6N = 99.9999 % " "

1 ppm of *impurities* :



100 atoms ~ 25 nm

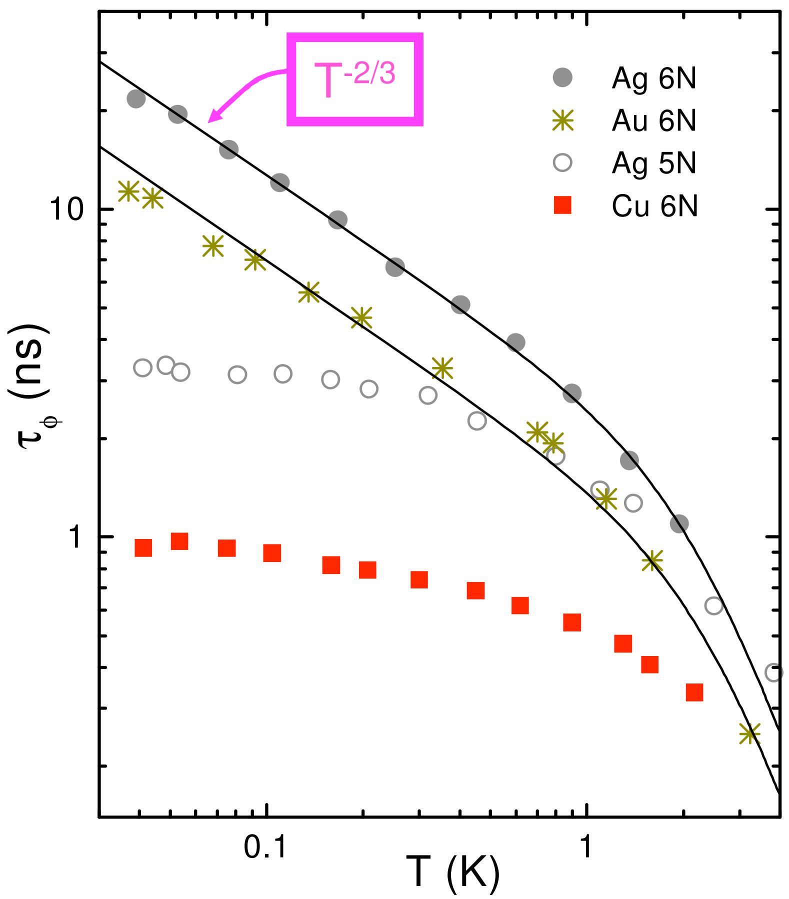
From these data, we extract

$$\tau_\phi(T) = 2.139...\tau_{in}(T)$$

Why not $\tau_\phi(T) = \tau_{in}(T)$?

Very precise measurement, we should be able to understand this discrepency.

$\tau_\phi(T)$ in Ag, Au & Cu wires



F. Pierre *et al.*,
PRB **68**, 0854213 (2003)

5N = 99.999 % source material purity
6N = 99.9999 % " " "

Intermediate summary:

Both the Quasiparticle states and the Cooperon have an exponential relaxation:

$$\frac{1}{P}\frac{dP}{dt} = -\frac{1}{\tau_{in}(T)}$$

$$\langle e^{i\Delta\phi}\rangle = e^{-t/\tau_{\phi(T)}}$$

Not satisfactory!

Our results: the relaxations are **not exponential**, but rather

$$\mathcal{P}(t) = e^{-\frac{2\sqrt{2}}{\pi}\left(\frac{t}{\tau_{in}}\right)^{3/2}}$$

with an identical behaviour for the Cooperon at small times, and

$$\tau_\phi(T) = \tau_{in}(T)$$

There is a distribution of relaxation times

The Fermi golden rule:

Claim: the relaxation rate $P(t)$ of a quasiparticle is not exponential so that we need to be careful with the use of the Fermi golden rule calculation that assumes :

$$\frac{1}{P(t)}\frac{dP}{dt} = -\frac{1}{\tau_{in}} = Cte$$

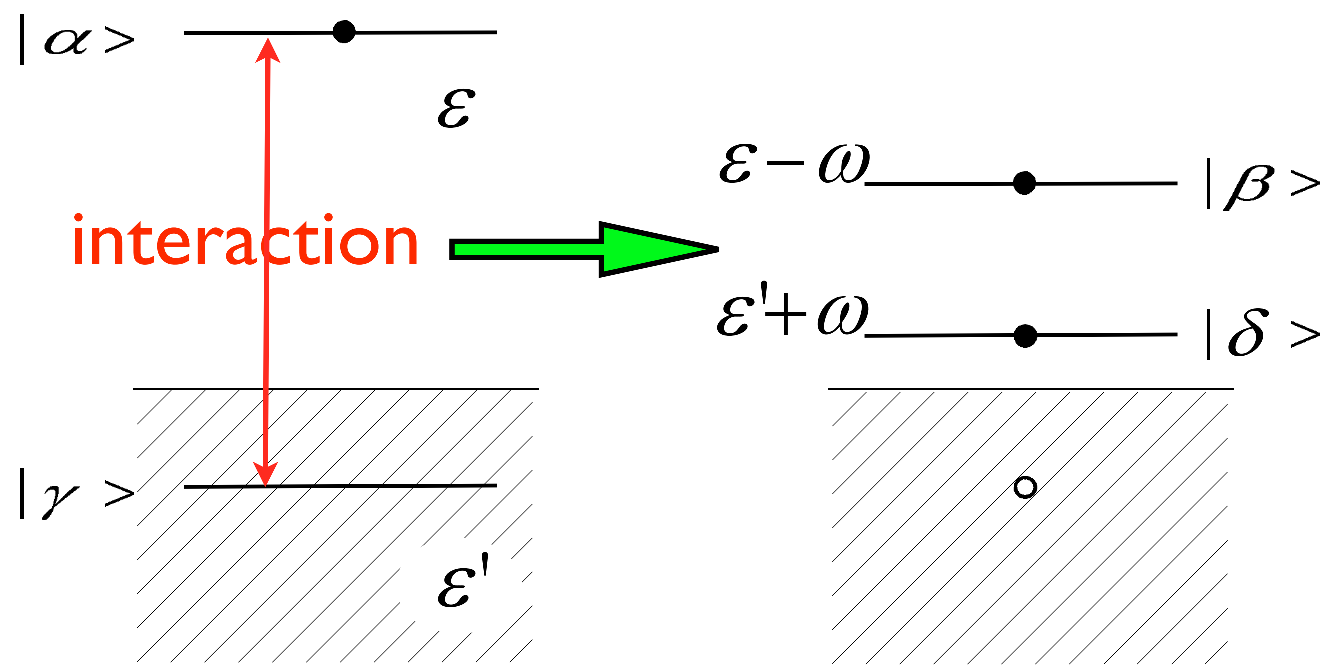
Consider a QP initially in a state $|0\rangle$ at the Fermi level. After a time t, the transition prob. $\mathcal{P}^{(2)}(t)$ to lowest order in perturbation is

$$\mathcal{P}^{(2)}(t) = \frac{e^2}{\hbar^2}\sum_{n\neq 0}\int_0^t d\tau \int_0^t d\tau' \langle 0|V_I(\tau)|n\rangle\langle n|V_I(\tau')|0\rangle$$

where $V_I(\tau) = V(r(\tau),\tau)$

Simplification: (Altshuler,Aronov,Khmelnitskii)

The overall effect of the Coulomb interaction on a given QP is described using a fluctuating potential $V(r,t)$ whose characteristics are determined by the fluctuation-dissipation theorem.



Transition prob. towards final states is

$$\mathcal{P}_{\alpha}^{(2)}(t) = \frac{2}{\hbar^2} \sum_{\beta\gamma\delta} |U_{\alpha\gamma,\beta\delta}|^2 f_t \left(\frac{\epsilon_\alpha + \epsilon_\gamma - \epsilon_\beta - \epsilon_\delta}{\hbar} \right)$$

where $U_{\alpha\gamma,\beta\delta}$ is the matrix element of the interaction, and

$$f_t(\Delta\omega) = \left(\frac{\sin \Delta\omega t/2}{\Delta\omega/2} \right)^2$$

can usually be approximated by $f_t(\Delta\omega) \simeq 2\pi t \delta(\Delta\omega)$, namely a linear decay of the probability.

This approximation is not always valid !

Due to the diffusive motion of electrons, we need to keep the full expression of $f_t(\Delta\omega)$

$$\mathcal{P}^{(2)}(t,T) = \frac{2e^2T}{\pi\hbar^2\sigma}\int_0^t d\tau d\tau' \int \frac{d\mathbf{q}}{(2\pi)^d q^2}\int_{1/t}^{T/\hbar} d\omega e^{i\mathbf{q}.(\mathbf{r}(\tau)-\mathbf{r}(\tau'))-i\omega(\tau-\tau')}$$

diffusive motion

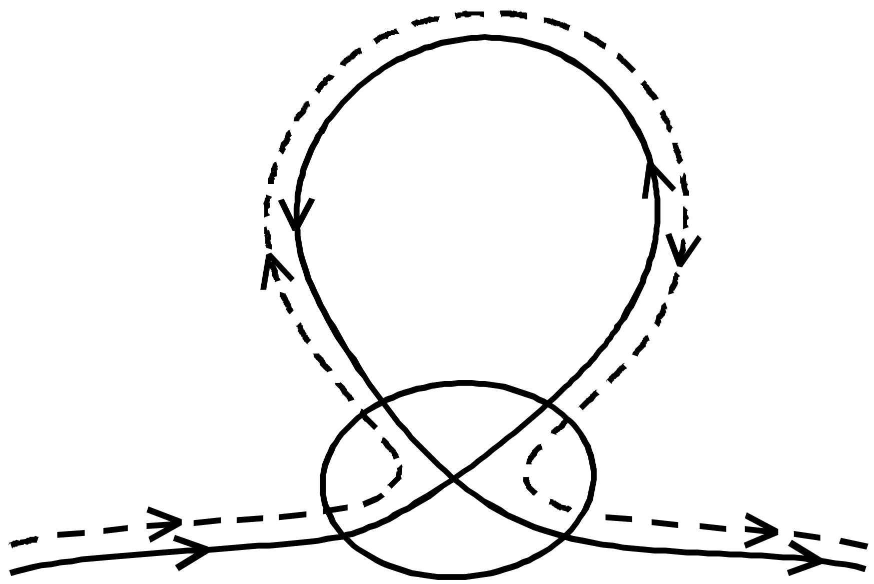
Averaging over disorder, $\langle e^{i\mathbf{q}.(\mathbf{r}(\tau)-\mathbf{r}(\tau'))}\rangle = e^{-Dq^2|\tau-\tau'|}$

so that

$$\boxed{\mathcal{P}(t) = 1 - \langle\mathcal{P}^{(2)}(t)\rangle \simeq 1 - \frac{2\sqrt{2}}{\pi}\left(t/\tau_{in}\right)^{3/2}}$$

Non exponential behavior !

Behavior of the phase shift and of the Cooperon



$$P_c(r,r',t) \to P_c(r,r',t)\langle e^{i\Delta\phi}\rangle$$

due to interactions

$$P_c(r,r,\gamma) = \int_0^\infty \frac{dt}{\sqrt{t}}\langle e^{i\Delta\phi}\rangle e^{-\gamma t} = -\sqrt{\pi\tau_{in}}\frac{Ai(\gamma\tau_{in})}{Ai'(\gamma\tau_{in})}$$

Inverse Laplace transform (analytic function):

$$\left\langle e^{i\Phi}\right\rangle_{T,\mathcal{C}} = \sqrt{\frac{\pi t}{\tau_{in}}}\sum_{n=1}^{\infty}\frac{e^{-|u_n|t/\tau_{in}}}{|u_n|} \simeq e^{-\frac{\sqrt{\pi}}{4}(t/\tau_{in})^{3/2}}$$

at small times

with $|u_n| = \left(\frac{3\pi}{2}(n-\frac{3}{4})\right)^{2/3}$

The origin of the factor 2.139... comes from the fact that although

$$\left\langle e^{i\Phi} \right\rangle_{T,\mathcal{C}} \simeq e^{-\frac{\sqrt{\pi}}{4}(t/\tau_{in})^{3/2}} \neq e^{-t/2\tau_{in}}$$

the integral of the two functions are close.