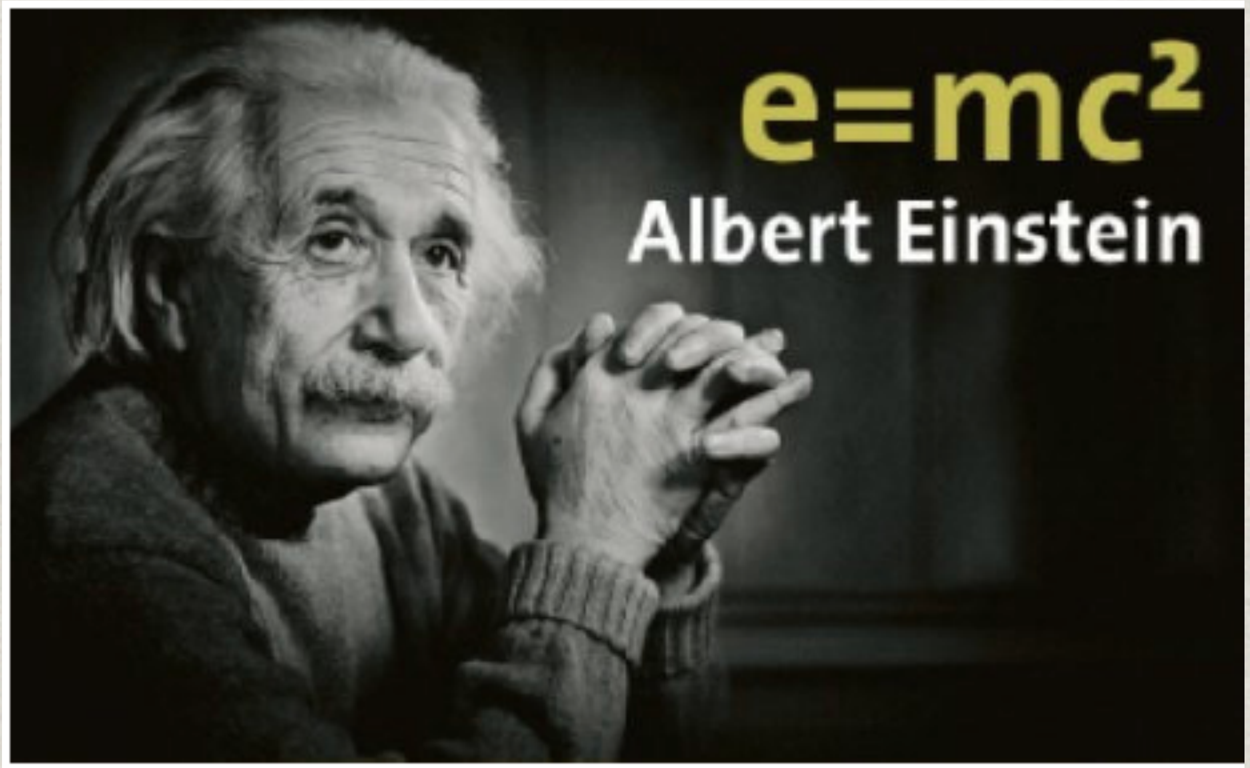
Universality- Scaling and measurement in Physics : the Einstein legacy

- Universality: Physics is a natural science. It tries to find universal laws common to the largest number of phenomena.
  To that purpose, we need to identify a small (?) number of generic (canonical) phenomena to understand (Rosetta's stones). From their solution, we must be able to extrapolate to the largest possible class of phenomena.
- Scaling (dimensional analysis) Identify the relevant quantities describing a phenomenon and their units.

We are very much used to this aspect of the work of A. Einstein



There is another aspect related to Universality, scaling and Thermodynamics.

This is the purpose of this talk

Einstein 1905 : the extraordinary year

- 18/03 Blackbody radiation and the discontinuous nature of light (Photons)
- 30/04 A new determination of molecular dimensions (Ph.D. thesis). His most cited work!
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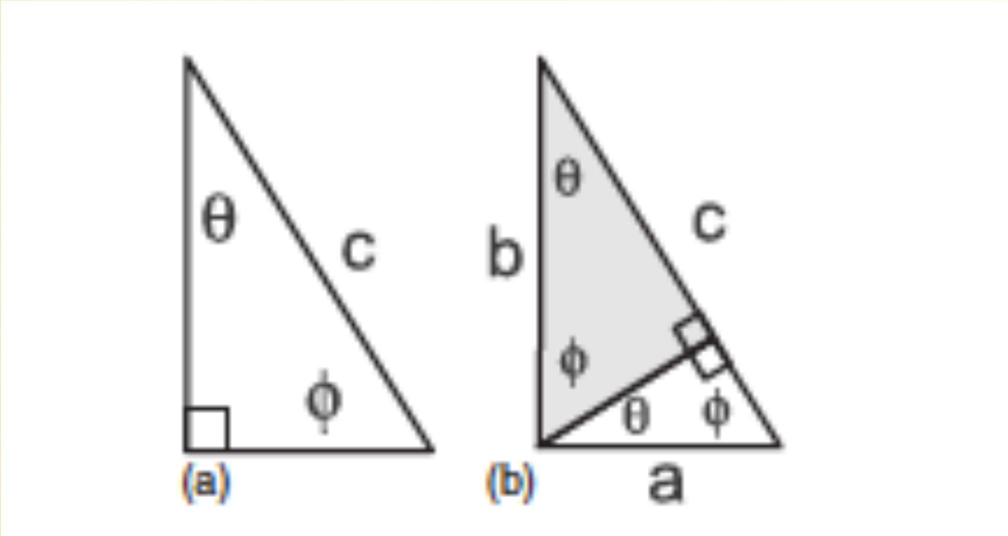
Scaling and Dimensional analysis

We do not need to always know everything about everything !

A simple example : The Pythagorean theorem



Area of the (a) triangle :

$$A_c = c^2\, f(\theta,\phi)$$

(θ,ϕ) are dimensionless and so is $f(\theta,\phi)$

<u>Remark</u>: Could also choose another side and another function f

Divide the triangle (a) into two similar triangles (b) and apply the same result to each:

$$A_a = a^2\, f(\theta,\phi)$$

$$A_b = b^2\, f(\theta,\phi)$$

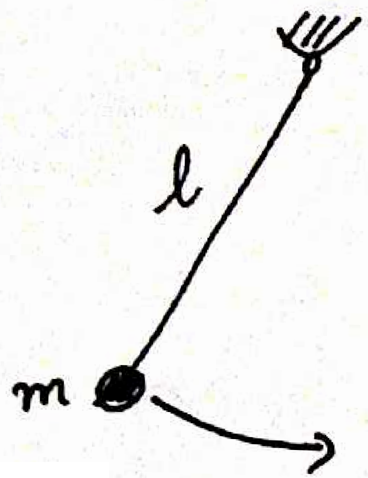
Since $A_c = A_a + A_b$ then $c^2 = a^2 + b^2$

No need of the full information (here the angular function f) to prove a result.

Dimensional analysis

The π - theorem of Buckingham

Simple pendulum :



Period of oscillations T ?

On what kind of quantities does T depend ?

1) length ℓ (Units L)
2) Mass m (" M)
3) gravitational acceleration: g (units $L.T^{-2}$)

out of (ℓ, m, g), there is only one single quantity with units of time :

$$T \propto \sqrt{\ell/g} \quad \text{independent of } m.$$

The quantity $\Pi \equiv T\sqrt{\frac{g}{\ell}}$ is dimensionless. It is a number

whose value is
- * Universal (independent of the system of units)
- * of order unity (complete calculation : 2π)
- * Unique.

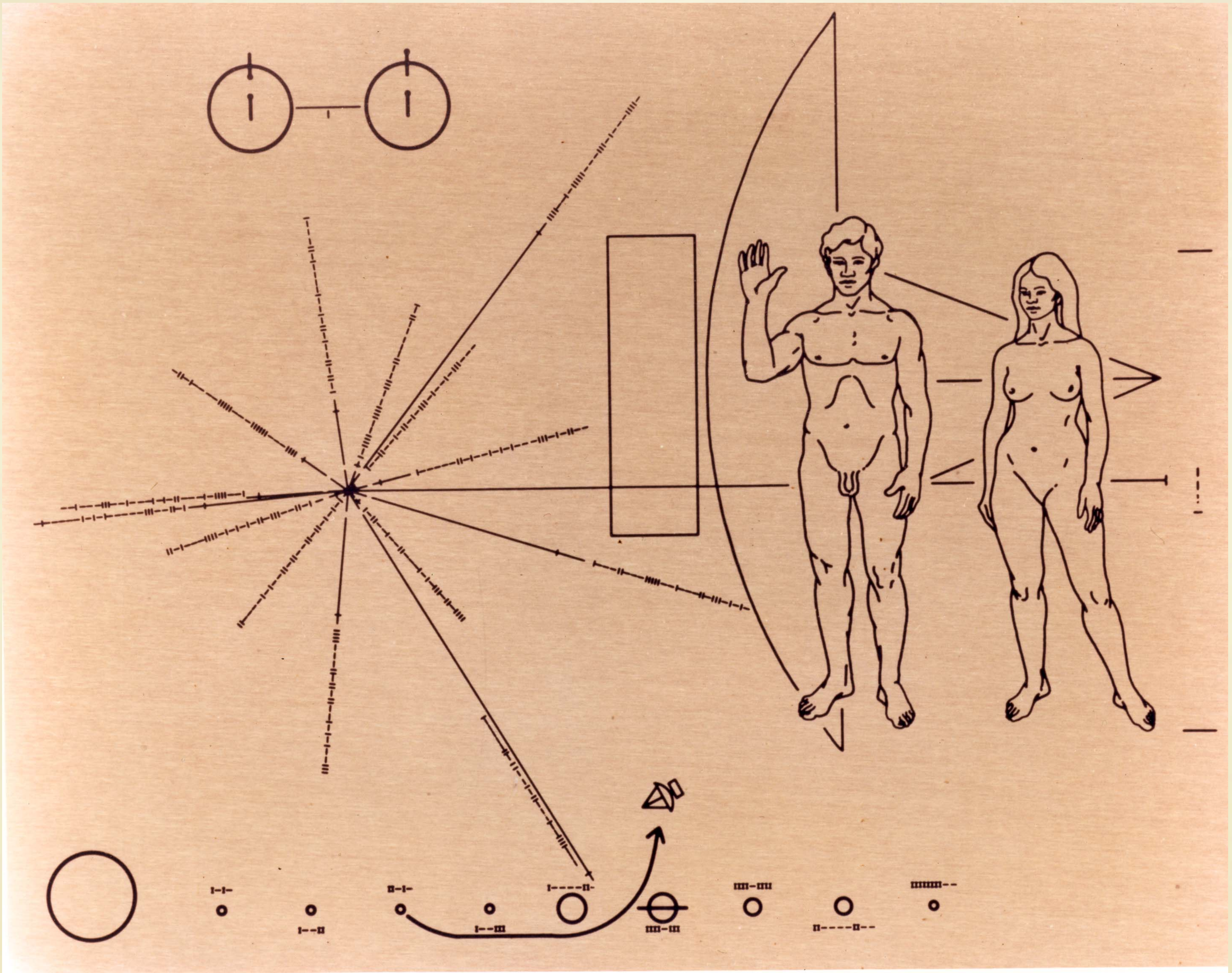
Universal laws and scaling are the two main tools needed to make further progress understanding our world at all scales without always having adequate instruments.

Example :

1. What is the mass of an atom of Hydrogen ?

2. How to measure the volume of the sun ?

No adequate rulers !



Pioneer plaques placed on board the 1972 Pioneer 10 spacecraft

Law of gases-
Is matter continuous or discontinuous ?

Observe a gas or a liquid, how to decide ?

Basic observations : (1787, Charles and Gay-Lussac)

Consider a gas in a volume V

At fixed pressure, $V = a\,T$ (a is a constant, which depends on pressure P)

At fixed temperature, $P\,V = b$ (b is a constant which depends on T).

Then,

$$PV = cT$$

But no answer to the question !

The Avogadro principle

In 1811, Amedeo Avogadro (after Bernoulli) states that gases are made of molecules (smallest and stable characteristic particle of a substance).

How big ? How small ? No clue ! But probably many...

Avogadro principle : Equal volumes of gases at fixed T and P contain the same number of molecules.

$$V = aN$$

Combined with the two previous laws, it gives PV = a N T

Need to fix the constant (choose units): For

$$T = 25^{\circ} C, P = 1 Atm \ and \ V = 22.4\,l$$

the number of molecules is N_A (Avogadro number, Perrin 1909)

SO THAT, WE HAVE THE LAW OF PERFECT GASES

$$PV = \frac{N}{N_A} RT$$

where $R = 8.314\, JK^{-1} mol^{-1}$ is the gas constant which fixes units (note that it is of order unity).

The Avogadro number N_A is a universal constant of Nature just like $c, \varepsilon_0, \mu_0, G, \ldots$ expressed in convenient units.

The message : whenever N_A shows up, there are molecules, i.e., discontinuity

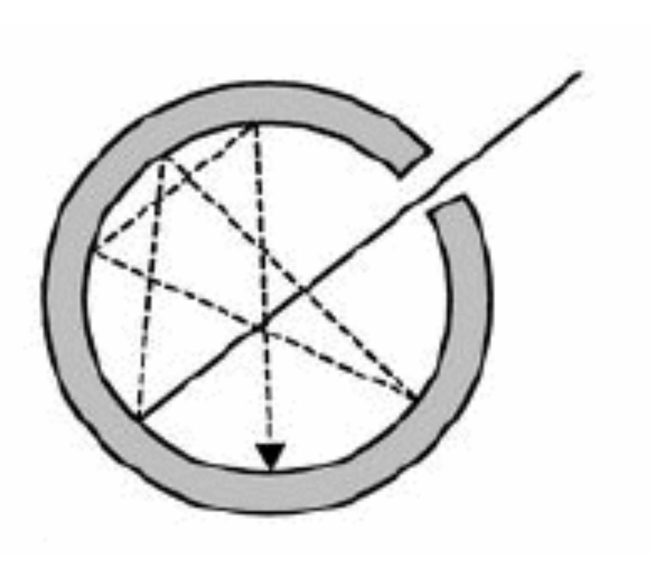
Taking $N_A \to \infty \Leftrightarrow$ Continuous description of matter.

Extrapolate to non ideal gases, liquids,...

Blackbody radiation

- The phenomenon
- Universality
- Scaling
- The puzzle
- The solution

The phenomenon



Any cavity enclosed within matter at equilibrium at temperature T is full of light (radiation) at equilibrium at the same temperature :

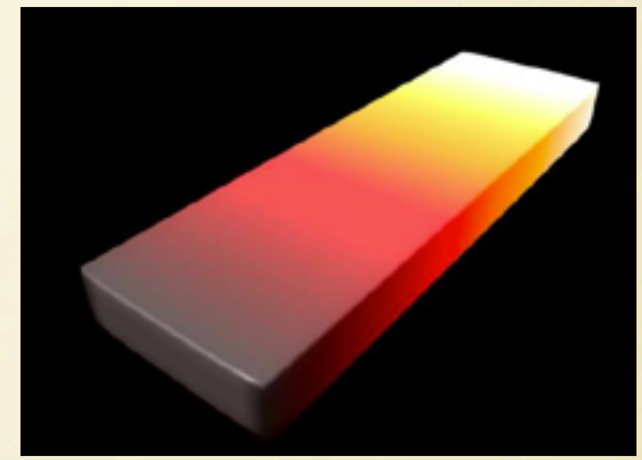
Black body radiation

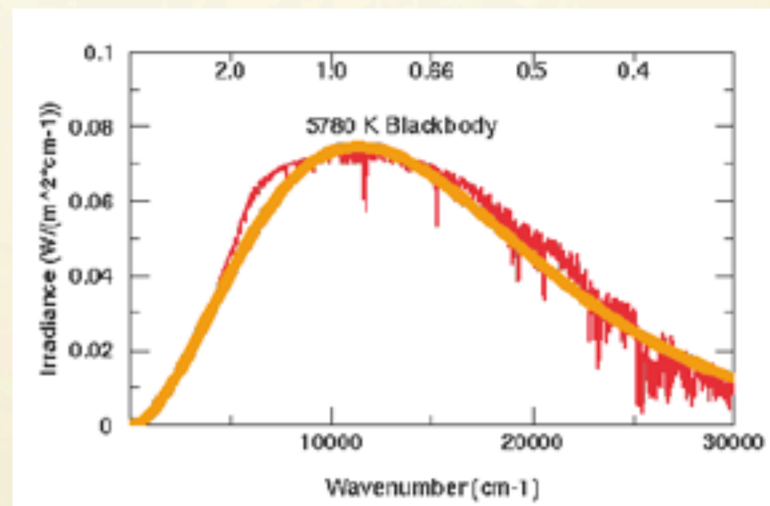
Black body radiation is universal

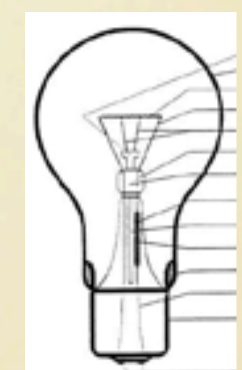
- It does not depend on the material of the cavity.
- It does not depend on its shape.
- It does not depend on its volume, surface or any geometric characteristics.
- It is not possible to focus the light in the cavity. Then, we cannot distinguish objects or shapes inside the cavity.
- The light travels at light velocity c inside the cavity.

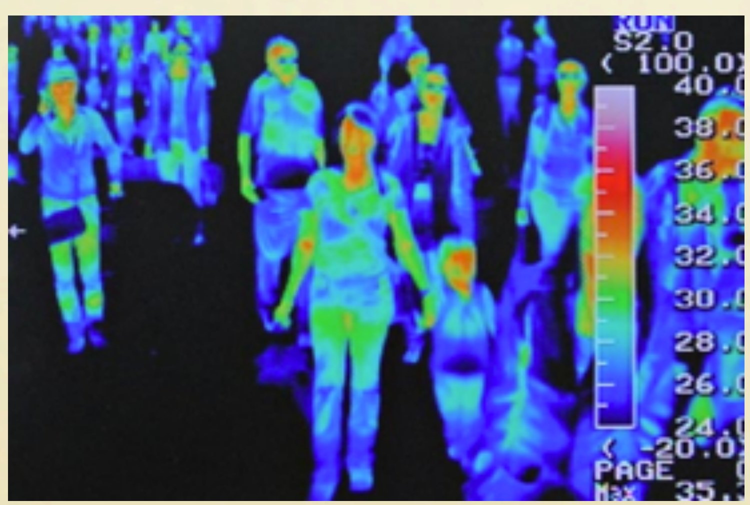
Examples

- A furnace
- A heated piece of metal.
- The Sun or any other star.
- An electric bulb
- The human body (infrared)

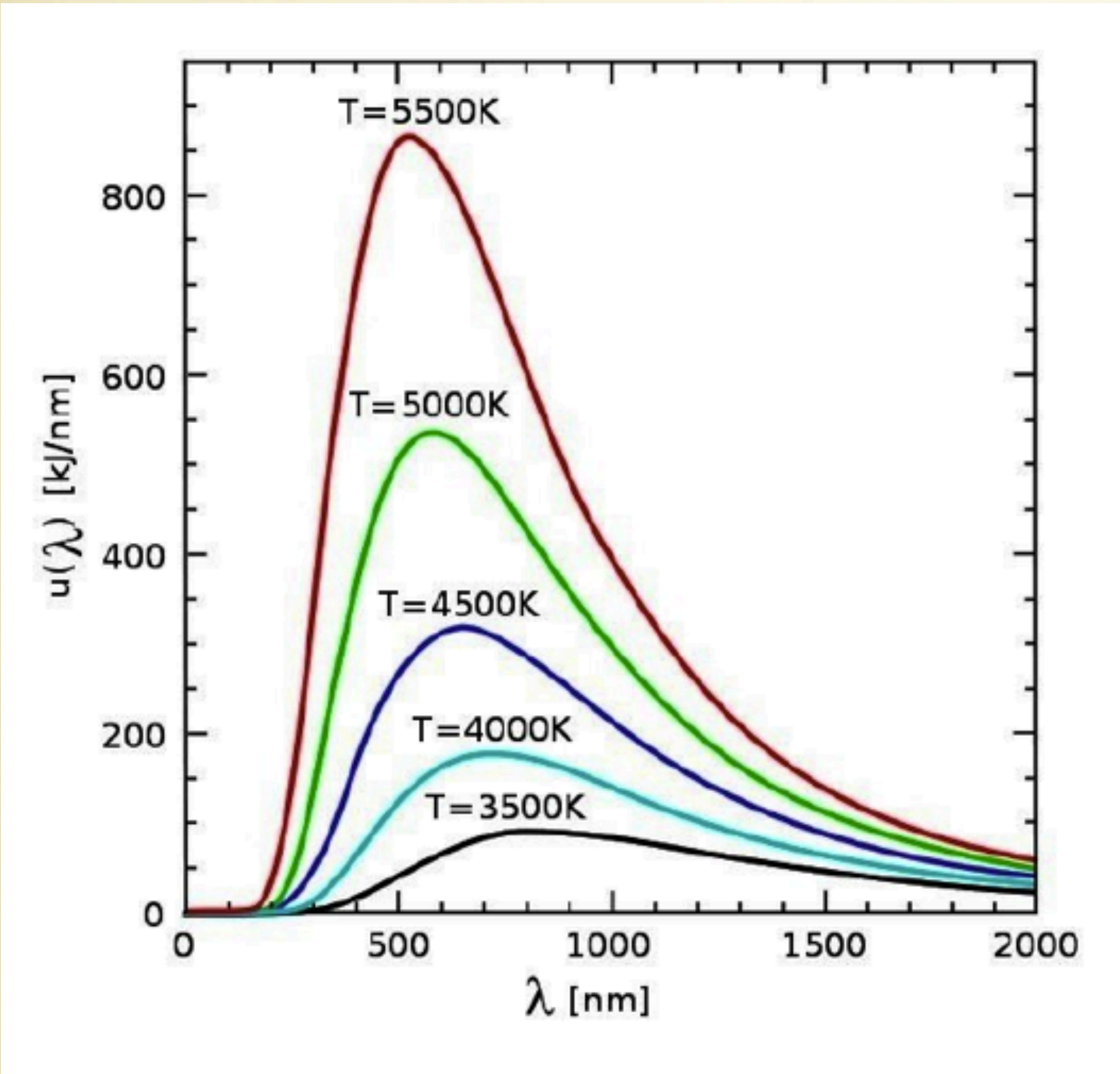


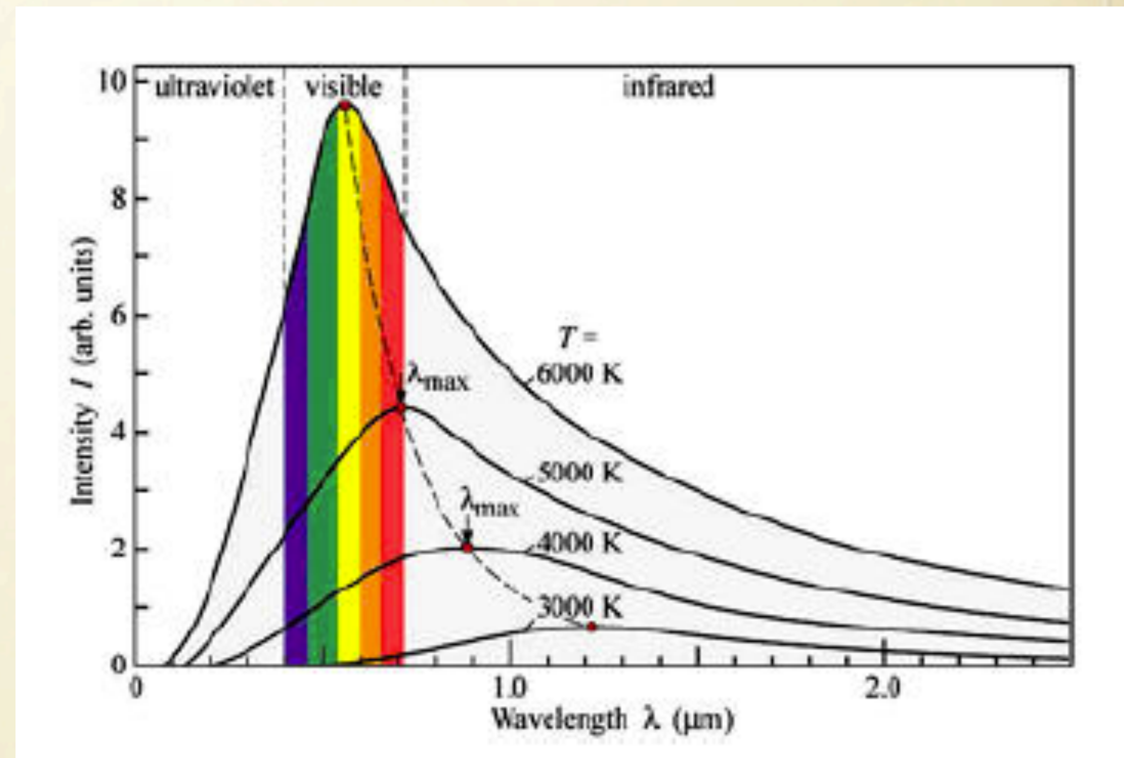






This light (radiation) is visible, or not, depending on the temperature. The energy density is distributed according to the wavelengths of the light.





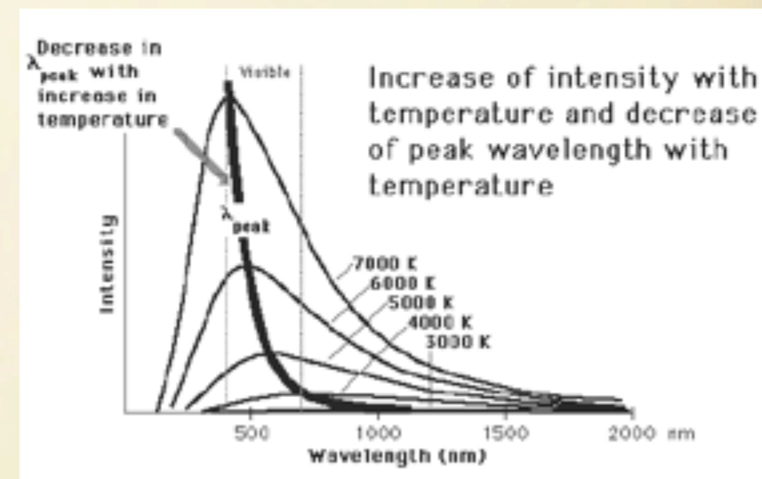
Universality

- Wien displacement law : At a given frequency and temperature, the energy per unit volume of the radiation is of the form:

$$u(\nu,T)=T^3 f\left(\frac{\nu}{T}\right)$$

- From this form, we obtain

$$\lambda_{\max} T = A$$



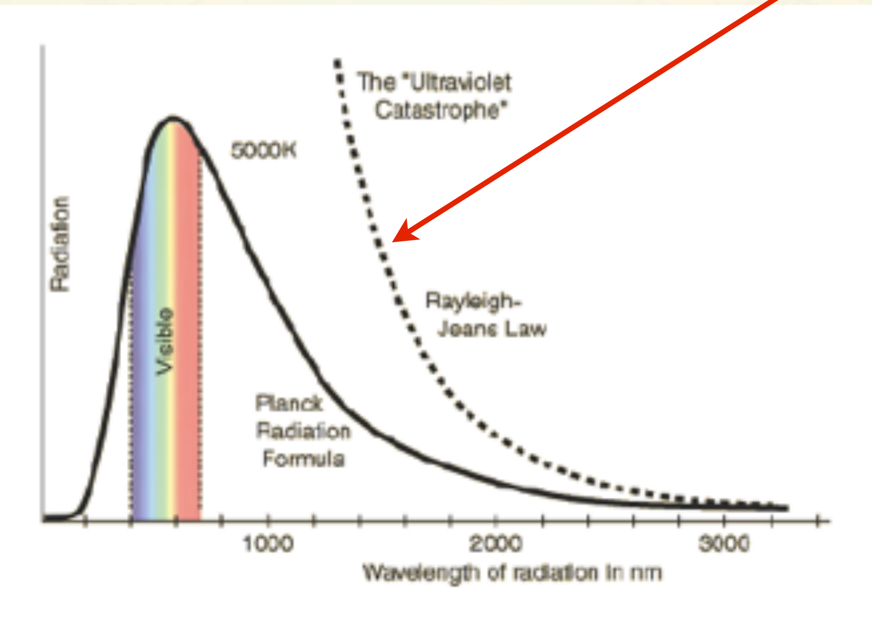
- Stefan law: Total energy radiated at fixed T and per unit volume

$$u(T)=\sigma T^4$$

where (A,σ) are universal constant.

The puzzle

- No theoretical explanation for the black body radiation curve.
  There is a classical theory which works only at high wavelengths.



The Planck solution (1901) : Matlab/Mathematica + thermodynamic intuition : find a good fit.

$$u(\nu,T)=\frac{\alpha\nu^3}{e^{\beta\nu/T}-1}$$

(α,β) are fitting constants.

The solution/controversy Einstein (1905)

Einstein considers (for some reason) the following dimensionless combination of (α, β)

$$\frac{8\pi R}{c^3} \cdot \frac{\beta}{\alpha} \simeq 6.17 \times 10^{23}$$

It looks rather close to the existing rough estimations of the Avogadro number N_A !

If N_A enters black body radiation, there are 2 options:

1. Planck explanation: It is related to the matter surrounding the radiation. Not so intuitive: for a gas of molecules, we describe the properties of the gas without reference to the container ! It contains the idea of "*quantization*"

2. Einstein (1905): Planck may be right, but this explanation is not elegant ! N_A is related to the radiation only.

Einstein 1905 : the extraordinary year

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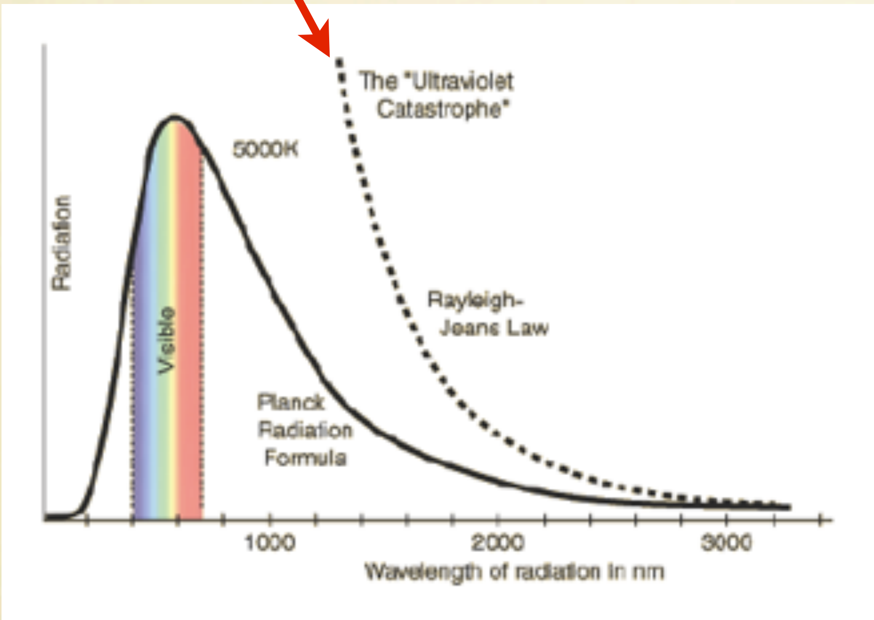
Weird ! It is known that radiation is a superposition of electromagnetic waves with a continuous distribution of frequencies (or wavelengths).

Perhaps, each frequency mode is a “molecule” of light, then we have a gas of modes (gas of musical notes).

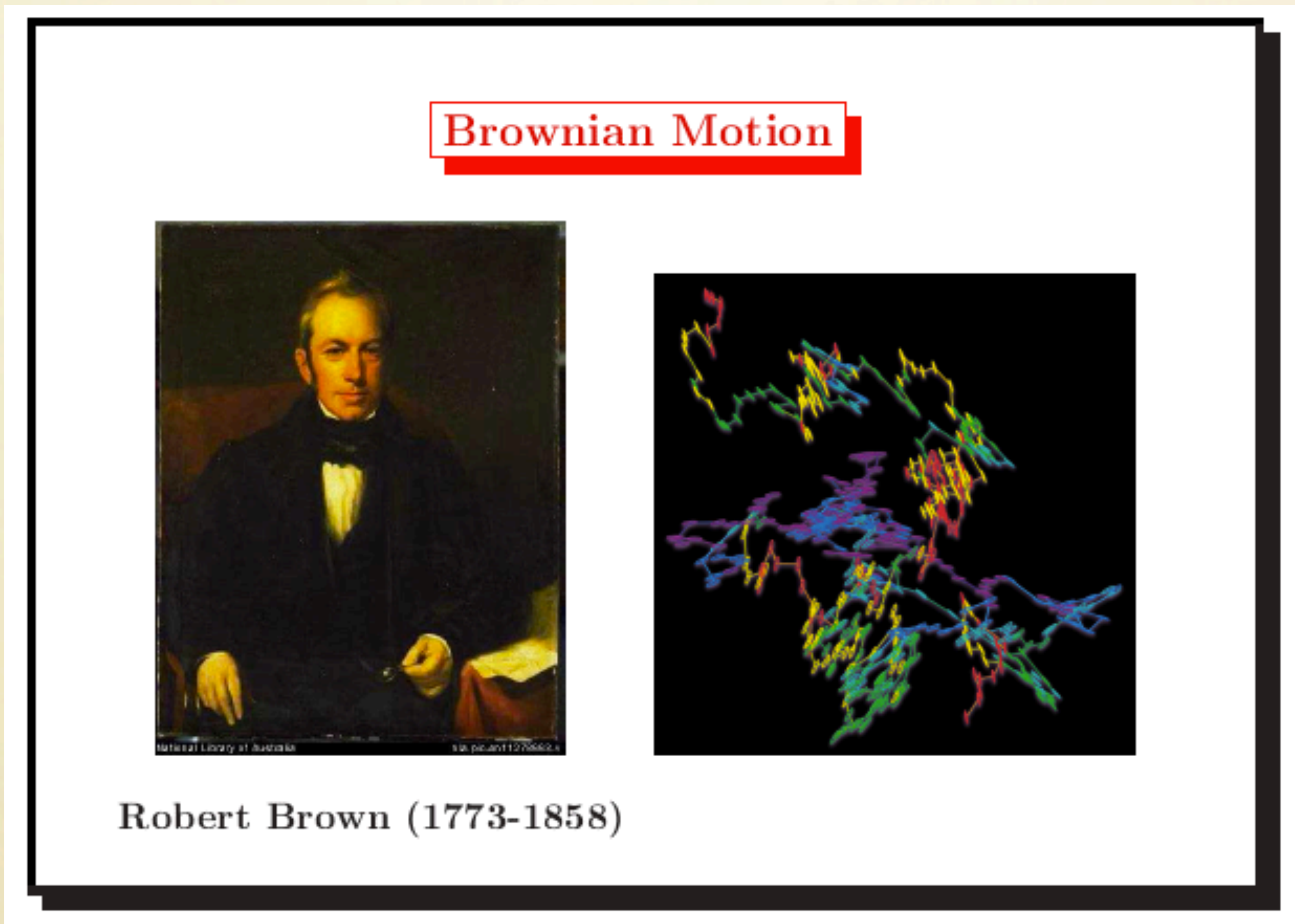
This is the classical theory which gives the wrong result.



Perhaps, $\frac{8\pi R}{c^3} \cdot \frac{\beta}{\alpha} \simeq N_A$ is fortuitous and wrong, since there is no independent measure of the Avogadro number to compare to.

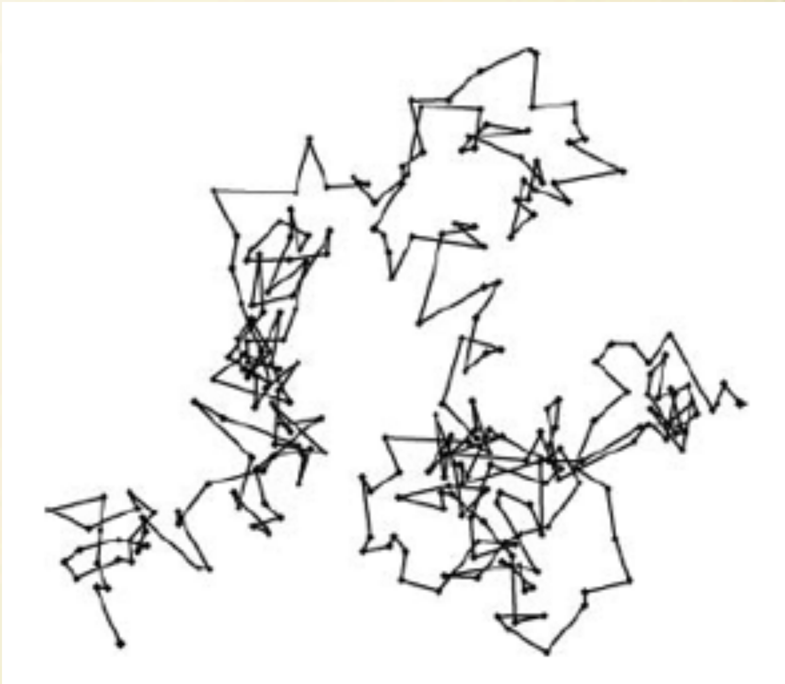
Intermezzo: The Brownian motion (Einstein 1905)

Pollen grain (very small but visible size) in suspension in a liquid (water)



Characteristics

- The motion is extremely irregular and the trajectories seems to have no tangent (today we know it is a fractal, Werner 2006).
- The smaller the particles, the livelier the motion.
- The nature and density of particles have no influence.
- The motion is most active in less viscous liquids.
- The motion is most active at higher temperatures.
- The motion never stops.



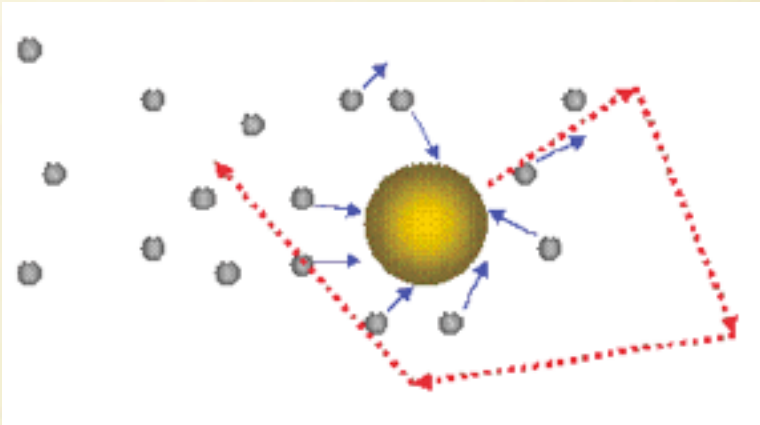
The puzzle : Why this restless motion ?

1. many strange answers...

2. Related to the underlying molecular nature of the liquid (they are also made of molecules)

(1860) Unlikely ! $\dfrac{Mass\ of\ a\ pollen\ grain}{Mass\ of\ a\ molecule} \simeq 10^8 - 10^9$

It cannot move it ! But what about collective motion of many "molecules" ?



3. Stokes : Liquids are continuous (no need of the molecular theory to explain them).

The first solution (Einstein, 1905, Ph.D. thesis).

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The Einstein solution

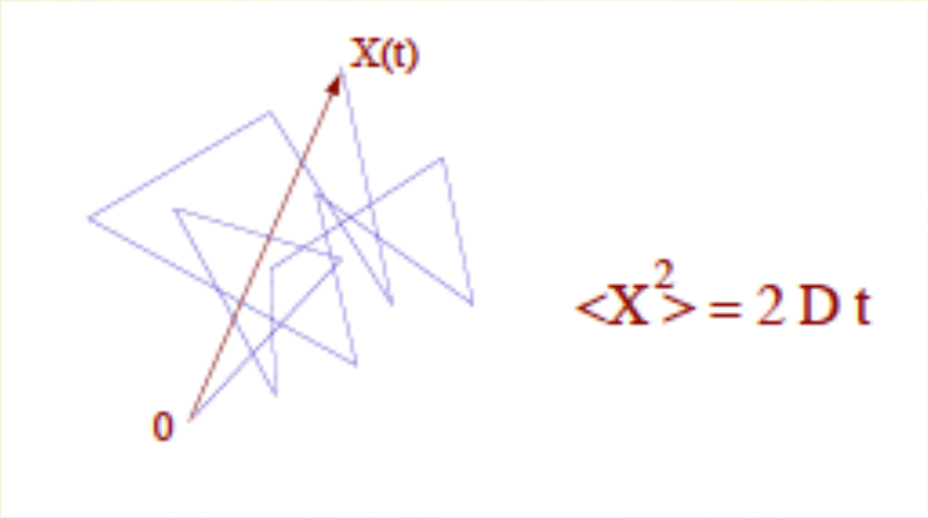
- The Stokes approach : Liquids are continuous. The friction force opposing a moving sphere of radius a in a liquid of viscosity η is macroscopic:

$$F = -\gamma v \quad with \quad \gamma = 6\pi\eta a$$

- Van't Hoff : A moving sphere is similar to any other molecule of the fluid and subject to the same molecular chaotic motion.

- Einstein : Both points of view are simultaneously valid for particles as big as brownian particles.

Brownian particles diffuse



The diffusion coefficient is given by the Einstein formula

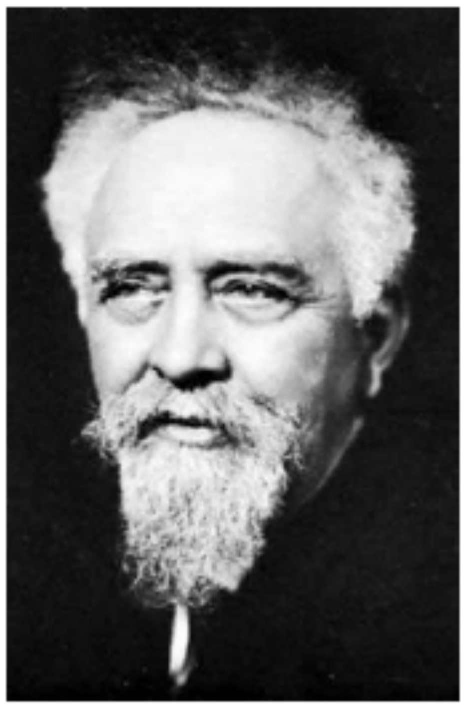
$$D = \frac{RT}{N_A} \frac{1}{6\pi \eta a}$$

Outstanding formula ! It does not fulfill the Buckingham π since there is a dimensionless number N_A not of order unity.

For $N_A \to \infty \Rightarrow D \to 0$: No Brownian motion for continuous matter

Weighing the Hydrogen atom

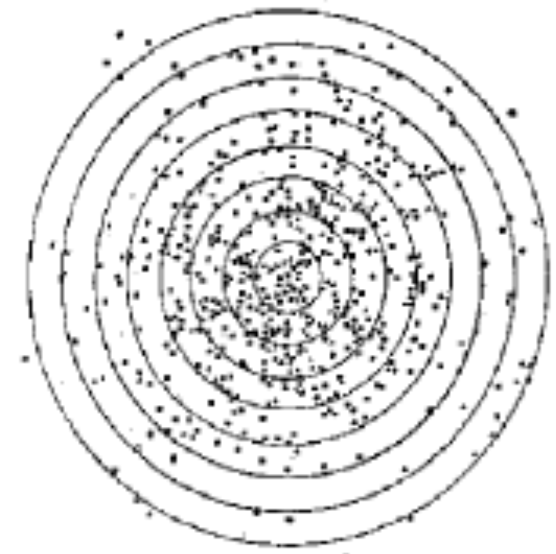
Jean Perrin (1909)



Jean Perrin (1870-1942)

$$\eta \sim 10^{-3} \; a \sim 10^{-7} \; R \sim 8.3$$

$$D \simeq 10^{-12} m^2/s$$



Mass of the Hydrogen atom

$$1.6 \times 10^{-24}\, g$$

Avogadro number

$$N_A = 6.3 \times 10^{23}$$

Last updated value : $N_A = 6.022 \times 10^{23}$

The solution/controversy Einstein (1905)

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2. Einstein (1905): Planck may be right, but this explanation is not elegant !
N_A is related only to the radiation.

End of the Intermezzo - Back to Black body radiation

Einstein seems to be correct : Radiation at thermodynamic equilibrium is composed of “molecules” (to be named Photons much later, 1926).

Using an elegant thermodynamic derivation, Einstein shows that the total energy of a gas of N molecules/photons can be written in terms of the constants (α, β) introduced by Planck and the frequency :

$$E = \frac{R}{N_A} \beta N \nu \triangleq N h\nu$$

Today, we use the so-called Planck constant : $h = \frac{R}{N_A}\beta \simeq 6.62 \times 10^{-34}\, Js$

so that $\alpha = \frac{8\pi}{c^3} h$

Black body radiation is an ideal gas of photons in the sense of Avogadro.

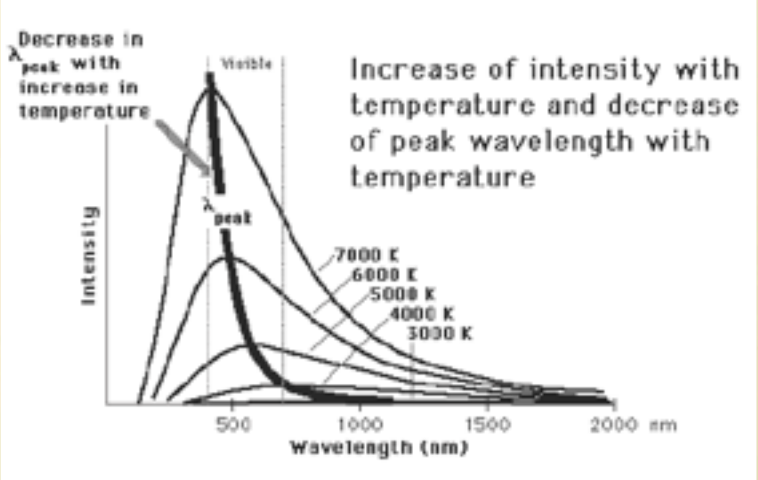
Back to Universality

- Wien displacement law : At a given frequency and temperature, the energy per unit volume of the radiation is of the form:

$$u(\nu,T) = T^3 f\left(\frac{\nu}{T}\right)$$

- From this form, we obtain

$$\lambda_{\max} T = A = \frac{N_A hc}{5R}$$



- Stefan law: Total energy radiated at fixed T and per unit volume

$$u(T) = \sigma T^4 = \frac{2\pi^5 R^4}{15 h^3 c^2 N_A^4}$$

Measuring the Sun : $R_S = 10\left(D_{TS} T_T^2\right)\left(\frac{\lambda_{\max} R}{hc N_A}\right)^2 \simeq 7\times 10^8\, m$

Epilogue

- How to observe a single "light particle" (photon) ? Serge Haroche Nobel Prize 2012

Seeing a single photon without destroying it

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Light detection is usually a destructive process, in that detectors annihilate photons and convert them into electrical signals, making it impossible to see a single photon twice. But this limitation is not fundamental—quantum non-demolition strategies[1–3] permit repeated measurements of physically observable quantities, yielding identical results. For example, quantum non-demolition measurements of light intensity have been demonstrated[4–14], suggesting possibilities for detecting weak forces and gravitational waves[3]. But such experiments, based on nonlinear optics, are sensitive only to macroscopic photon fluxes. The non-destructive measurement of a single photon requires an extremely strong matter–radiation coupling; this can be realized in cavity quantum electrodynamics[15], where the strength of the interaction between an atom and a photon can overwhelm all dissipative couplings to the environment. Here we report a cavity quantum electrodynamics experiment in which we detect a single photon non-destructively. We use atomic interferometry to measure the phase shift in an atomic wavefunction, caused by a cycle of photon absorption and emission. Our method amounts to a restricted quantum non-demolition measurement which can be applied only to states containing one or zero photons. It may lead to quantum logic gates[16] based on cavity quantum electrodynamics, and multi-atom entanglement[17].

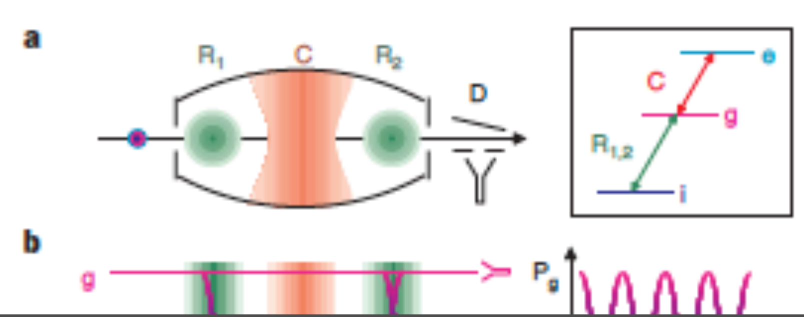
In optical quantum non-demolition (QND) measurements, a 'signal' beam is coupled to a 'meter' beam in a nonlinear medium whose refractive index depends on light intensity[4]. The phase shift of the meter resulting from the non-resonant couplings of the beams with the medium is measured by optical interferometry. The meter beam is split into two parts, one of which is coupled to the signal, the other being used as reference. The two parts are recombined before reading the meter output intensity. The signal intensity remains unaltered. In the single-photon quantum non-

$C_g e^{i\pi}|g,1\rangle + C_i|i,1\rangle$ while, in the empty-cavity case, $C_g|g,0\rangle + C_i|i,0\rangle$ is unchanged. In short, the atomic coherence changes its phase by π if there is 1 photon in C.

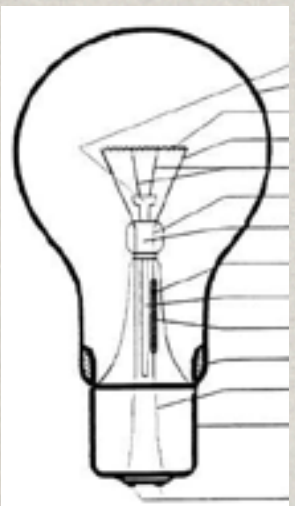
The meter wavefunction phase-shift is measured by Ramsey interferometry[21]. The meter, initially in g, undergoes two interactions with a classical auxiliary field (frequency ν), before and after crossing the cavity mode (zones R_1 and R_2 in Fig. 1a). This field is nearly resonant with the g ⇒ i transition at frequency ν_{gi}. The first pulse prepares the coherent superposition $(1/\sqrt{2})(|g\rangle+|i\rangle)$ and the second mixes the states again, probing after C the superposition phase-shift. The final atomic populations are measured downstream with a state-selective detector D. The probability of finding the meter in g or i results from a quantum interference between two paths in which the atom crosses C in either g or i. It is modulated when ν is tuned, resulting in sinusoidal 'Ramsey fringes'.

When C contains 1 photon, the amplitude associated with one of the paths (atom in g) is phase-shifted by π. The corresponding fringe pattern should thus be π out of phase with the one obtained with C empty. The interfering quantum paths and the corresponding fringes are depicted in Fig. 1b,c. Setting ν at a fringe extremum (for instance at $\nu - \nu_{gi}=0$) results in a perfect correlation between the state of the meter (i or g) and the photon number (0 or 1).

This SP-QND scheme can measure only two photon numbers (0 and 1) in a non-demolition way. For an n-photon field with $n > 1$, the Rabi frequency is $\Omega\sqrt{n}$ and n-conservation cannot be enforced for all ns. By slightly detuning the cavity from the e ⇒ g transition, it is however possible to suppress the meter absorption, thus avoiding completely photon demolition. The dispersive phase shift experienced by the meter in the non-resonant case is dependent upon the photon number. This shift could also be measured by Ramsey interferometry, leading now to an unrestricted QND measurement of the photon number. This dispersive method was described in our original cavity quantum electrodynamics-quantum



Do it yourself



Probing Planck's law at home

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Received 14 July 2010, in final form 9 September 2010
Published 8 October 2010
Online at stacks.iop.org/EJP/31/1463

Abstract
We report on the physics around an incandescent lamp. Using a consumer-grade digital camera, we combine electrical and optical measurements to explore Planck's law of black-body radiation. This simple teaching experiment is successfully used to measure both Stefan's and Planck's constants. Our measurements lead to a strikingly accurate value for Planck's constant: $h = 6.7 \pm 0.4 \times 10^{-34}\,\mathrm{kg\,m^2\,s^{-1}}$. A digital camera is thus sufficiently good equipment to measure a constant directly related to quantum mechanics. The simplicity of the proposed experiments makes this paper appropriate for undergraduate students interested in the experimental aspects of fundamental physics.

(Some figures in this article are in colour only in the electronic version)

$$h = 6.7 \pm 0.4 \times 10^{-34}\ kg\, m^2\, s^{-1}$$