Physique mesoscopique des electrons et des photons -
Structures fractales et quasi-periodiques

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Aux frontieres de la physique mesoscopique,
Mont Orford Quebec, Canada,

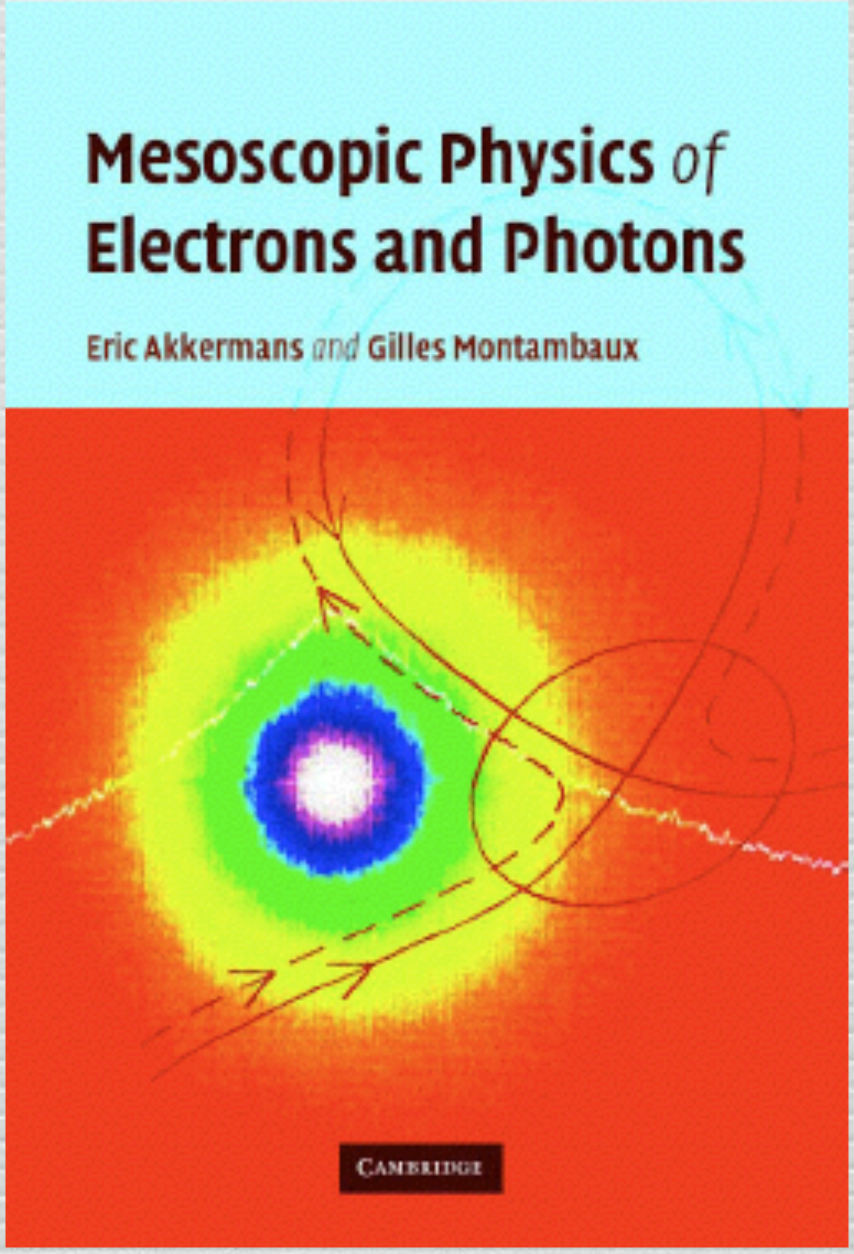
Part 1

What is quantum in mesoscopic physics ?

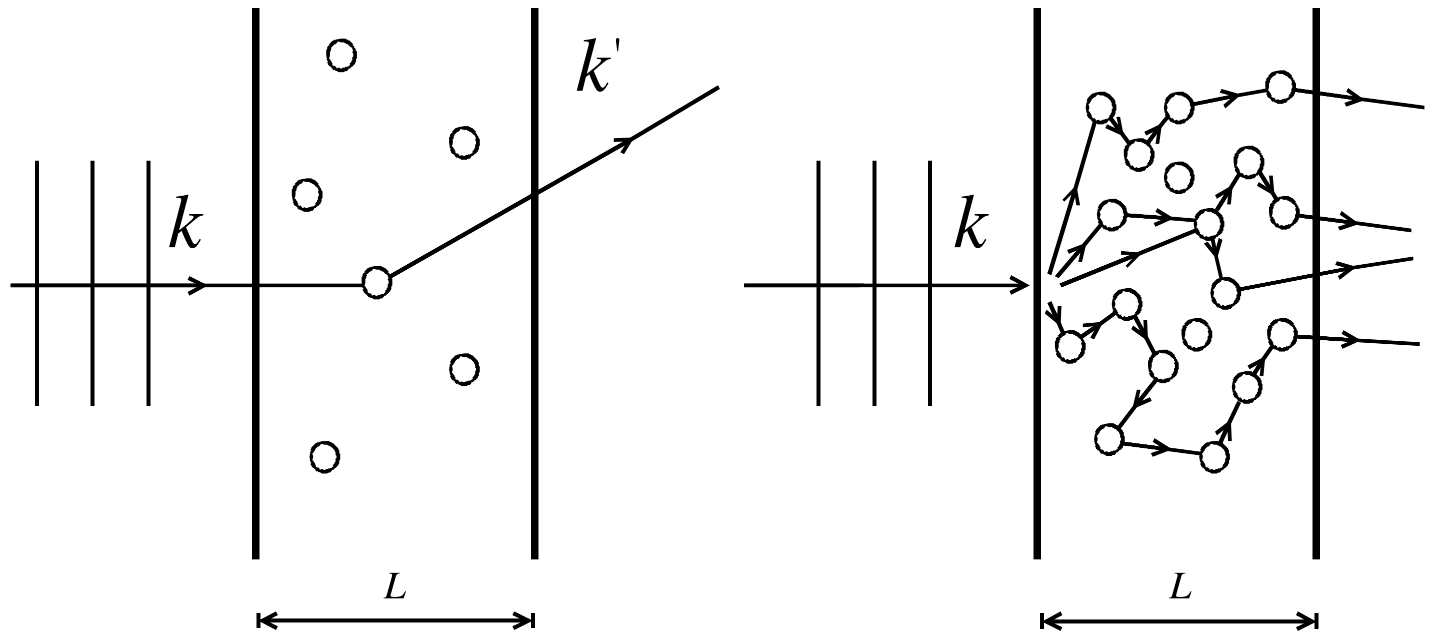
Transport and interferences

(E.A., G. Montambaux)

more details in:



Multiple scattering of electrons



2 characteristic lengths:

Wavelength: $\lambda_F = k_F^{-1}$

Elastic mean free path: l (Disorder - Origin ?)

Weak disorder $\lambda_F \ll l$: independent scattering events

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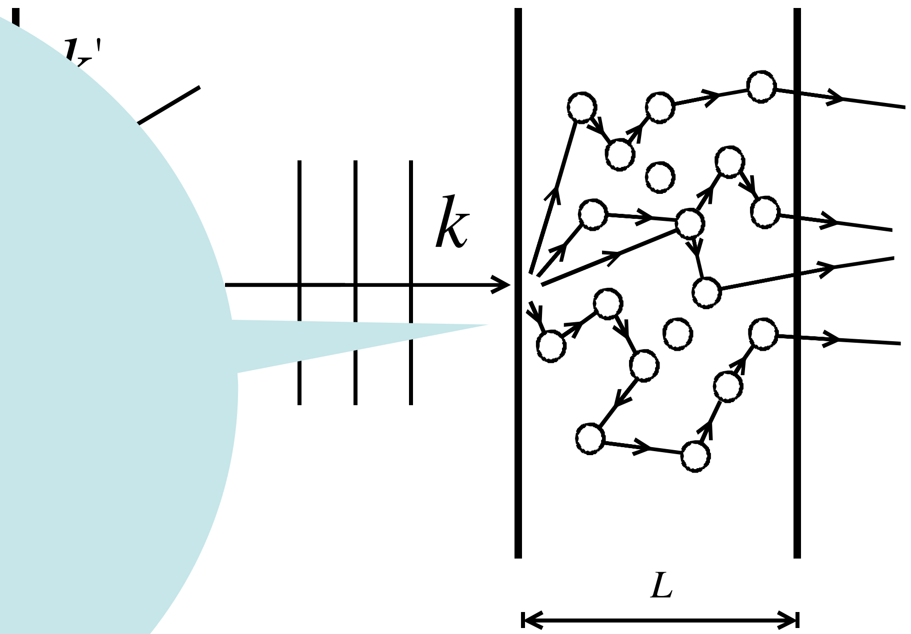

We shall be interested only by this limit

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Some “canonical” mesoscopic effects

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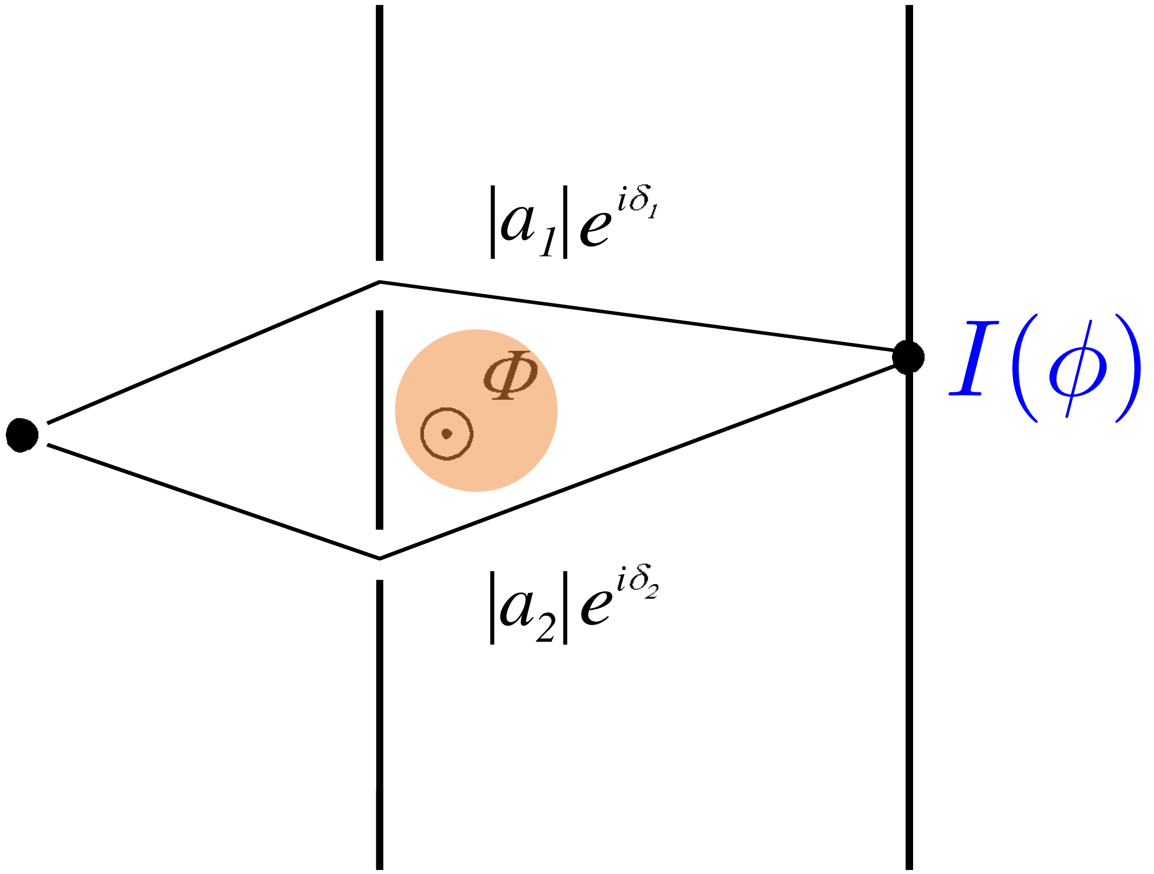
The Aharonov-Bohm effect

Aharonov-Bohm (1959)

The setup : Young slits

No magnetic field on the electrons : no Lorentz force and no orbital motion.



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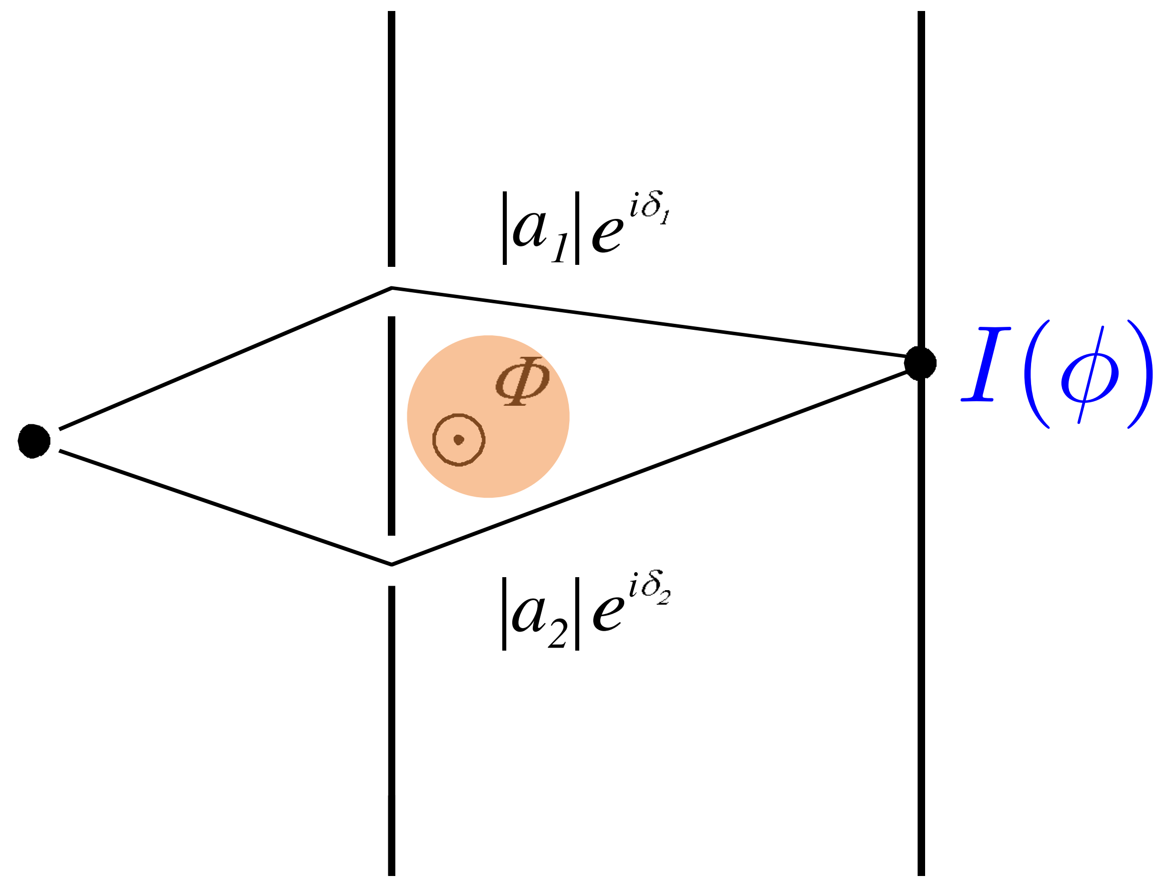


The quantum amplitudes $a_{1,2} = |a_{1,2}|e^{i\delta_{1,2}}$ have phases:

$$\delta_1 = \delta_1^{(0)} - \frac{e}{\hbar}\int_1 \mathbf{A}.d\mathbf{l} \quad \text{and} \quad \delta_2 = \delta_2^{(0)} - \frac{e}{\hbar}\int_2 \mathbf{A}.d\mathbf{l}$$

The intensity $I(\phi)$ is given by

$$I(\phi) = |a_1 + a_2|^2 = |a_1|^2 + |a_2|^2 + 2|a_1 a_2| \cos(\delta_1 - \delta_2)$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\delta_1 - \delta_2)$$

The phase difference $\Delta\delta(\phi) = \delta_1 - \delta_2$ is modulated by the magnetic flux ϕ :

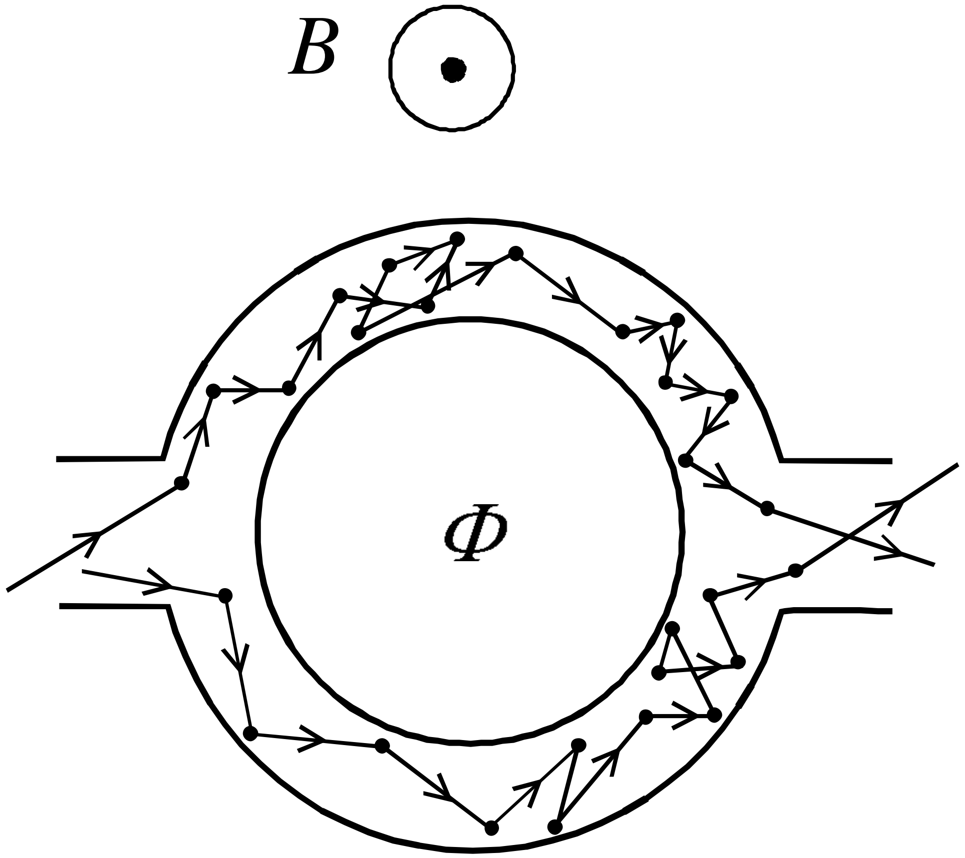
$$\Delta\delta(\phi) = \delta_1^{(0)} - \delta_2^{(0)} + \frac{e}{\hbar} \oint \mathbf{A}.d\mathbf{l} = \Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0}$$

where $\phi_0 = h/e$ is the quantum of magnetic flux.

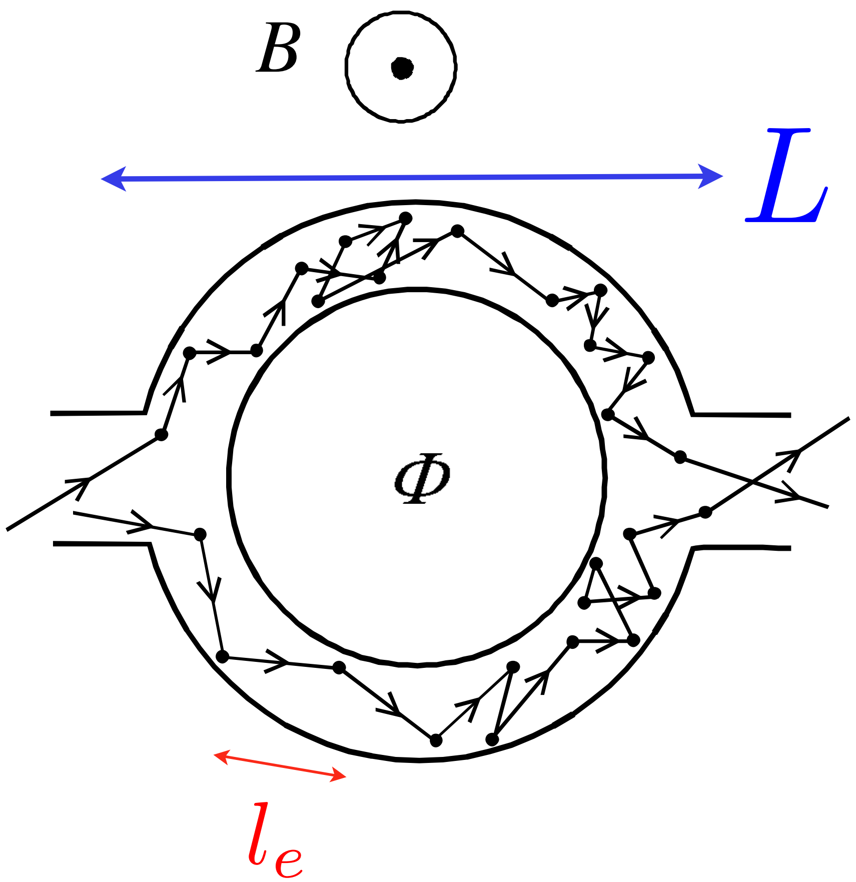
There is a continuous change of the state of interference:

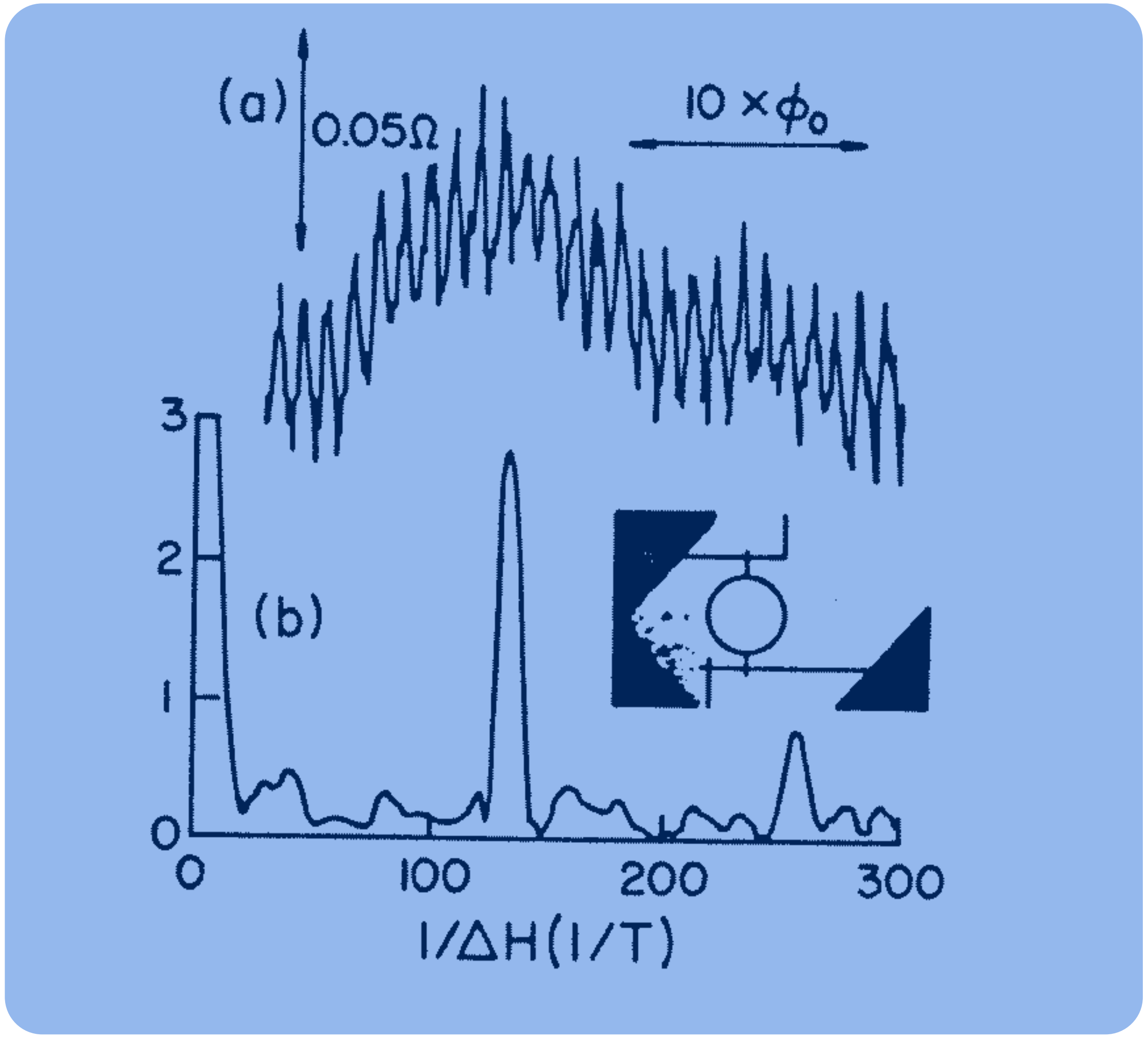
Aharonov-Bohm effect (1959)

- Aharonov-Bohm effect in disordered metals



Implementation in metals : the conductance $G(\phi)$ is the analog of the intensity.





Webb et al. 1985

$$G(\phi) = G_0 + \delta G \cos(\Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0})$$

Phase coherent effects subsist in ***disordered*** metals.

Reconsider the Drude theory.

Phase coherence and effect of disorder

The *Webb* experiment has been realized on a ring of size $L \simeq 1\mu$.

For a macroscopic normal metal, coherent effects are washed out.

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Quantum coherence: gas of quantum particles in a finite volume

Quantum states of the gas are coherent superposition of single particle states and they extend over the total volume (*ex. superconductivity, superfluidity, free electron gas*).

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Quantum coherence: gas of quantum particles in a finite volume

Quantum states of the gas are coherent superposition of single particle states and they extend over the total volume (*ex. superconductivity, superfluidity, free electron gas*).

For the electron gas, coherence disappears at non zero temperature so that we can use a classical description of transport and thermodynamics

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Vanishing of quantum coherence results from the existence of incoherent and irreversible processes associated to the coupling of electrons to their surrounding (additional degrees of freedom) :

Coupling to a bath of excitations : thermal excitations of the lattice (phonons)

Chaotic dynamical systems (large recurrence times, Feynman chain)

Impurities with internal degrees of freedom (magnetic impurities)

Electron-electron interactions,....

The understanding of decoherence is difficult.

It is a great challenge in quantum mesoscopic physics.

The phase coherence length L_ϕ accounts in a generic way for decoherence processes.

The observation of coherent effects requires

$$L << L_\phi$$

Averaging over disorder ?

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Expect to wash up interference effects

Average coherence and multiple scattering

What is the role of <u>**elastic**</u> disorder ? Does it erase coherent effects ?

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Phase coherence leads to interference effects for a *given realization of disorder.*

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Phase coherence leads to interference effects for a *given realization of disorder.*

The *Webb* experiment corresponds to a fixed configuration of disorder.

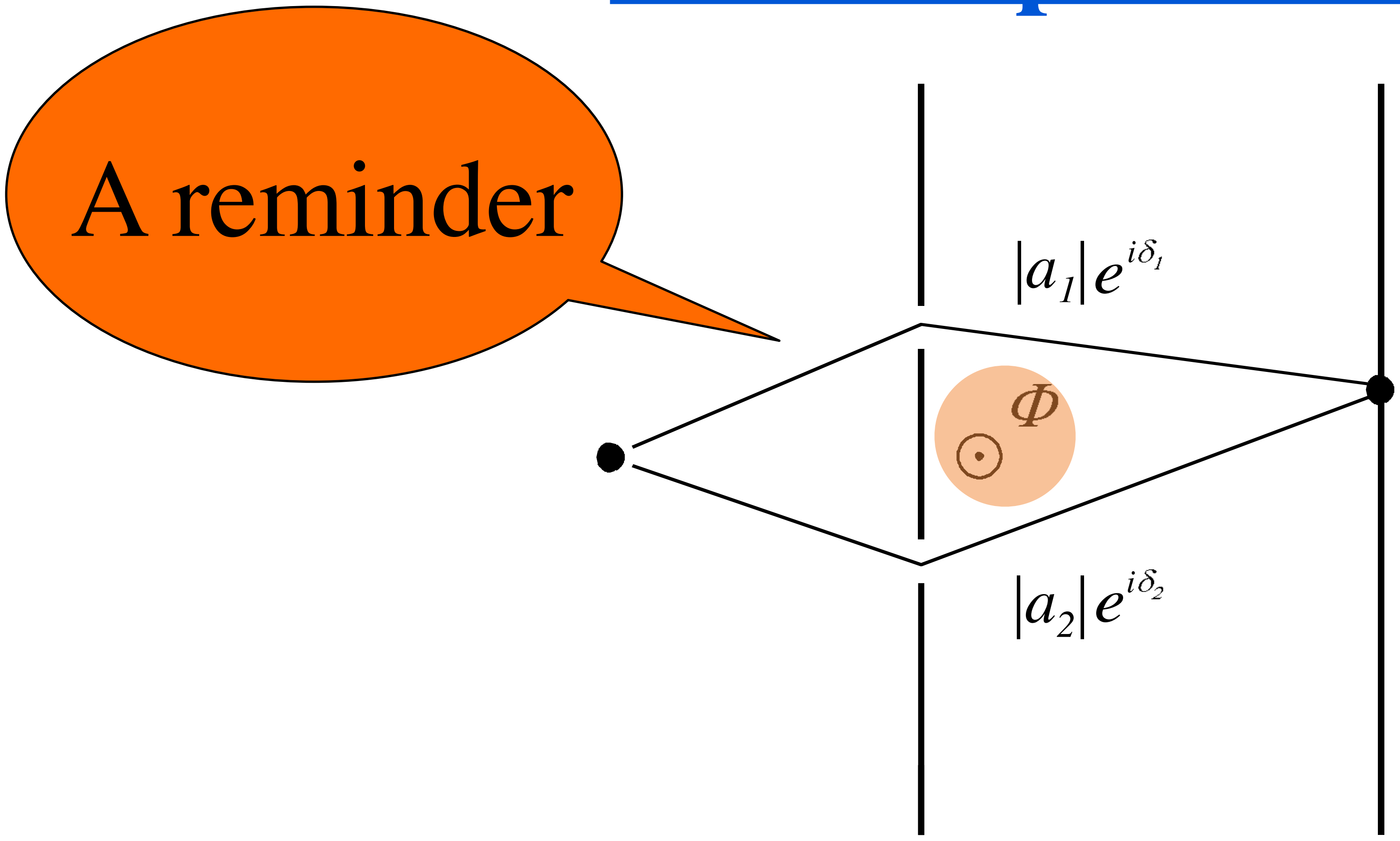
Averaging over disorder ⇒ vanishing of the Aharonov-Bohm effect

$$G(\phi) = G_0 + \delta G \cos(\Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0})$$

$$\Rightarrow \quad \langle G(\phi) \rangle = G_0$$

Disorder seems to erase coherent effects....

The setup : Young slits



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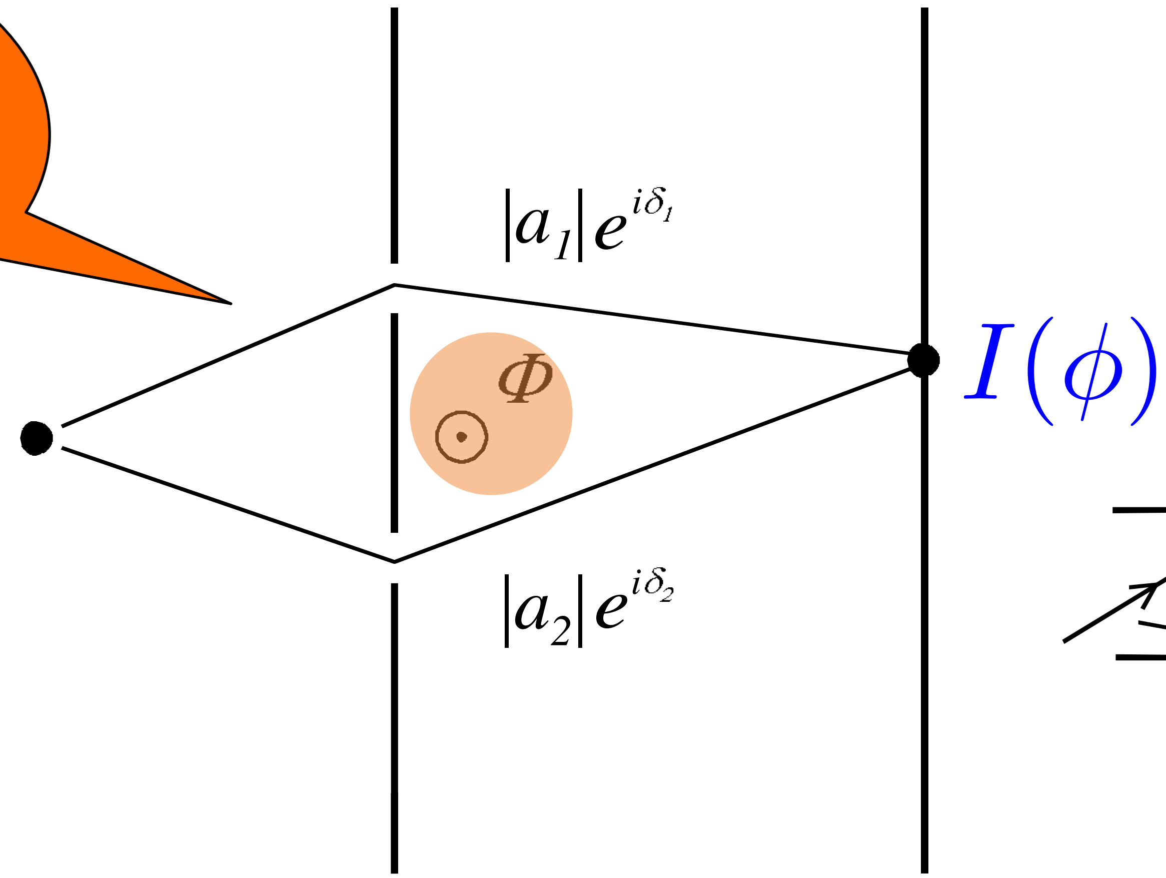
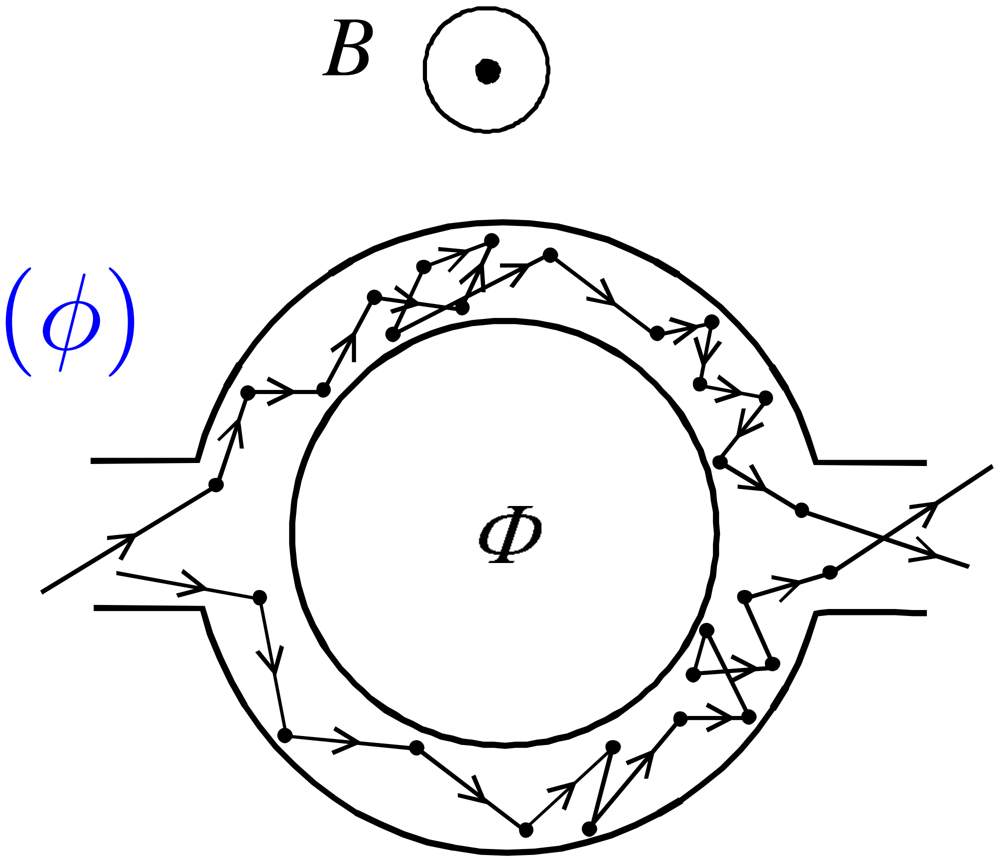
A reminder





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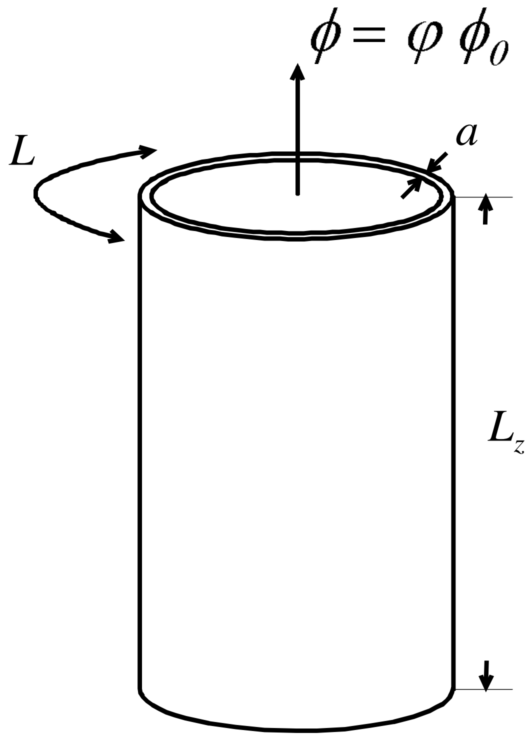
The Sharvin^2 experiment



Experiment analogous to that of *Webb* but performed on a hollow cylinder of height larger than L_ϕ pierced by a Aharonov-Bohm flux.

Ensemble of rings identical to those of *Webb* but incoherent between themselves.

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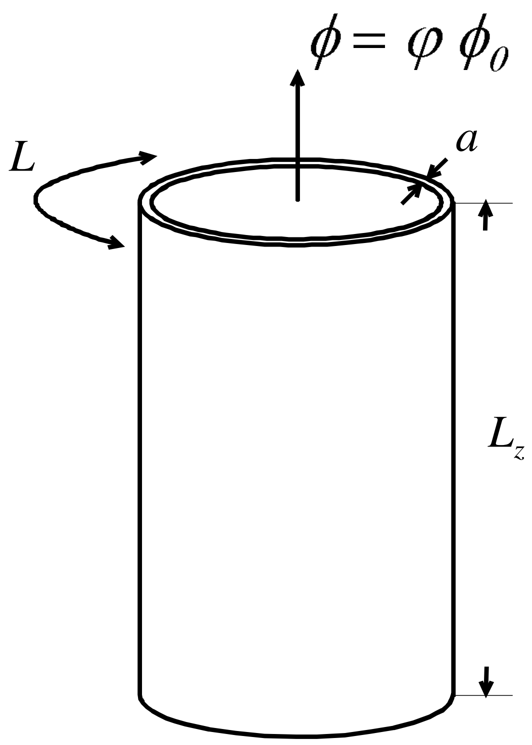

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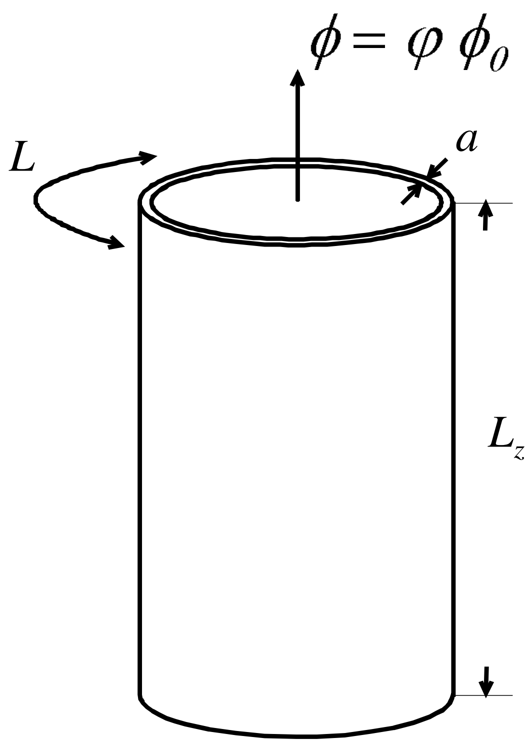

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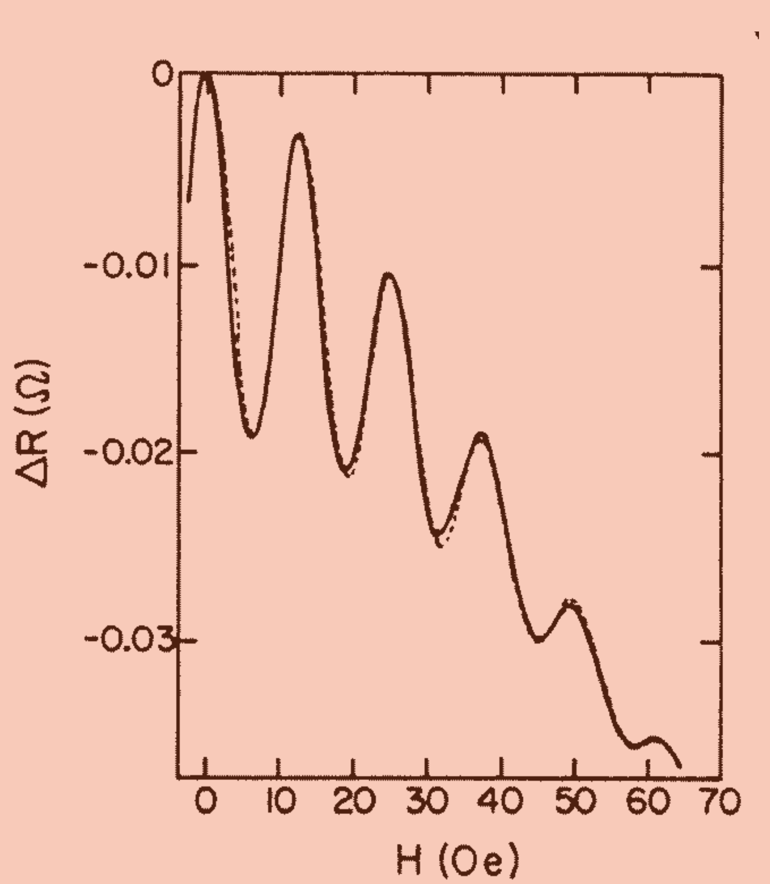


The signal modulated at ϕ_0 *disappears* but, instead, it appears a new contribution modulated at $\phi_0/2$

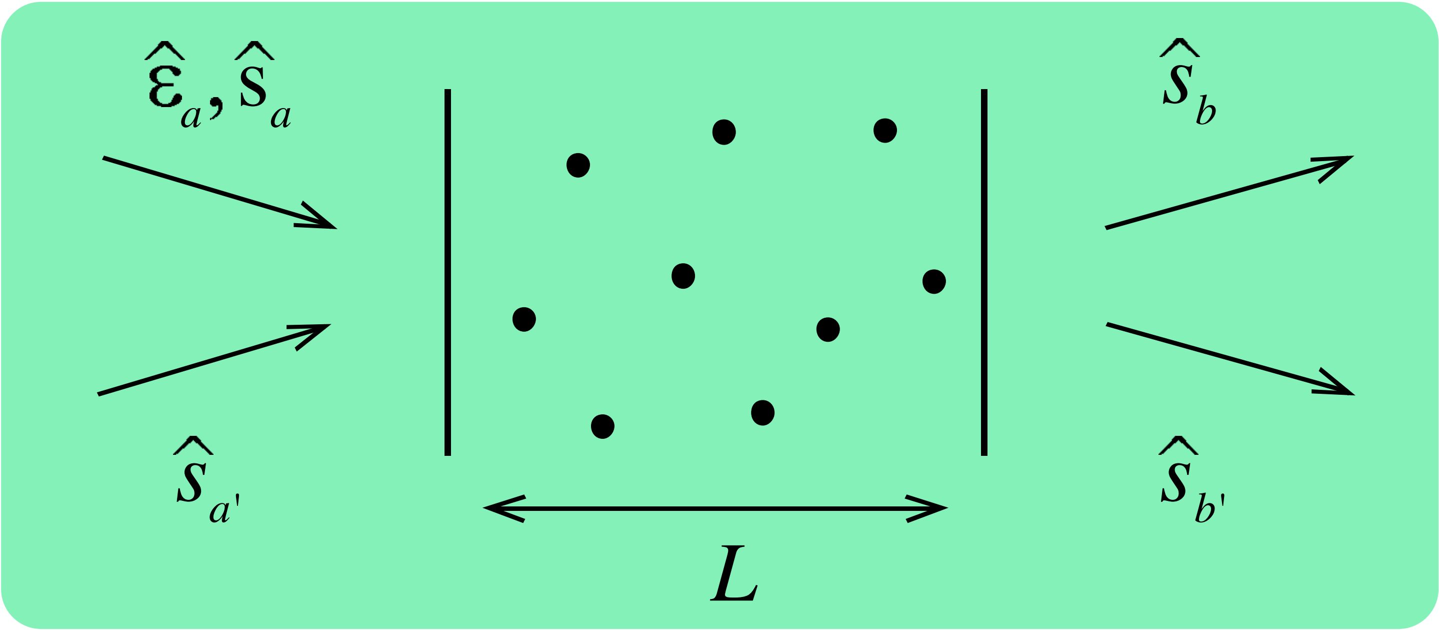
After all, disorder does not seem to erase coherent effects, but to modify them....

Some “canonical” mesoscopic effects

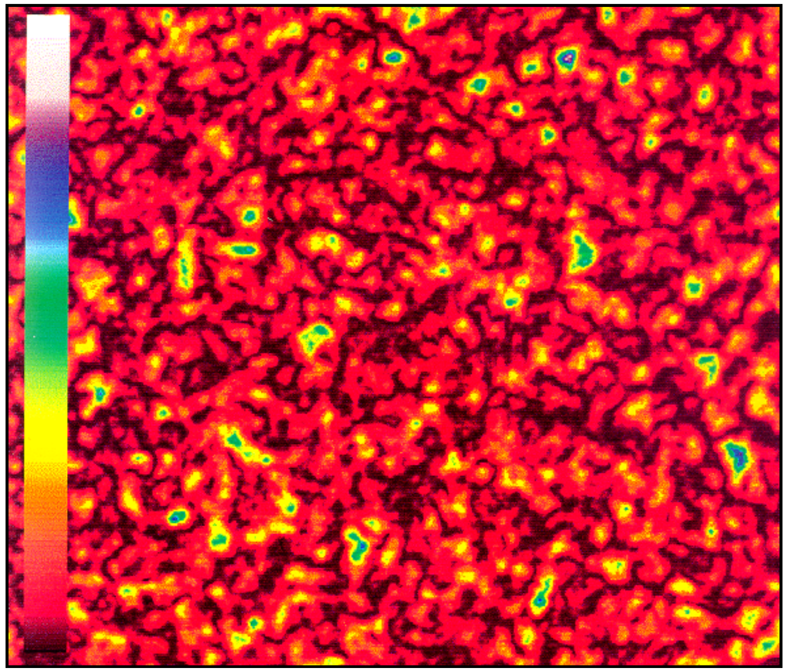
Coherent backscattering in optics

An analogous problem: *Speckle patterns in optics*

Consider the elastic multiple scattering of light transmitted through a fixed disorder configuration.

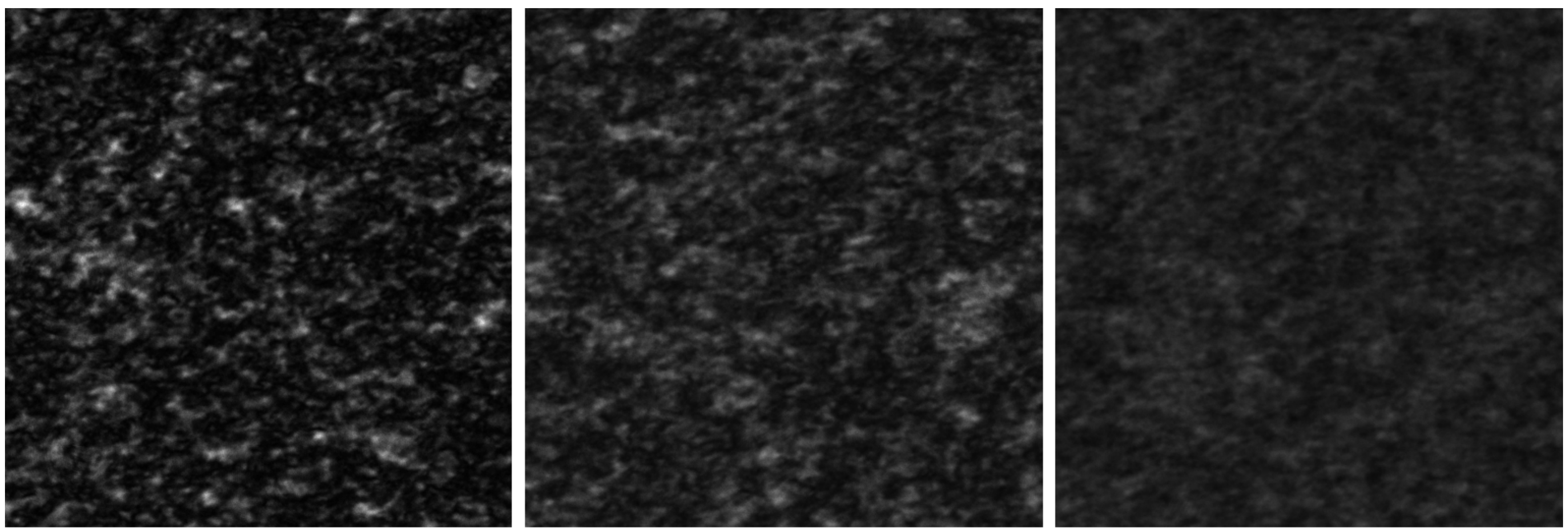


Outgoing light builds a speckle pattern *i.e.*, an interference picture:



Averaging over disorder erases the speckle pattern:

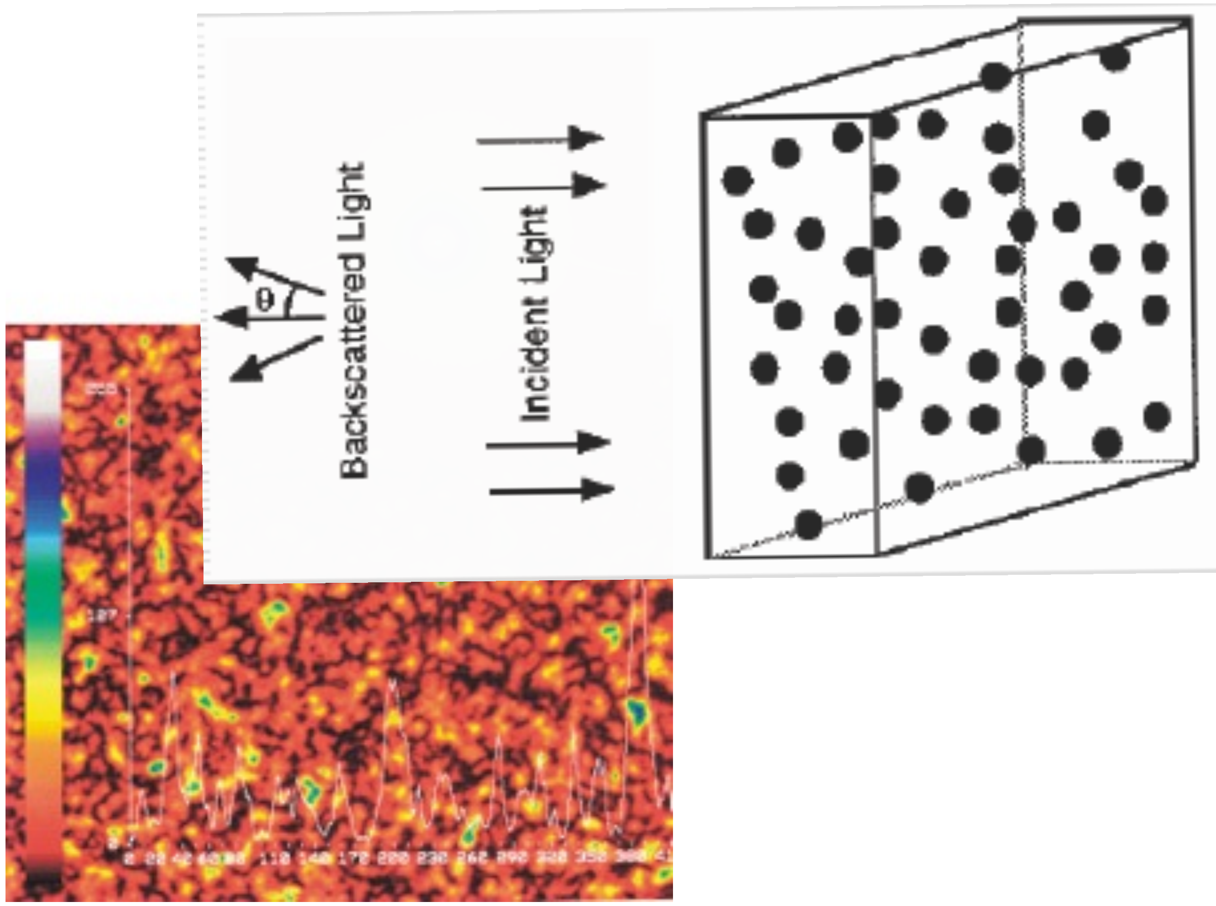
Integration over the motion of the scatterers leads to self-averaging

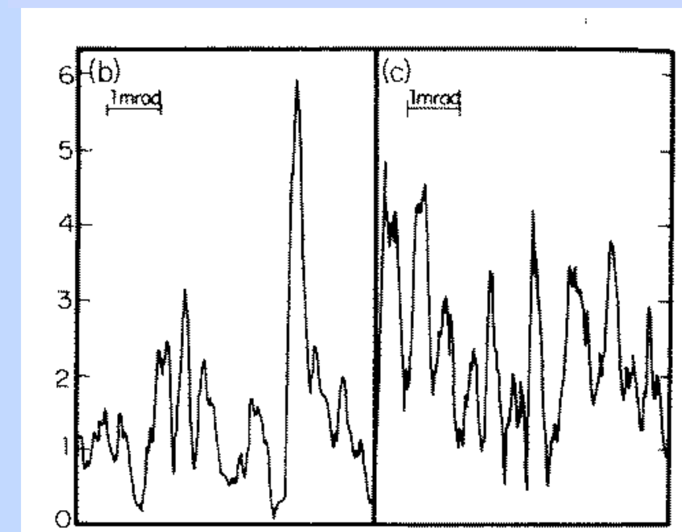


(a) (b) (c)

Time averaging

Does it erase all interferences ?





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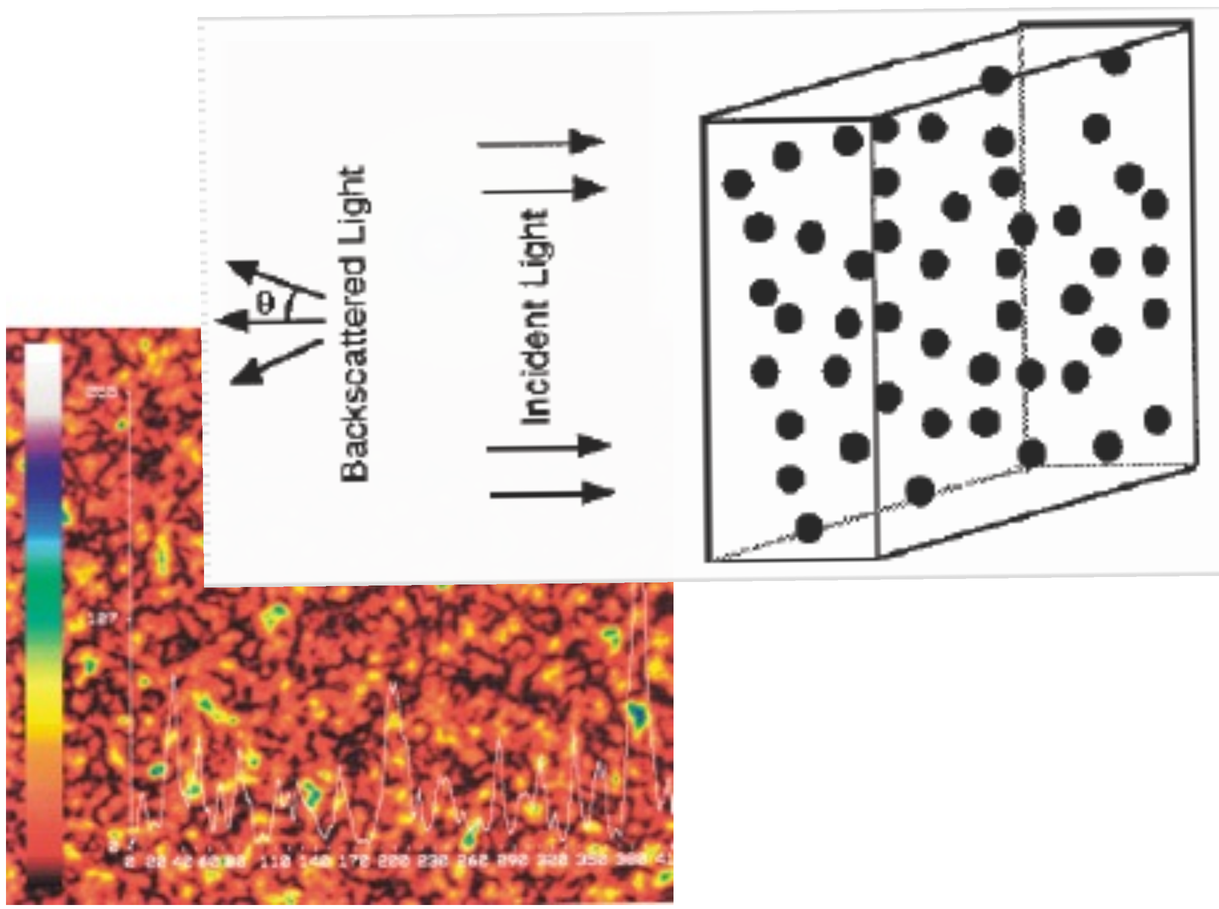
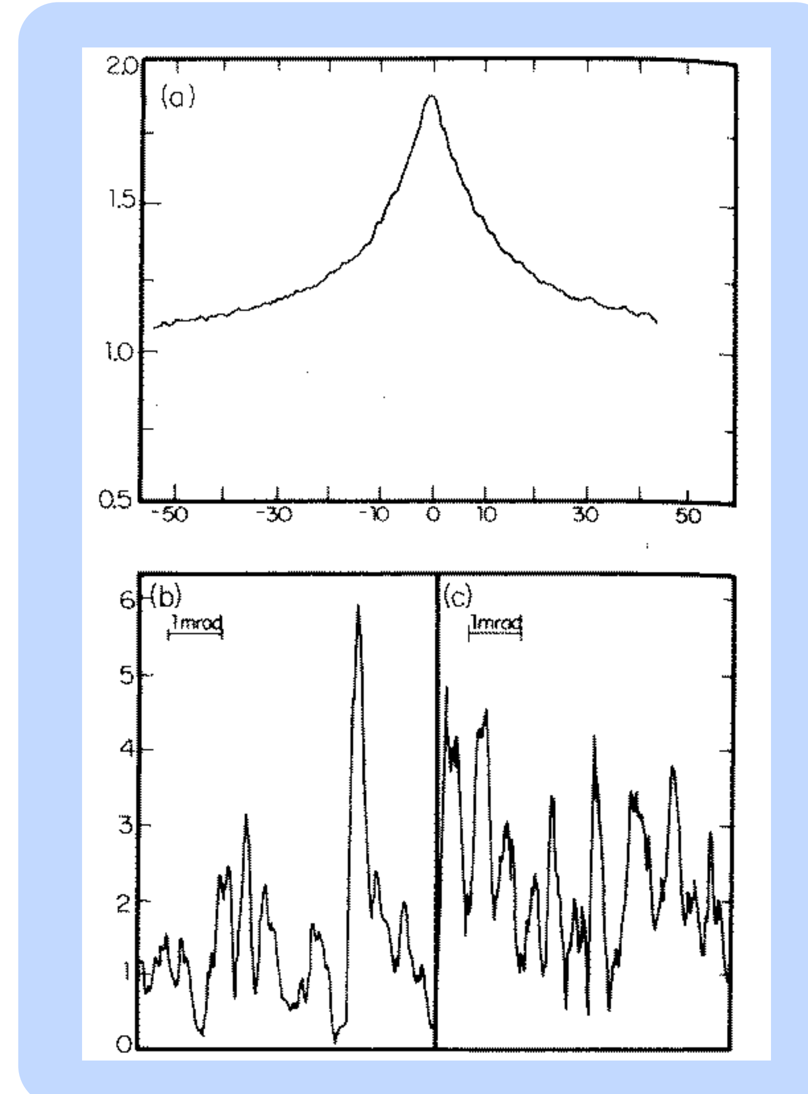
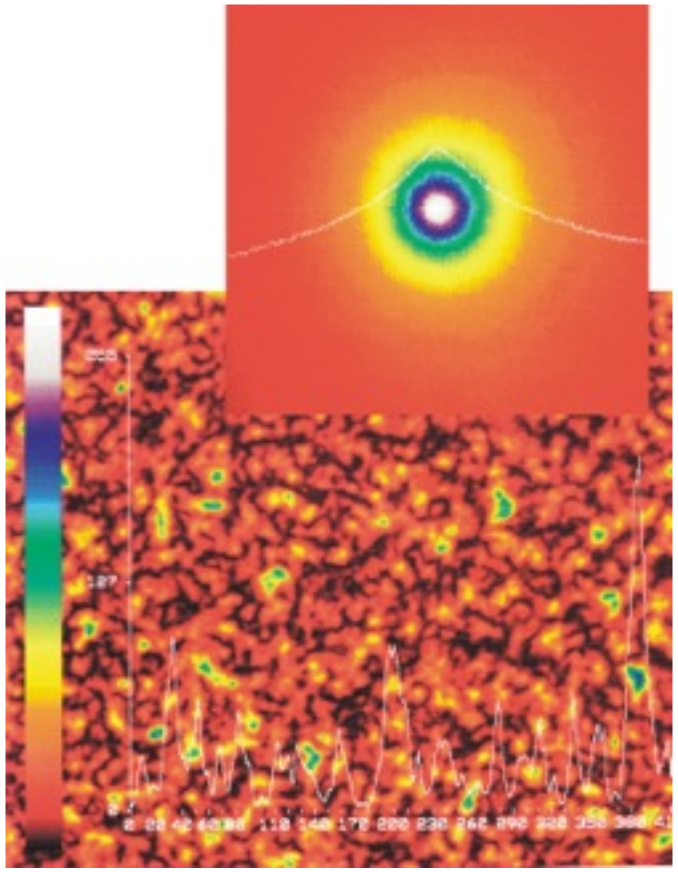


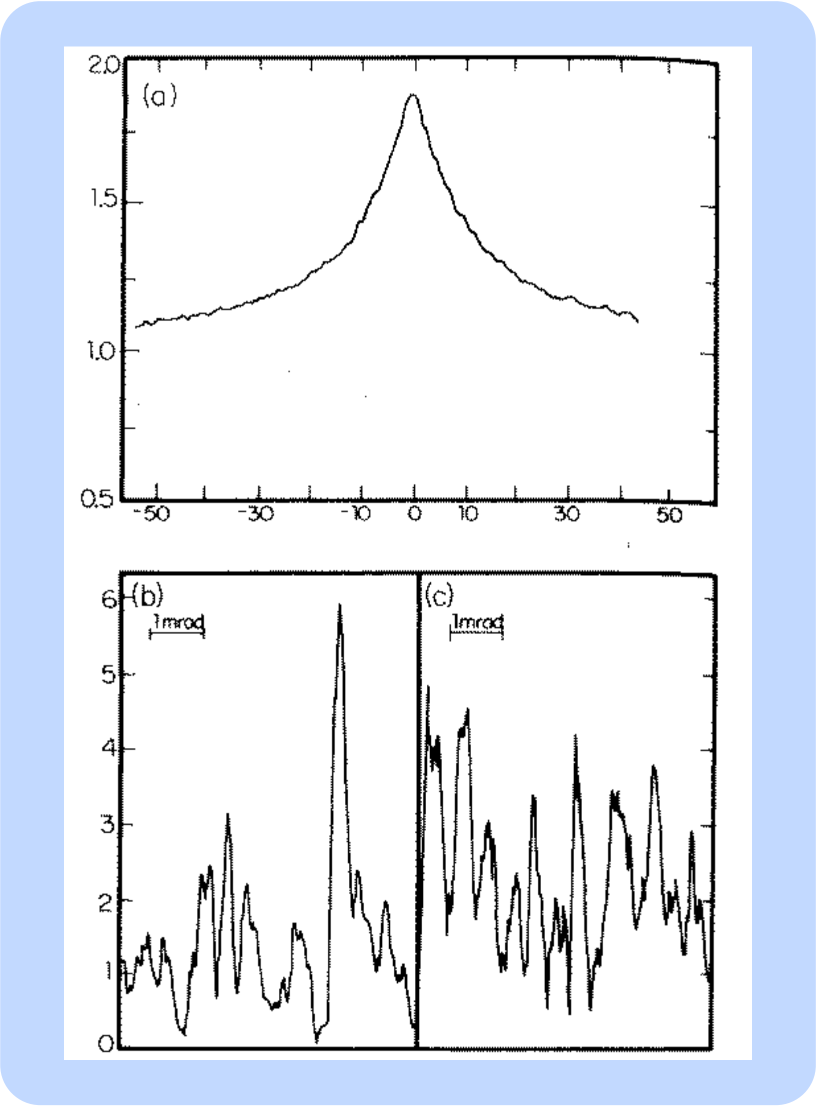


Averaging over disorder does not produce incoherent intensity only, but also an angular dependent part, the <u>coherent backscattering</u>, which is a coherence effect. We may conclude:

Elastic disorder is not related to decoherence : disorder does not destroy phase coherence and does not introduce irreversibility.

Does it erase all interferences ?





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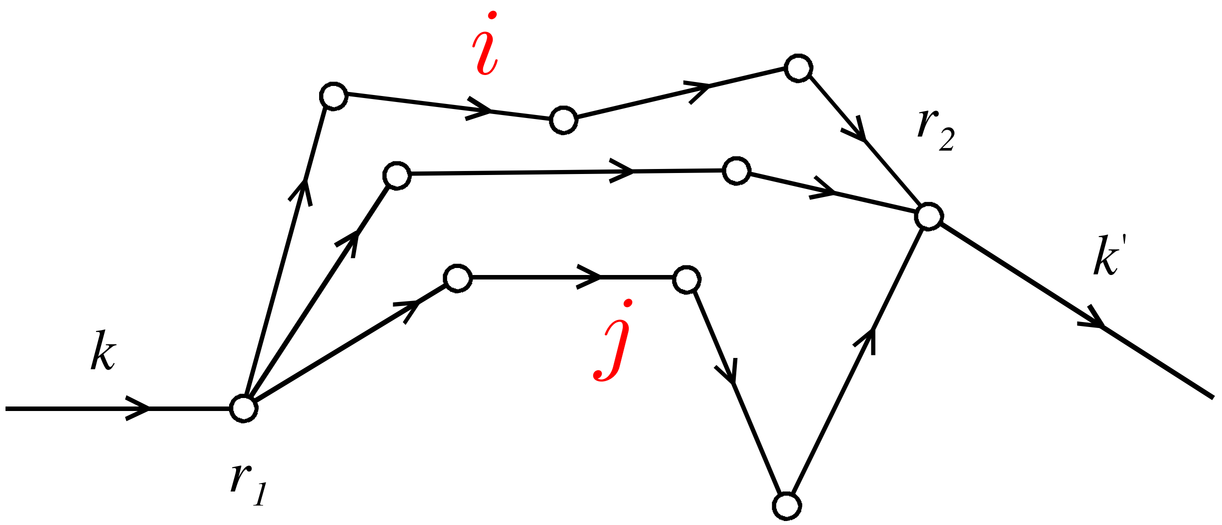
How to understand average coherent effects ?



Complex amplitude $A(\mathbf{k},\mathbf{k}')$ associated to the multiple scattering of a wave (electron or photon) incident with a wave vector $\mathbf{k}$ and outgoing with $\mathbf{k}'$

$$A(\mathbf{k},\mathbf{k}') = \sum_{\mathbf{r_1},\mathbf{r_2}} f(\mathbf{r_1},\mathbf{r_2}) e^{i(\mathbf{k}.\mathbf{r_1} - \mathbf{k}'.\mathbf{r_2})}$$

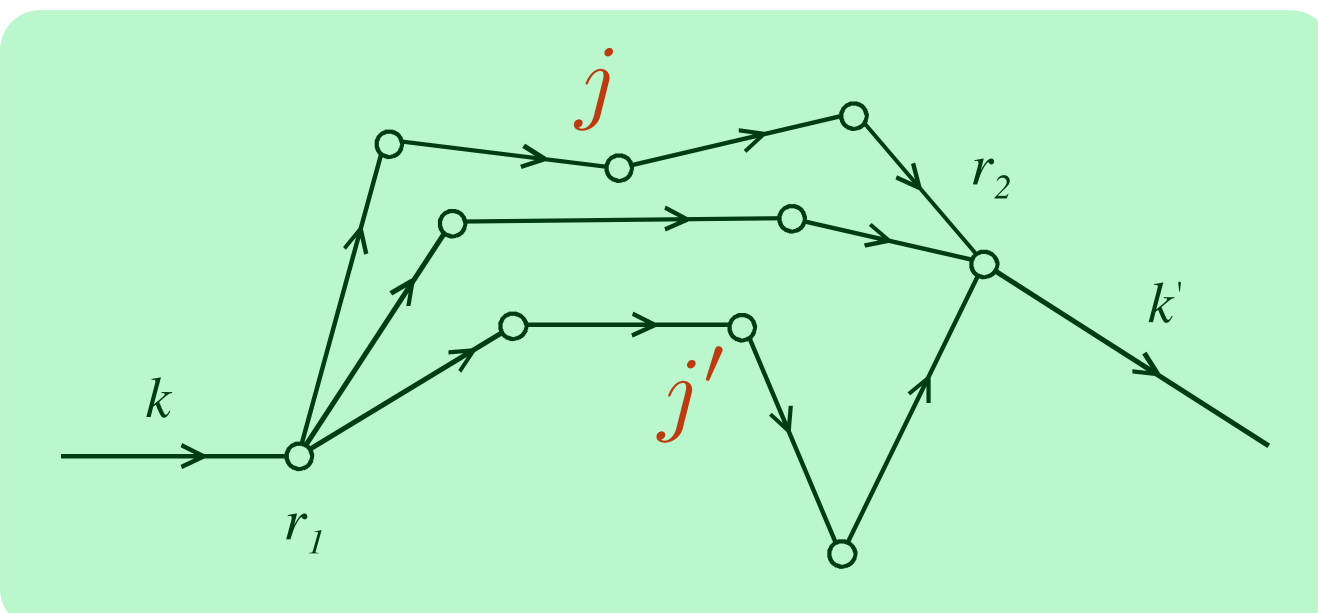
the complex amplitude $f(\mathbf{r_1},\mathbf{r_2}) = \sum_j |a_j| e^{i\delta_j}$ describes the propagation of the wave between $\mathbf{r_1}$ and $\mathbf{r_2}$.

The corresponding intensity is

$$|A(\mathbf{k},\mathbf{k}')|^2 = \sum_{\mathbf{r_1},\mathbf{r_2}} \sum_{\mathbf{r_3},\mathbf{r_4}} f(\mathbf{r_1},\mathbf{r_2}) f^*(\mathbf{r_3},\mathbf{r_4}) e^{i(\mathbf{k}.\mathbf{r_1}-\mathbf{k}'.\mathbf{r_2})} e^{-i(\mathbf{k}.\mathbf{r_3}-\mathbf{k}'.\mathbf{r_4})}$$

with

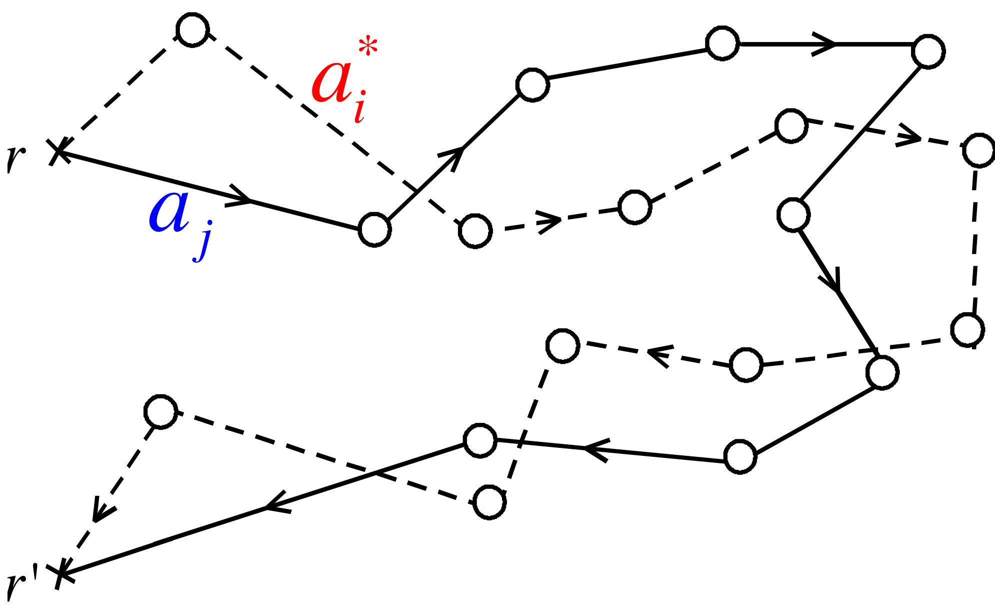
$$f(\mathbf{r_1},\mathbf{r_2}) f^*(\mathbf{r_3},\mathbf{r_4}) = \sum_{j,j'} a_j(\mathbf{r_1},\mathbf{r_2}) a^*_{j'}(\mathbf{r_3},\mathbf{r_4}) = \sum_{j,j'} |a_j||a_{j'}| e^{i(\delta_j - \delta_{j'})}$$



On average over disorder, most contributions to $f f^*$ disappear since the dephasing $\delta_j - \delta_{j'} \gg 1$

The only remaining contributions to the intensity correspond to terms with zero dephasing, *i.e.*, to identical trajectories.

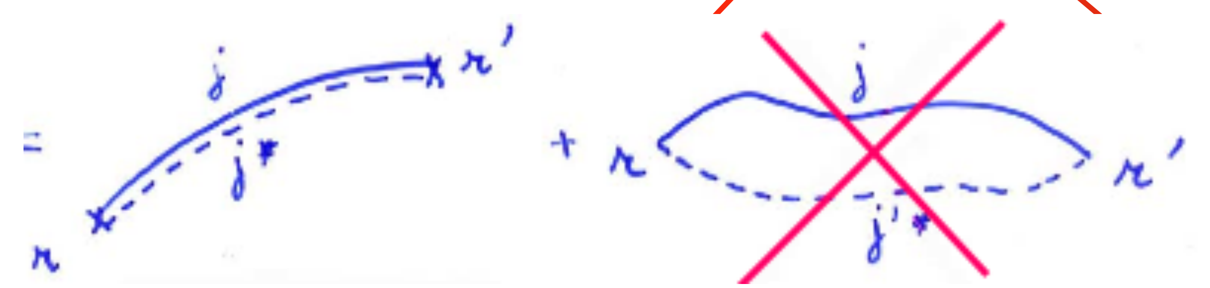
Quantum probability for propagation between two points



$$P(r,r') = \sum_{i,j} \overline{a_i^*(r,r')\, a_j(r,r')}$$

$$P(r,r') = \sum_j \overline{\left|a_j(r,r')\right|^2} + \sum_{i \neq j} \overline{a_i^*(r,r') a_j(r,r')}$$

$$a_i^* a_j = |a_i| |a_j| e^{i(\delta_i - \delta_j)}$$

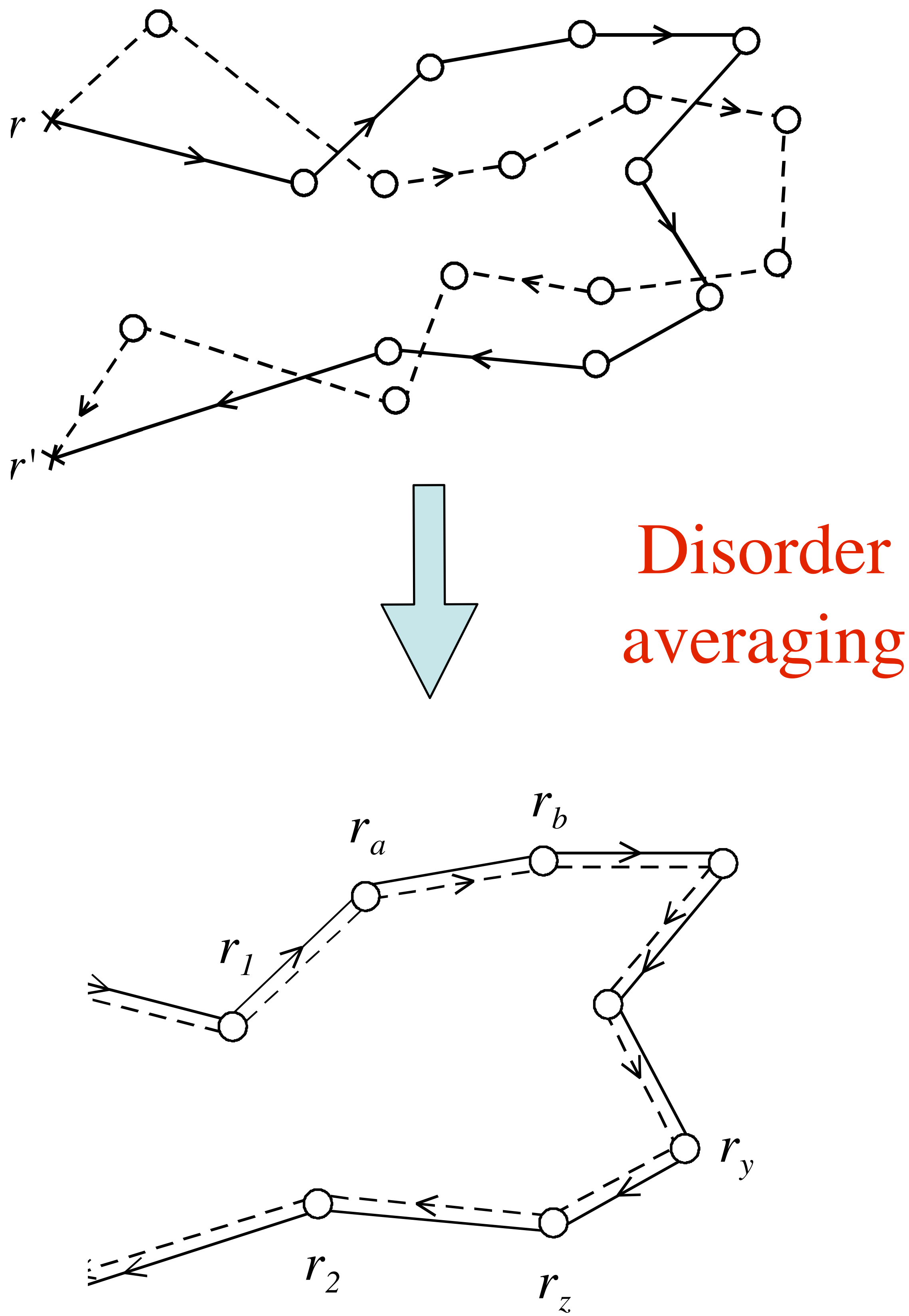


$$\delta_i - \delta_j \gg 1$$

Incoherent propagation !

vanishes on average

Some useful design



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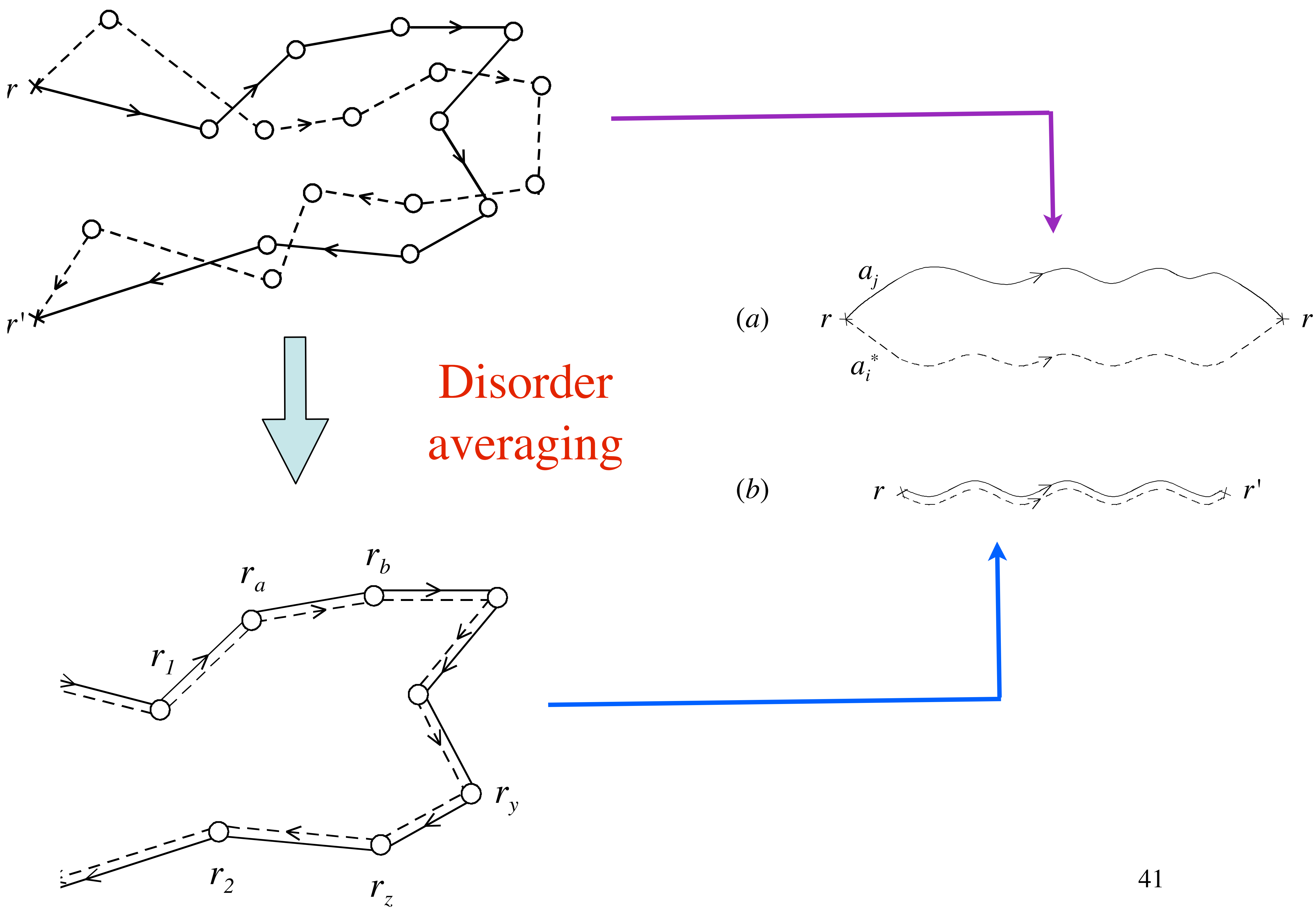
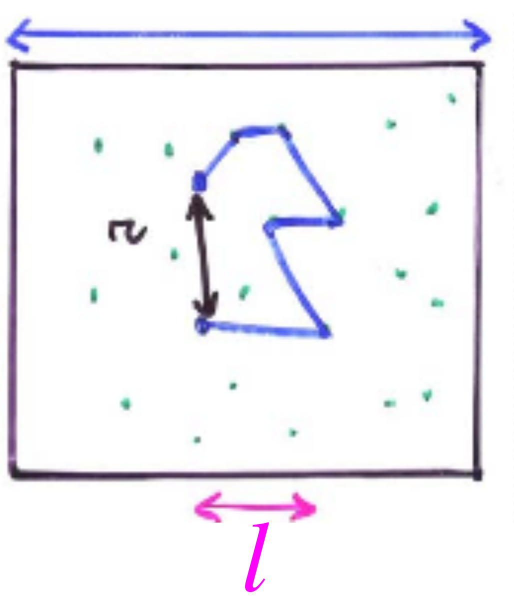


To a good approximation, the incoherent contribution obeys a classical diffusion equation

$$\left(\frac{\partial}{\partial t} - D\Delta\right)P(r,r',t) = \delta(r-r')\delta(t) \quad \Leftrightarrow \quad \left(-i\omega + Dq^2\right)P(q,\omega) = 1$$

Incoherent electrons diffuse in the conductor with a *diffusion coefficient D* *(Drude theory)*



$l \ll L$

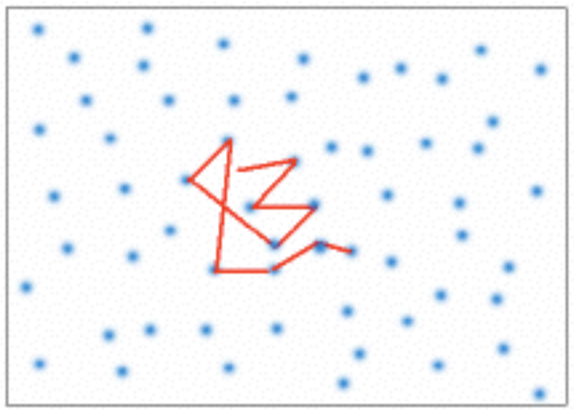
$$\langle r^2 \rangle = 2d\ Dt$$

space dimensionality

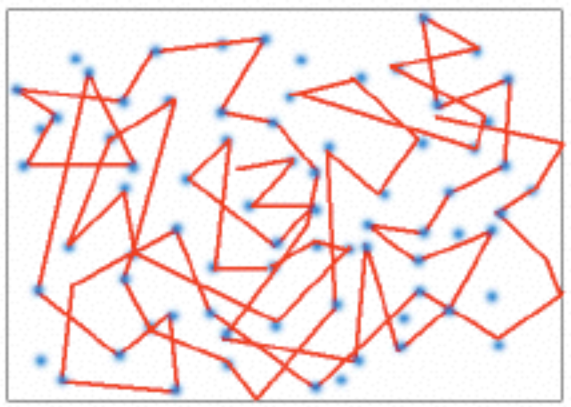
Thouless time

$$L^2 = D\tau_D$$

$t \ll \tau_D$

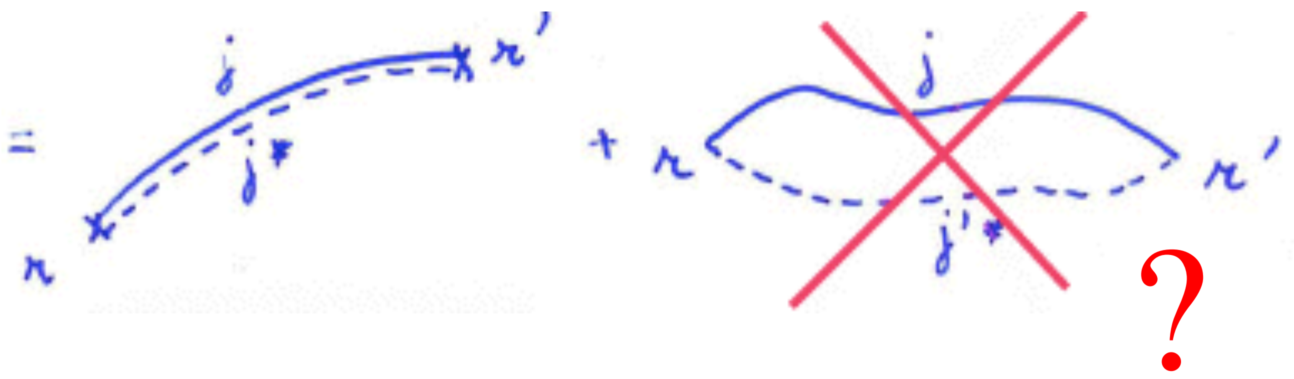


$t \gg \tau_D$



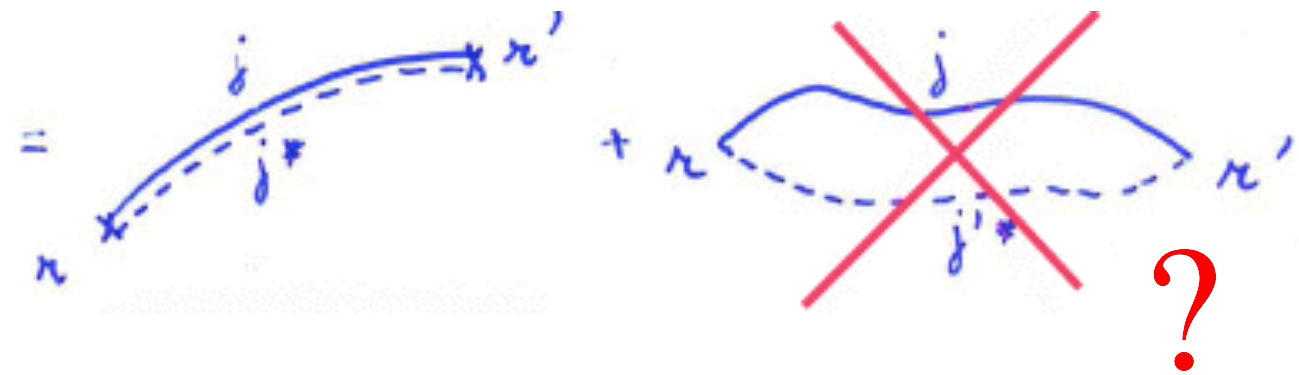
Beyond incoherent diffusion (qualitative description)

Coherent effects



What is the first correction *i.e.*, with the *smallest phase shift* ?

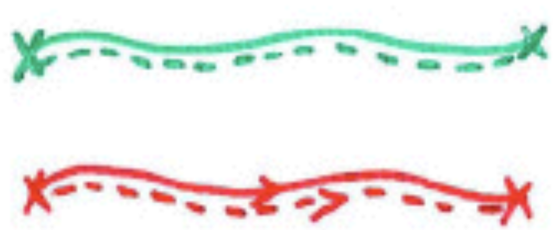
Coherent effects



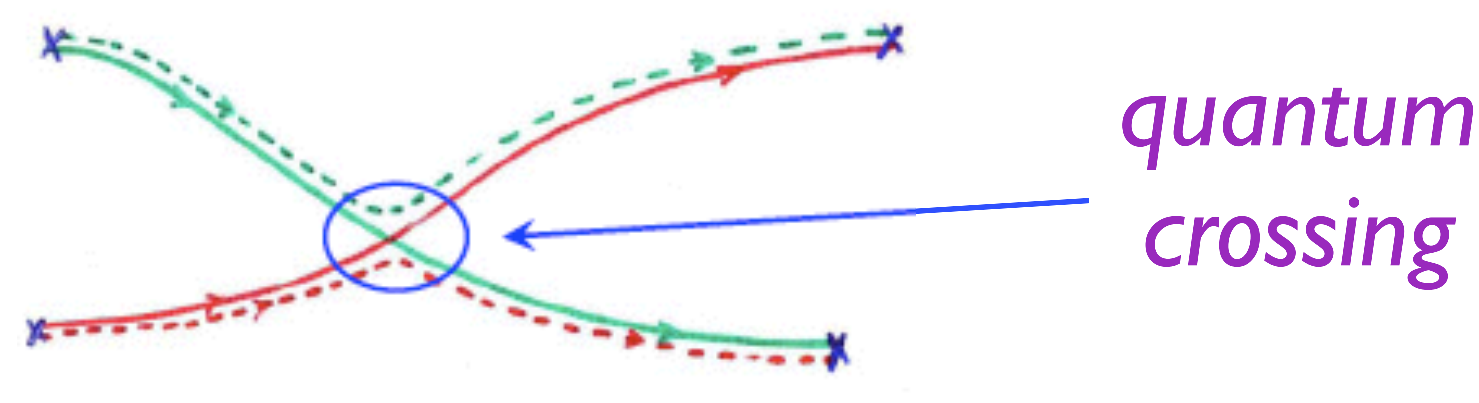
What is the first correction *i.e.*, with the *smallest phase shift* ?

When amplitude paths cross

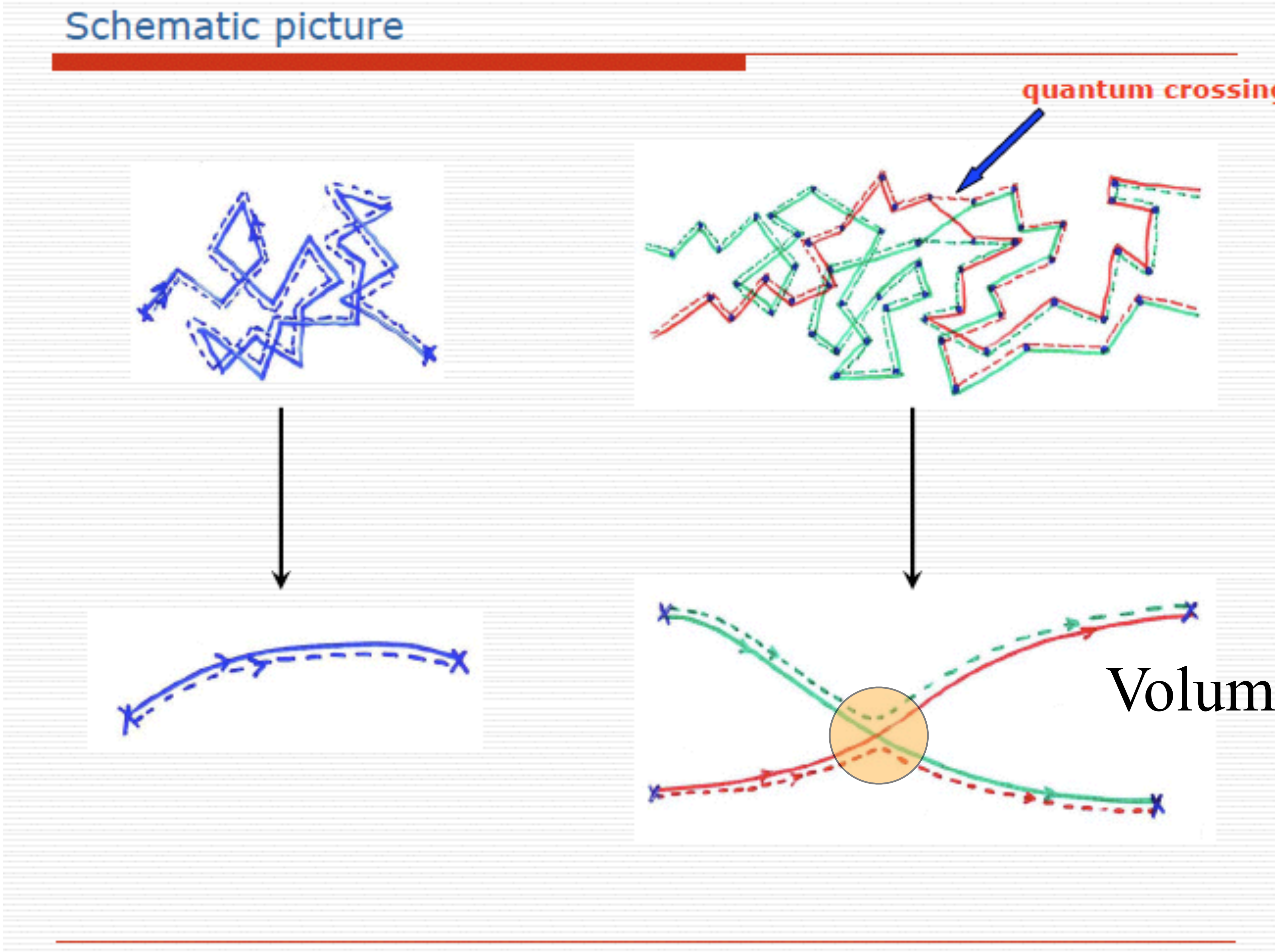
Example :



Classical diffusion



Exchange of amplitudes



Volume $\lambda^{d-1}\, l$

Quantum crossings decrease the diffusion coefficient D :

weak localization

λ : Fermi wavelength

Occurrence of a *quantum crossing* after a time t for a electron diffusing in a volume L^d

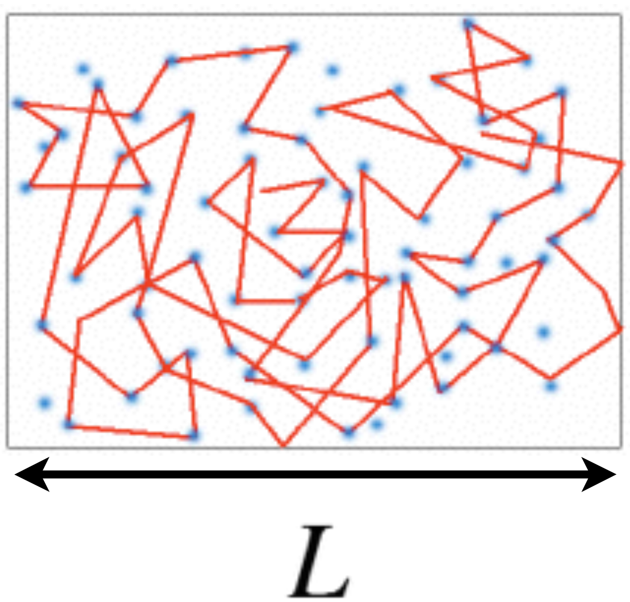
$$p_\times(t) = \frac{\lambda_F^{d-1} v_F t}{L^d}$$

v_F : Fermi velocity

The time spent by a diffusing electron is $\tau_D = L^2/D$ so that

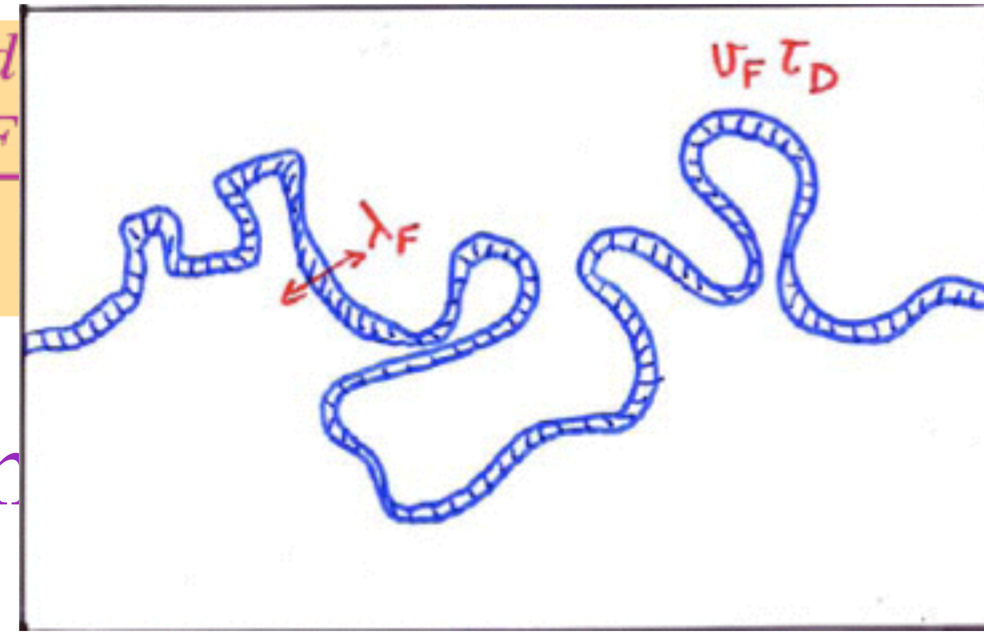
$$p_\times(\tau_D) = \frac{\lambda_F^{d-1} v_F \tau_D}{L^d} \equiv \frac{1}{g}$$

$$g = \frac{D}{v_F \lambda_F^{d-1}} L^{d-2}$$



Occurrence of a *quantum crossing* after a time t for a electron diffusing in a volume L^d

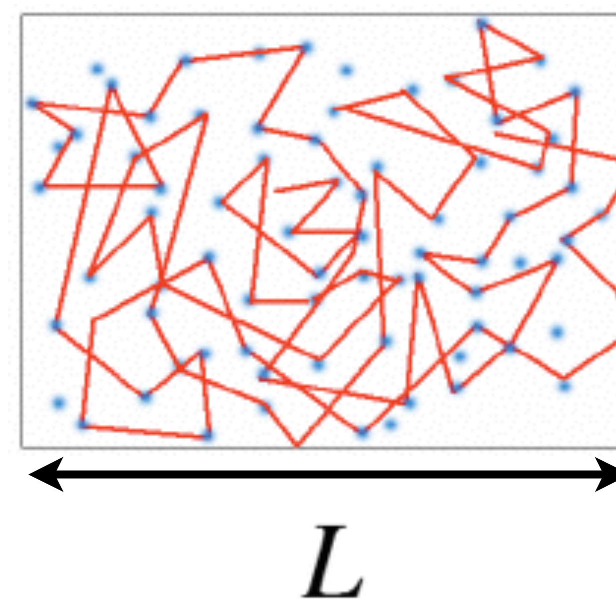
$$p_\times(t) = \frac{\lambda_F^{d}}{\ldots}$$



mi velocity

The time spent by a diffusin … o that

$$g \sim \frac{V}{\lambda_F^{d-1}\, v_F \tau_D}$$



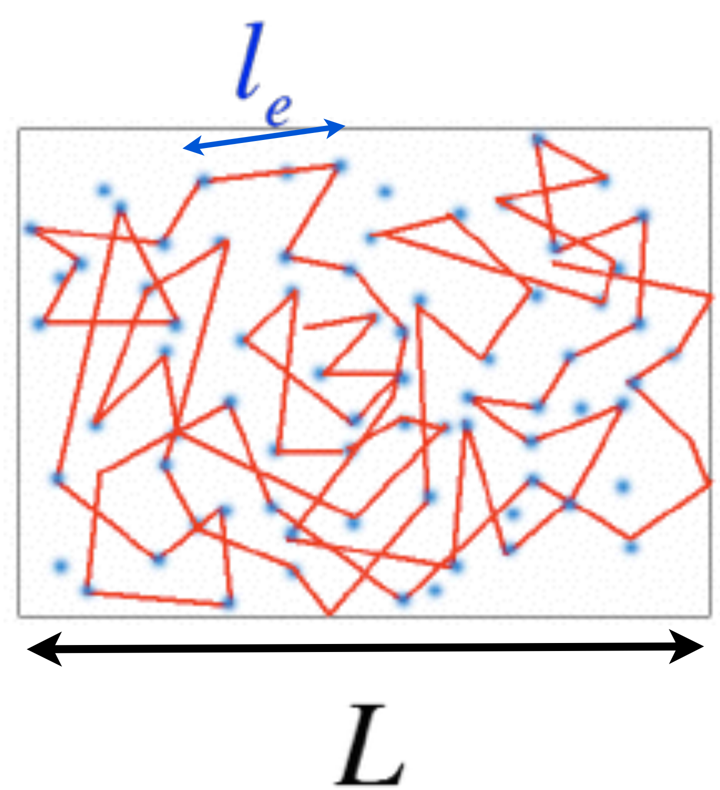
$$g = \frac{D}{v_F \lambda_F^{d-1}} L^{d-2}$$

Physical meaning of this parameter ?

Electrical conductance of a metal

A metal can be modeled as a quantum gas of electrons scattered by an elastic disorder l_e .

Classically, the conductance of a cubic sample of size L^d is given by Ohm's law: $G = \sigma L^{d-2}$ where σ is the conductivity.



Conductance = ratio of two volumes

Conductivity (Drude) $$\sigma = \frac{ne^2\tau_e}{m} = e^2 D\rho$$ $$D = \frac{v_F l_e}{3}$$

Conductance (Ohm's law) $$G = \sigma\frac{S}{L}$$ $$S = W^{d-1}$$

Dimensionless conductance $$G = g\frac{e^2}{h}$$

$$g \sim \frac{D}{v_F\lambda^{d-1}}\frac{W^{d-1}}{L} = \frac{D}{v_F\lambda^{d-1}}\frac{V}{L^2} \sim \frac{V}{\lambda^{d-1}v_F\tau_D}$$

A direct consequence: quantum corrections to electrical transport

Classical transport : $G_{cl} = g \times \frac{e^2}{h} \, \mathrm{with} \, g \gg 1$

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so that $\Delta G \simeq \frac{e^2}{h}$ is universal

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Quantum corrections: Δ

What does it mean ?

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Classical transport : $G_{cl} = g \times \frac{e^2}{h}$ with $g \gg 1$

Independent of the microscopic (and often unknown) disorder - Depends only on the geometry

Quantun

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A direct consequence: quantum corrections to electrical transport

Cla $G_{cl} = g \times \frac{e^2}{h}$ with $g \gg 1$

Not that simple !
numbers... Need to sum up Feynman diagrams.

Quantum correc

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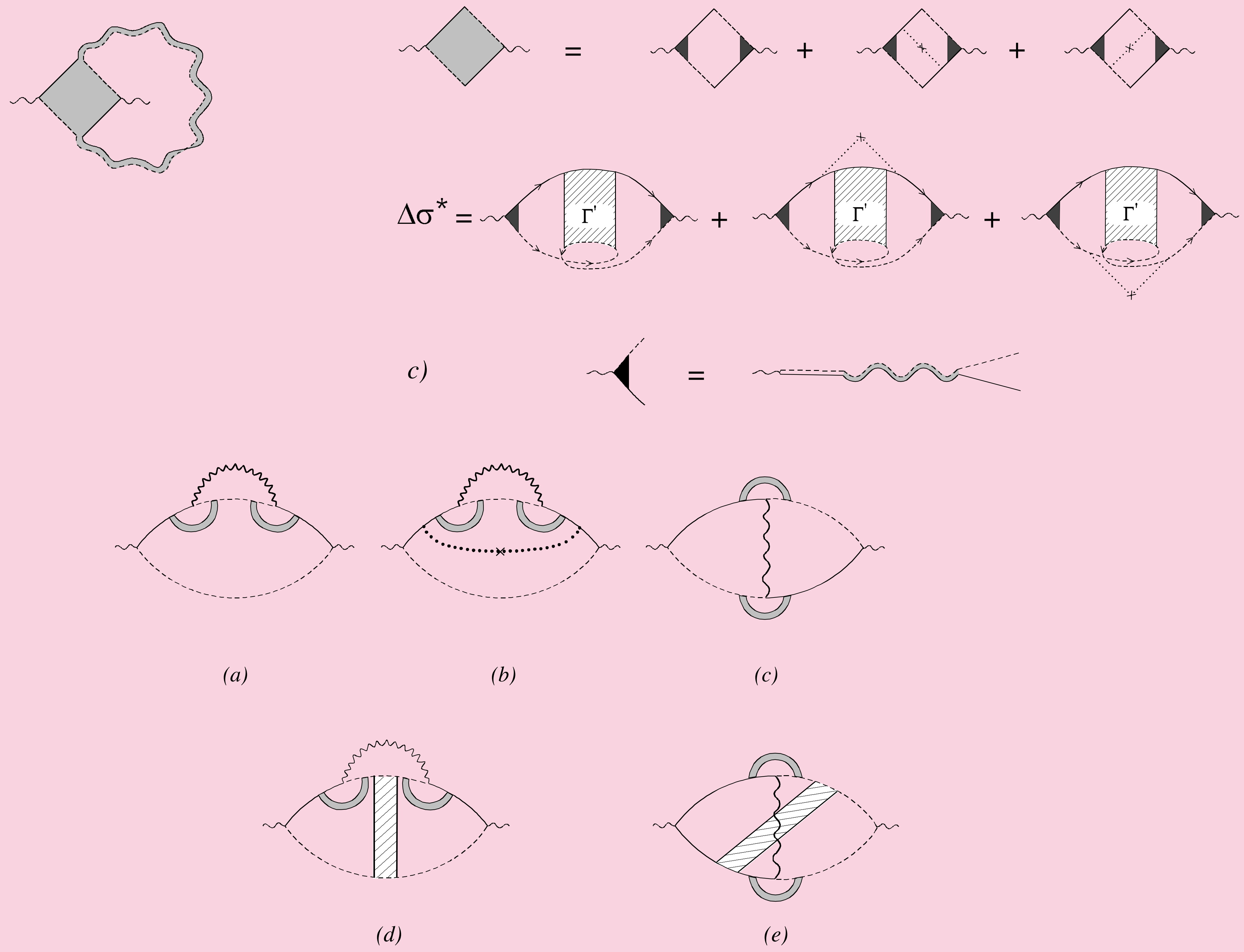
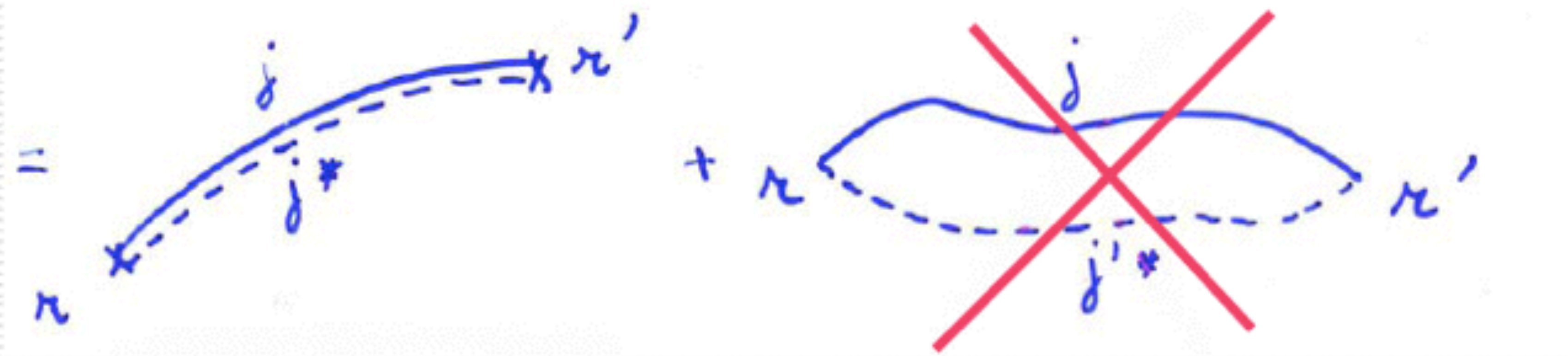
A direct consequence: quantum corrections to electrical



Summary

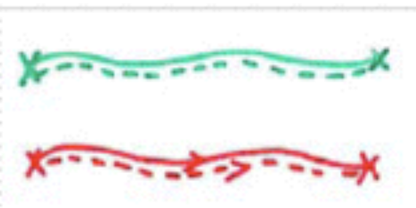
conductance ~ transmission ~ probability

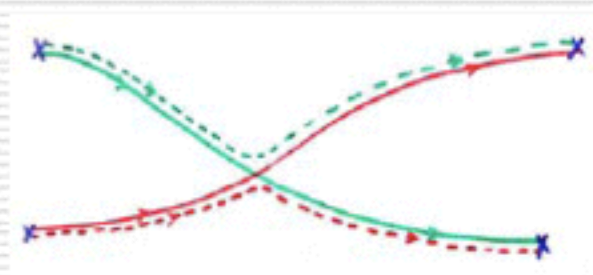
$P_{cl}(r,r') =$



classical diffuson

quantum corrections





quantum crossing → 1/g correction

$$p_{\times}(\tau_D) \sim \frac{\lambda_F^{d-1} v_F \tau_D}{V} \sim \frac{1}{g}$$

classical transport $\propto g \frac{e^2}{h}$

quantum effects $\propto \frac{e^2}{h}$

Complexity of a quantum mesoscopic system

Elastic disorder does not break phase coherence and it does not introduce irreversibility

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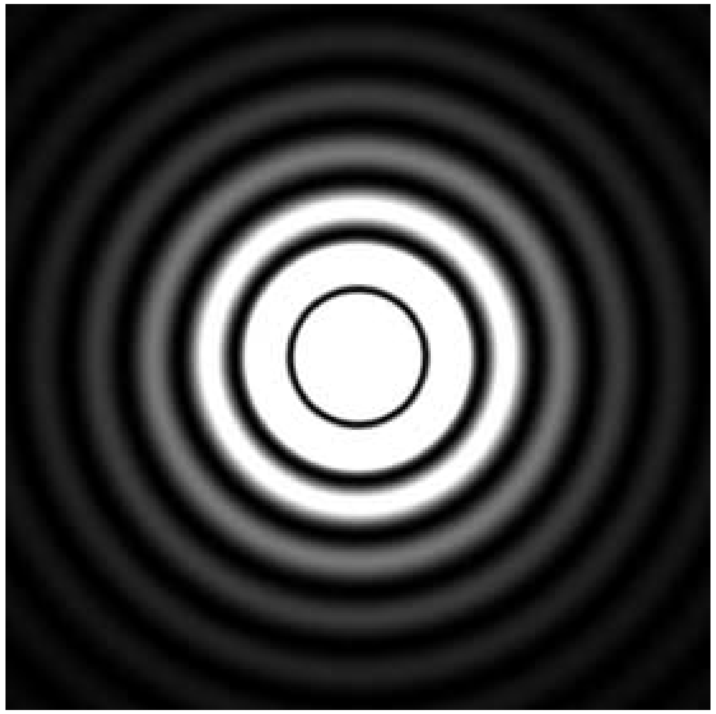
Disorder introduces randomness and complexity:

All symmetries are lost, there are no good quantum numbers.

Exemple: speckle patterns in optics

Diffraction through a circular aperture: order in interference



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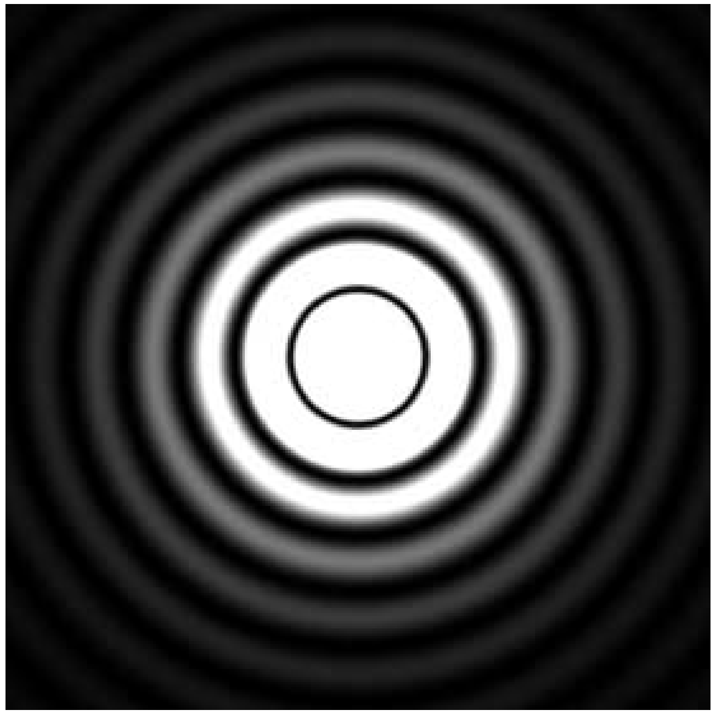

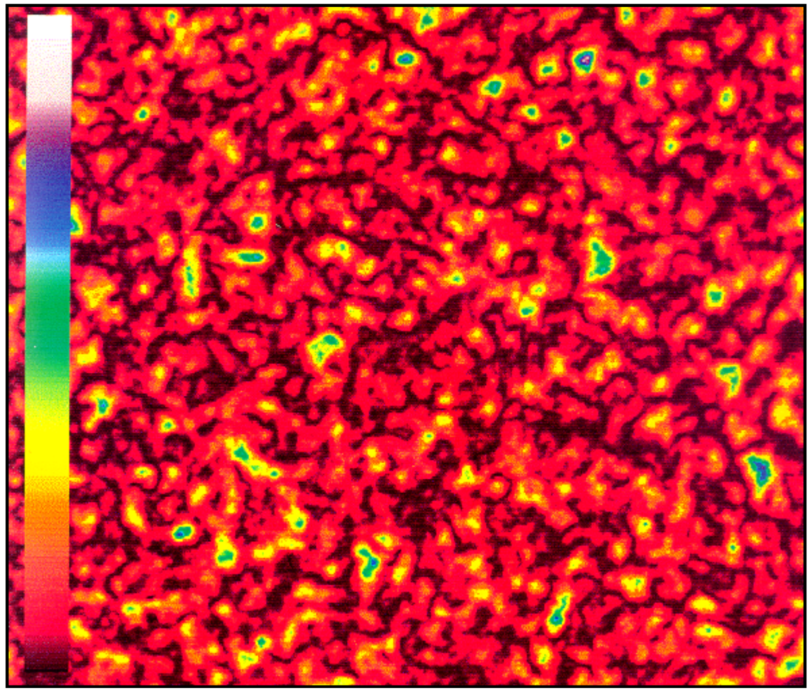

Transmission of light through a disordered suspension: complex system

Mesoscopic quantum systems

- Most (all ?) quantum systems are complex
- Complexity (randomness) and decoherence are separate and independent notions.
- Complexity
  quantum numbers)
- Decoherence
  coherence $L \gg L_\varphi$

A mesoscopic quantum system is a coherent complex quantum system with $L \leq L_\varphi$

Phase coherence and self-averaging: universal fluctuations.

Classical limit : $L \gg L_\varphi$

The system is a collection of $N = (L/L_\varphi)^d \gg 1$ statistically independent subsystems.

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Law of large numbers: any macroscopic observable is equal with probability one to its average value.

The system performs an average over realizations of the disorder.

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For $L \ll L_\varphi$, we expect deviations from self-averaging which reflect the underlying quantum coherence.

Quantum conductance fluctuations

Classical self-averaging limit : $\frac{\delta G}{\overline{G}} = \frac{1}{N} = \left(\frac{L_\varphi}{L}\right)^{d/2}$

where $\delta G = \sqrt{\overline{G^2} - \overline{G}^2}$ and $\overline{G} = \sigma L^{d-2}$

$\overline{\cdots}$ is the average over disorder.

$$\delta G^2 \propto L^{d-4}$$

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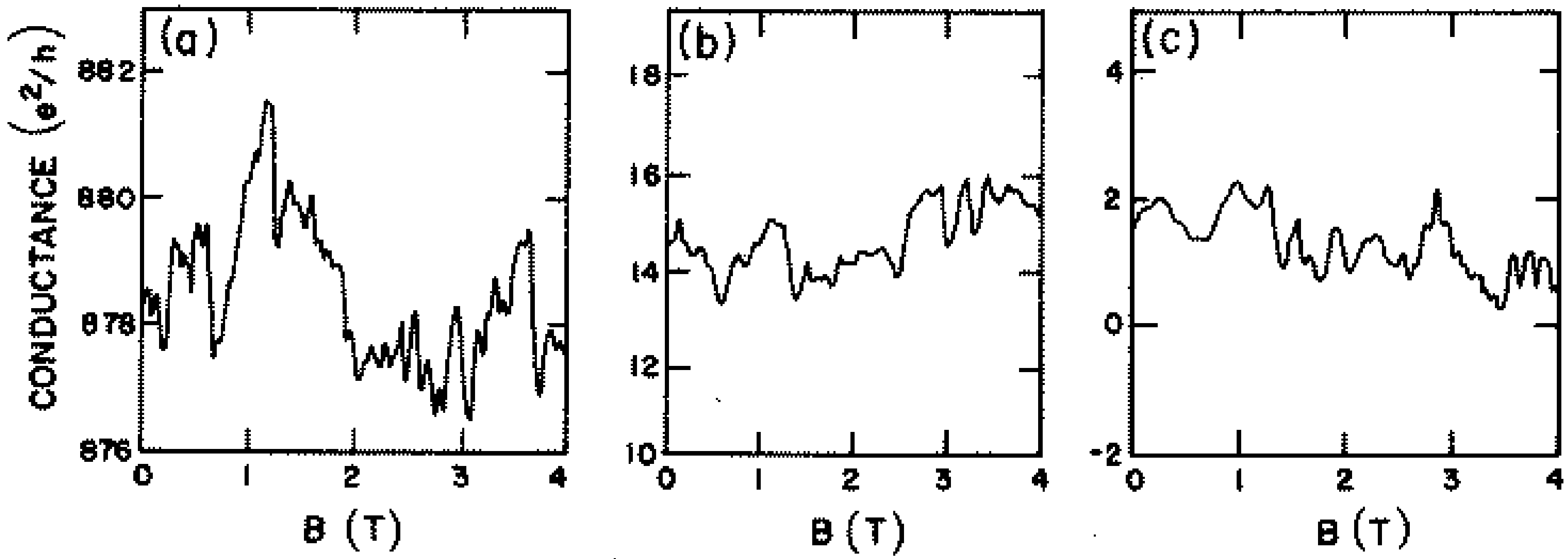
In the mesoscopic limit, the electrical conductance is not self-averaging.



Gold ring

Si-MOSFET

NUMERICS ON
THE ANDERSON MODEL

Summary : key ideas and concepts in quantum mesoscopic physics

1. Classical diffusion

2.

3.
crossings) are propagated over long distances by means of classical diffusion.

3. Complexity
from decoherence.