Large deviations in the Symmetric Simple Exclusion Process (SSEP) on graphs

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PHYSICS-TECHNION

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The 6th KIAS conference on statistical physics

Seoul, July 08-11, 2014

הקרן הלאומית למדע
ISRAEL SCIENCE FOUNDATION

MOTIVATION OF THIS WORK:

SHOT NOISE IN QUANTUM MESOSCOPIC SYSTEMS

Charge fluctuations in quantum mesoscopic conductors

Current that flows in an electric conductor fluctuates due to the stochastic nature of electron transport

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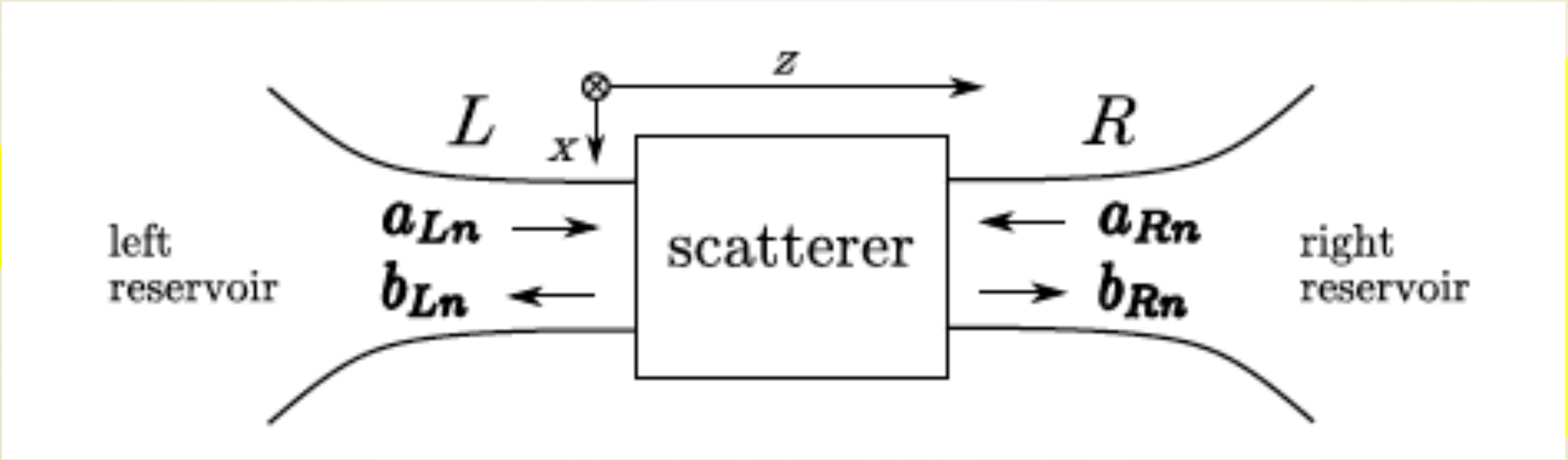
Current that flows in an electric conductor fluctuates due to the stochastic nature of electron transport

Study of Transport and Noise allows to characterize basic physical mechanisms at work.

Transport and shot noise

Two-terminal conductors



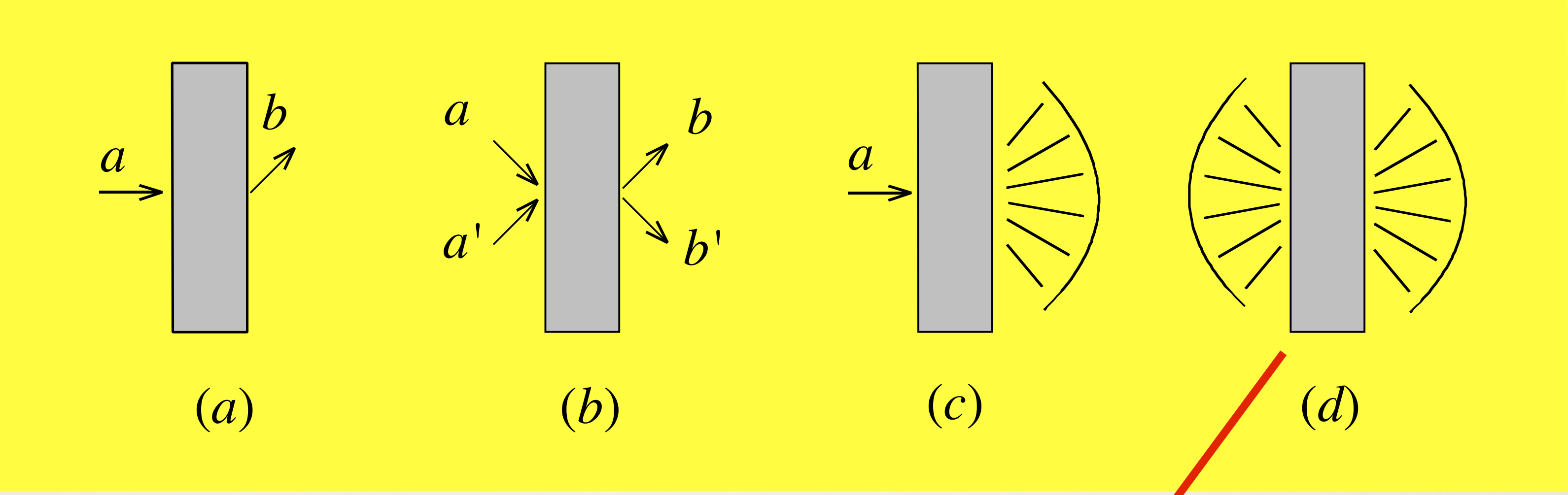
Electric conductance (Landauer)

$$G = \frac{e^2}{h} Tr\, t\, t^{\dagger}$$

Transport and shot noise

Two-terminal conductors

$$T_{ab} = |t_{ab}|^2$$



Electric conductance (Landauer)

$$G = \frac{e^2}{h} Tr\, t\, t^\dagger$$

Noise power is a current-current correlation

$$S(\omega,V)=\int dt\, e^{i\omega t}\left\langle \delta\hat{I}(t)\delta\hat{I}(0)+\delta\hat{I}(0)\delta\hat{I}(t)\right\rangle$$

where $\delta\hat{I}(t)=\hat{I}(t)-\langle I\rangle$ are electronic current operators

<u>Equilibrium noise (V=0)</u>

$$S(\omega,0)=2G\omega\ \coth\left(\frac{\omega}{2T}\right)$$

(Nyquist fluctuation-dissipation)

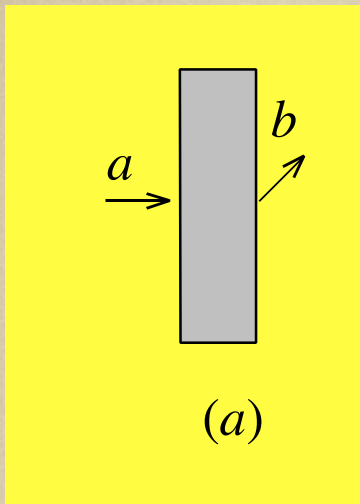
Non-equilibrium noise $V \neq 0$ at $T = 0$

$$S(0,V) - S(0,0) = \frac{e^2}{h}|2eV| Tr\, tt^\dagger \left(1 - tt^\dagger\right)$$

Excess noise measures the second cumulant of charge fluctuations :

$$S(0,V) - S(0,0) \propto \langle Q_t^2 \rangle - \langle Q_t \rangle^2$$

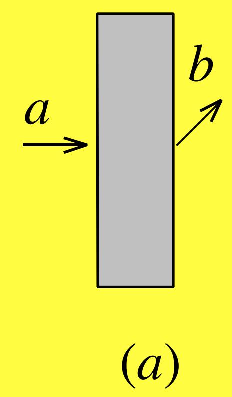
Fano factor



$$F = \frac{S(0,V) - S(0,0)}{e\,I} = \frac{\sum_{ab} T_{ab}(1 - T_{ab})}{\sum_{ab} T_{ab}}$$

T_{ab} is the transmission coefficient along the channel ab

FANO FACTOR



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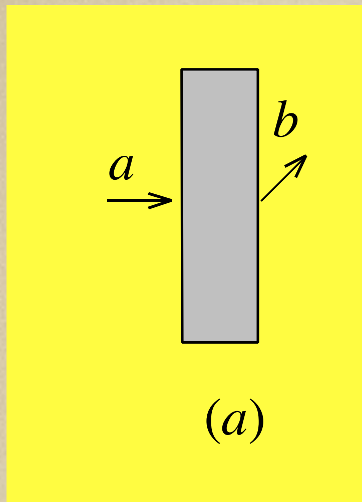
Fano factor



$$Tr\, tt^{\dagger}\left(1 - tt^{\dagger}\right)$$

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F has a universal value 1/3 for weakly disordered "one-dimensional" conductors

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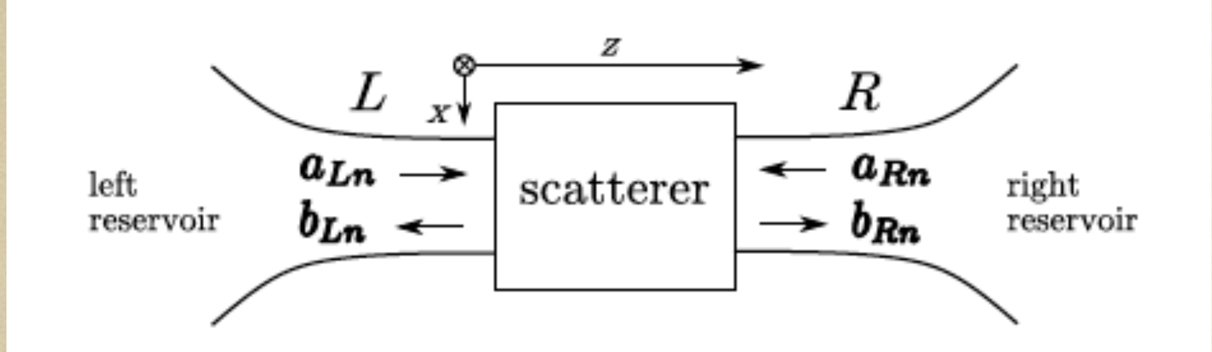
Is this result universal ?

nature of disorder, geometry, space dimensionality, extends to higher order cumulants,...

Classical description of a quantum conductor



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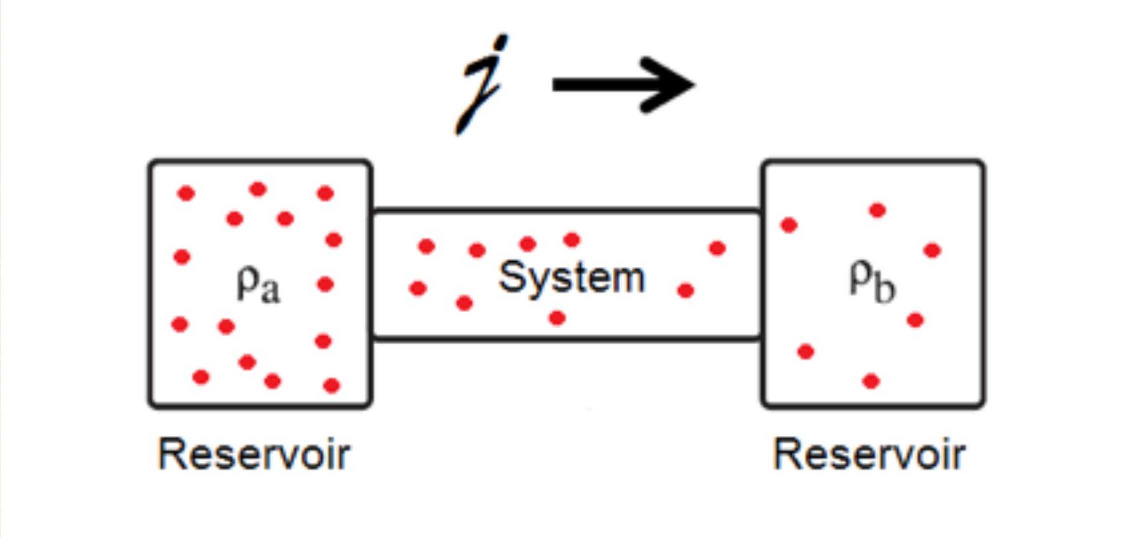
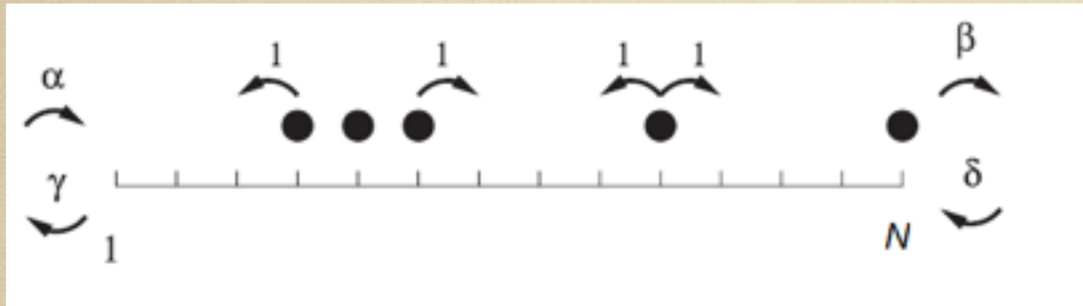
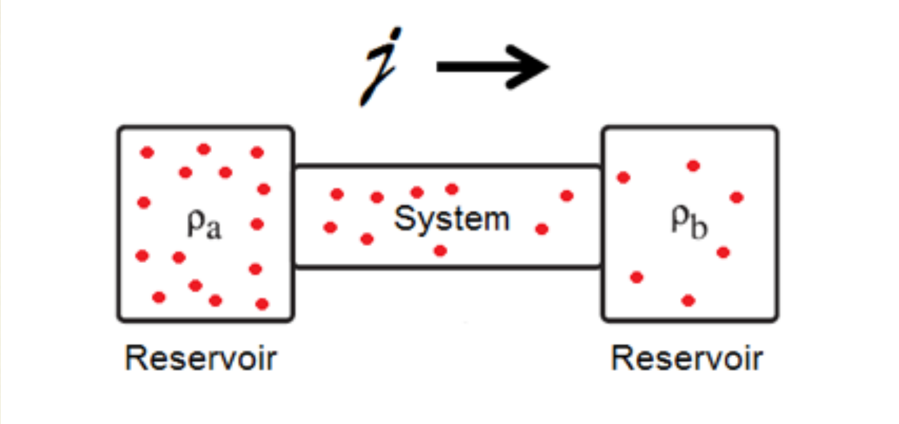


Same physical content : particles cannot pile up on the same site (exclusion principle)

Defines the **Symmetric Simple exclusion process** (SSEP)

The SSEP





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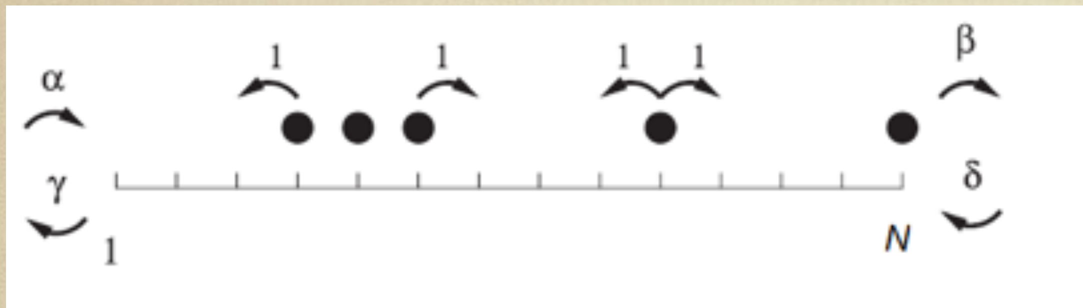

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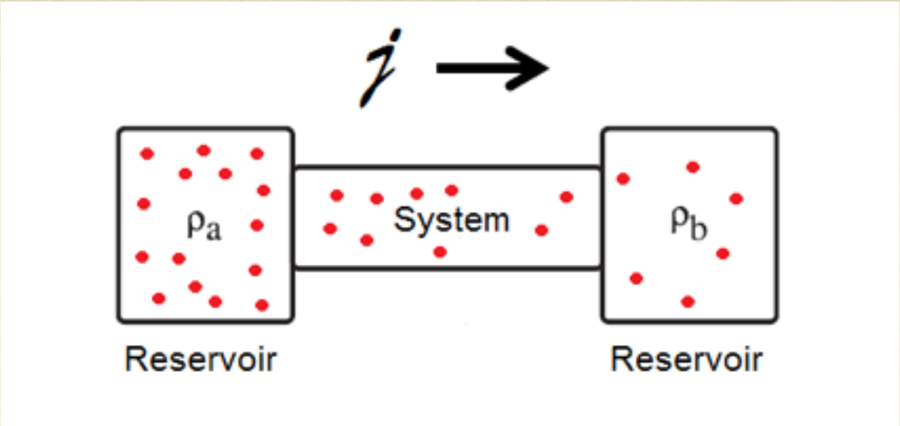

For $N \gg 1$, $\rho_a = \dfrac{\alpha}{\alpha+\gamma}, \rho_b = \dfrac{\delta}{\beta+\delta}$

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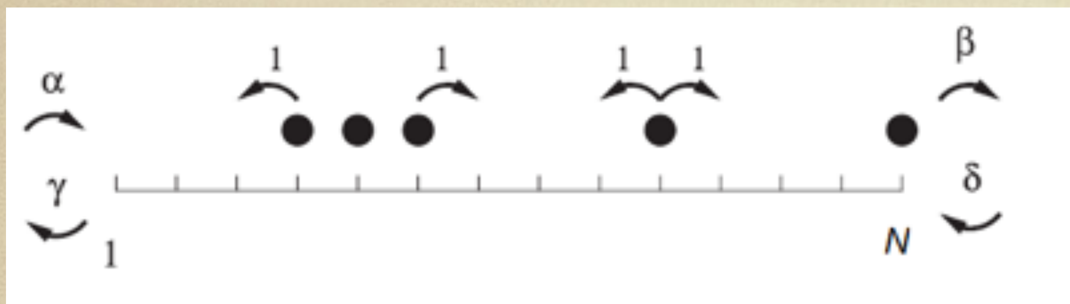

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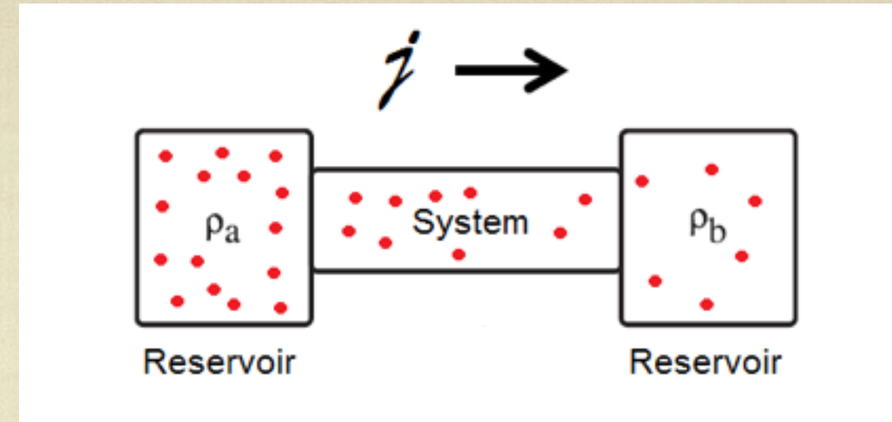

For $N \gg 1$, $\rho_a = \dfrac{\alpha}{\alpha+\gamma}, \rho_b = \dfrac{\delta}{\beta+\delta}$

For large enough time, the system is in a **steady state**.

Define the probability $P(Q_t)$ of observing Q_t particles flowing through the system during a time interval t and for 2 reservoirs at densities ρ_a and ρ_b

All the cumulants are known for arbitrary densities ρ_a and ρ_b

The generating function

$$\lim_{N\to\infty}\lim_{t\to\infty}\frac{N}{t}\log\left\langle e^{\lambda Q_t}\right\rangle=\left(\sinh^{-1}\left(\sqrt{\omega}\right)\right)^2$$

depends on a single scaling variable

$$\omega=\rho_a\left(e^{\lambda}-1\right)+\rho_b\left(e^{-\lambda}-1\right)-\rho_a\left(e^{\lambda}-1\right)\rho_b\left(e^{-\lambda}-1\right)$$

The Fano factor is

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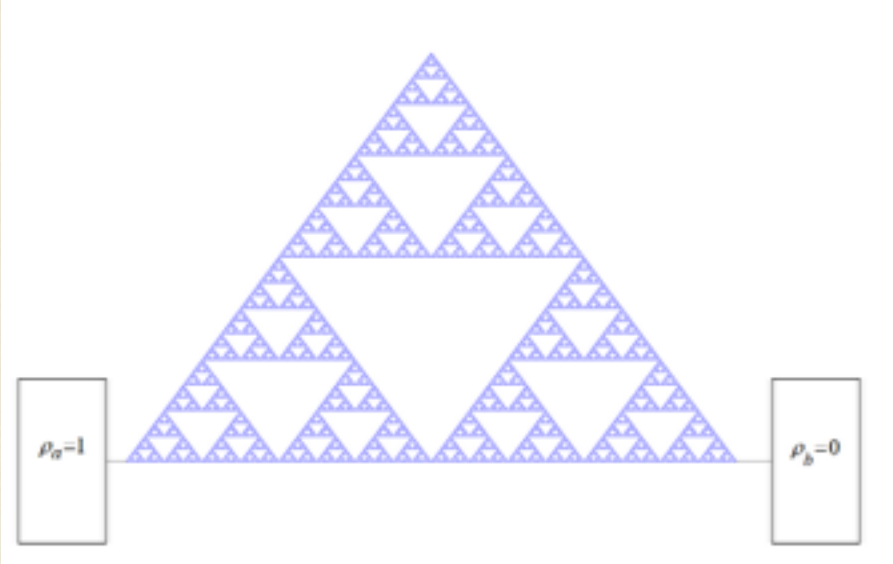
All cumulants are identical to those calculated in the quantum mesoscopic case.

How these results generalize to higher space dimensions ?



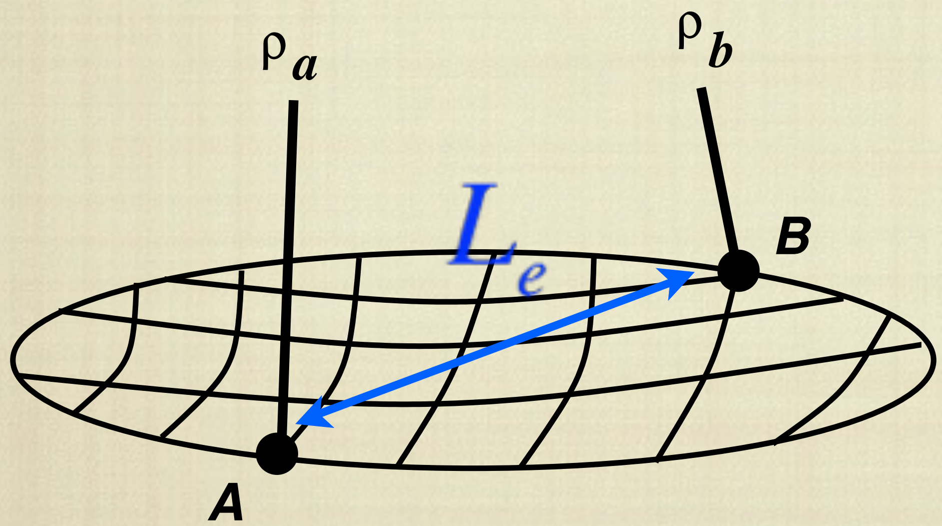
Numerical results on a Sierpinski gasket fractal network suggests a Fano factor $F = \frac{1}{3}$

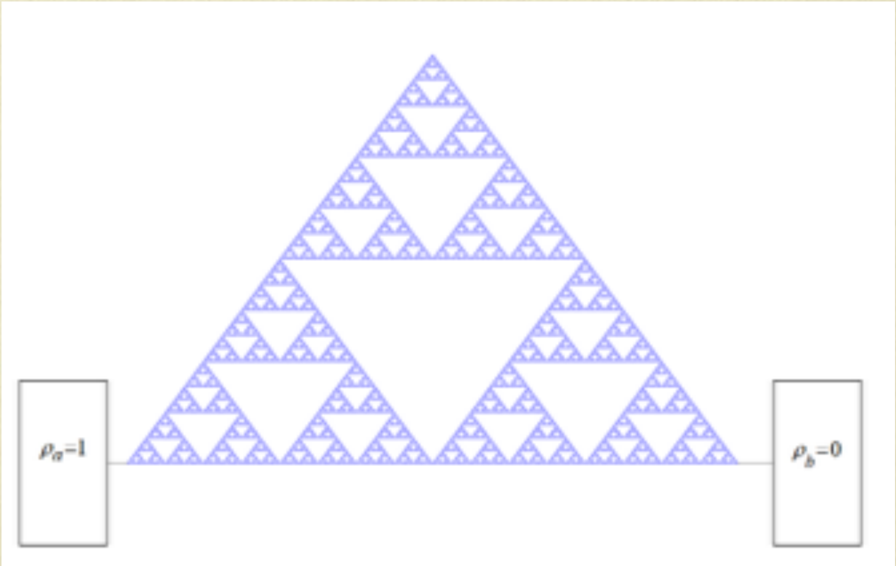
(Groth et al. PRL 2008)

Our Results:

(T. Bodineau, B. Derrida, O. Shpielberg, E.A, 2013)

1. Large classes of graphs (including fractals) are characterized by an effective length L_e





2. For large values of L_e, the generating function of the cumulants of the **SSEP** is the same as for a linear chain, up to a multiplicative function

$$\lim_{t\to\infty}\frac{1}{t}\log\left\langle e^{\lambda Q_t}\right\rangle=\kappa\left(L_e\right)\left(\sinh^{-1}\left(\sqrt{\omega}\right)\right)^2$$

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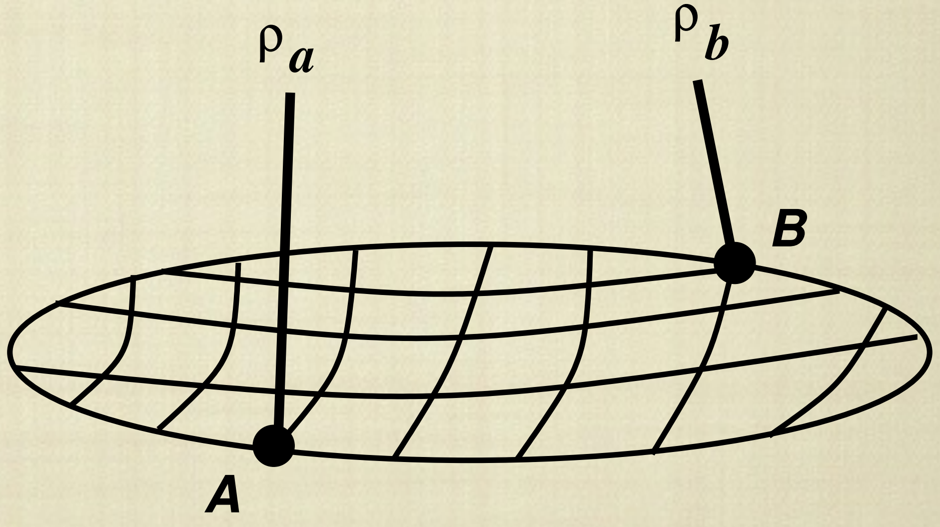
$$\kappa(L_e)\equiv L_e^{d-2}\int d\vec{r}\,\left(\vec{\nabla}v(\vec{r})\right)^2$$

$$\Delta v(\vec{r})=0,\ v(\partial A)=1, v(\partial B)=0$$



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$$\lim_{t\to\infty}\frac{1}{t}\log\left\langle e^{\lambda Q_t}\right\rangle = \kappa(L_e)\left(\sinh^{-1}\left(\sqrt{\omega}\right)\right)^2$$

Thus, the ratio between any pair of cumulants of Q_t is the same as for the linear chain. Then,

$$F = \frac{1}{3}$$

Elements of the proof

- Use the macroscopic fluctuation theory of Bertini et *al*. and the additivity principle.

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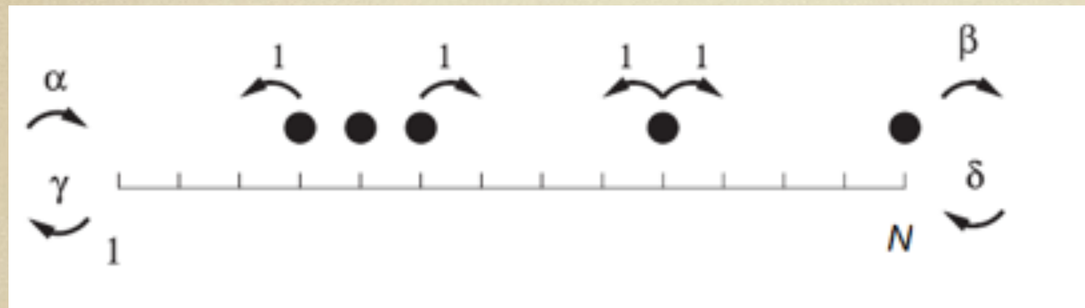
- Alternative description based on Energy/Dirichlet forms: allows to characterize the SSEP and to provide a derivation of the additivity principle.

The macroscopic fluctuation theory

Basic definitions and results

(Bertini, De Sole, Gabrielli, Jona-Lasinio, and Landim)

The SSEP



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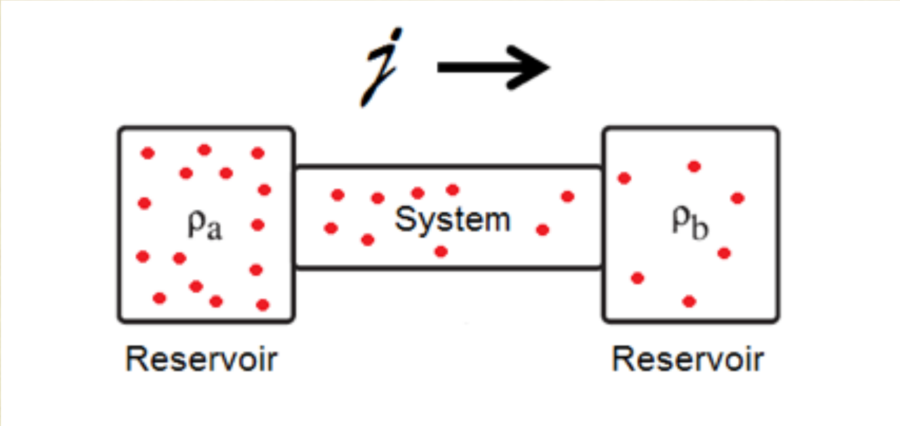

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The <u>large deviation function</u> F_L is defined from the probability

$$P_L\left(Q_t = j t, \rho_a, \rho_b\right) \equiv e^{t F_L(j, \rho_a, \rho_b)}$$

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It is the Legendre transform of $\mu(\lambda)$

$$\mu(\lambda) = \max_j \left(\lambda j + F_L(j(\lambda))\right)$$

Scaling - Electrical conductance

The large deviation is a scaling function :

$$F_L(j) = \frac{1}{L^{2-d}} G\left(jL^{2-d}\right)$$

F_L scales like an electrical conductance.

Additivity principle and large deviation function

General diffusive system (e.g. SSEP) s.t., $\rho_a = \rho \quad , \rho_b = \rho + \Delta\rho, \quad \Delta\rho \ll \rho$

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Weak current through the system : use Fick's law $\frac{\langle Q_t \rangle}{t} = \frac{D(\rho)}{L} \Delta\rho$

+ fluctuations : $\frac{\langle Q_t^2 \rangle}{t} = \frac{\sigma(\rho)}{L}$

For SSEP, $D(\rho) = 1, \quad \sigma(\rho) = 2\rho(1-\rho)$

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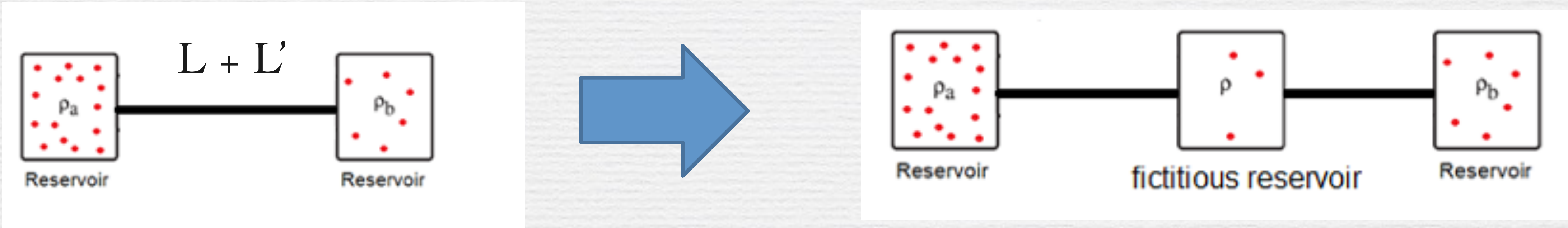
For SSEP, $D(\rho) = 1, \quad \sigma(\rho) = 2\rho(1-\rho)$

$F_L(j)$ has its maximum for $j = \frac{\langle Q_t \rangle}{t}$. Close to equilibrium : Gaussian distribution for the probability,

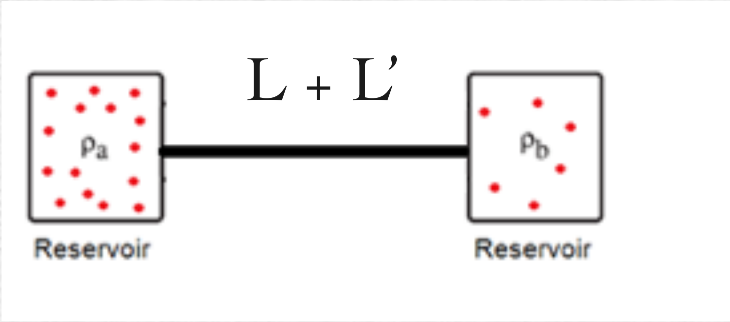
$$F_L(j) = -\frac{\left(j - \langle Q_t \rangle / t\right)^2}{2 \langle Q_t^2 \rangle / t} = -\frac{\left(j - \frac{\rho_i - \rho_{i+1}}{l} D(\rho_i)\right)^2}{2 \sigma(\rho_i) / l}$$

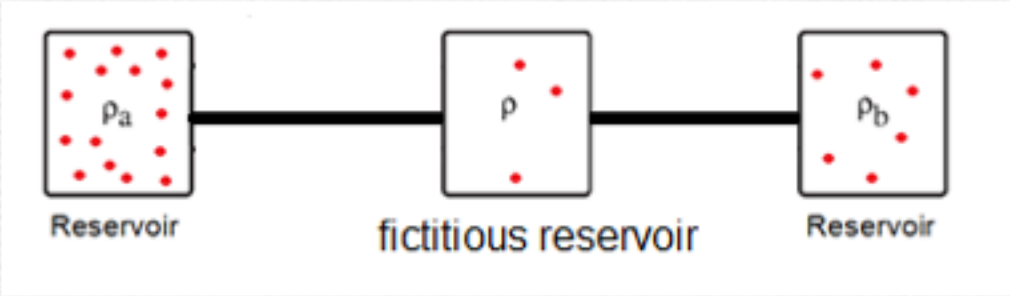
Additivity principle + scaling



$$F_{L+L'}(j,\rho_a,\rho_b) = \max_{\rho}\left\{F_L(j,\rho_a,\rho) + F_{L'}(j,\rho,\rho_b)\right\}$$

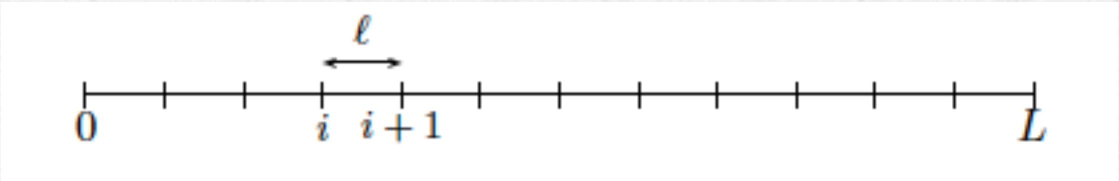
Additivity principle + scaling





$$F_{L+L'}\left(j,\rho_a,\rho_b\right) = \max_{\rho}\left\{F_L\left(j,\rho_a,\rho\right) + F_{L'}\left(j,\rho,\rho_b\right)\right\}$$

$$F_L(j) = -\frac{\left(j - \left\langle Q_t\right\rangle / t\right)^2}{2\left\langle Q_t^2\right\rangle / t} = -\frac{\left(j - \frac{\rho_i - \rho_{i+1}}{l} D\left(\rho_i\right)\right)^2}{2\sigma\left(\rho_i\right)/l}$$



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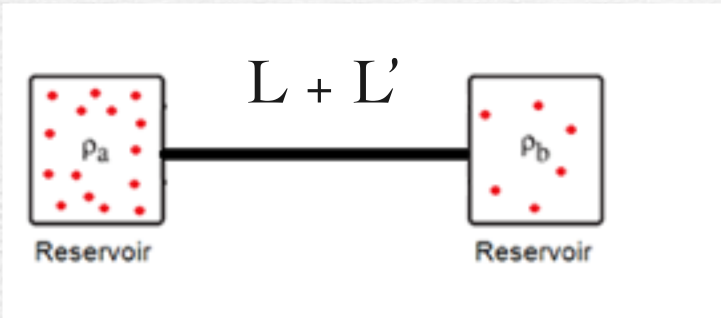
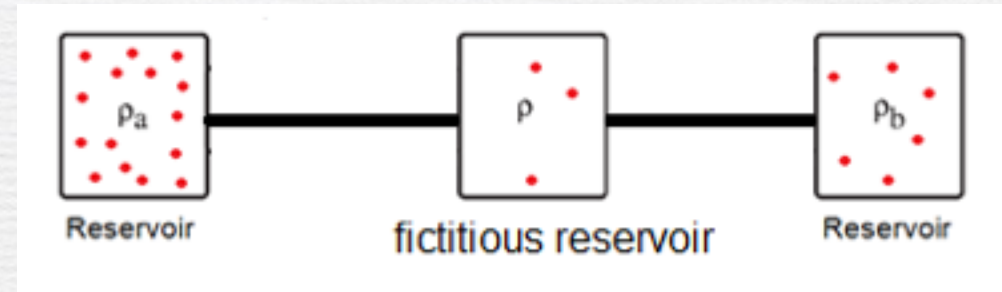


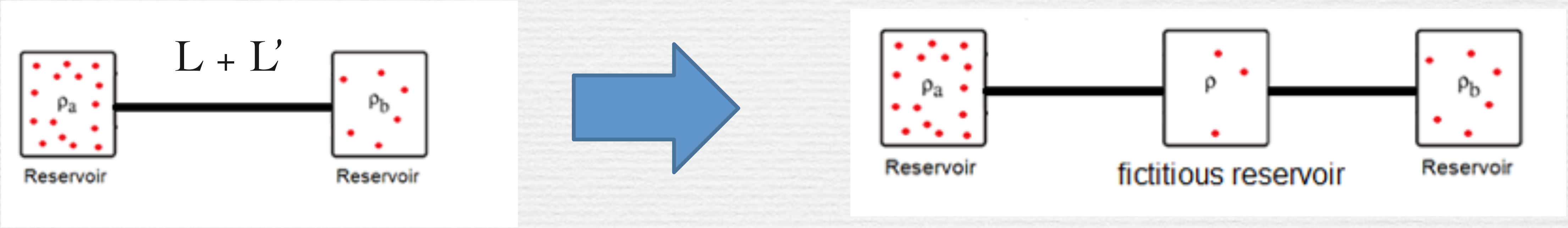


$$F_{L+L'}(j,\rho_a,\rho_b) = \max_{\rho}\left\{F_L(j,\rho_a,\rho) + F_{L'}(j,\rho,\rho_b)\right\}$$

so that,

$$F_L(j,\rho_a,\rho_b) = \max_{\rho(x)}\left\{-\int_0^1 dx \frac{\left(jL + D(\rho(x))\rho'(x)\right)^2}{2\sigma(\rho(x))}\right\}$$

Additivity principle + scaling



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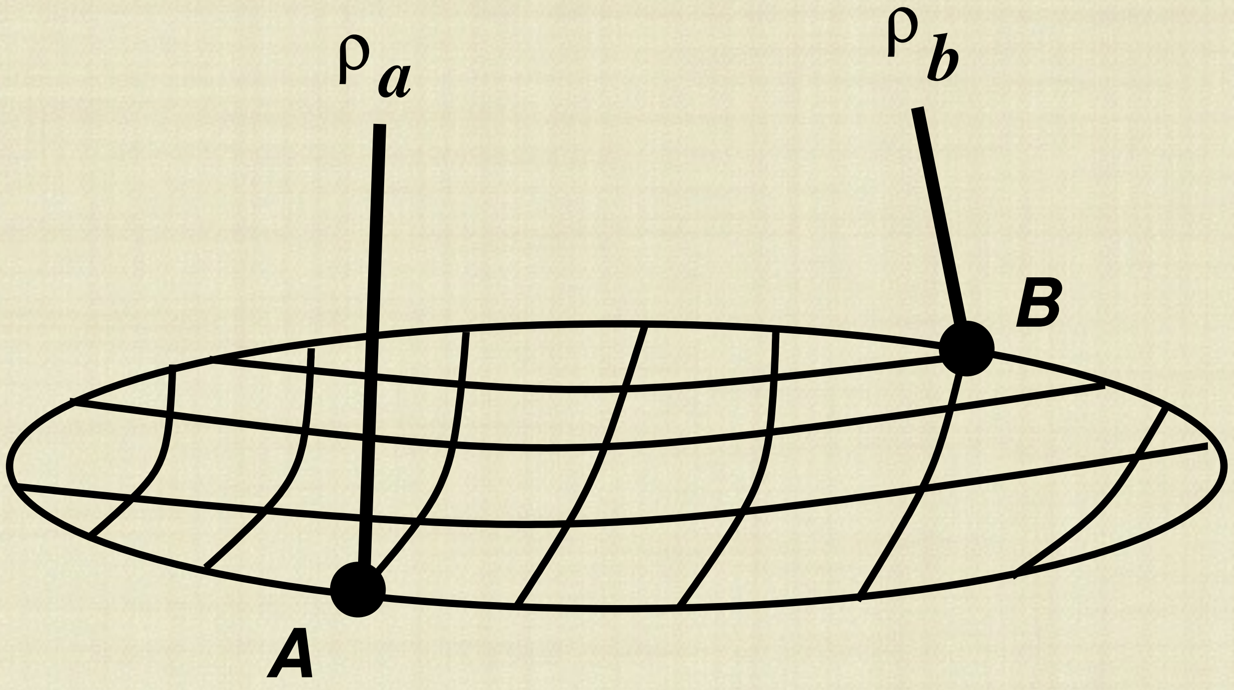
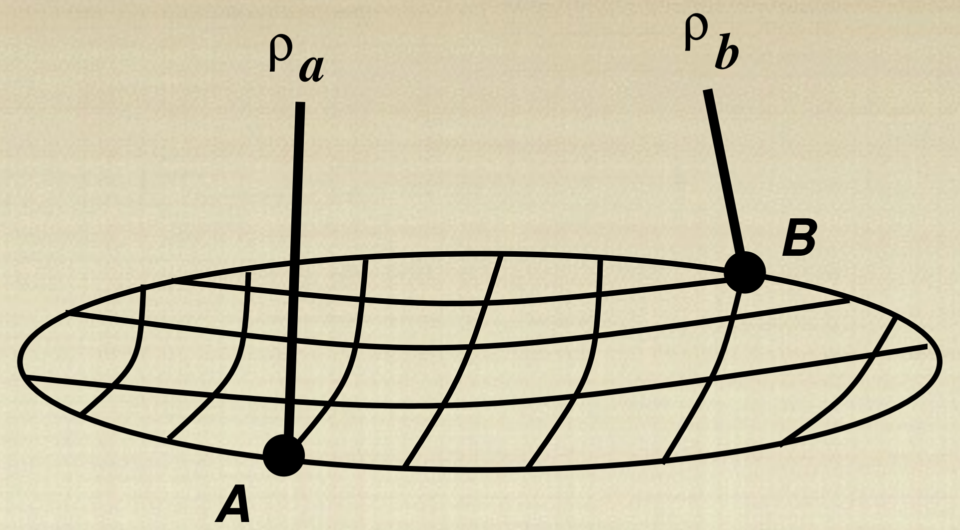
and

$$\mu(\lambda) = \max_j\left(\lambda j + F_L(j(\lambda))\right)$$

Macroscopic fluctuation theory for SSEP on a d-dimensional domain

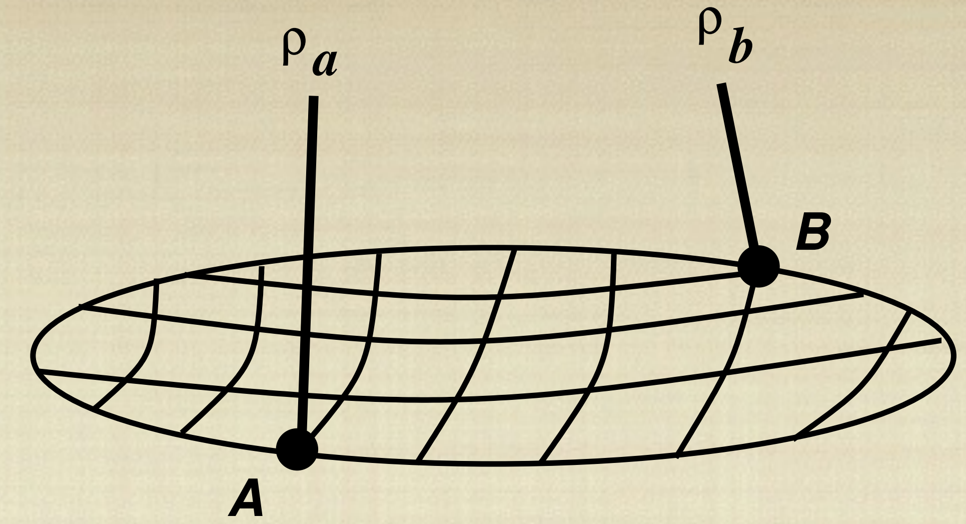




Define the number Q_t of particles flowing between the 2 reservoirs :

$$Q_t = \frac{1}{2}\sum_{i,j}\left(V_i - V_j\right)q_{i,j}(t)$$

where $q_{i,j}(t)$ is the number of particles transferred from i to j during t and V_i is an arbitrary function on site i except for $V_A = 1, V_B = 0$



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Nothing depends on the choice of the V_i's. We take it a solution of the Laplace eq. $\Delta V_i \equiv \sum_{j \sim i} V_j - V_i = 0$

Continuous version: $$Q_t = -L^d \int_0^{t/L^2} d\tau \int d\vec{r}\, \vec{\nabla} v(\vec{r}) \cdot \vec{j}(\vec{r})$$

where $$\Delta v(\vec{r}) = 0\,,\ v(\partial A) = 1, v(\partial B) = 0$$

The minimization in the generating function

$$\mu(\lambda) = -L^{d-2} \min_{\{\vec{j},\rho\}} \int d\vec{r} \left(\lambda \vec{\nabla} v(\vec{r}) \cdot \vec{j}(\vec{r}) + \frac{\left[\vec{j}(\vec{r}) + D(\rho(\vec{r}))\, \vec{\nabla}\rho(\vec{r})\right]^2}{2\sigma(\rho(\vec{r}))} \right)$$

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leads to

$$\vec{\nabla} \cdot \left(D(\rho(\vec{r}))\, \vec{\nabla}\rho(\vec{r}) \right) = \vec{\nabla} \cdot \left(\sigma(\rho(\vec{r})) \vec{\nabla} H(\vec{r}) \right)$$

$$D(\rho(\vec{r}))\, \Delta H(\vec{r}) = -\frac{\sigma'(\rho(\vec{r}))}{2} \left(\vec{\nabla} H(\vec{r}) \right)^2$$

where $H(\vec{r})$ is a Lagrange multiplier field associated to current conservation.

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$$D(\rho(\vec{r}))\,\Delta H(\vec{r})=-\frac{\sigma'(\rho(\vec{r}))}{2}\left(\vec{\nabla}H(\vec{r})\right)^2 \qquad (1)$$

in $d=1$ (chain of length L), then we know the solution in any dimension and for any domain !

The link between $d=1$ and higher dimensions:

If one knows the solution of

$$\vec{\nabla}\cdot\left(D\left(\rho(\vec{r})\right)\vec{\nabla}\rho(\vec{r})\right)=\vec{\nabla}\cdot\left(\sigma\left(\rho(\vec{r})\right)\vec{\nabla}H(\vec{r})\right)$$

$$D\left(\rho(\vec{r})\right)\Delta H(\vec{r})=-\frac{\sigma'\left(\rho(\vec{r})\right)}{2}\left(\vec{\nabla}H(\vec{r})\right)^2 \qquad (1)$$

in $d=1$ (chain of length L), then we know the solution in any dimension and for any domain !

This results from $\Delta v(\vec{r})=0,\ v(\partial A)=1, v(\partial B)=0$

so that $H(\vec{r})=H_{d=1}\left(v(\vec{r})\right),\ \rho(\vec{r})=\rho_{d=1}\left(v(\vec{r})\right)$

solve (1)

THE GENERATING FUNCTION IN d DIMENSIONS IS

$$\mu(\lambda) = L^{d-2} \int d\vec{r} \left(\left(\vec{\nabla} v(\vec{r}) \right)^2 \Phi\left(v(\vec{r}) \right) \right)$$

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Since

$$L \mu_{d=1}(\lambda) = \lim_{t \to \infty} \frac{L_e}{t} \log \left\langle e^{\lambda Q_t} \right\rangle \Big|_{d=1} = \left(\sinh^{-1}\left(\sqrt{\omega} \right) \right)^2$$

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Then,

$$\mu(\lambda)=\lim_{t\to\infty}\frac{1}{t}\log\left\langle e^{\lambda Q_t}\right\rangle=\kappa(L_e)\times\left(\sinh^{-1}\left(\sqrt{\omega}\right)\right)^2$$

with

$$\kappa(L_e)\equiv L_e^{d-2}\int d\vec{r}\,\left(\vec{\nabla}v(\vec{r})\right)^2$$

The generating function $\mu(\lambda)$ for an arbitrary domain in d-dimensions is the same as the $d=1$ generating function $\mu_{d=1}(\lambda)$ for the effective length L_e up to a multiplicative function independent of $(\lambda, \rho_a, \rho_b)$

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Therefore, for any d-dimensional domain, the ratio of any pair of cumulants is the same as in $d=1$

Analogies between SSEP on a graph and resistor networks

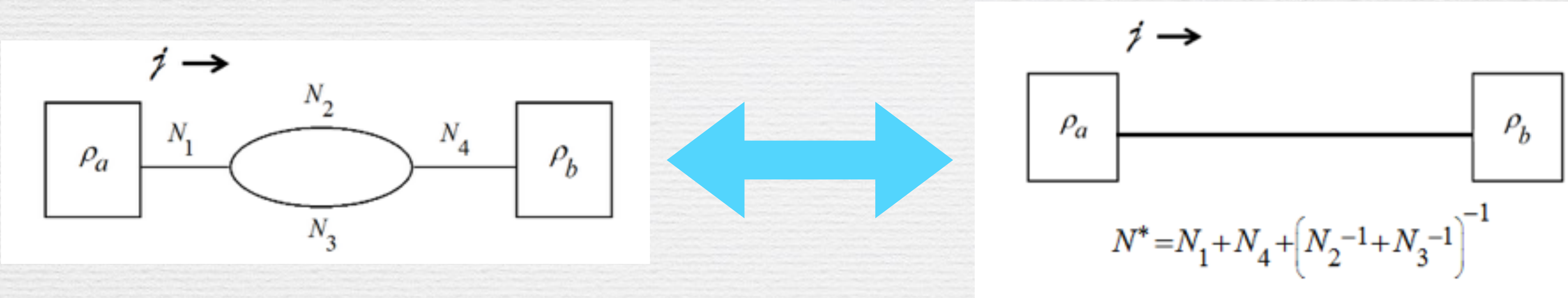
Scaling - Electrical conductance

The large deviation is a scaling function :

$$F_L(j) = \frac{1}{L^{2-d}} G\left(jL^{2-d}\right)$$

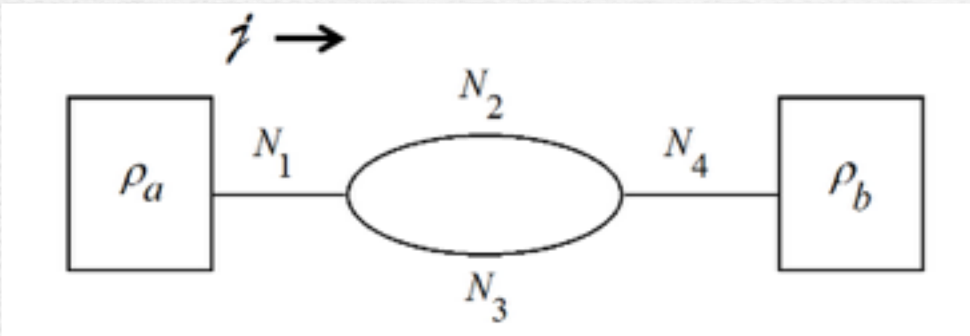
F_L scales like an electrical conductance.

Kirchhoff's rules - Addition in series and in parallel



(Derrida, Bodineau PRL 2004)

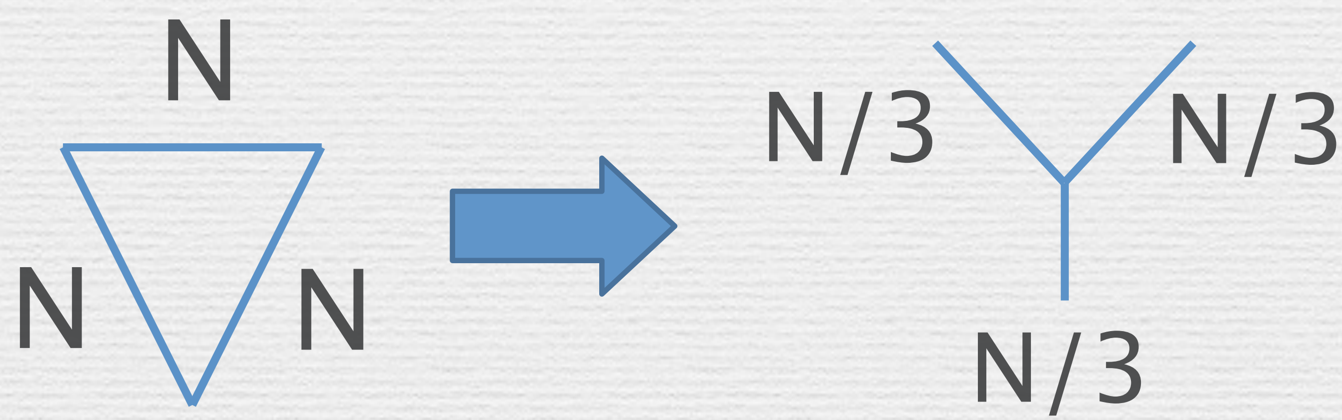
Kirchhoff's rules – Addition in series and in parallel



$$N^* = N_1 + N_4 + \left(N_2^{-1} + N_3^{-1}\right)^{-1}$$

(Derrida, Bodineau PRL 2004)

More generally, using the Δ–Y transform



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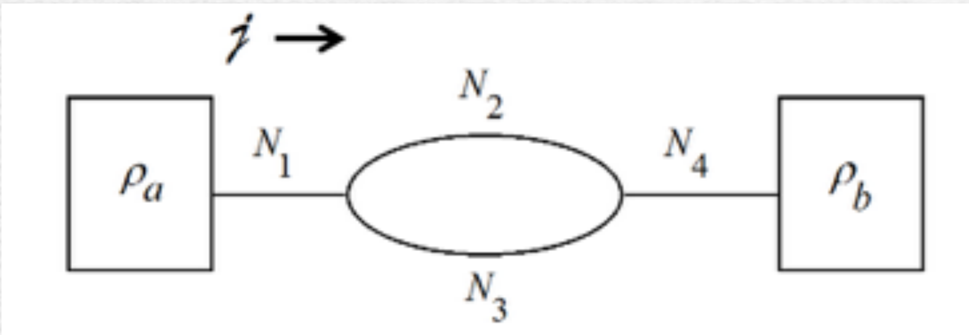

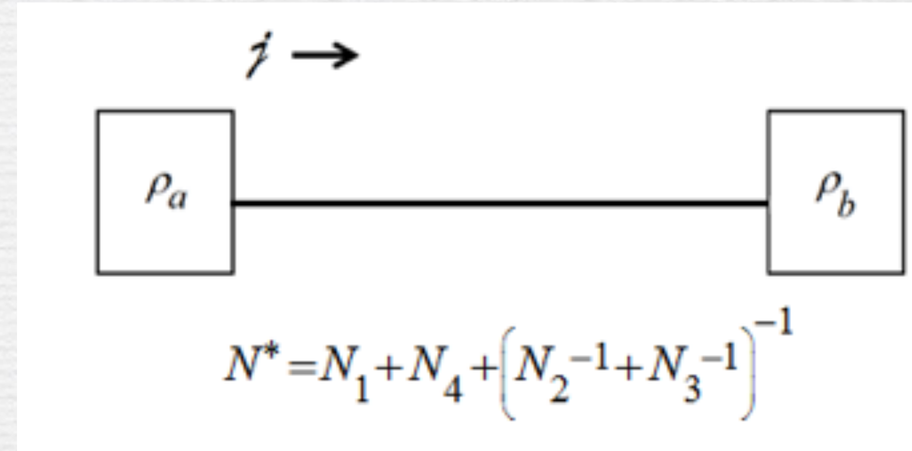

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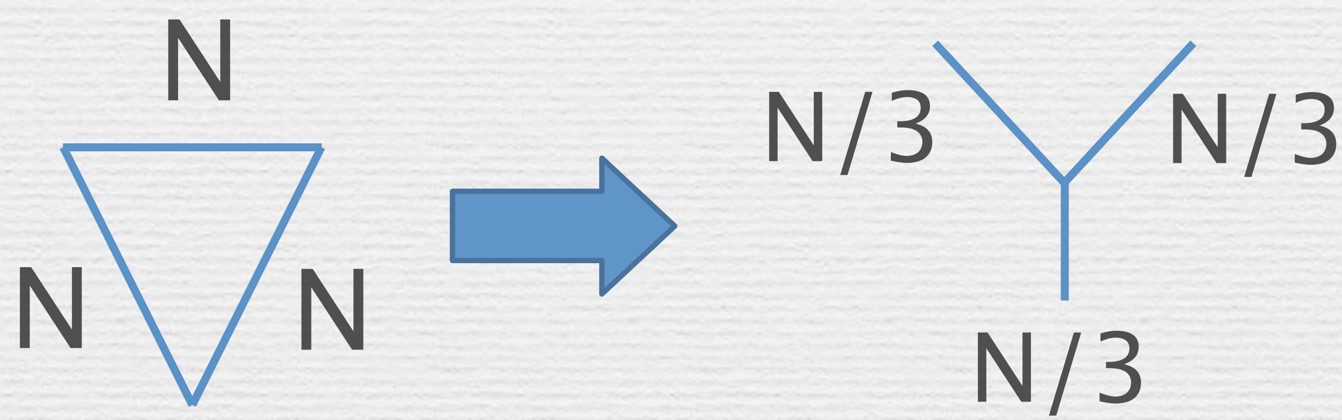
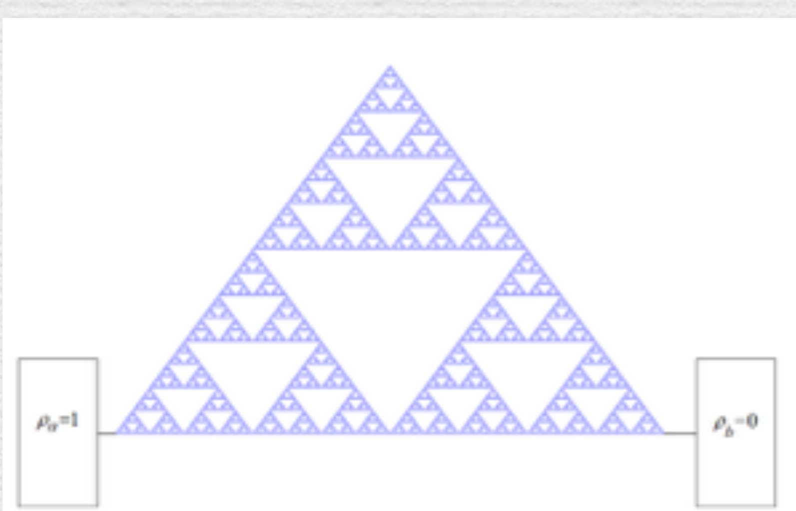
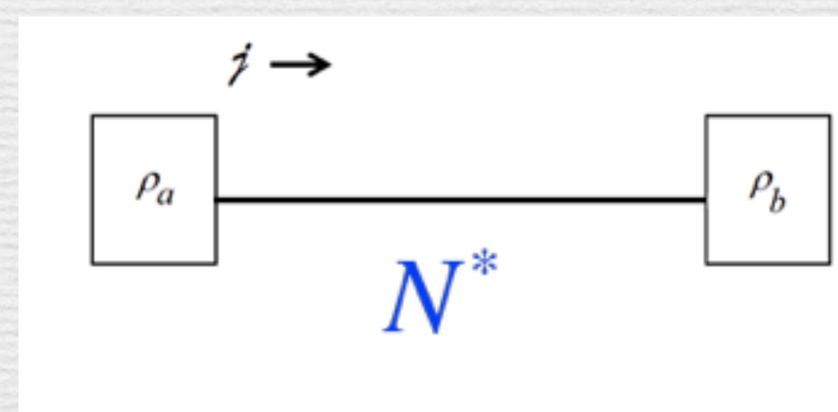
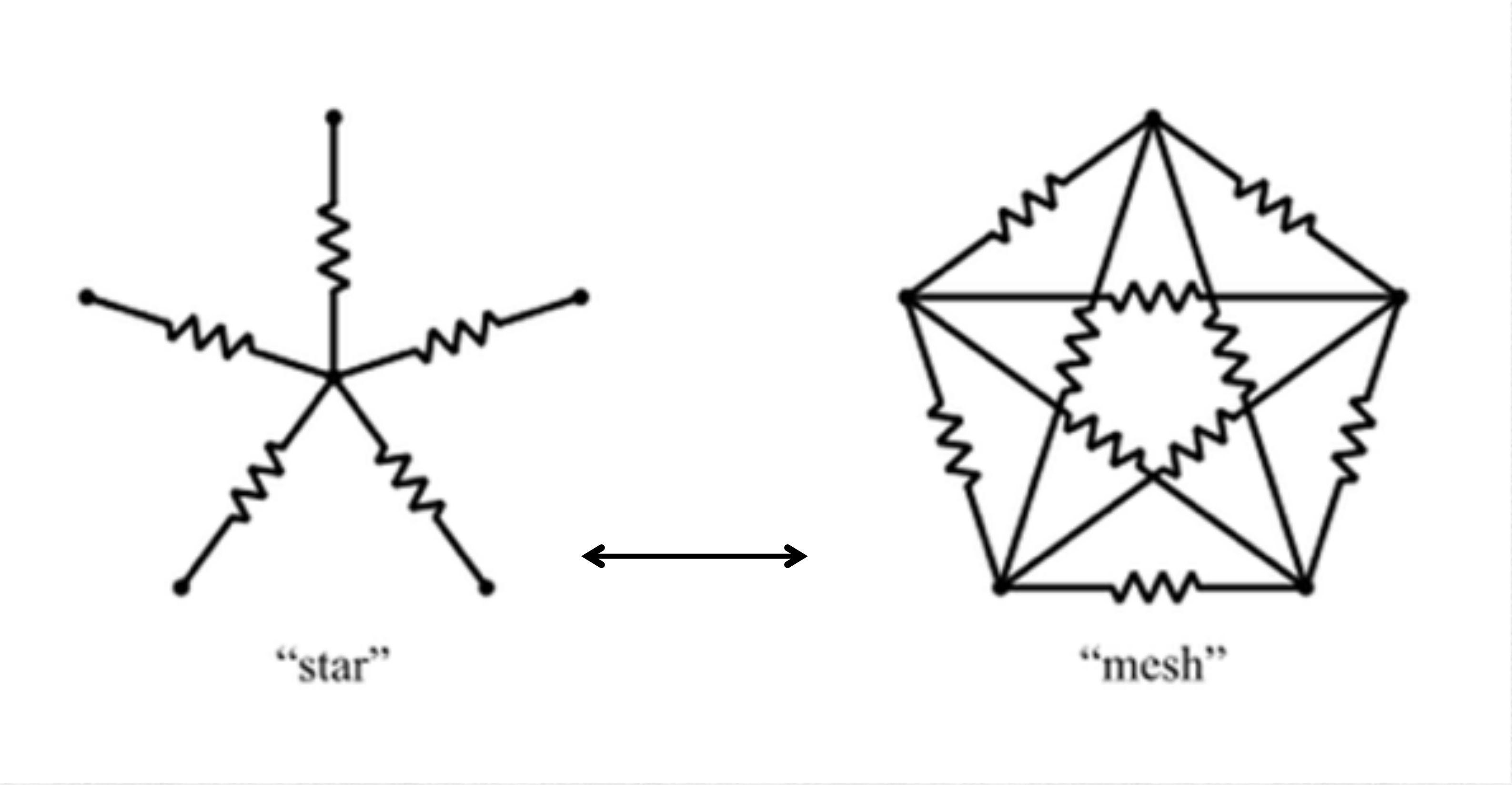






Star – mesh transform



- A two-terminal resistor network always has an equivalent resistor
- The equivalent resistor is obtained through repeated use of the star-mesh transform.

The same applies to any SSEP graph

The SSEP resistor theorem

- For any graph G, $F_G\left(j,\rho_a,\rho_b\right)=F_{N^*}\left(j,\rho_a,\rho_b\right)$

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The SSEP resistor theorem

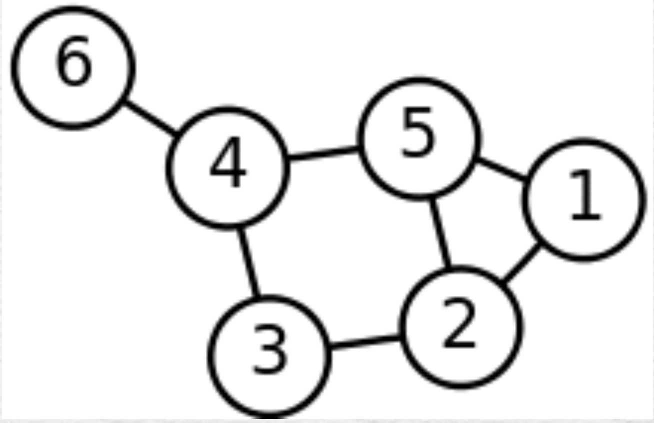
- For any graph G, $F_G(j, \rho_a, \rho_b) = F_{N^*}(j, \rho_a, \rho_b)$
- N^* can be obtained by Kirchhoff's resistor rules
- The theorem applies for any non-eq. process given that :

1. The additivity principle applies
2. The scaling assumption applies
3. There is a steady state

Energy/Dirichlet forms

Energy forms

A graph with sites and bonds



Each bonds carries a weight

Energy forms

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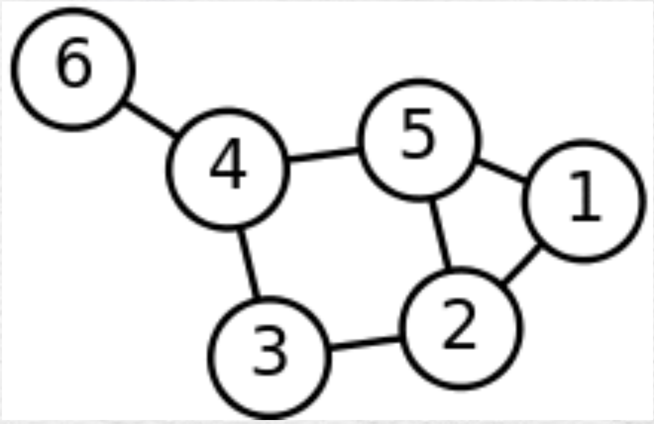

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We define the energy function $E_G(u) = \sum_{x \sim y} \frac{1}{r_{xy}} \left[u(x) - u(y) \right]^2$

Energy forms

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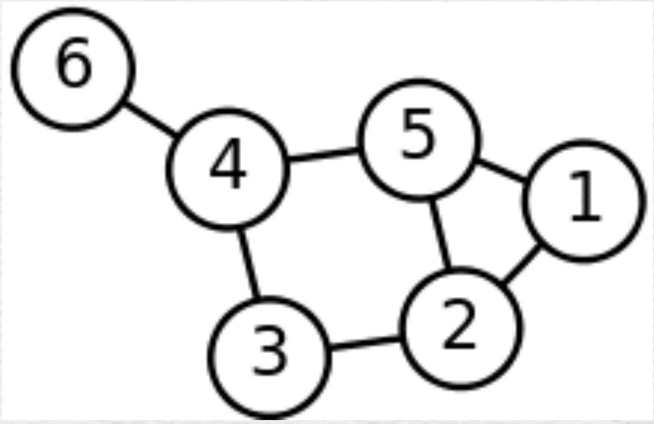

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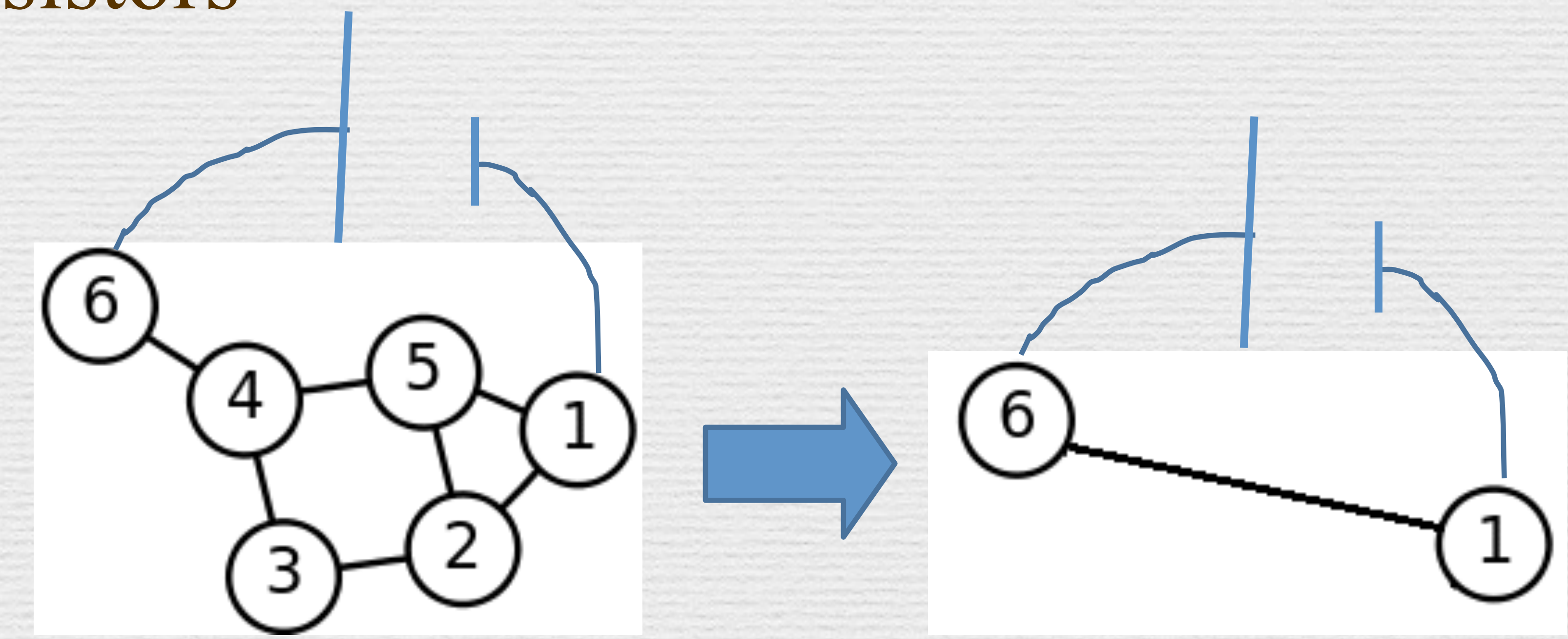
Connect the network of resistors
to a battery

$$E_G(h) = \inf_u E_G(u)$$

h - harmonic function



Well known exact mapping between electric networks of resistances and random walk on a lattice

(Doyle & Snell)

A theorem by Beurling and Deny extends this mapping to the equivalence between energy forms and symmetric Markov processes.

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This theorem allows to describe the ________ an effective conductance network

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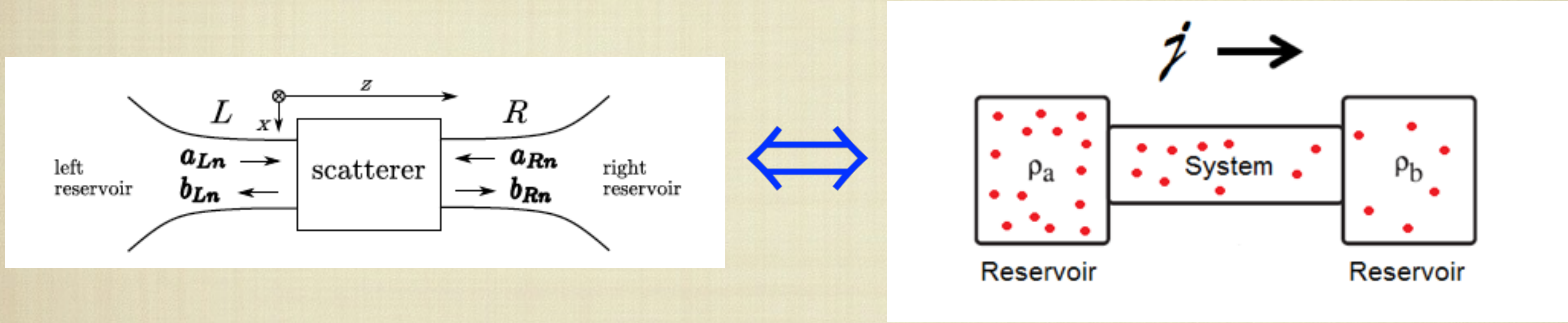
electric energy is the large deviation function.

Moreover, ______________________________ principle

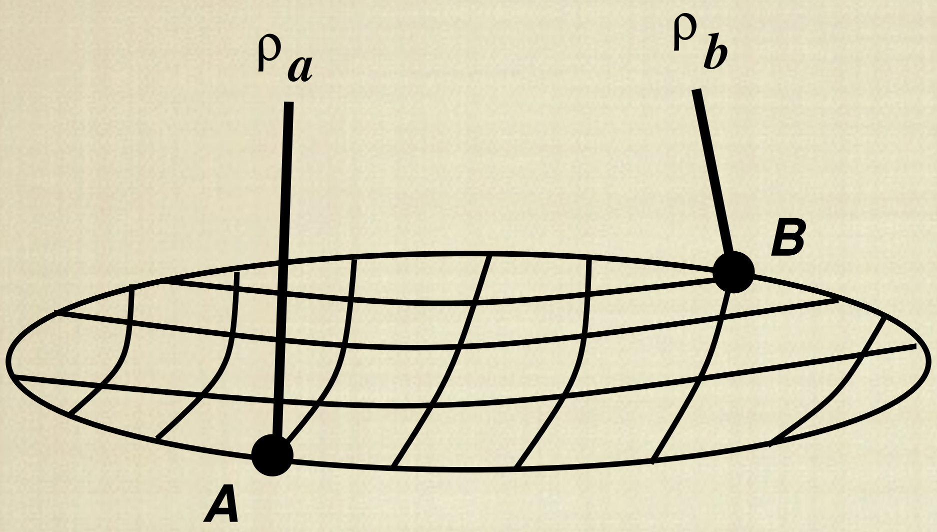
the minimum energy

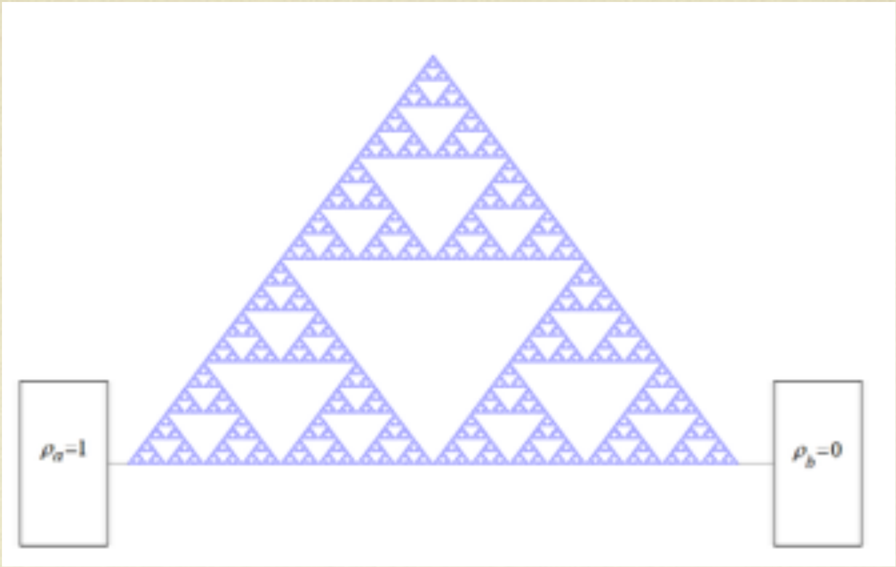
Summary - further issues

- Full counting statistics of quantum mesoscopic conductors is well described by the classical 1D SSEP model:



- For large system sizes, the generating function of the cumulants of the current of the d-dim. **SSEP** is the same as for a linear chain, up to a multiplicative function





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$$\kappa\left(L_e\right)\equiv L_e^{d-2}\int d\vec{r}\left(\vec{\nabla}v\left(\vec{r}\right)\right)^2$$

$$\Delta v\left(\vec{r}\right)=0\,,\ v\left(\partial A\right)=1, v\left(\partial B\right)=0$$

- SSEP - resistor theorem : analogy between electric networks and non-equilibrium stochastic processes.
- Energy forms : useful framework to derive the large deviation function of symmetric Markov processes.
- The additivity principle results from the energy form description.
- Extension to stochastic processes (ASEP) with phase transitions.
- More than 2 reservoirs ?
- Random graphs