Topology and Physics

The 2016 Physics Nobel Prize :
D. Thouless, D. Haldane
and J. Kosterlitz

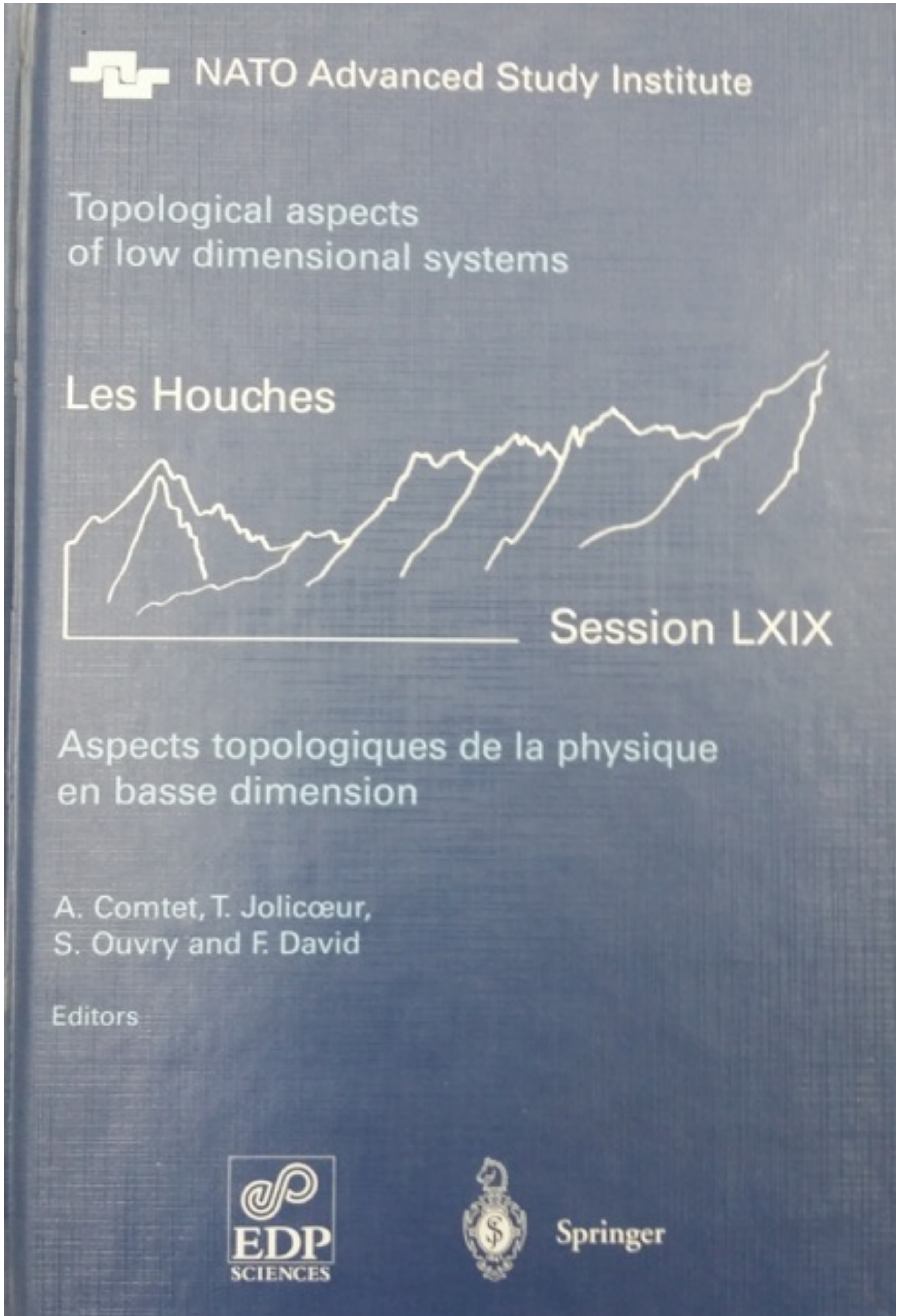
Technion
Israel Institute of Technology

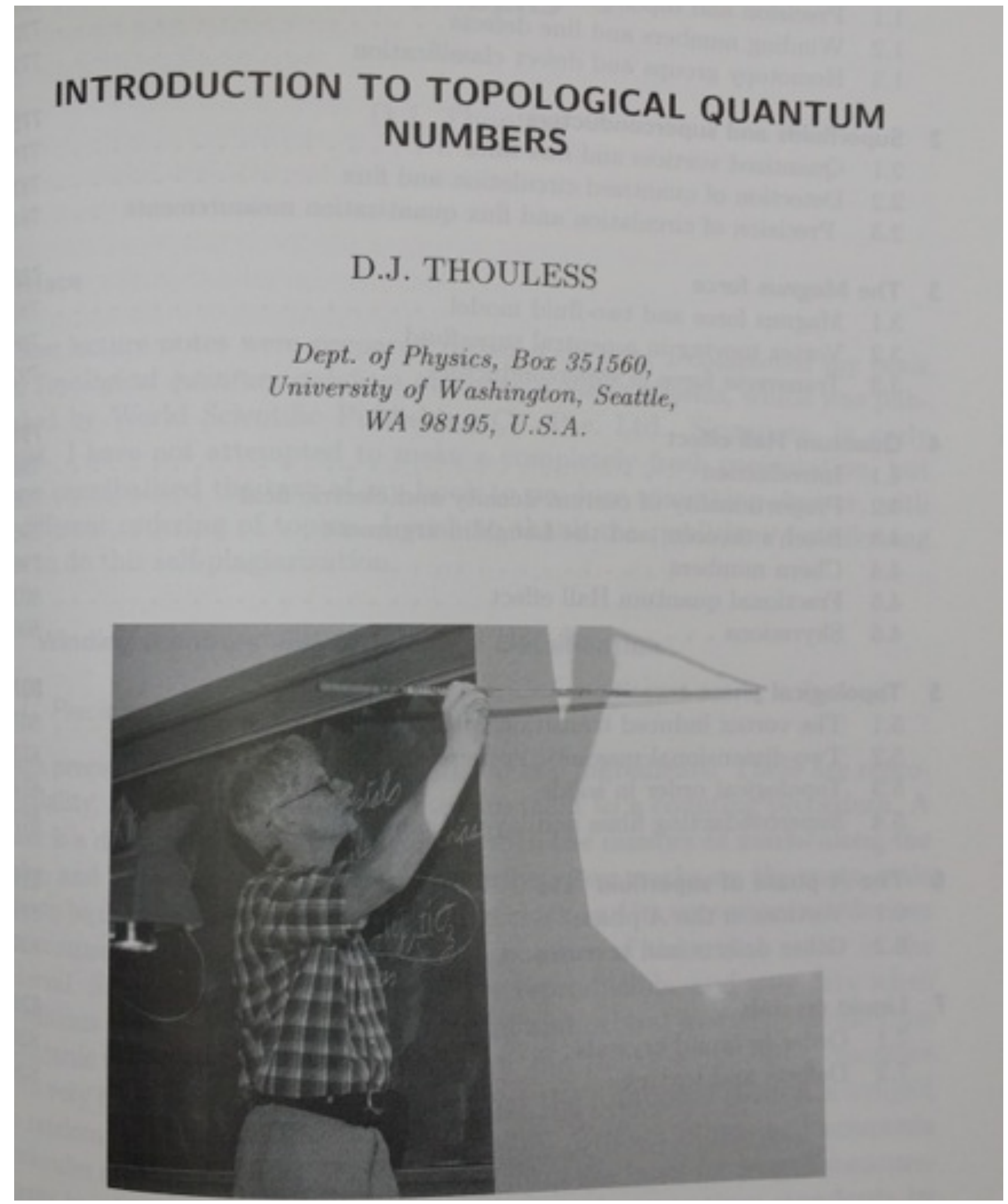
Eric Akkermans
Physics-Technion

Topological Phase Transitions - Topological Phases of Matter

In case you think that physicists are always very focused and serious…

A Nobel prize in action…





while others….

GEOMETRICAL DESCRIPTION OF VORTICES IN GINZBURG-LANDAU BILLIARDS

E. AKKERMANS

Laboratoire de Physique des Solides
and LPTMS, 91405 Orsay Cedex,
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Institute of Technology, Haifa 32000,
Israel

Topological Phase Transitions - Topological Phases of Matter

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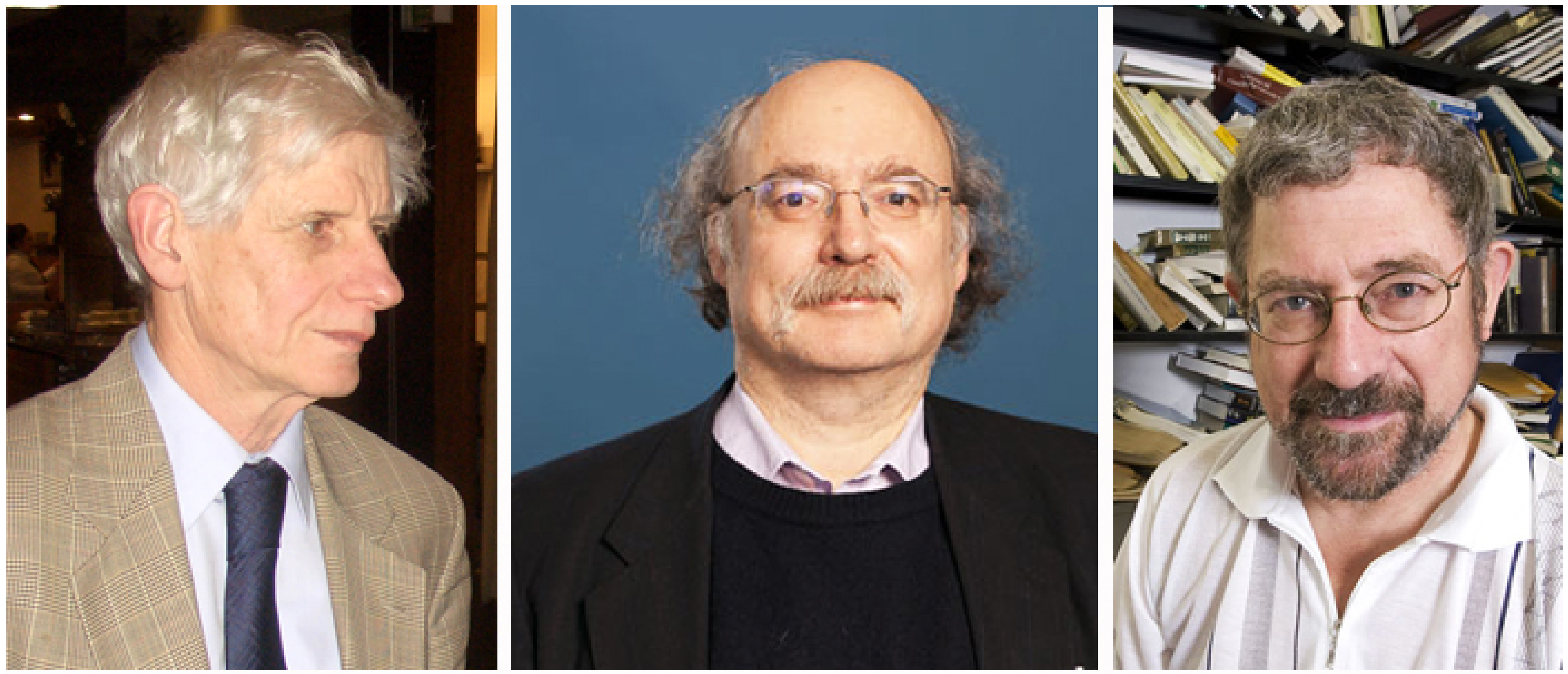


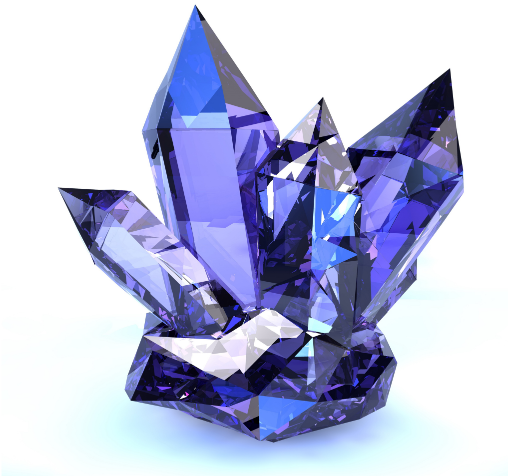
Thouless Haldane Kosterlitz

Topological Phase Transitions - Topological Phases of Matter

Phases of Matter ?

What are Phases of Matter ?

Different forms of solid state matter :

Crystal $\Leftrightarrow$ Symmetry

What are Phases of Matter ?

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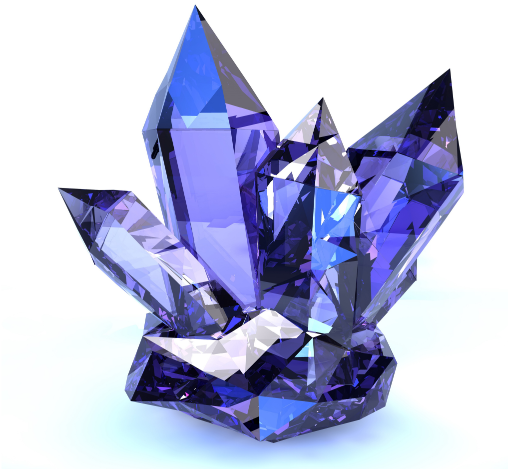

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Amorphous

What are Phases of Matter ?

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Crystal $\Leftrightarrow$ Symmetry

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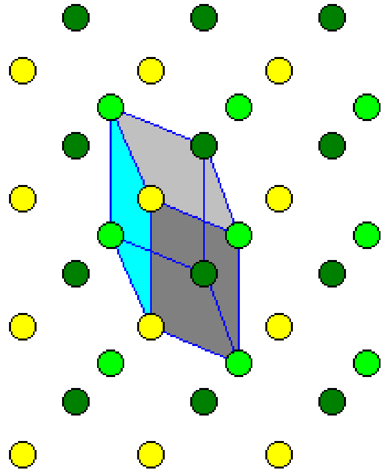
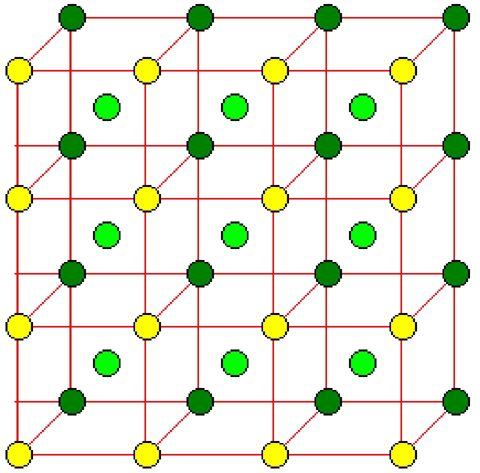
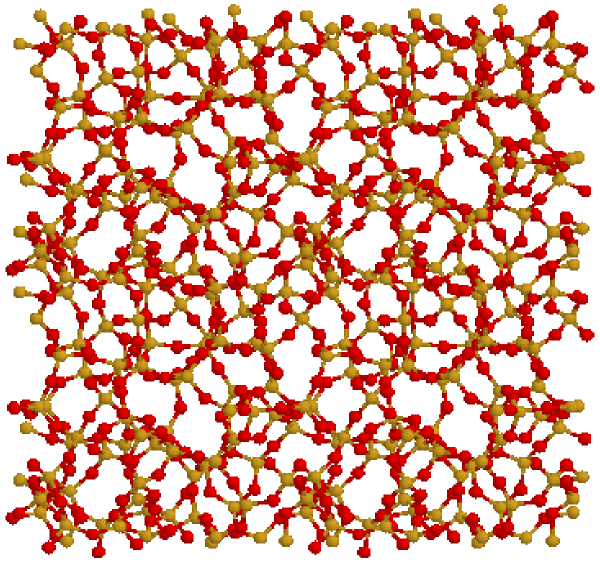
Under the "microscope"







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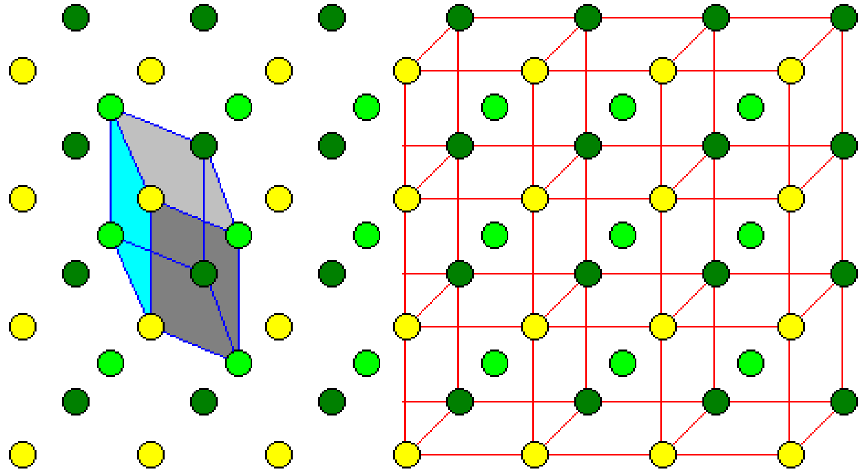

A macroscopic symmetry reflects a microscopic one

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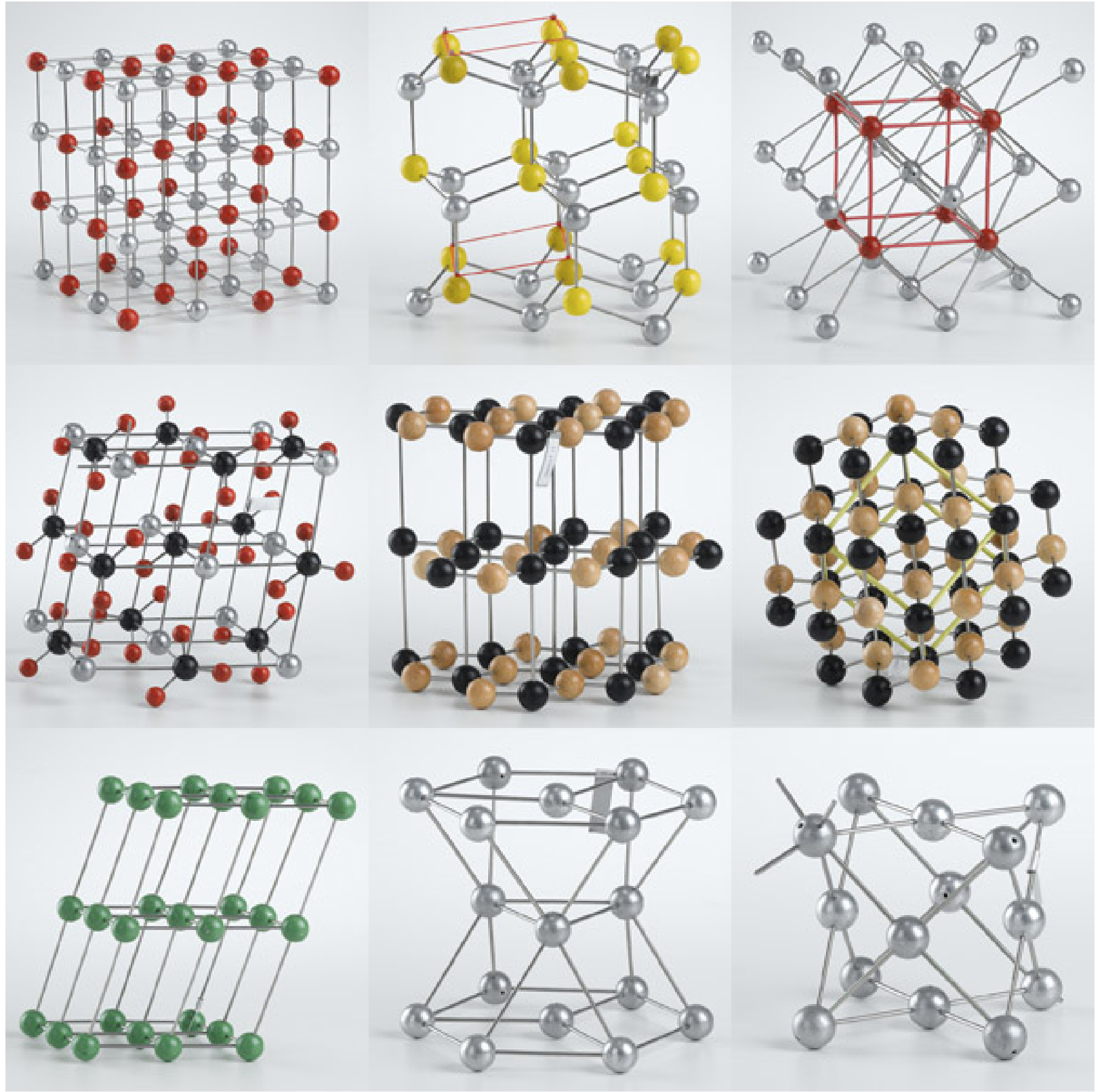
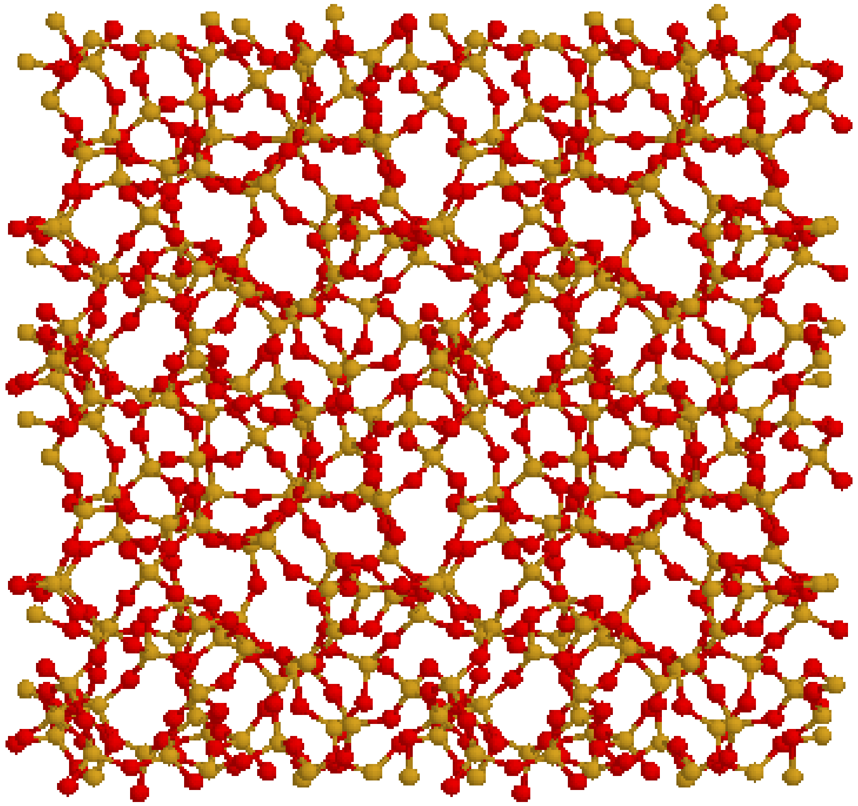


Crystal



Amorphous

Building blocks : Atoms

A bit primitive but
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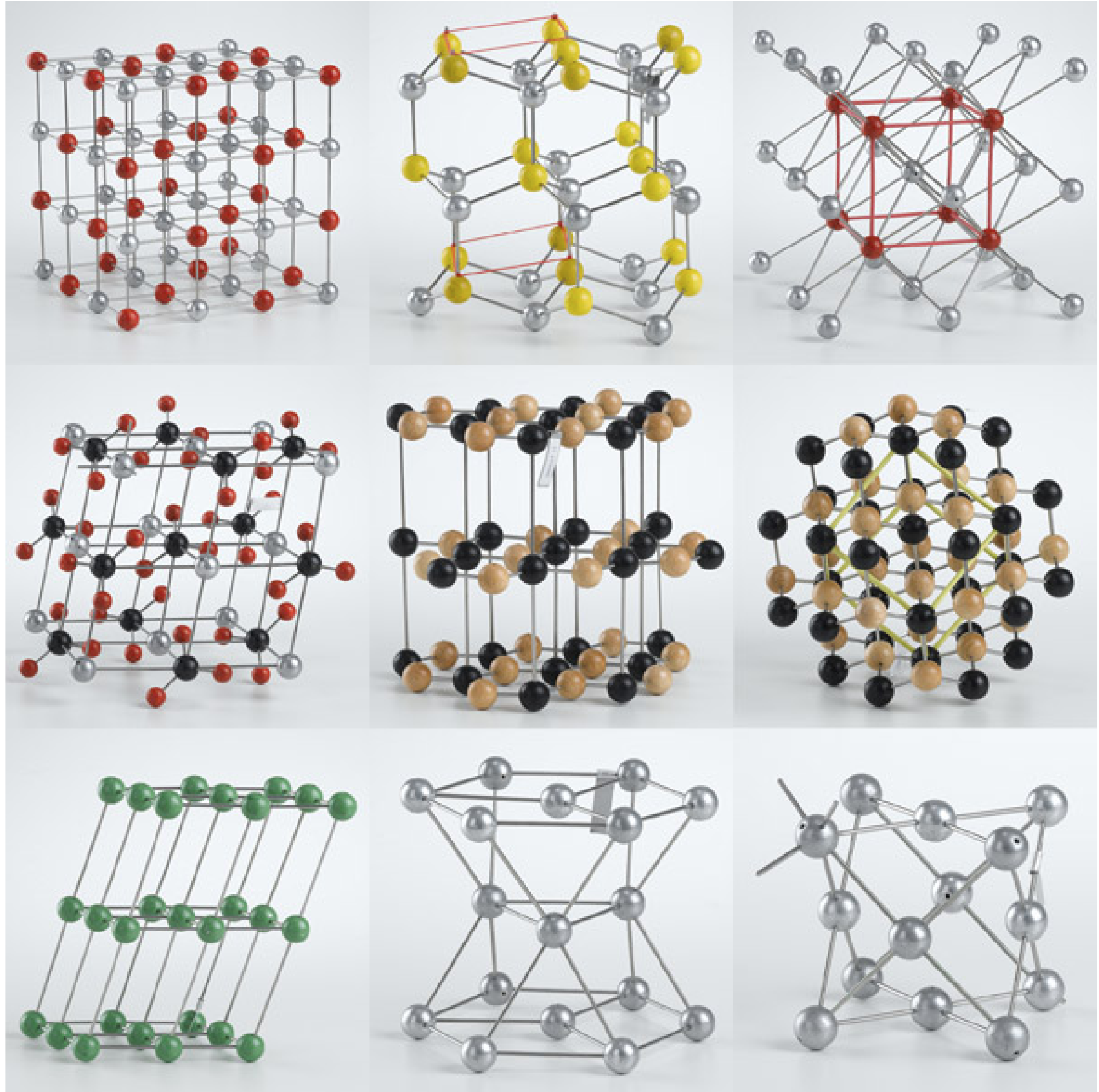
What tights atoms together ?



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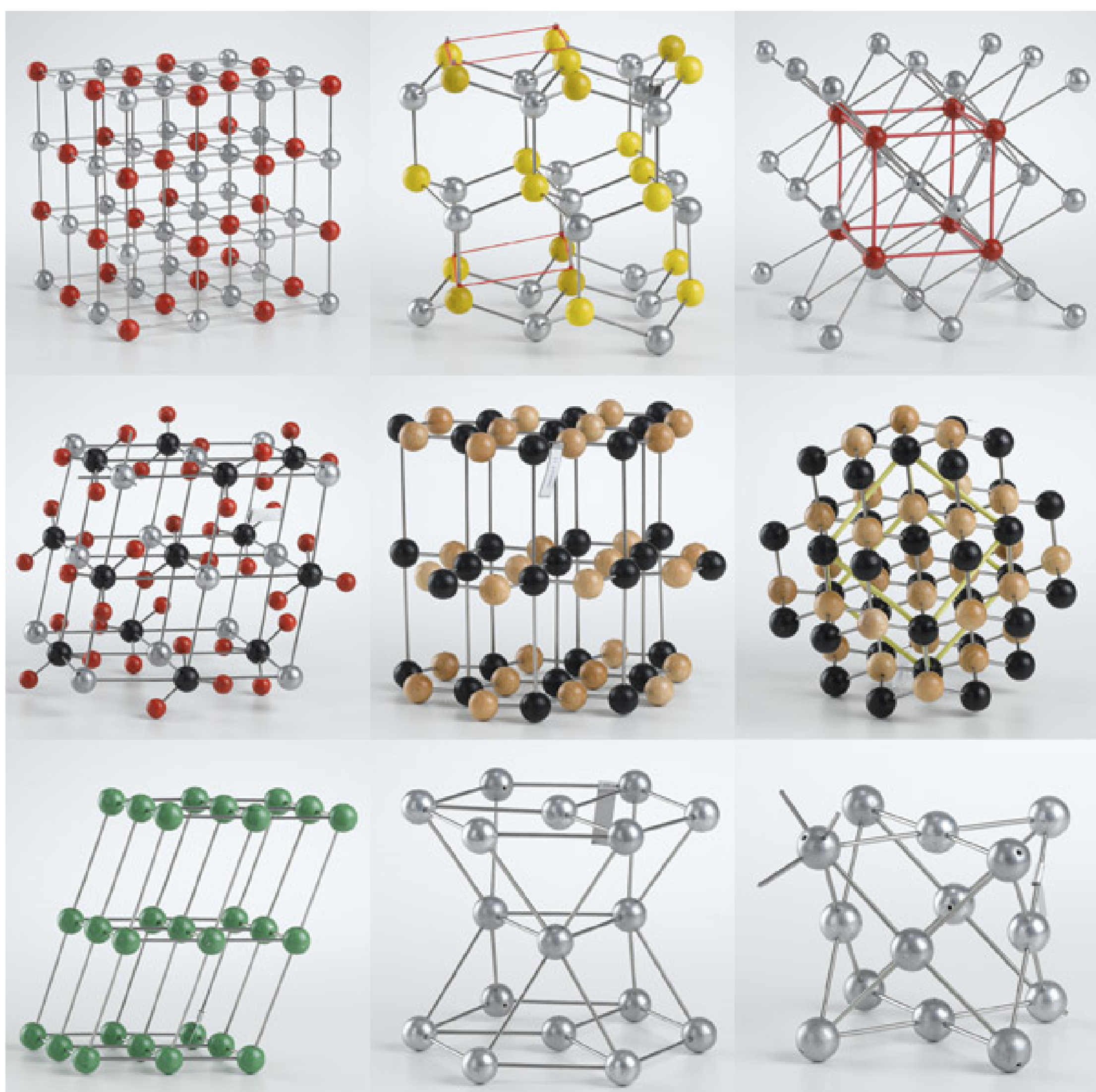
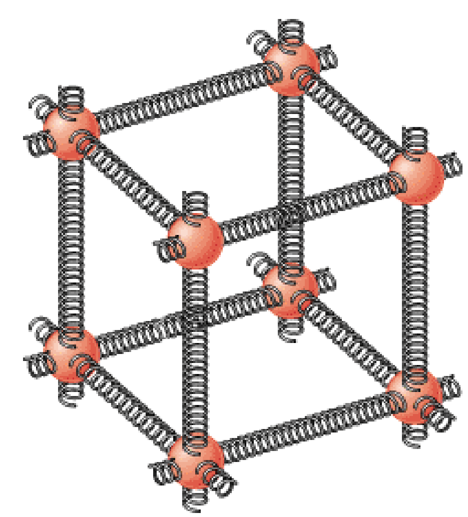
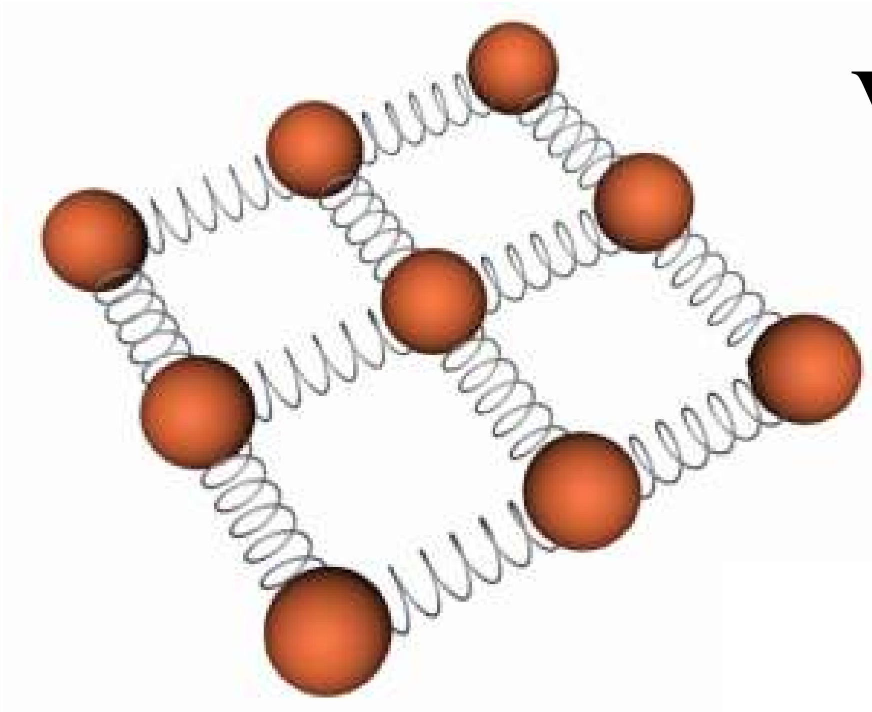


Usual picture : *Atoms are tight together by springs.*

Useful (melting, freezing,..), but limited and adhoc.

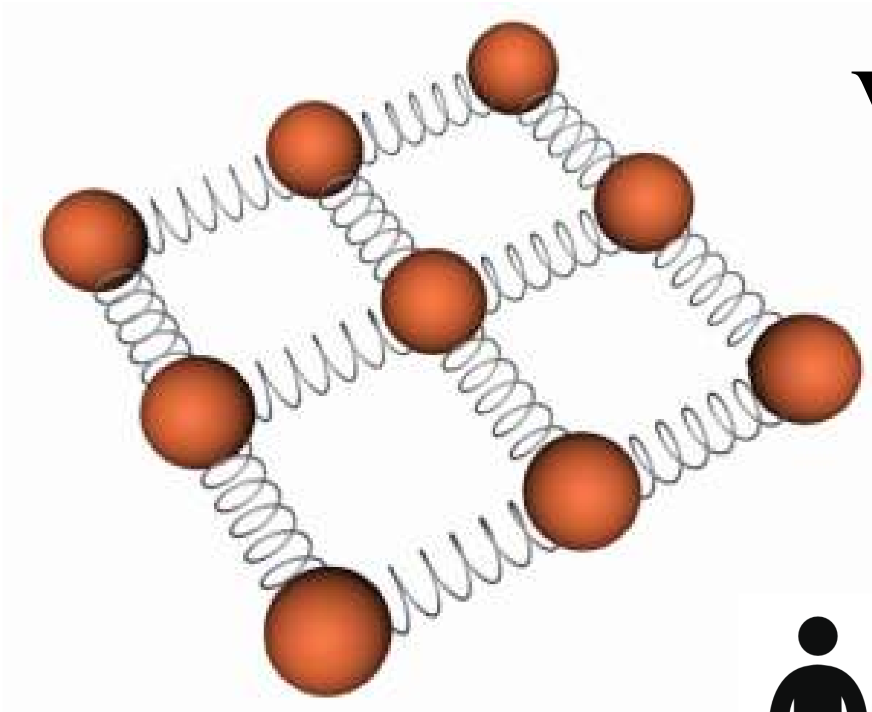


Freezing - Melting



Very low temperature T=0 $\left(-273^{\circ}\right)$

Freezing - Melting



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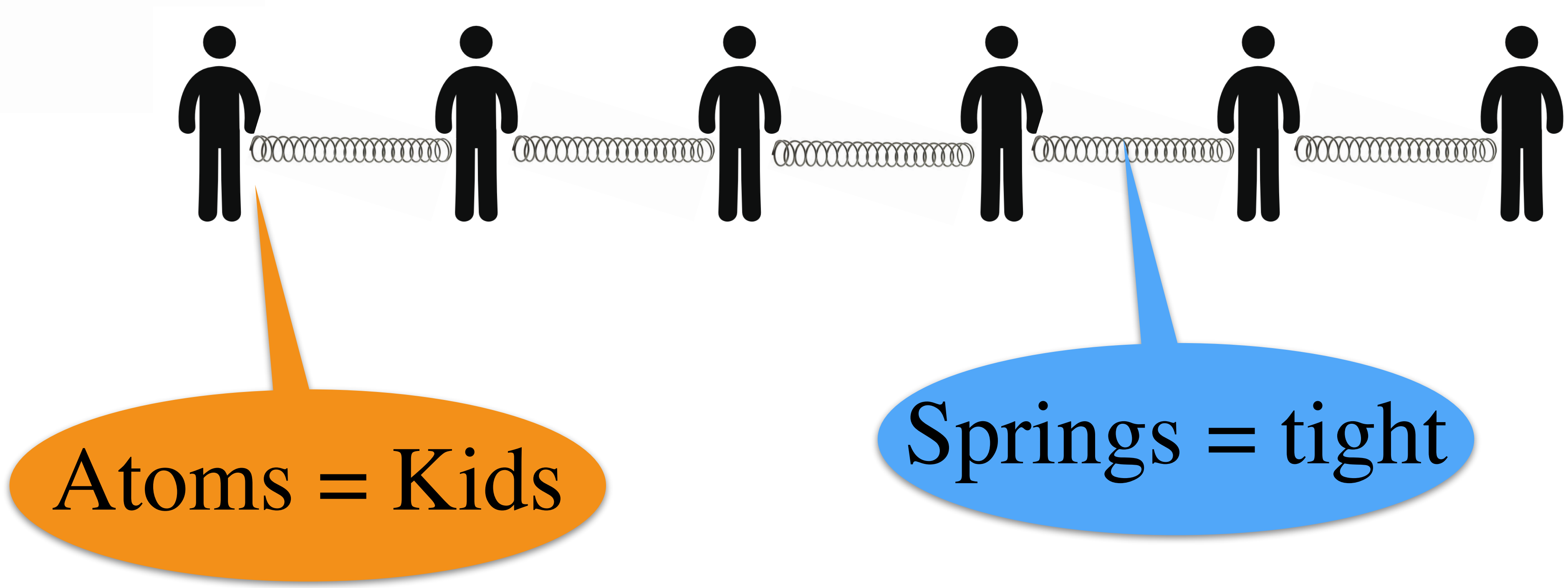

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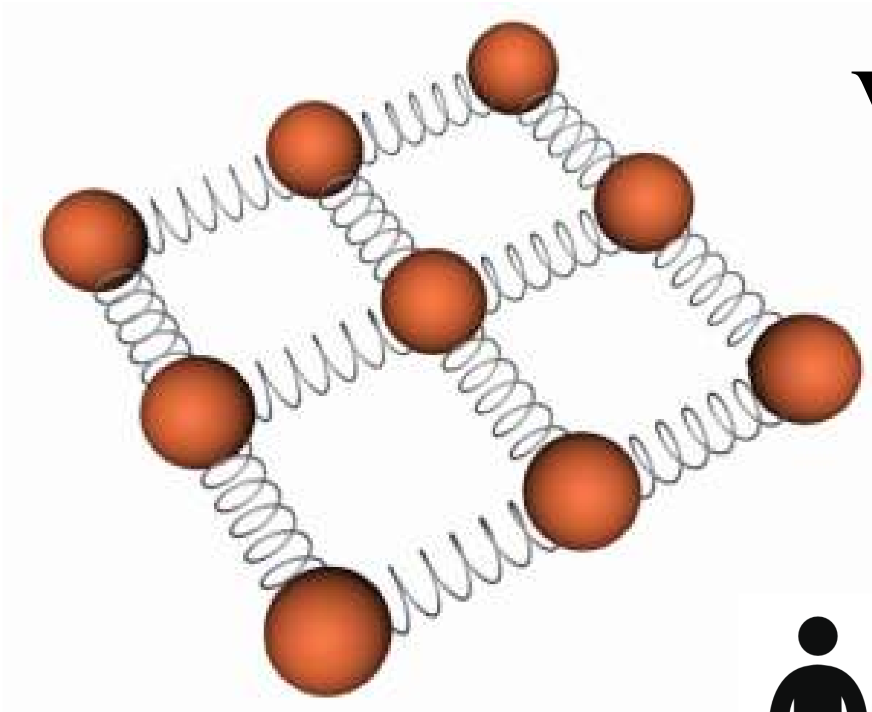

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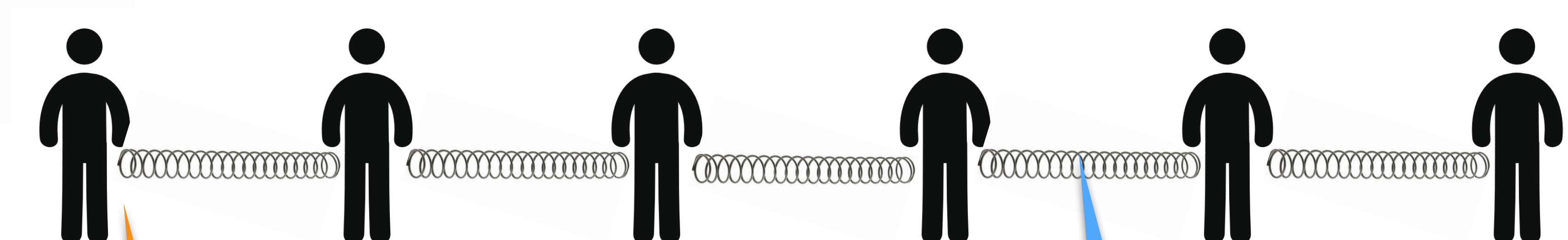


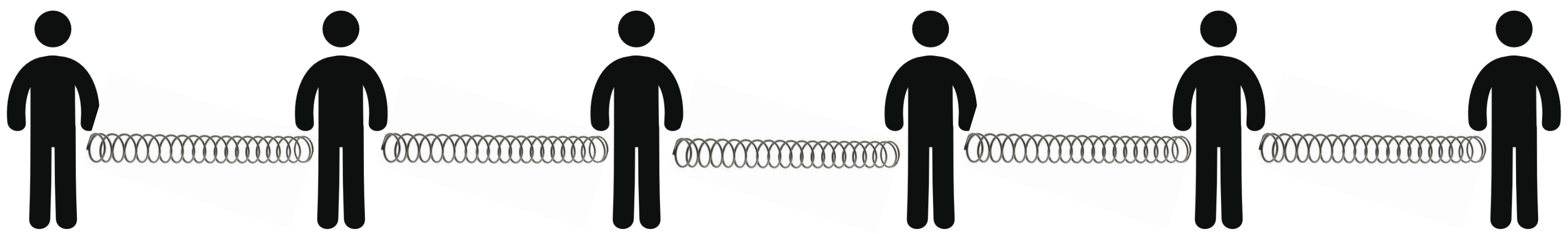
Atoms = Kids

Springs = tight

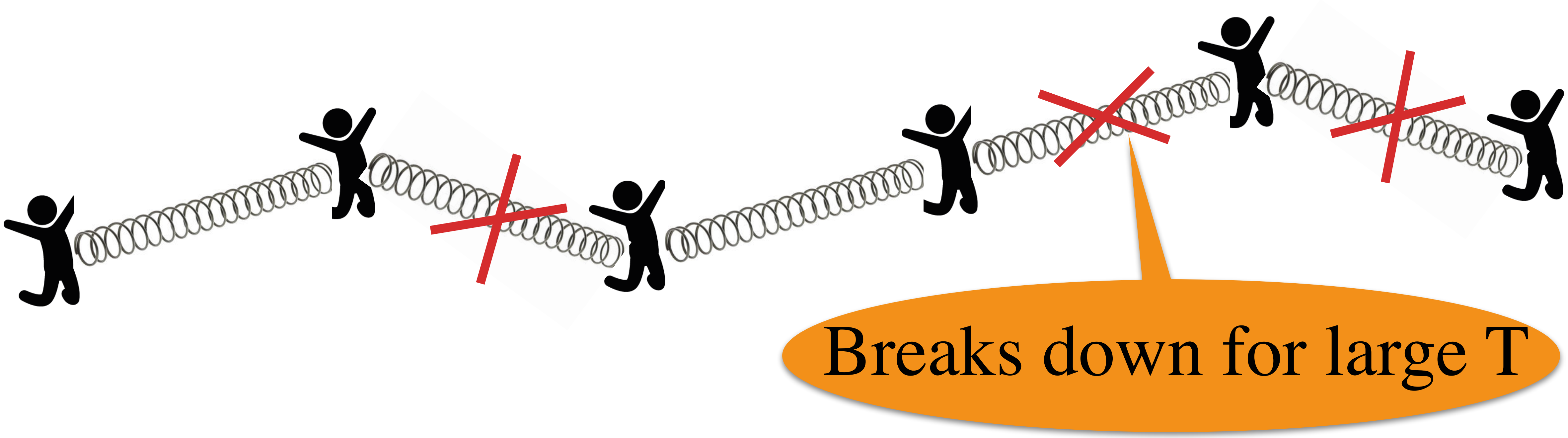
Freezing

Freezing - Melting

Very low temperature T=0



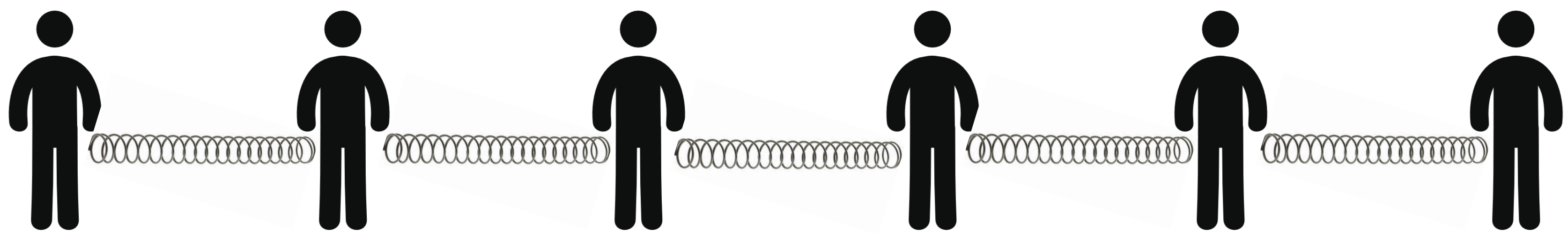
Large temperature $T \geq T_M$



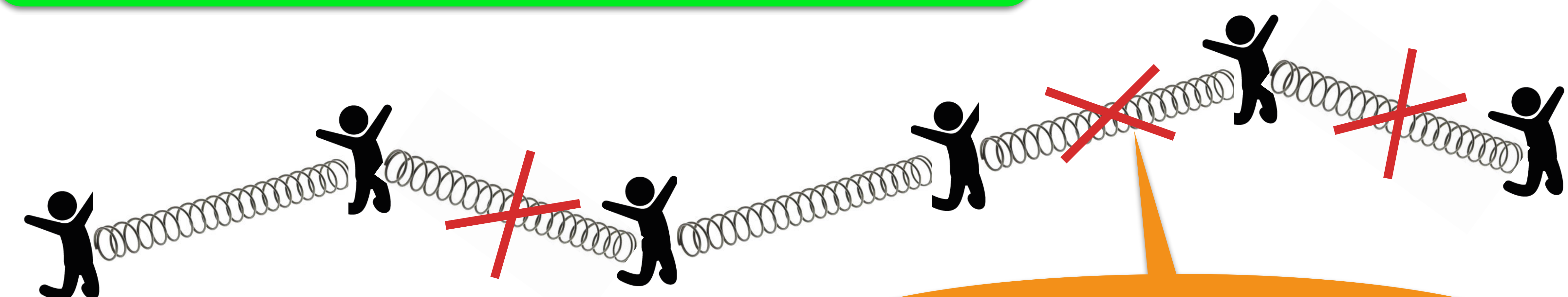
Melting - No crystal symmetry anymore

Freezing - Melting

Very low temperature T=0



Breaking the crystal symmetry



Break down for large T

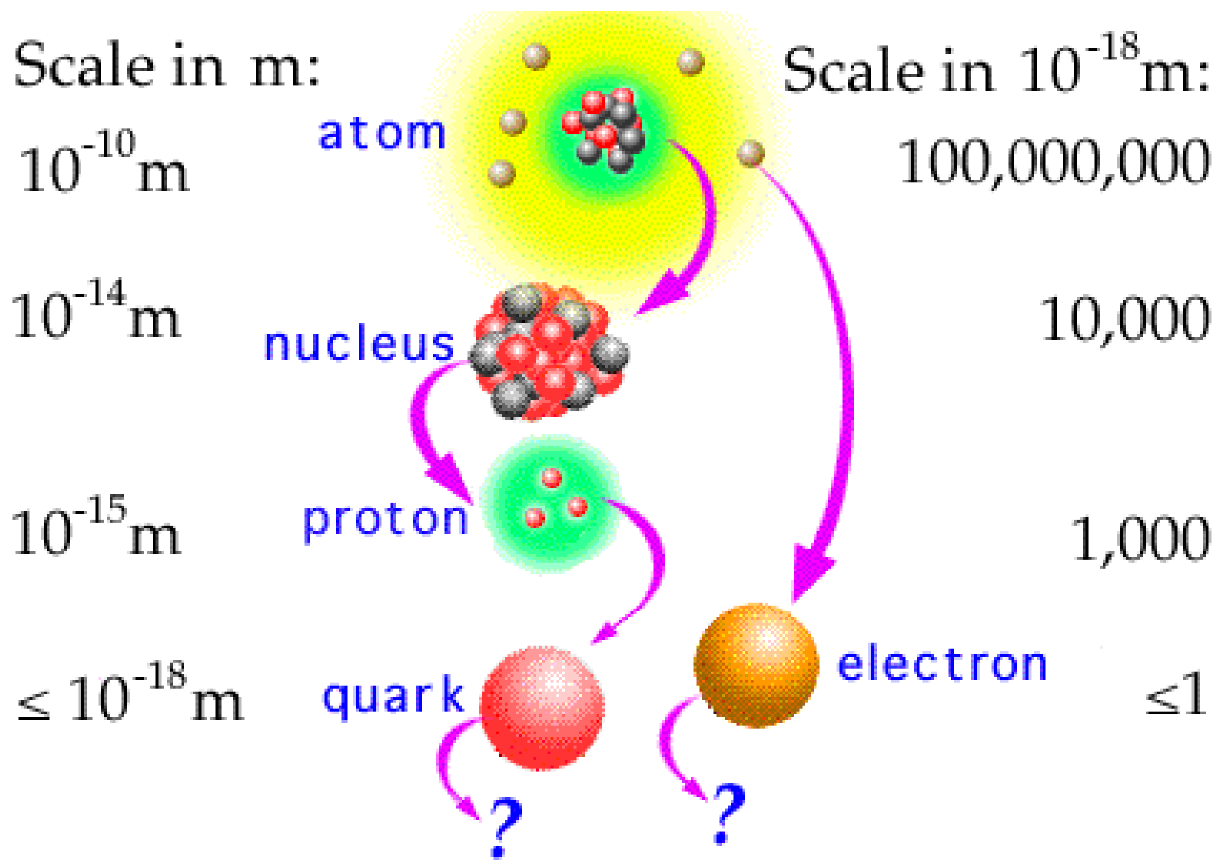
Melting - No crystal symmetry anymore

What determines the characteristics of the atoms springs ?

Difficult ! It is Quantum Physics

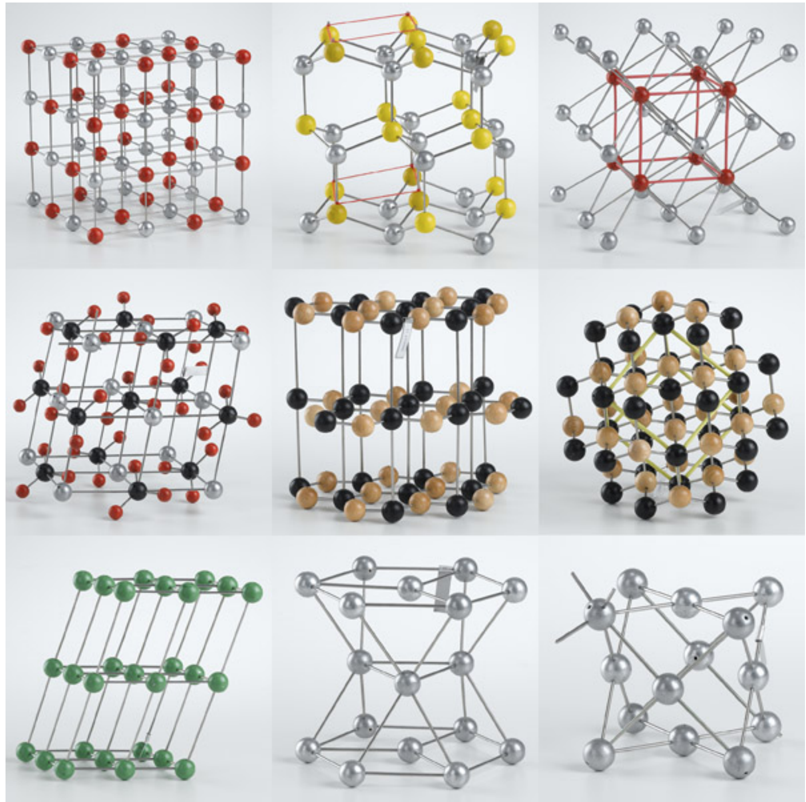
Atoms $\neq$ billiard balls

Rich and complicated structure



Bringing atoms together leads to different types of

and different symmetries.



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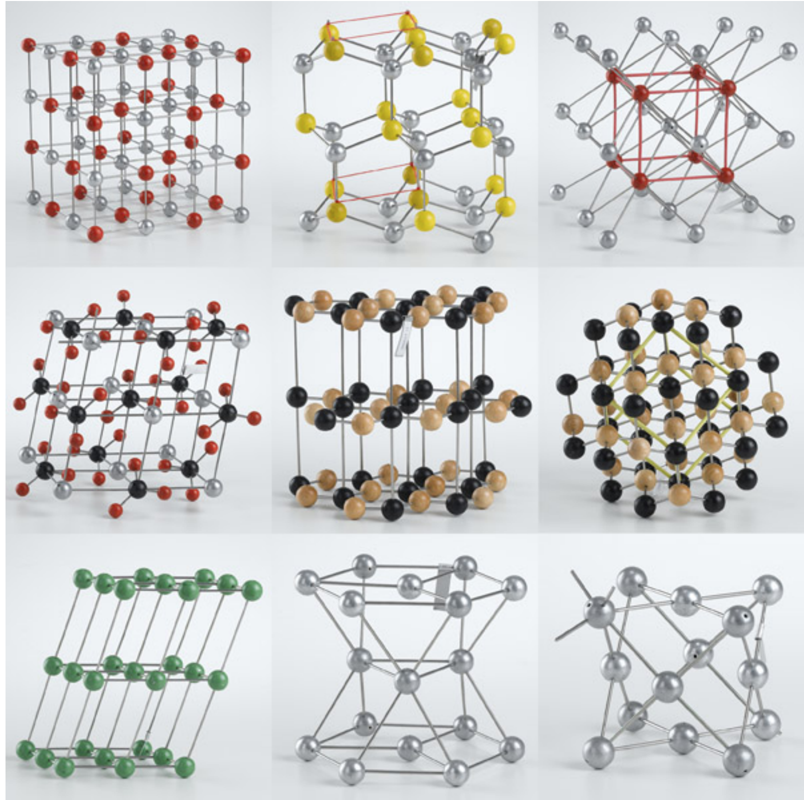

All possible symmetries have been listed and studied (X-Ray crystallography).

Allows to characterize the melting phase transition breaking the crystal symmetry (Landau)

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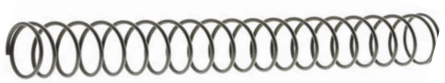
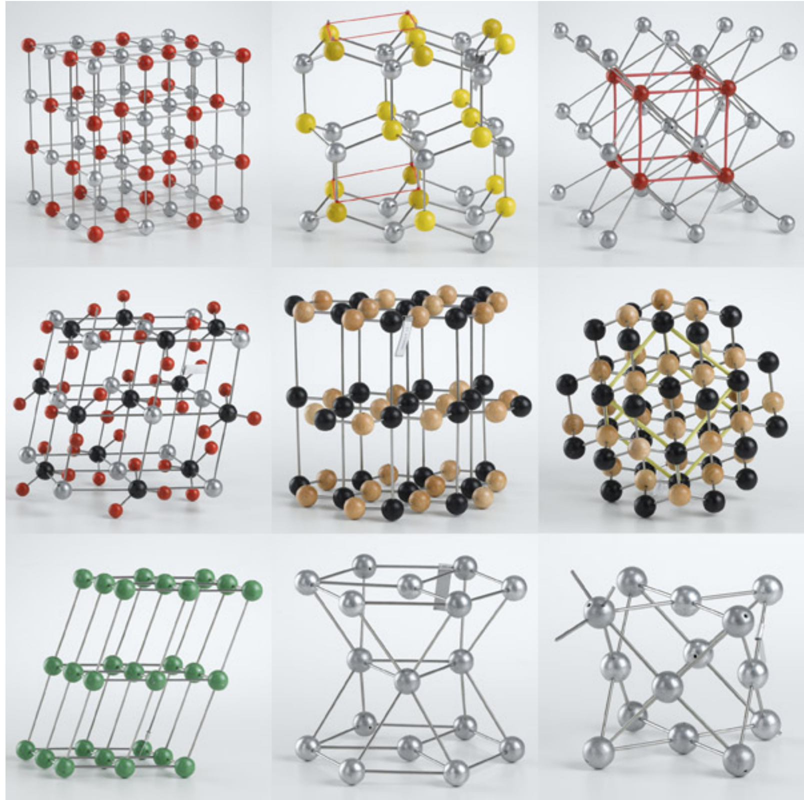


and different symmetries.



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Still a bit primitive ! Essential properties are missing

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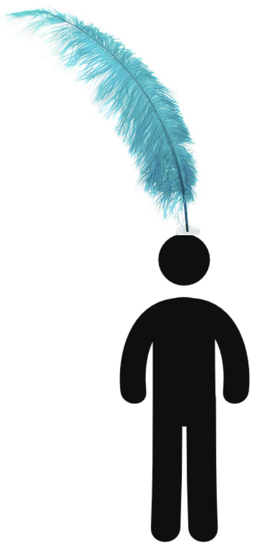
Magnetism

Metal / Insulator

Generalise the atoms/kids model

The Spin



Bosons (Bose-Einstein)

Fermions (Fermi-Dirac)

The Spin

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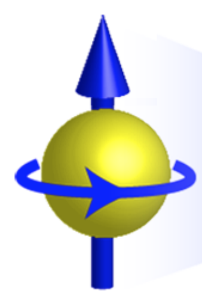
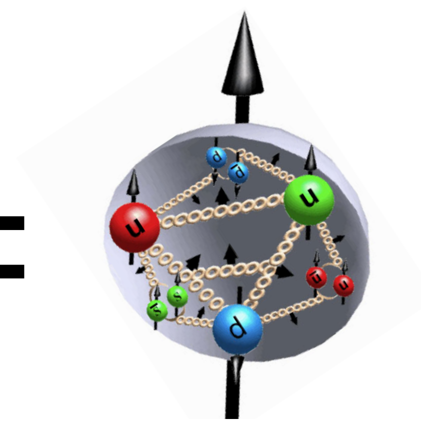
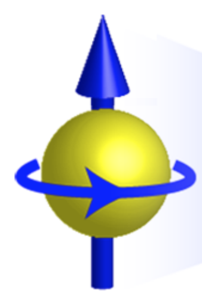
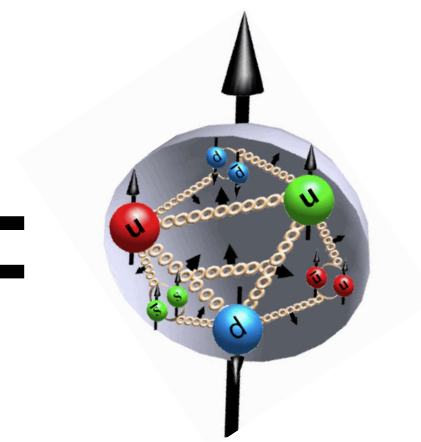
 =  =  (Spin)

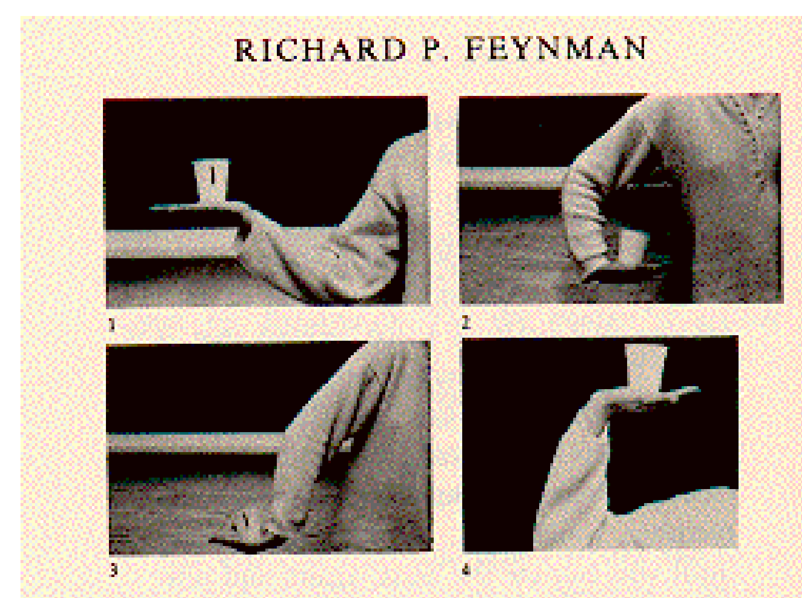
The Spin

Bosons (Bose-Einstein) Fermions (Fermi-Dirac)

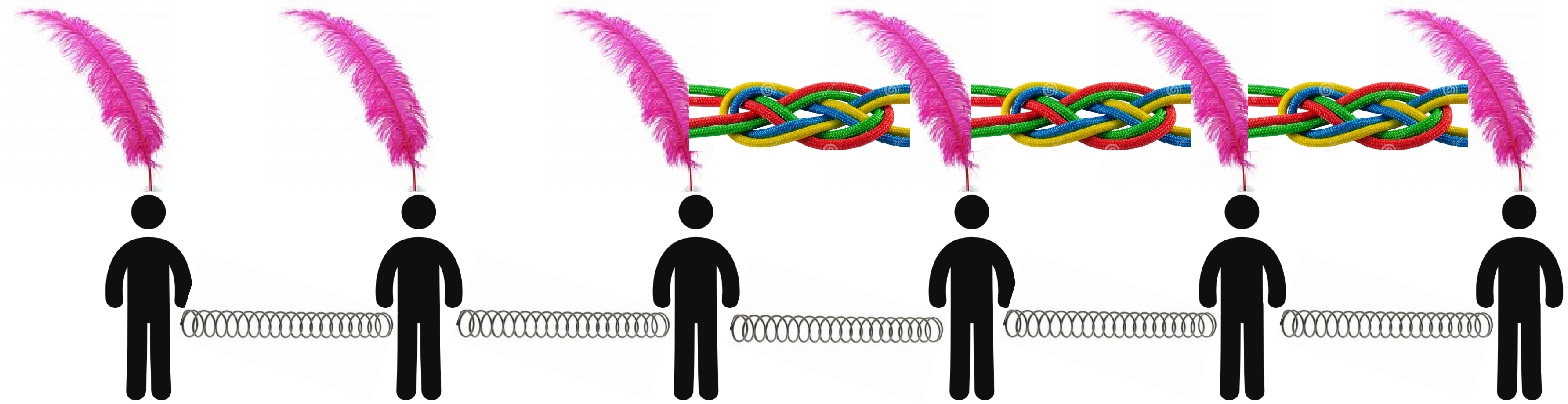
 =  =  (Spin)

Not a simple arrow - A magic one !

Not 360° but $2 \times 360^{\circ}$!



Interaction between spins/feathers results from Quantum Mechanics



Ferromagnetic order : MAGNET

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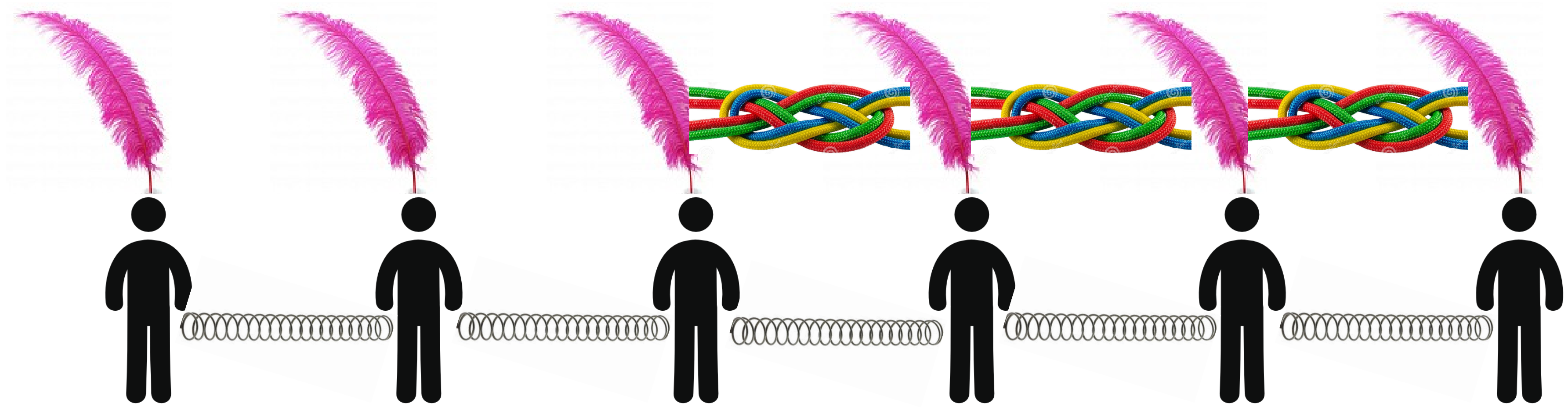
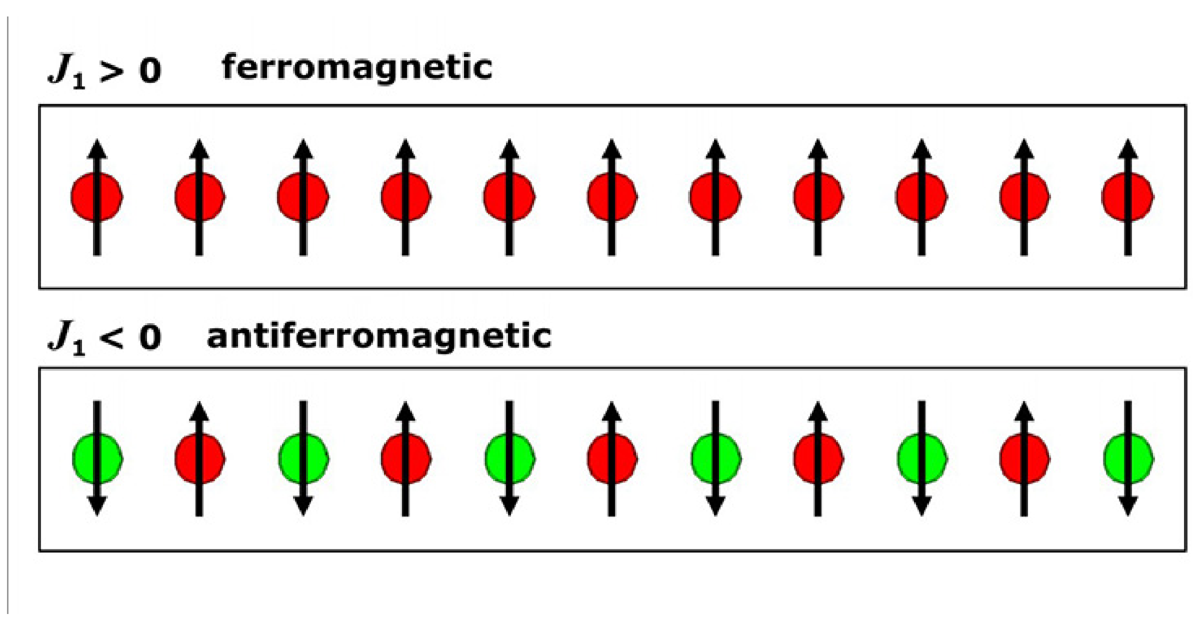
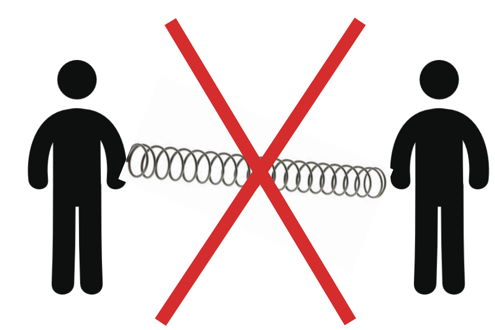
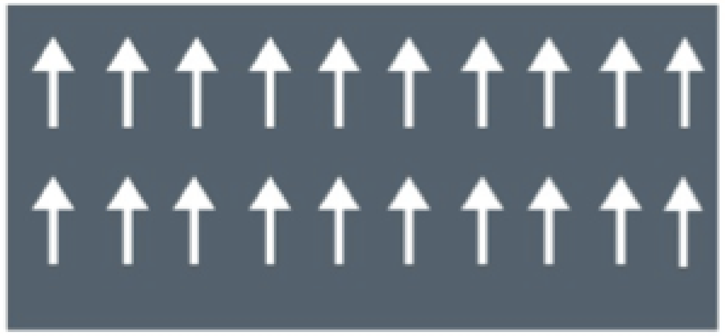


Ferromagnetic order : MAGNET





Heating a magnet destroys the magnetic order



B = ferromagnetic

$T \nearrow$

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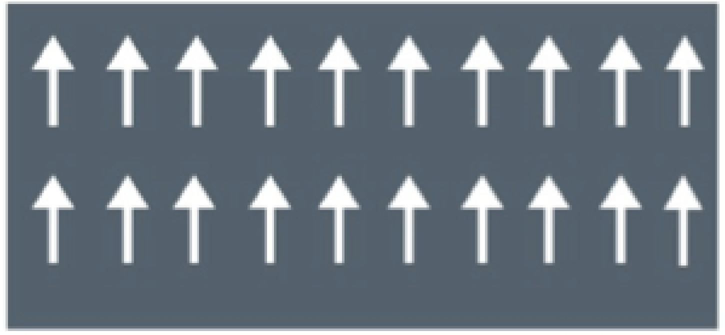
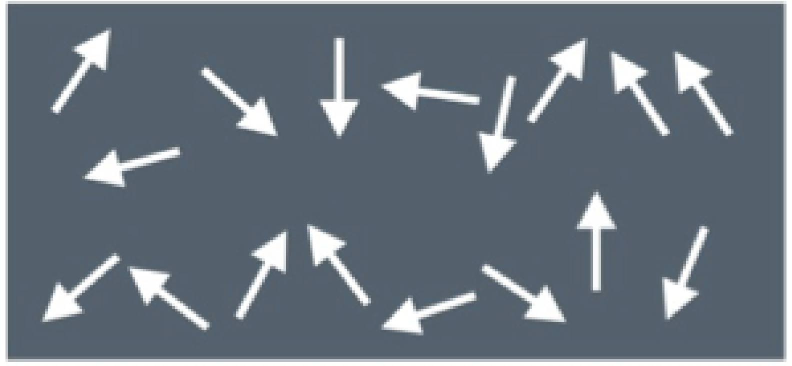


B = ferromagnetic

$$T \geq T_c$$



A = paramagnetic

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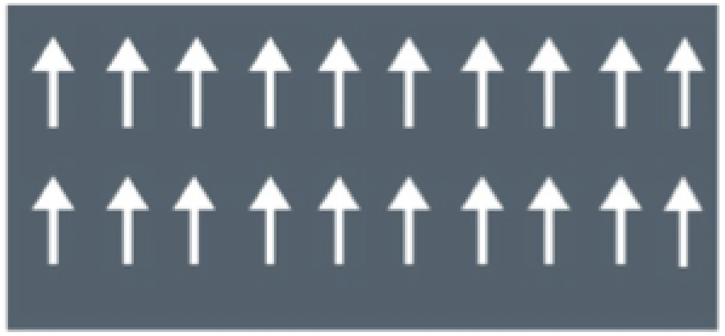

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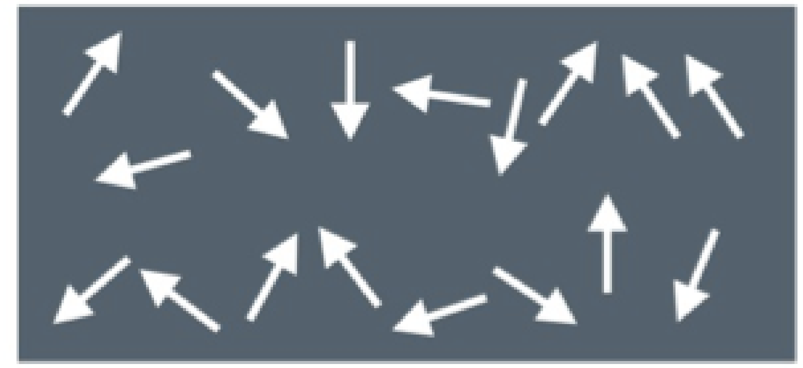
"c" for Curie
(Pierre not Marie)



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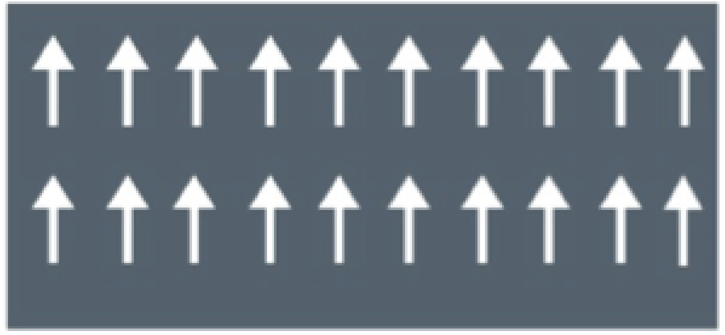

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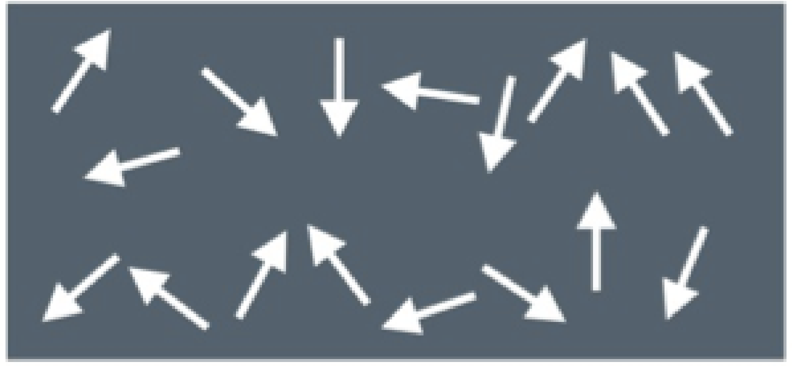

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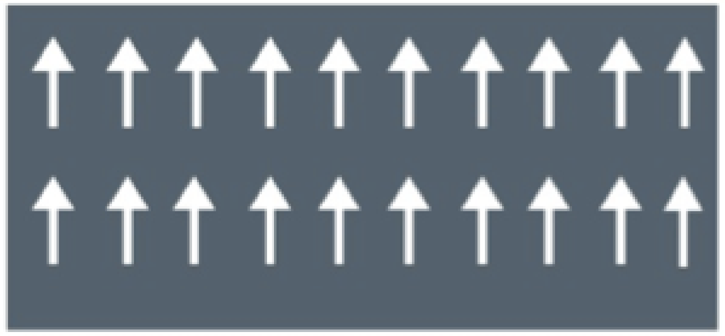
Breaking the symmetry of the spins

Heating a magnet destroys the magnetic order



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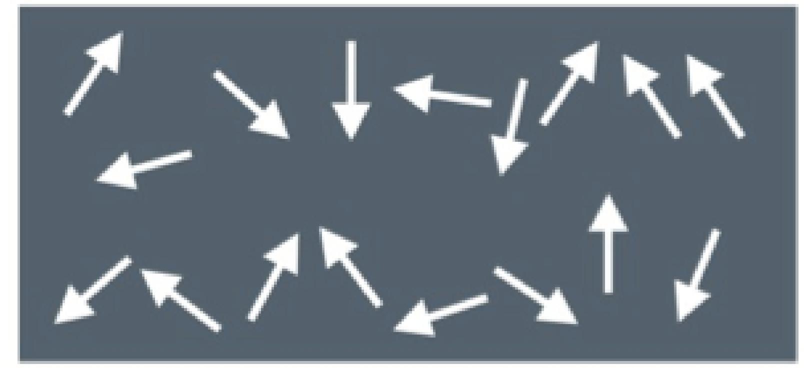

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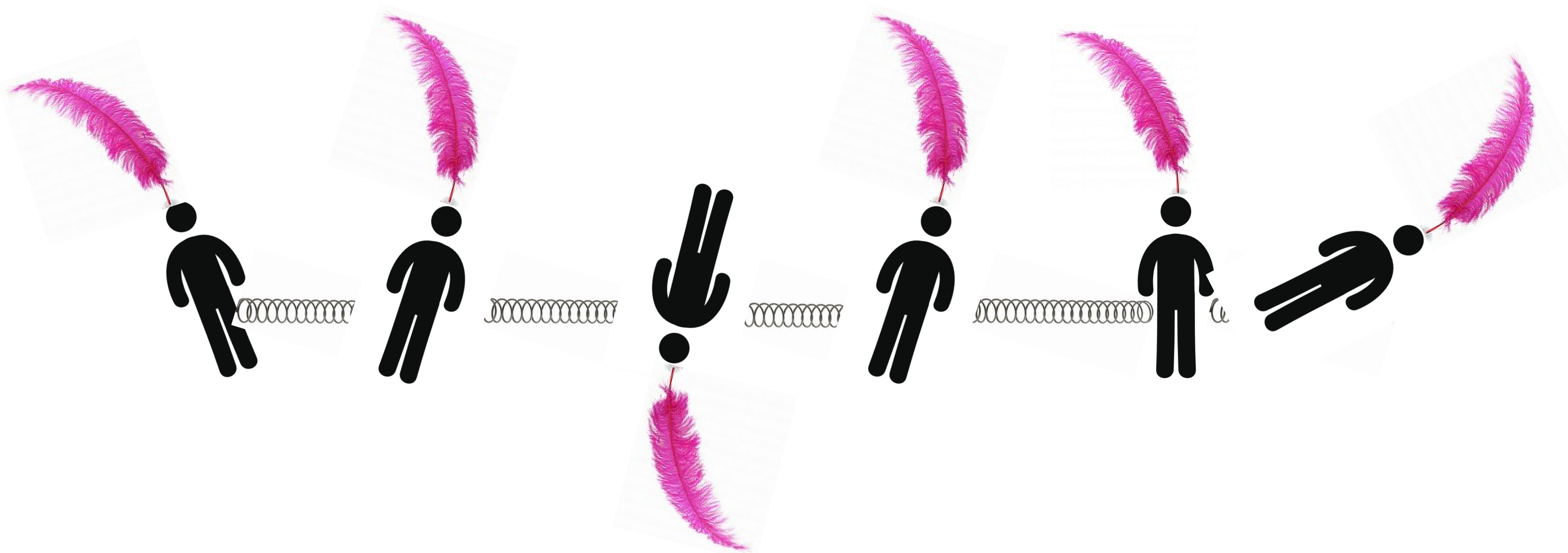

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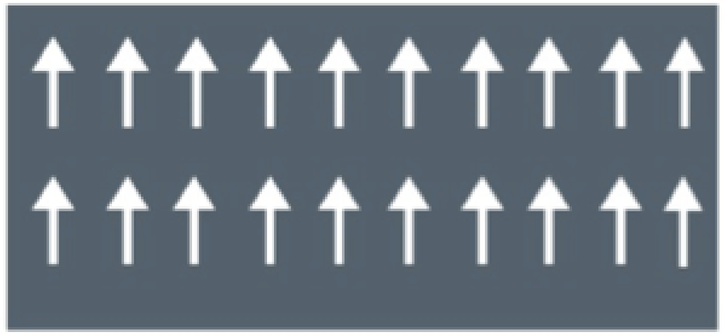
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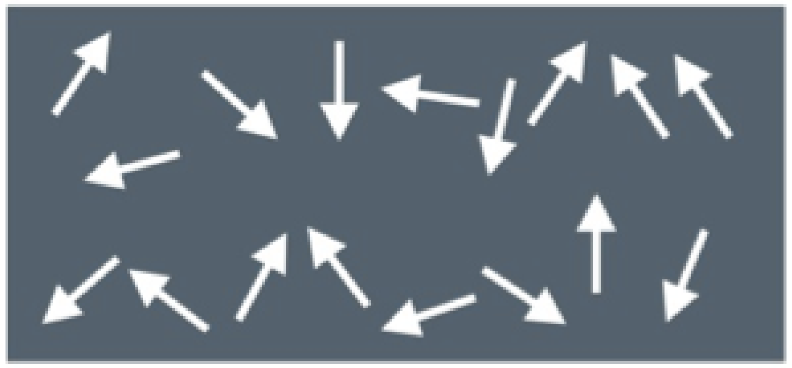
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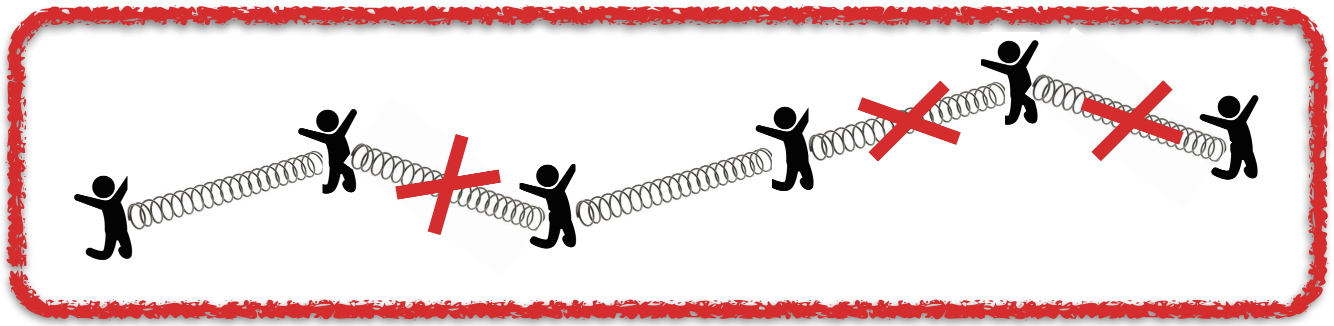
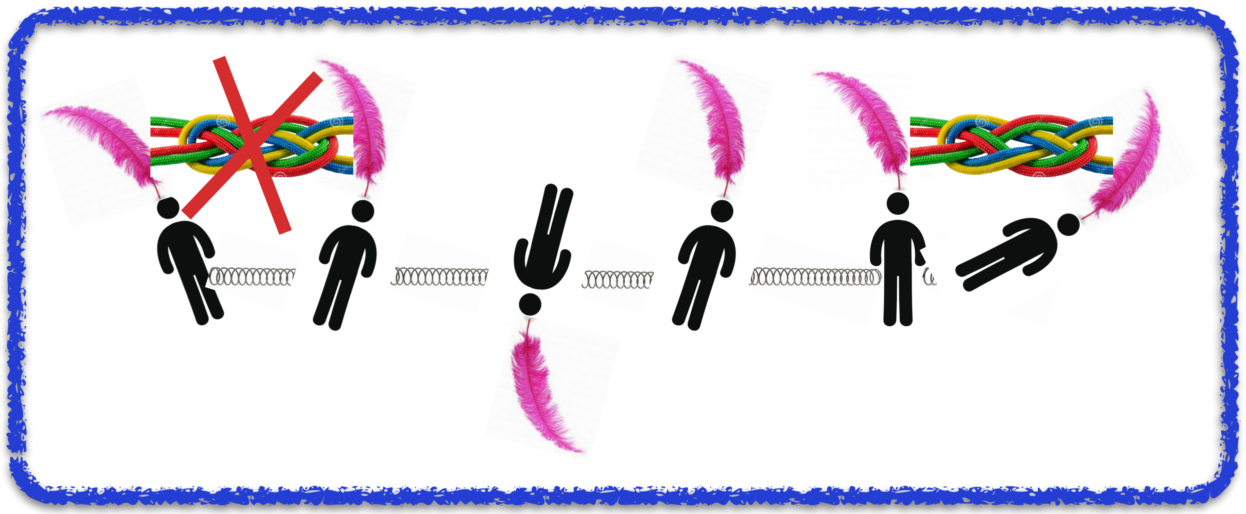
Breaking the symmetry of the spins

This Magnetic Phase Transition between a magnet and non magnet is analogous to Melting but the origin and mechanisms <u>are very different</u>.

But in both cases : breaking of a symmetry



Melting $T \geq T_M$



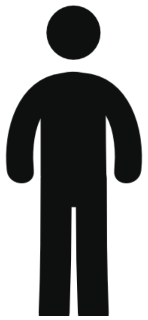
Magnetic phase transition $T \geq T_c$

$$T_c \neq T_M$$

Different mechanisms - Open problem until today !

Universality

Notion of Universality : Each solid state system is a collection of a huge amount of details : Atoms, interactions, Spin, Mass,…

   …

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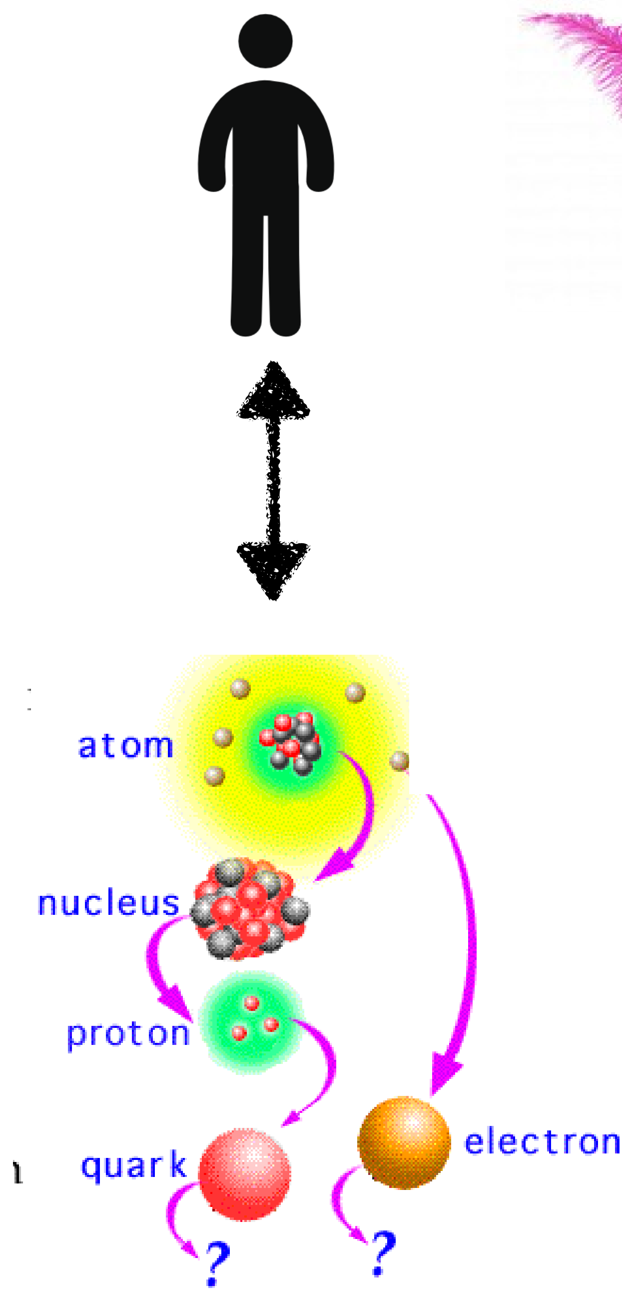
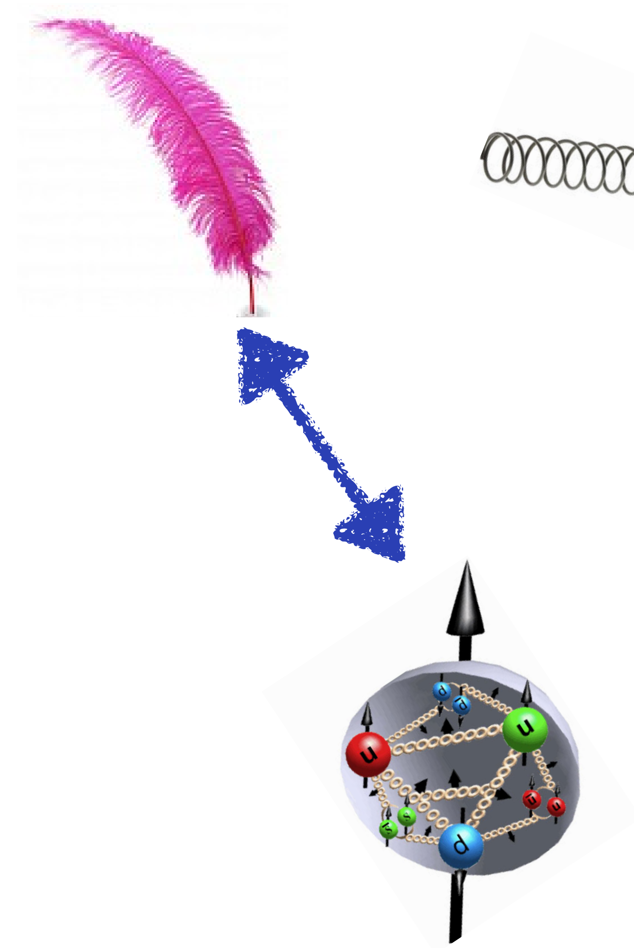


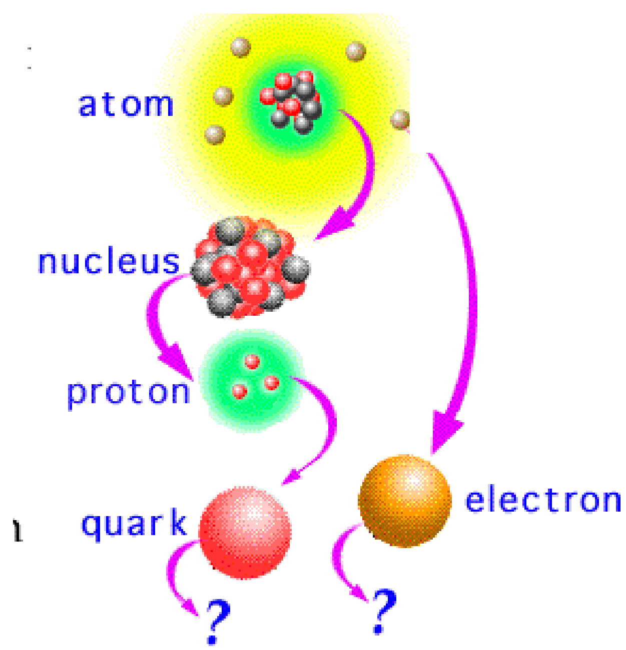


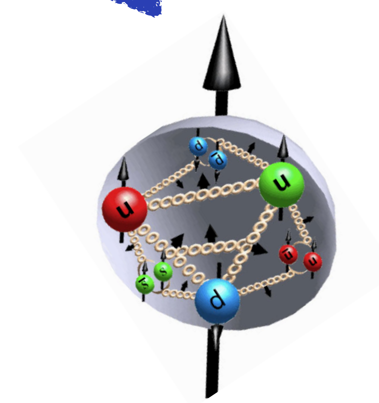
(Spin)

. . .

Notion of Universality : Each solid state system is a collection of a huge amount of details : Atoms, interactions, Spin, Mass,…

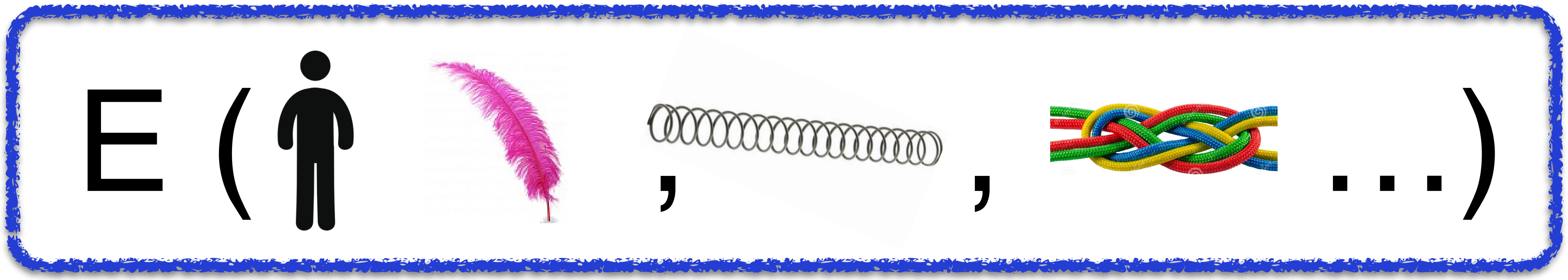
Properties/Behaviour of solids depend on all these details. Hopeless !





(Spin)

Model : Identify a minimal (and small) number of relevant quantities and insert them into an *energy function.*



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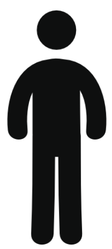
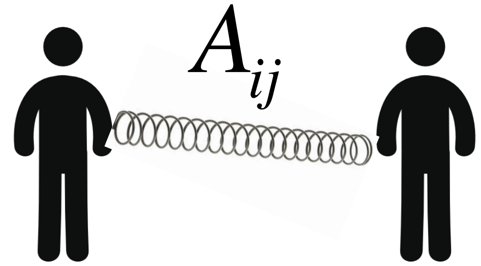
E (  ,  ,  ...)

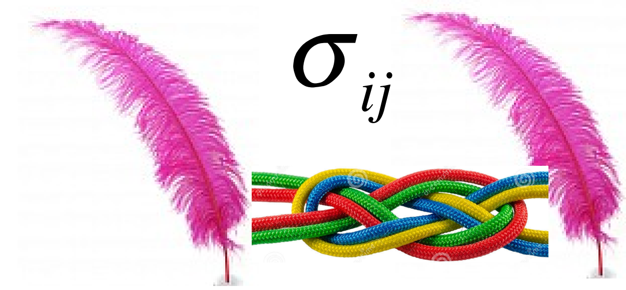
Right combination of these quantities so that E is minimum.

Short list of relevant Models/Energies.

$E = \sum_{i,j}\Big($ A_{ij}



$+$ σ_{ij}



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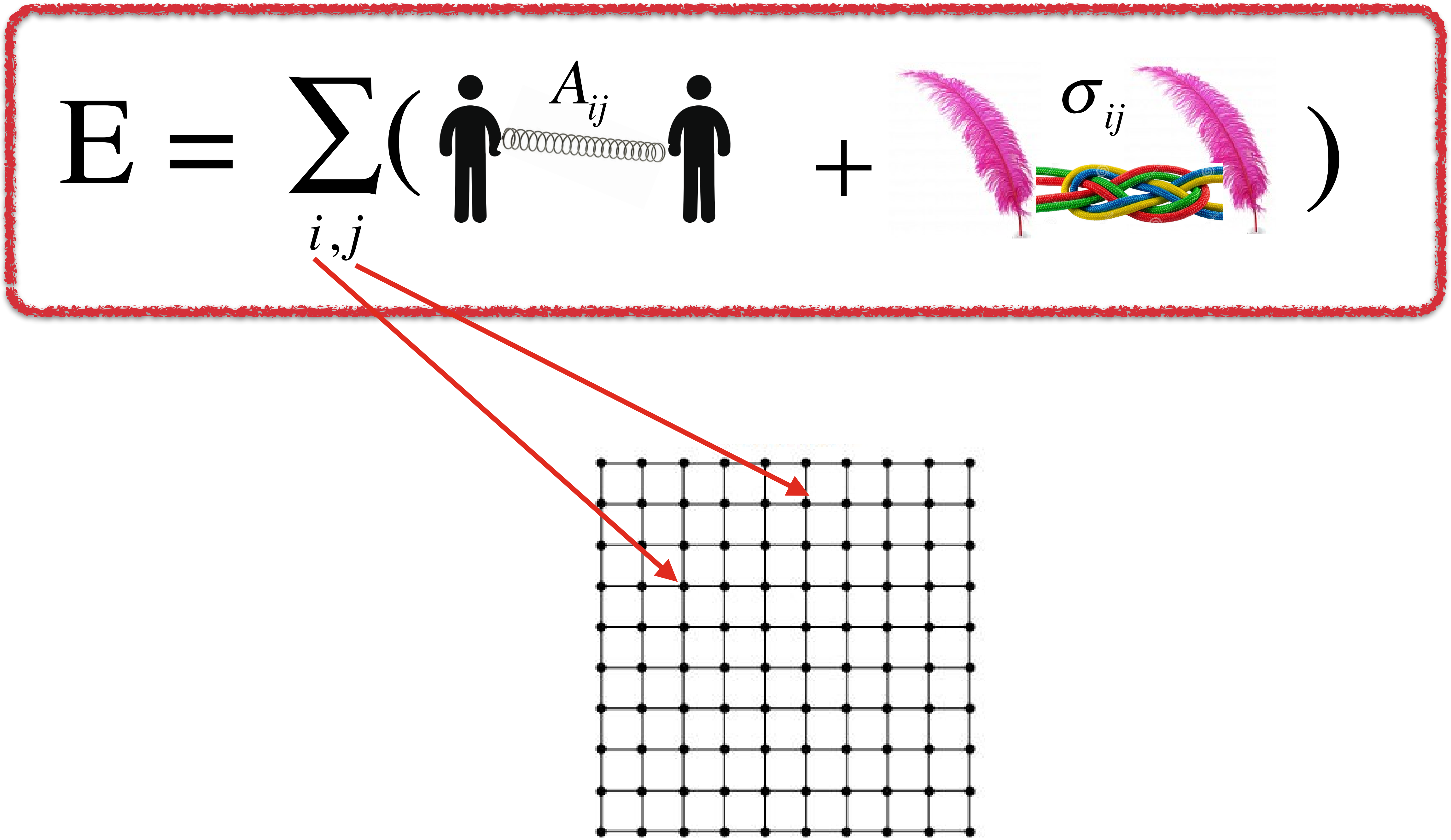
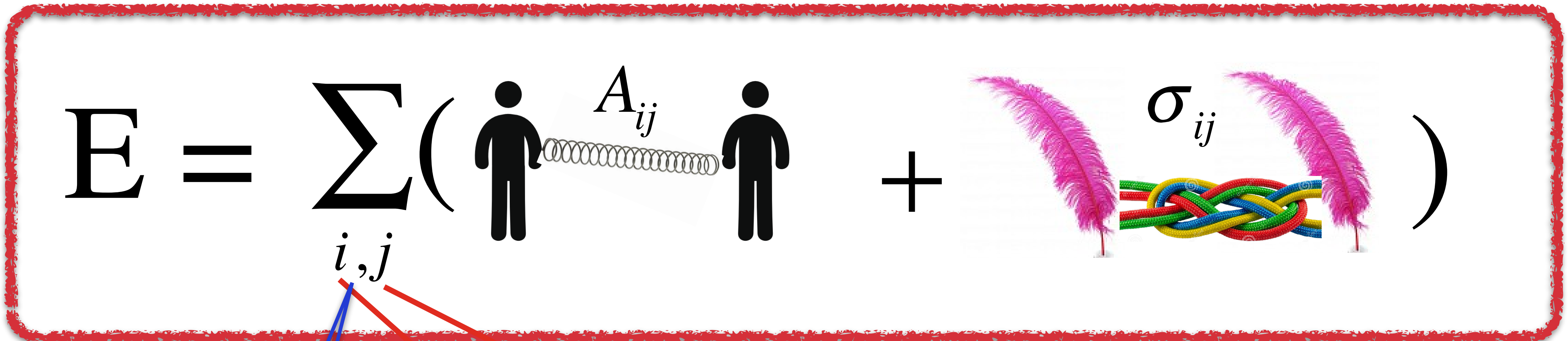
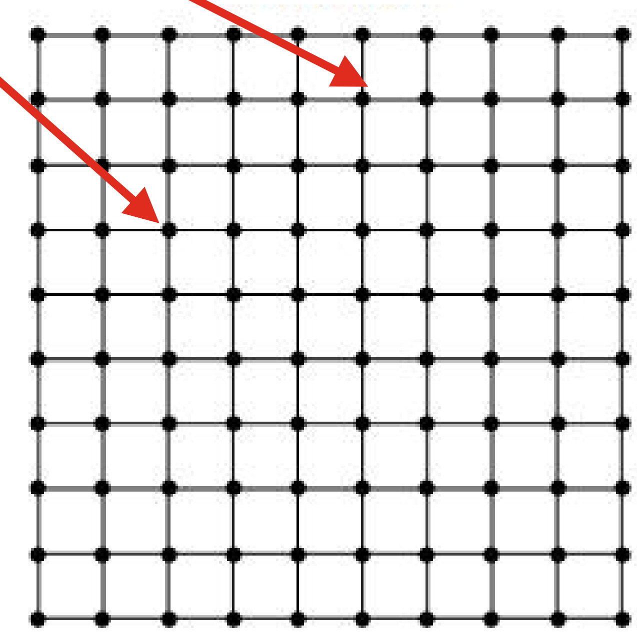


Short list of relevant Models/Energies.



symmetry



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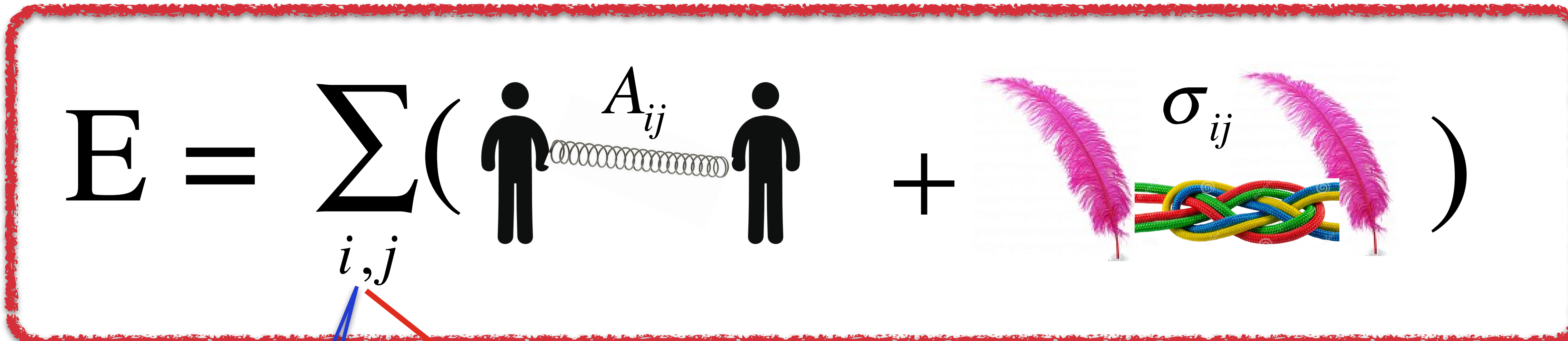
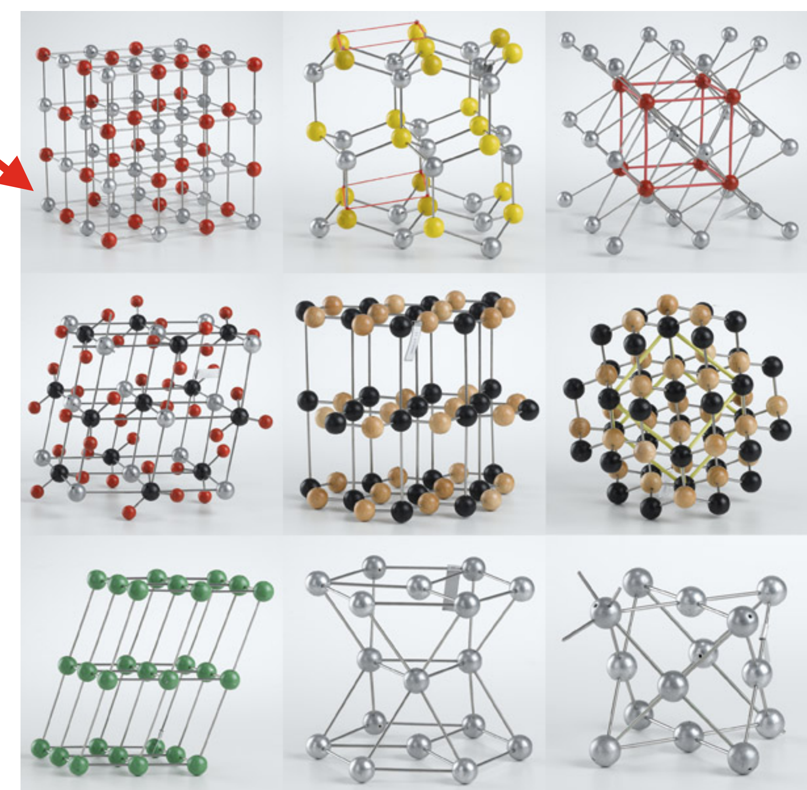


symmetry



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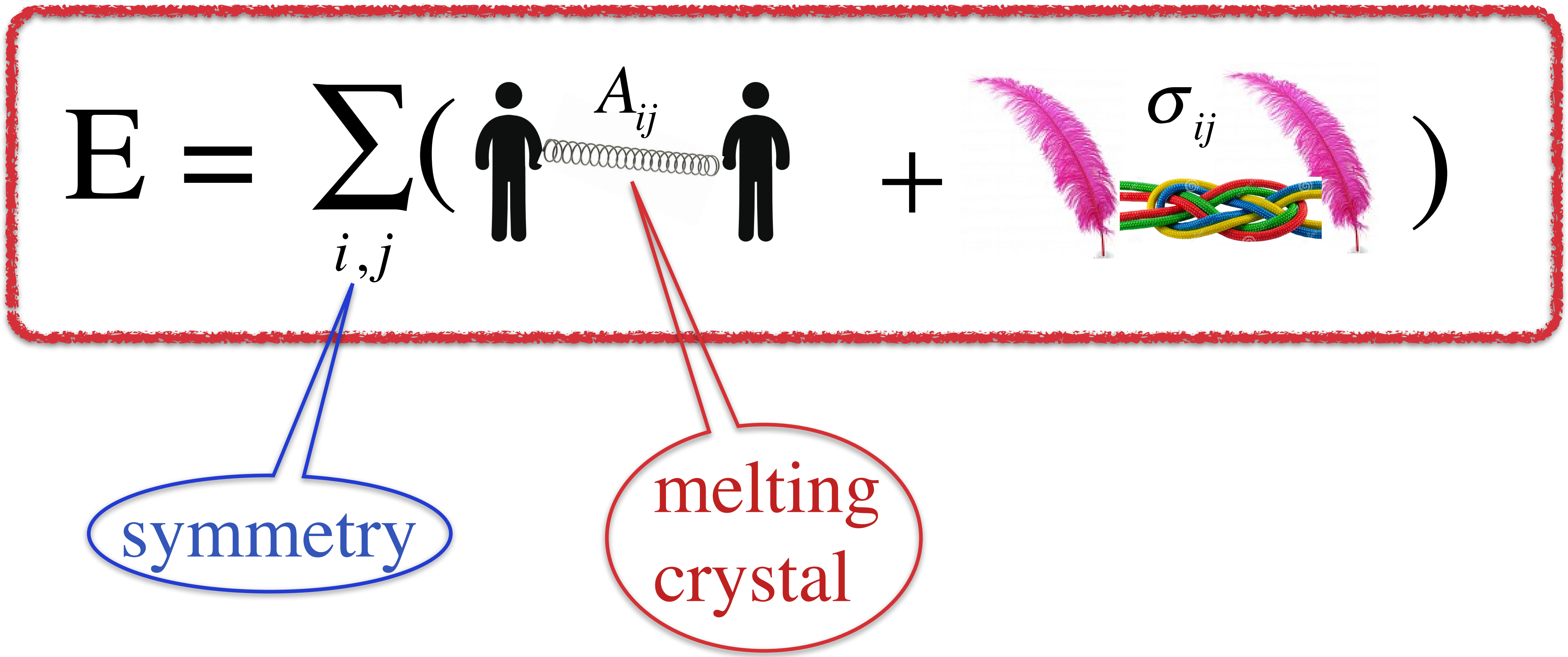

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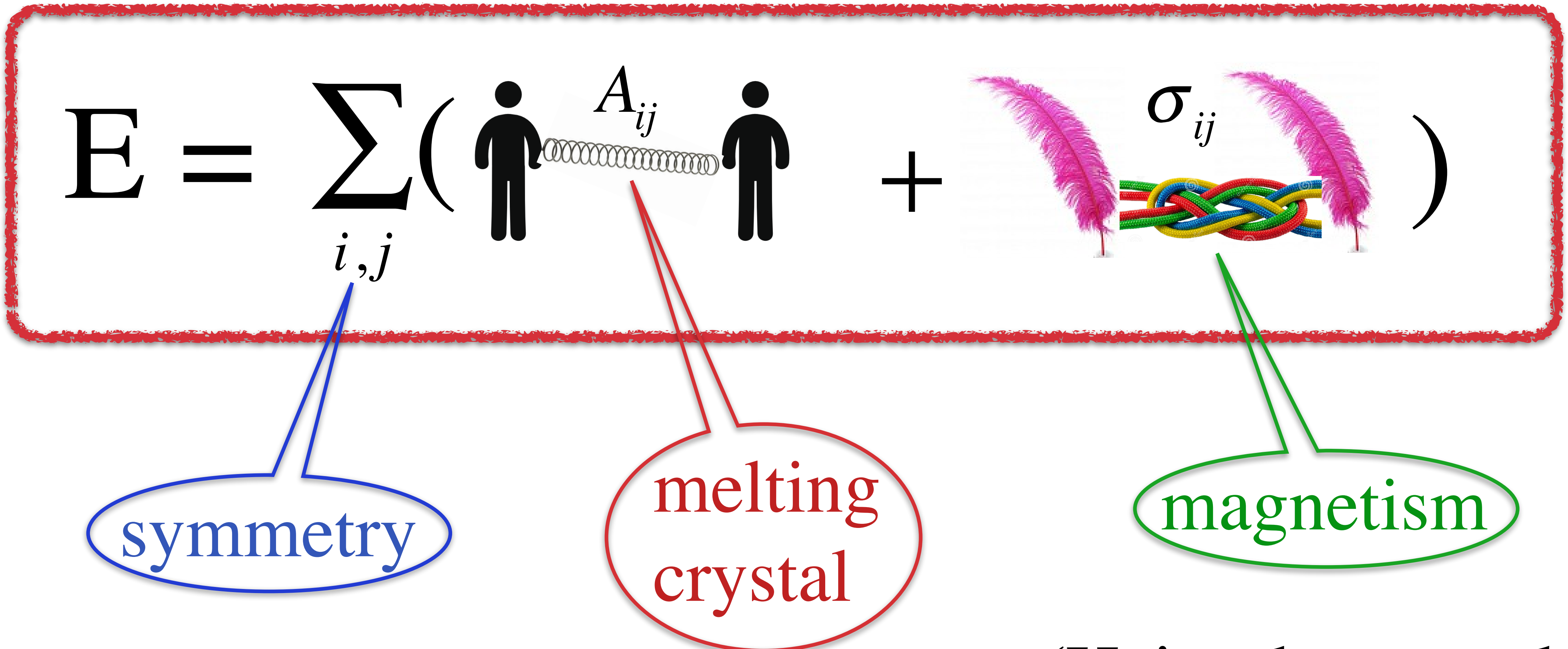
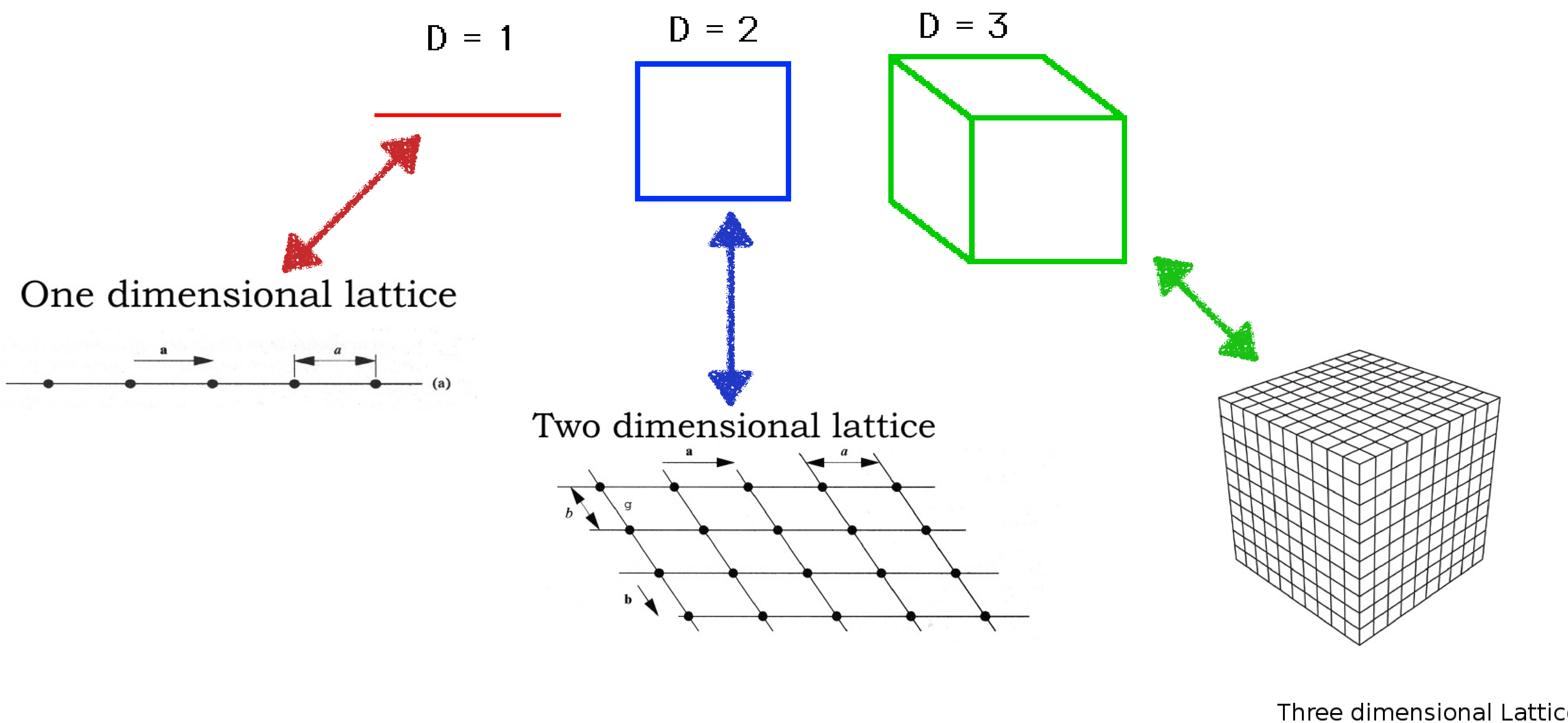


(Heisenberg model)

Importance of the spatial dimension



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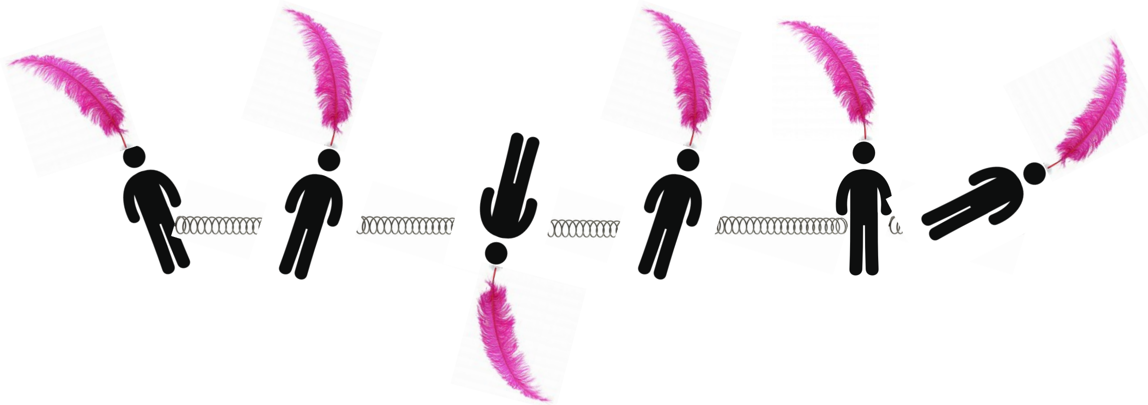
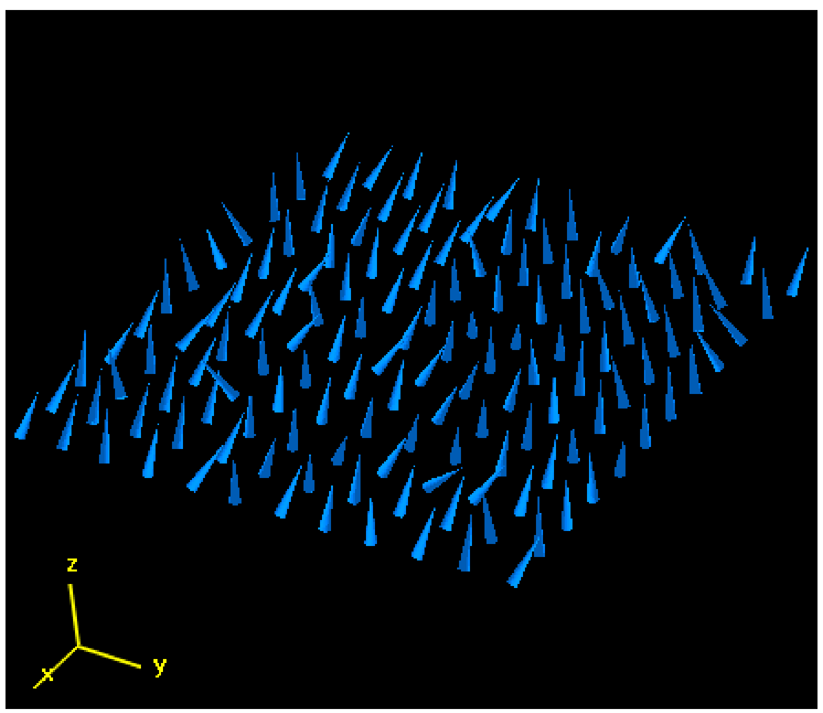
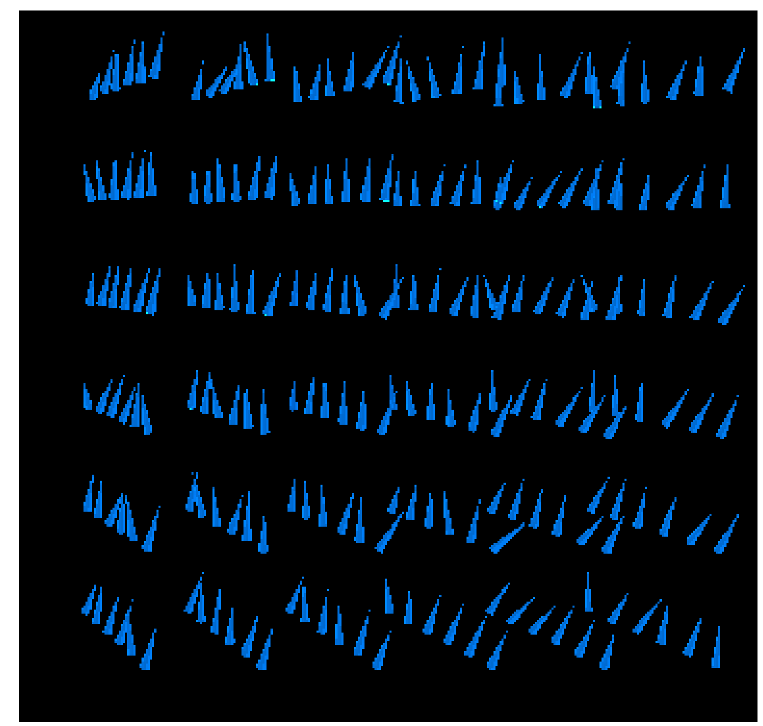


$d = 1$

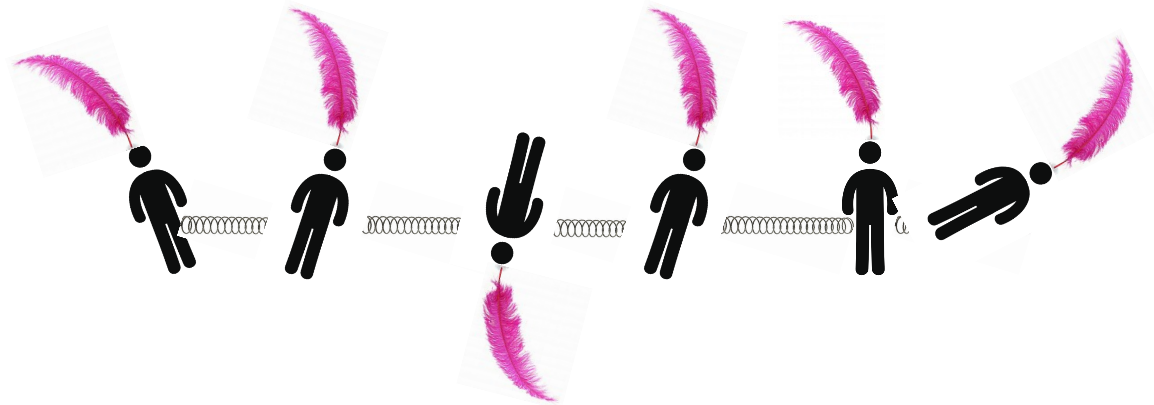


$d = 2$

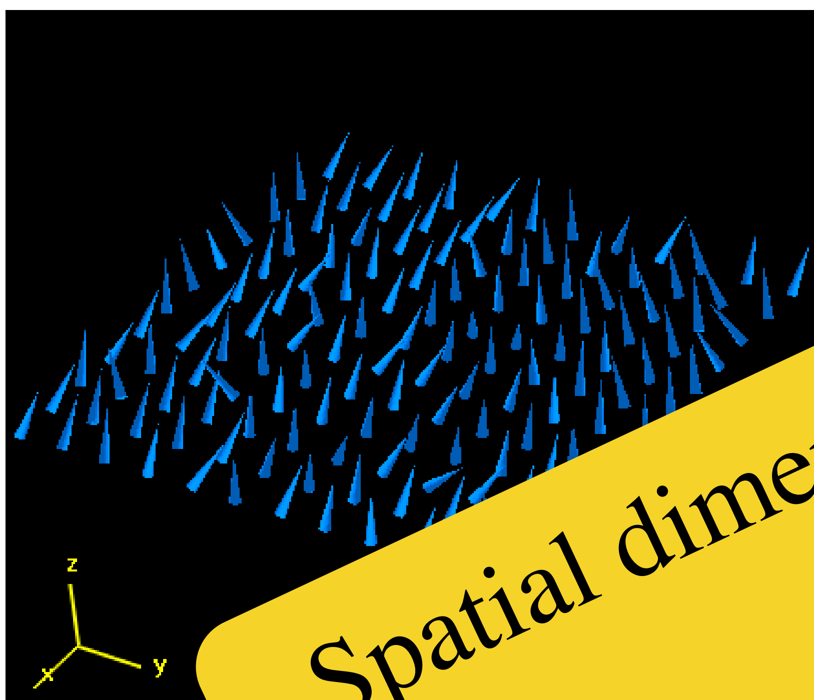


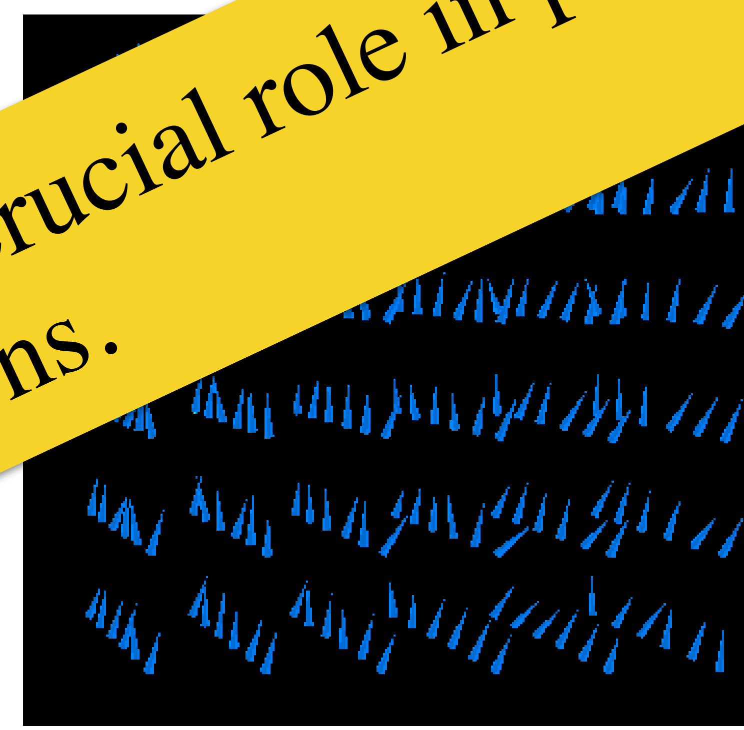
$d = 3$

Importance of the spatial dimension



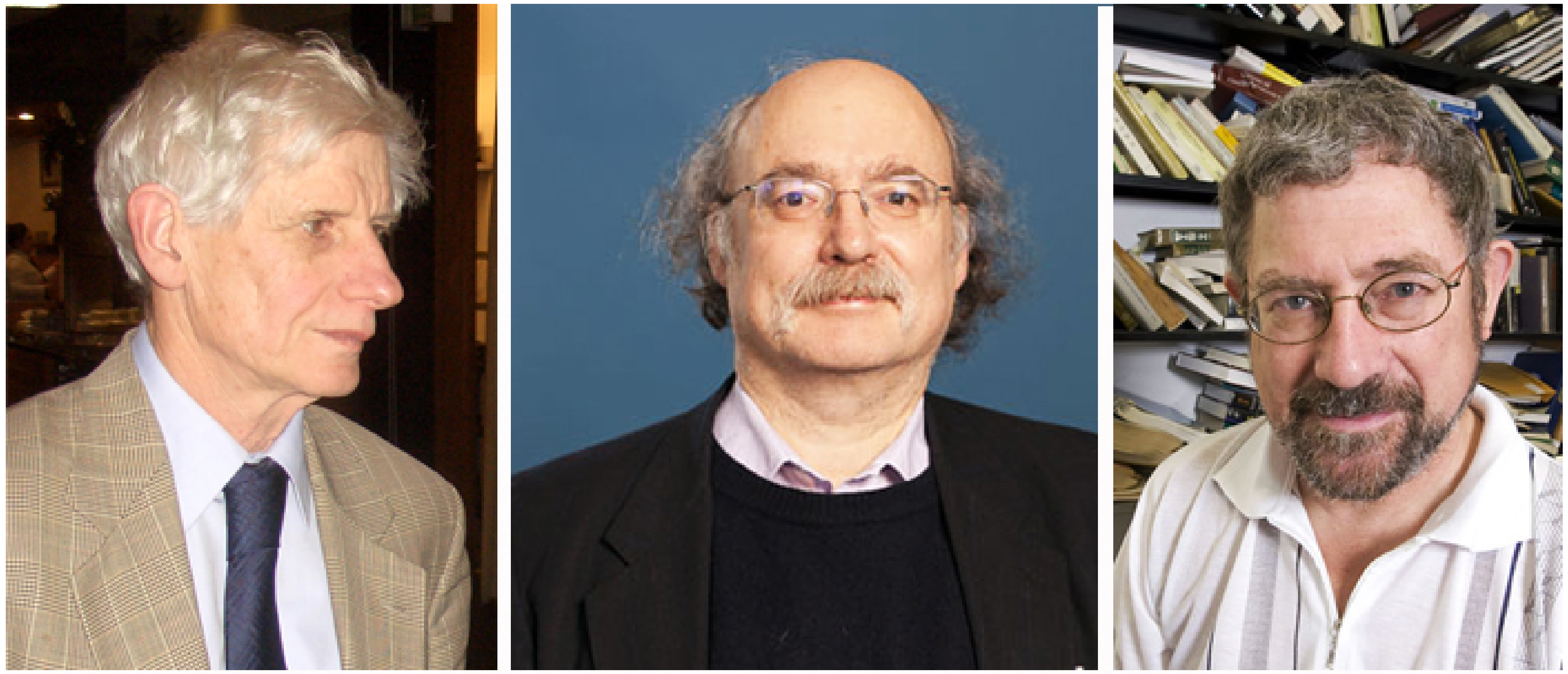
$d = 1$





$d = 3$

Spatial dimension plays a crucial role in phase transitions.



Thouless Haldane Kosterlitz

Topological Phase Transitions - Topological Phases of Matter

Phase transition :

?

Symmetry , Space Dimension, Topology.

New Paradigm ?

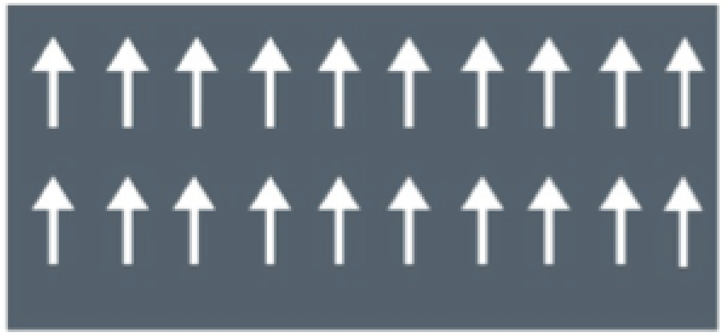
What is the revolutionary idea ?

New Paradigm ?

What is the revolutionary idea ?

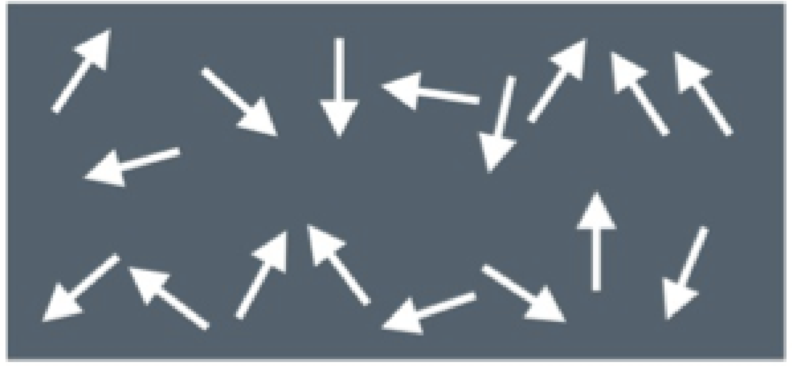
Phase Transition = Breaking a symmetry in D=3

Heating a magnet destroys the magnetic order



B = ferromagnetic

$$T \geq T_c$$



A = paramagnetic

Breaking the symmetry of the spins

This Magnetic Phase Transition between a magnet and non magnet is analogous to Melting but the origin and mechanisms <u>are very different</u>.

But in both cases : breaking of a symmetry

New Paradigm ?

What is the revolutionary idea ?

Phase Transition = Breaking a symmetry in D=3

Phase Transition results from Topology in D=1,2
without breaking a symmetry

New Paradigm ?

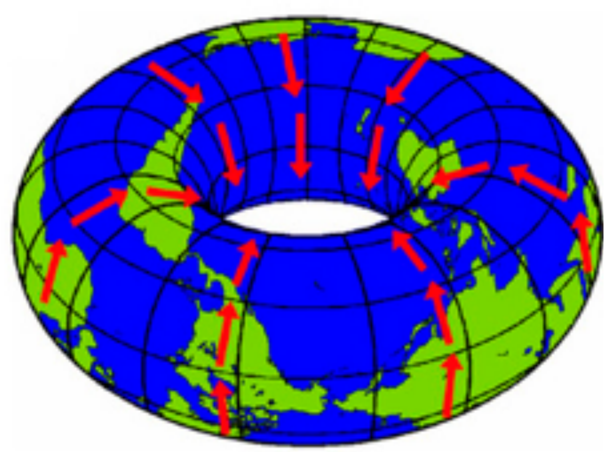
What is the revolutionary idea ?

Phase Transition = Breaking a symmetry in D=3

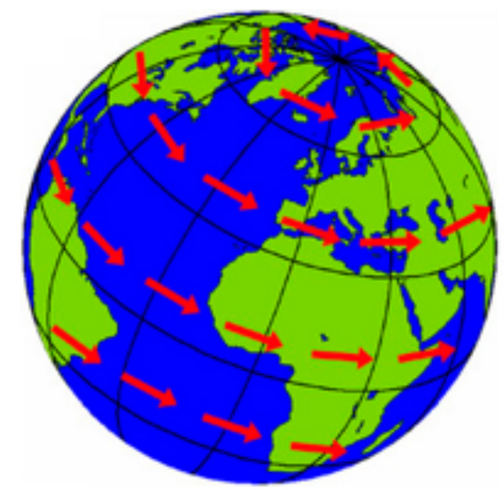
Phase Transition results from Topology in D=1,2
without breaking a symmetry

Topology.

It is a branch of Mathematics which formalises a way to say that :



$\neq$

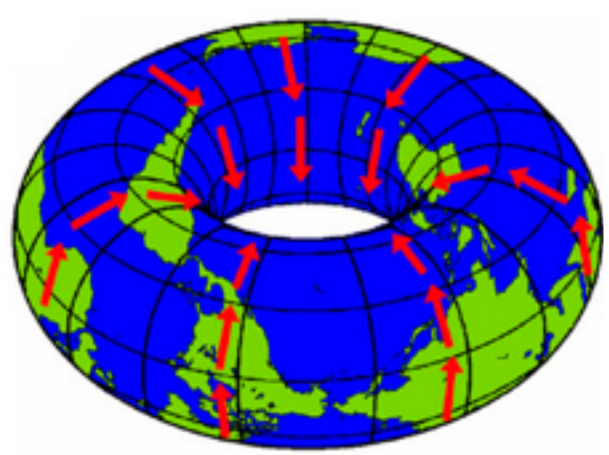
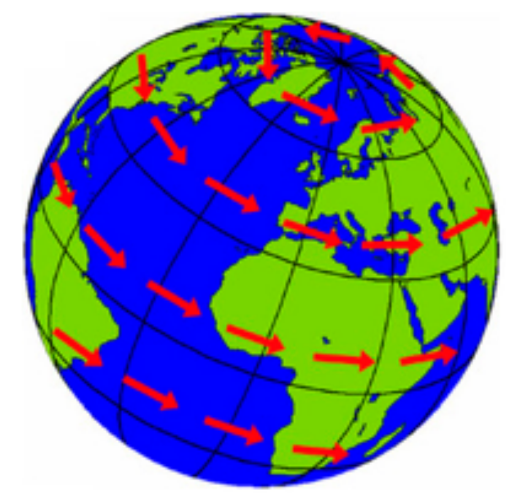


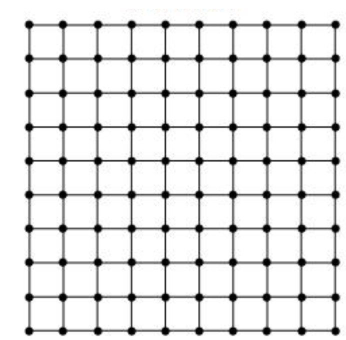
Torus

Sphere

Topology.

It is a branch of Mathematics which formalises a way to say that :

 $\neq$ 

Meaning? locally same symmetry 

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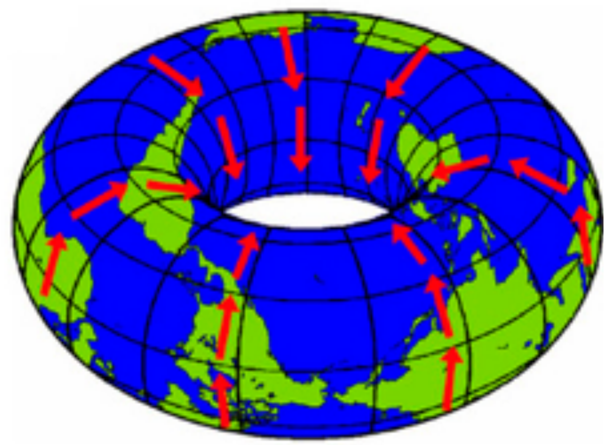
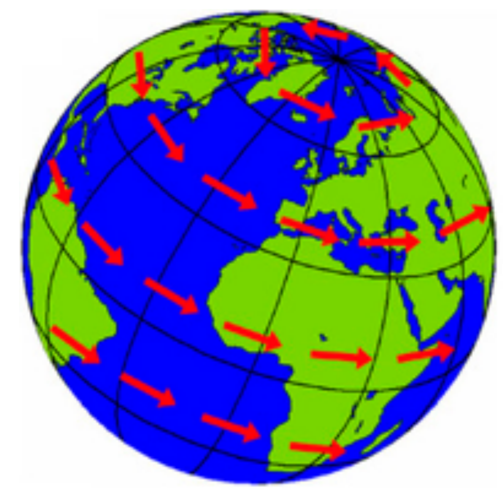
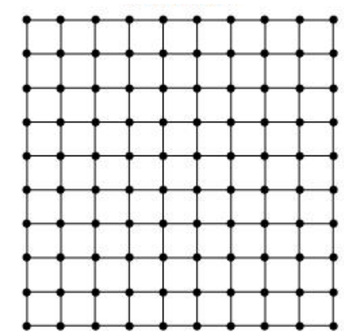


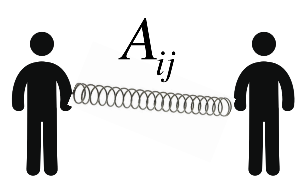
$\neq$



Meaning? locally same symmetry



$$E = \sum_{i,j}$$



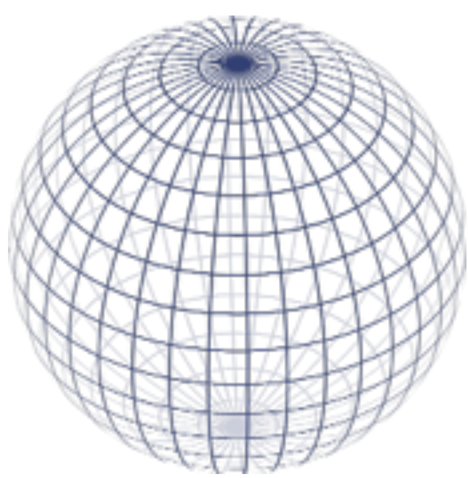
$+$

independent of topology

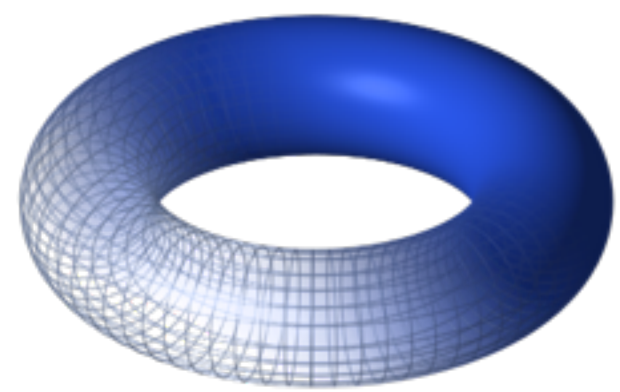
Euler-Poincare characteristics

Different ways to characterise the topology :

1. Count holes $\quad \chi(S) = 2(1-h) \quad h$: number of holes

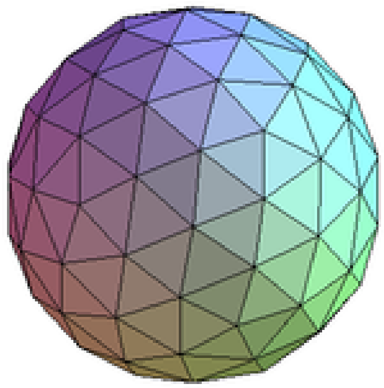


Sphere $\chi(S_2) = 2$

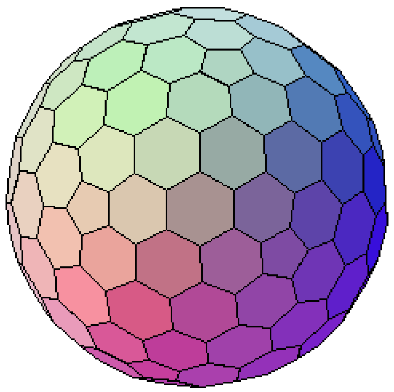


$\chi(T_2) = 0$ Torus

2. Triangulation



or



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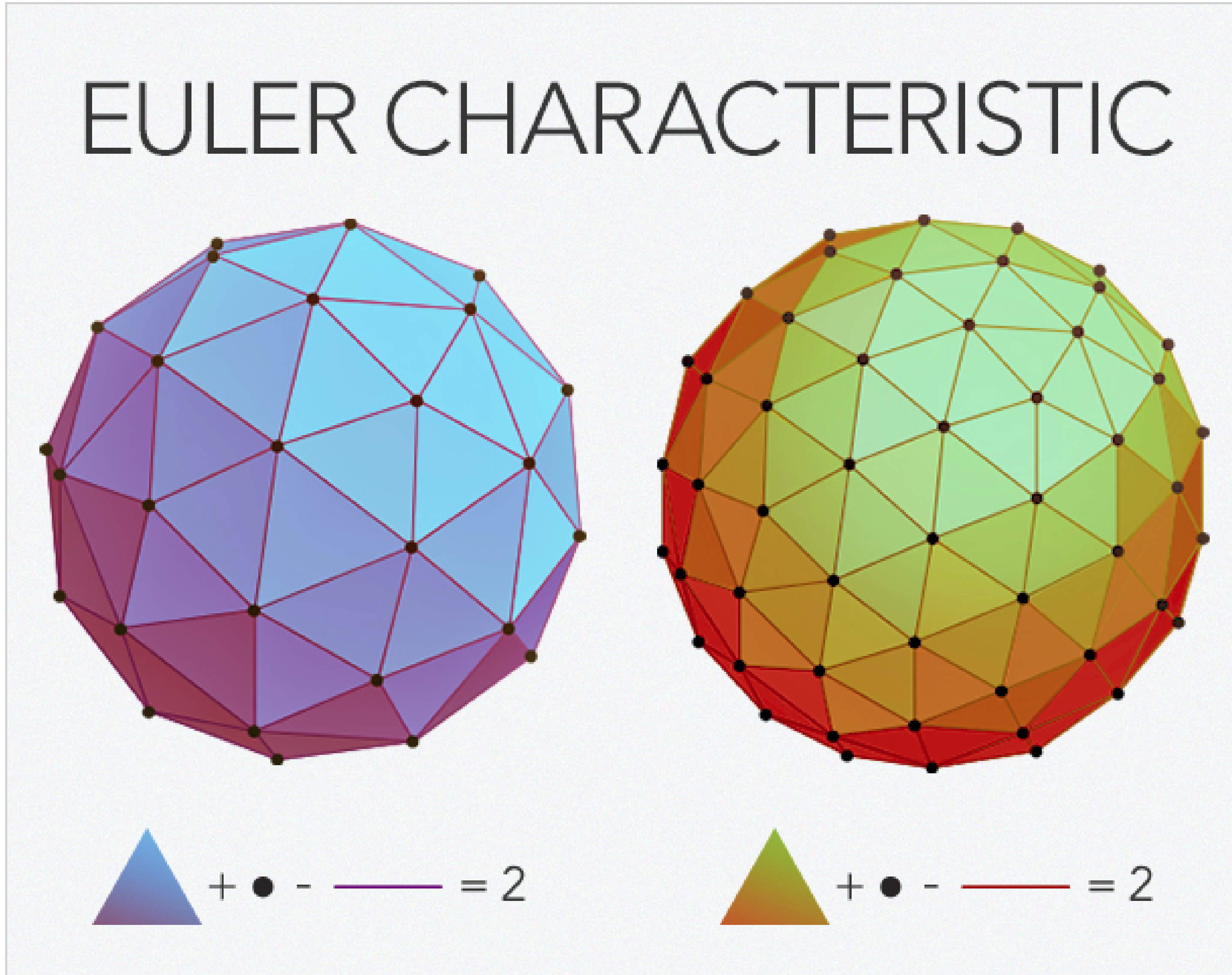
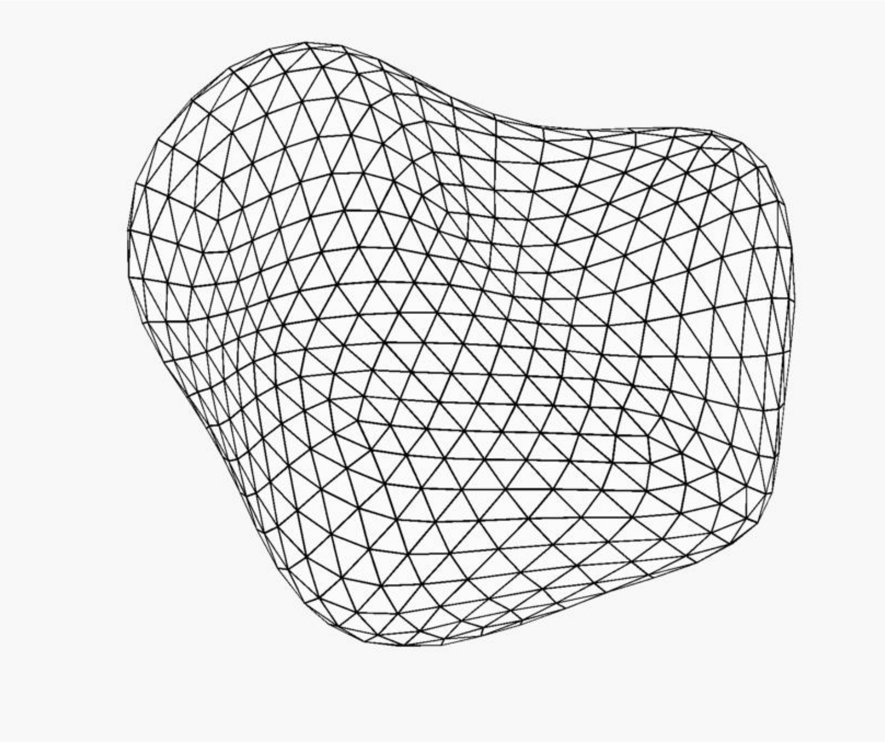


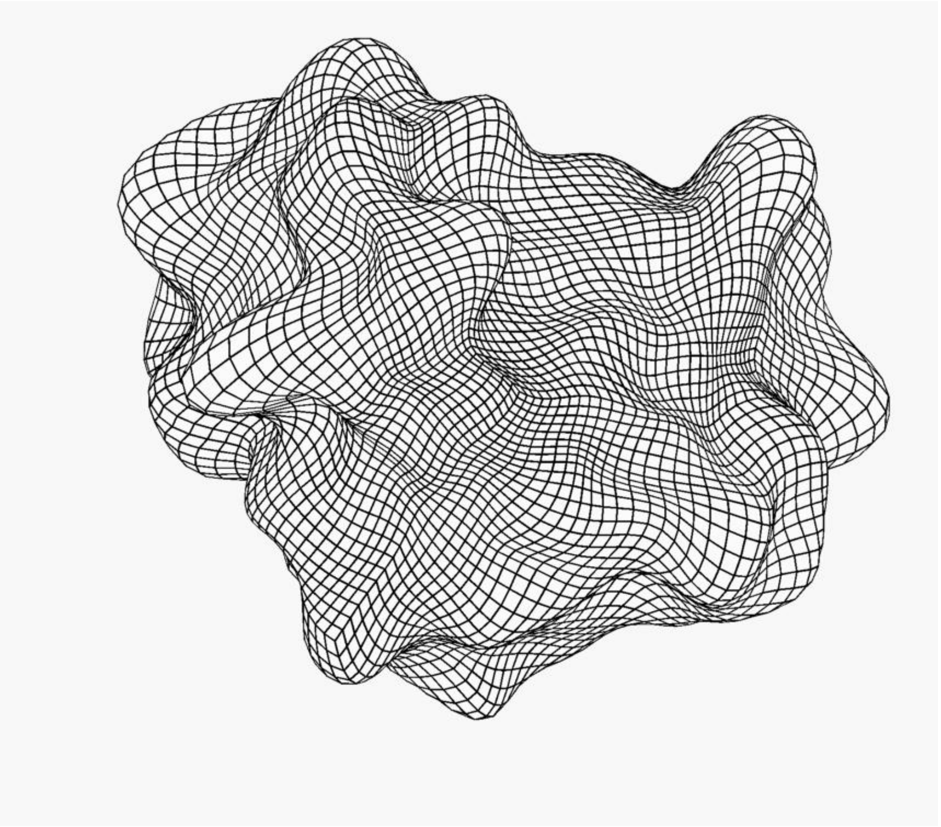
Euler : $\chi(S) = V - E + F$

V=# of vertices ; E = # of edges and F = # of faces

3. Triangulation of potatoes



or



$$\chi(S_2) = 2$$

Euler : $\chi(S) = V - E + F$

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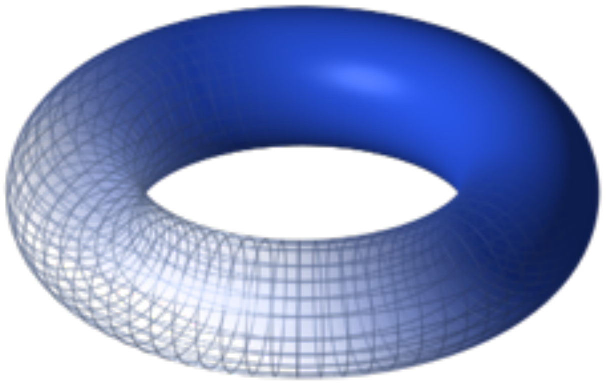
Topological invariance

4. Torus



$$\chi(T_2) = 0$$

Euler : $\chi(S) = V - E + F$

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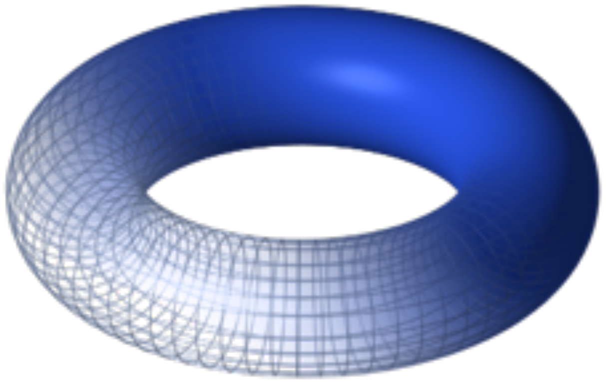
4. Torus



$$\chi(T_2) = 0$$

Topological invariance

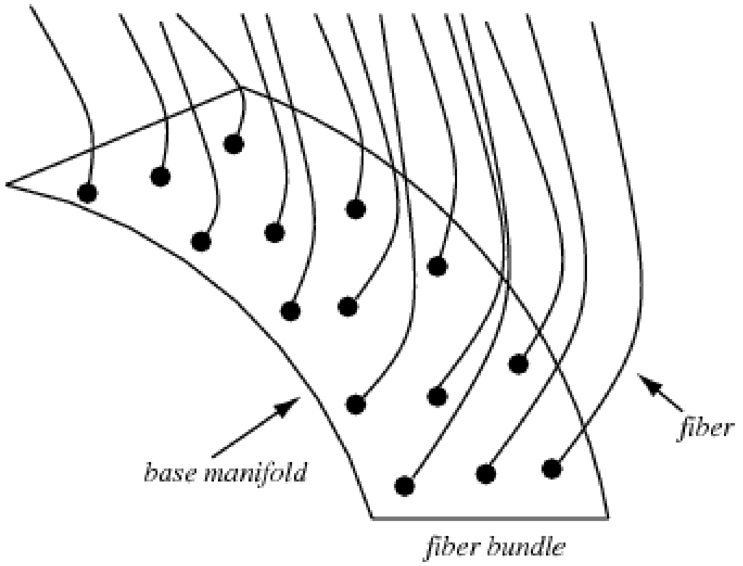
Topology of more complicated shapes

$$\chi(S) = 2(1-h)$$

Guggenheim Bilbao (F. Gehry)

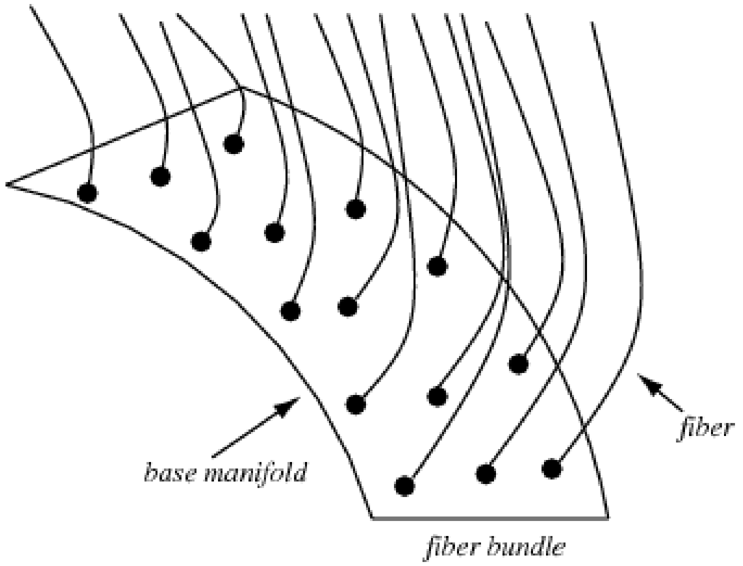
Hairy manifolds (fiber bundles)

Define a field on a manifold



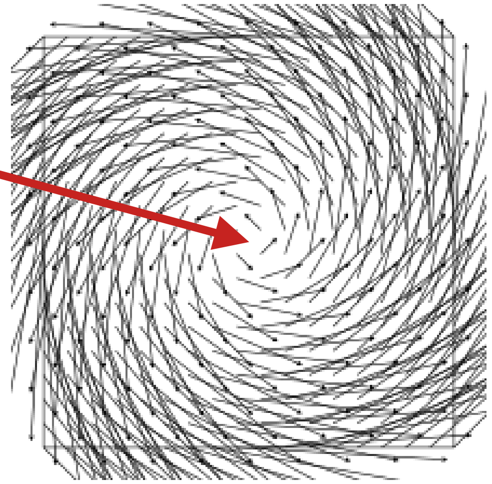
Hairy manifolds (fiber bundles)

Define a field on a manifold



You cannot comb a sphere

Always be a singularity - vortex



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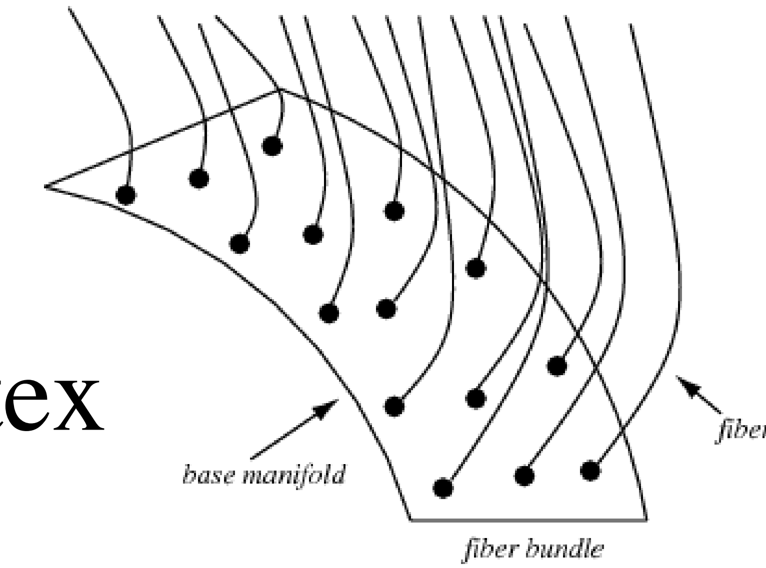
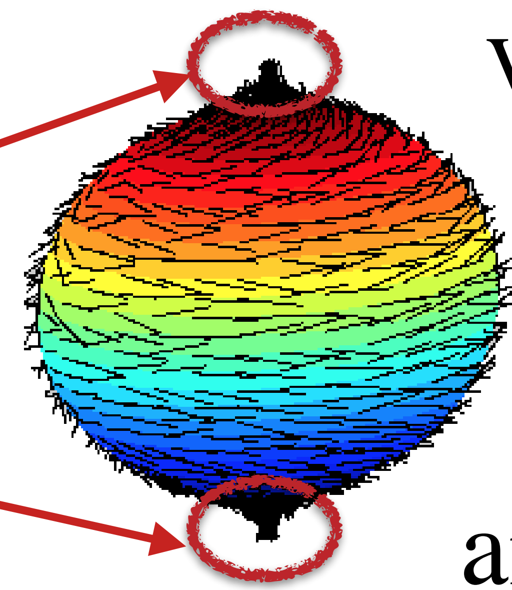


<u>You cannot comb a sphere</u>



vortex

$\chi(S_2) = 2$

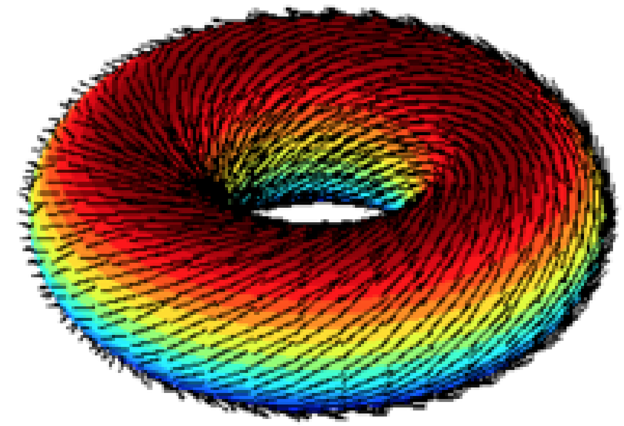
anti-vortex

Cyclone + Anticyclone

Topological result !

You can comb a torus

$$\chi(T_2) = 0$$



$$\chi(S) = 2(1-h)$$

No vortex - No cyclone on a torus Earth !

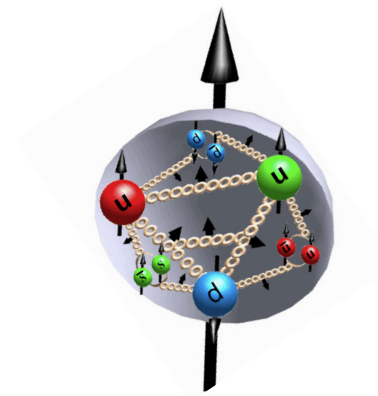
Topological result !

Establishes a deep relation between two different branches of Mathematics : topology and analysis.

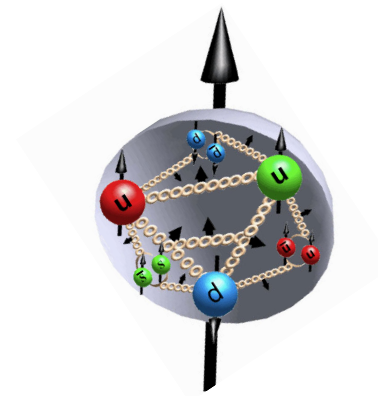
The Kosterlitz - Thouless transition
A topological phase transition

(1972-73)

The Kosterlitz - Thouless transition
A topological phase transition

Back to spins :  =  = 

The Kosterlitz - Thouless transition
A topological phase transition

Back to spins :  =  = 

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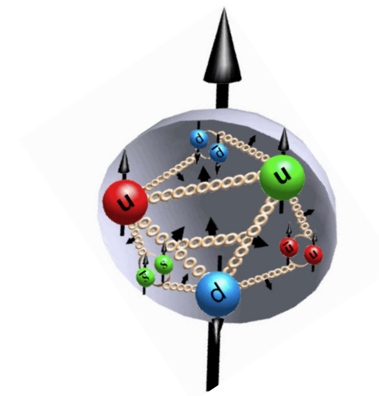
Back to spins :  =  = 

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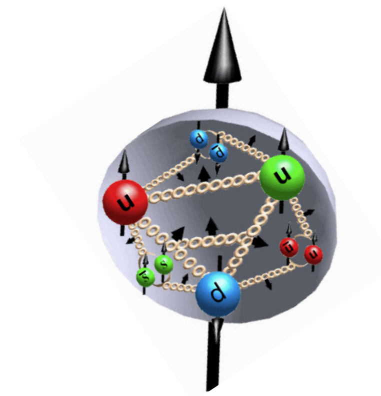
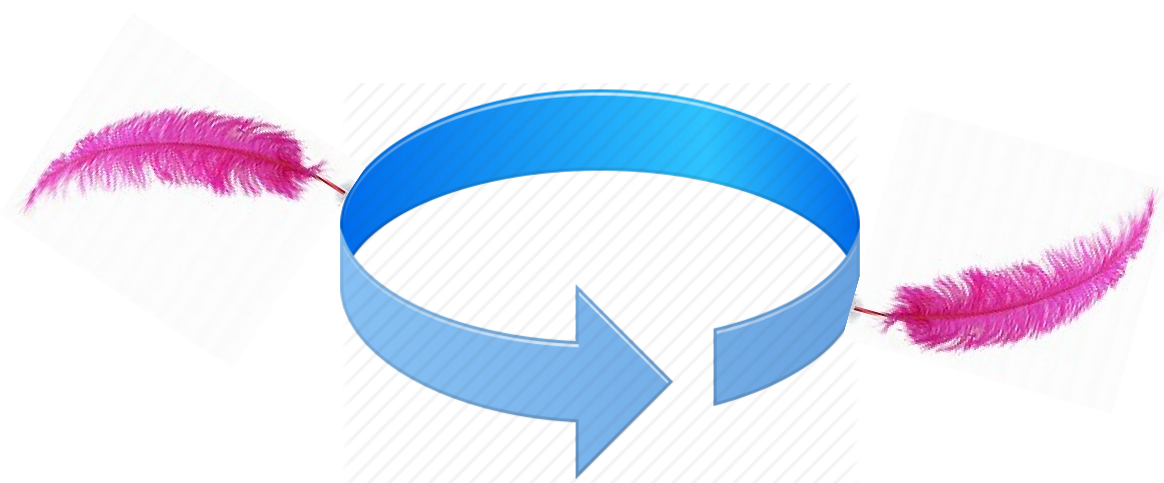
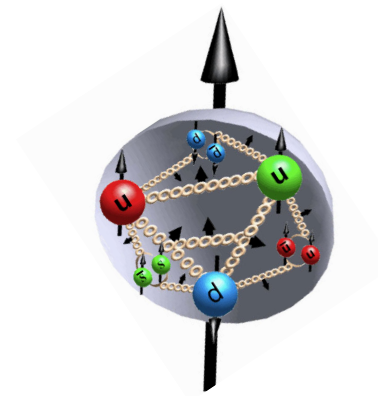
Back to spins :  =  = 

$d = 2$

planar spin

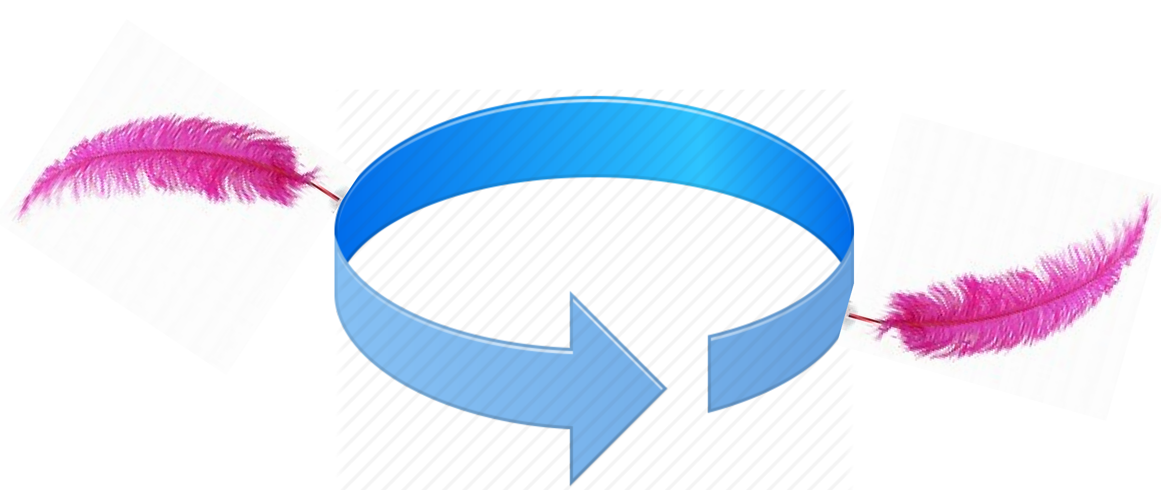


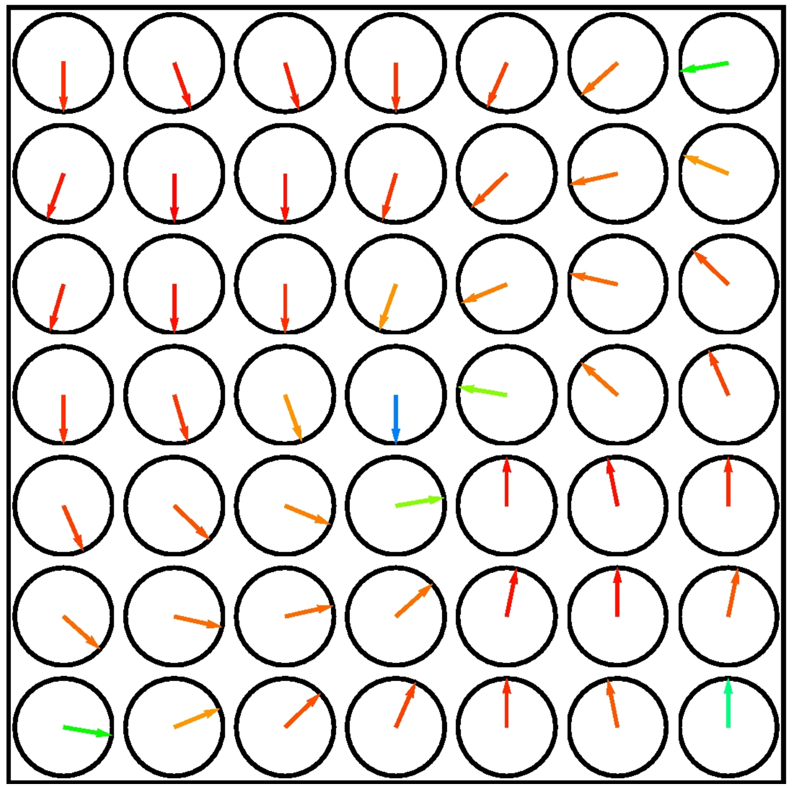
The Kosterlitz - Thouless transition
A topological phase transition

Back to spins :  =  = 

$d = 2$

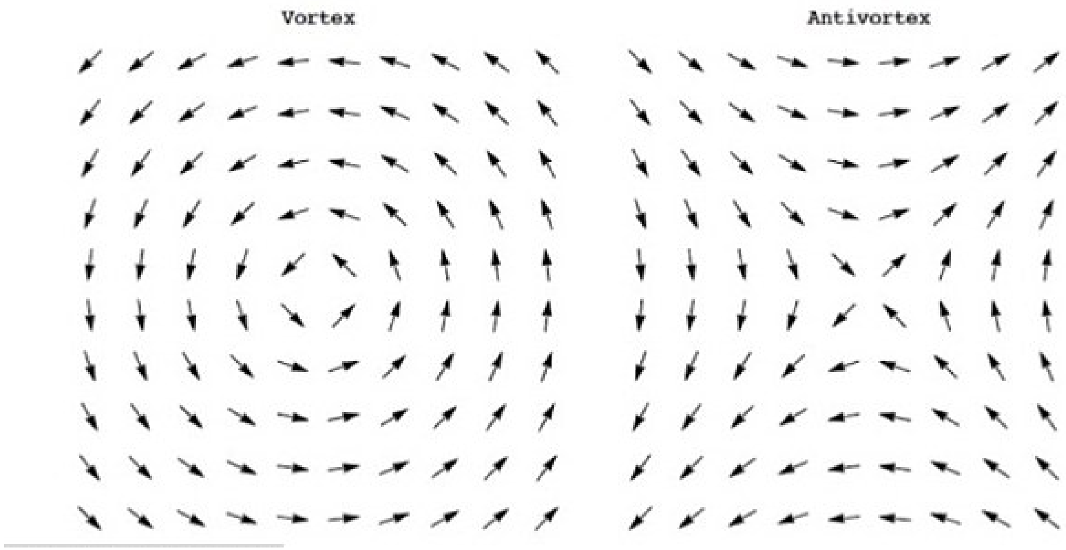
planar spin

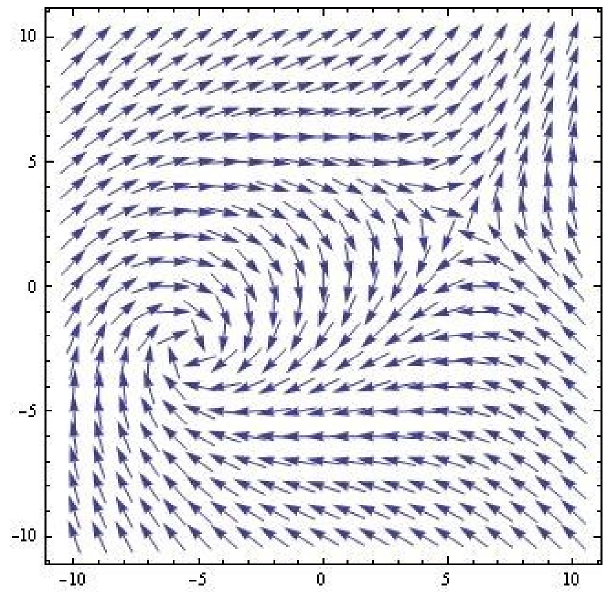




Topology : no possible order of the spins even at T = 0

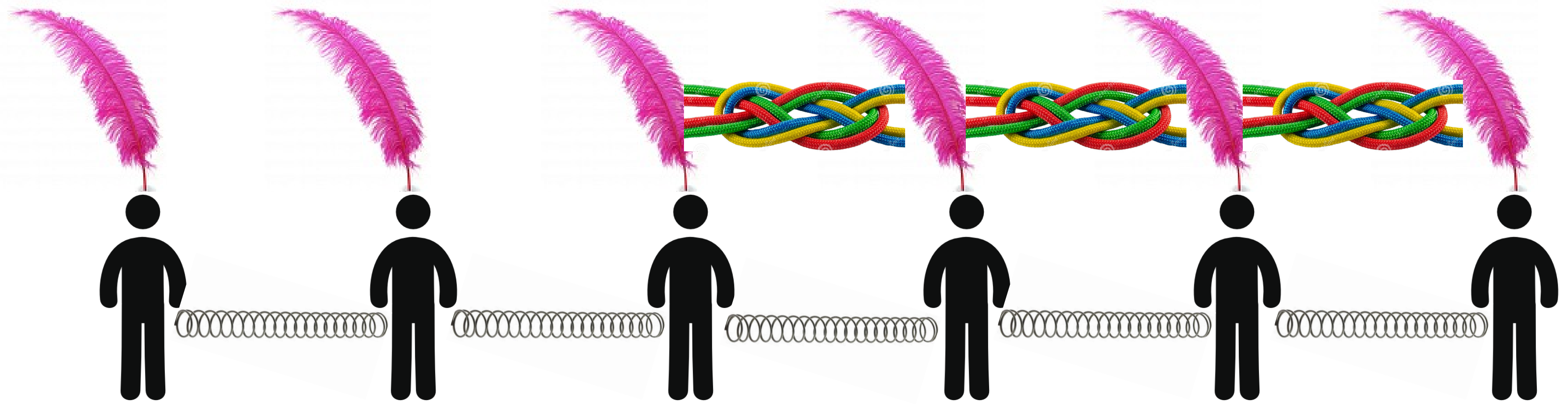
There will be always local defects (vortex)



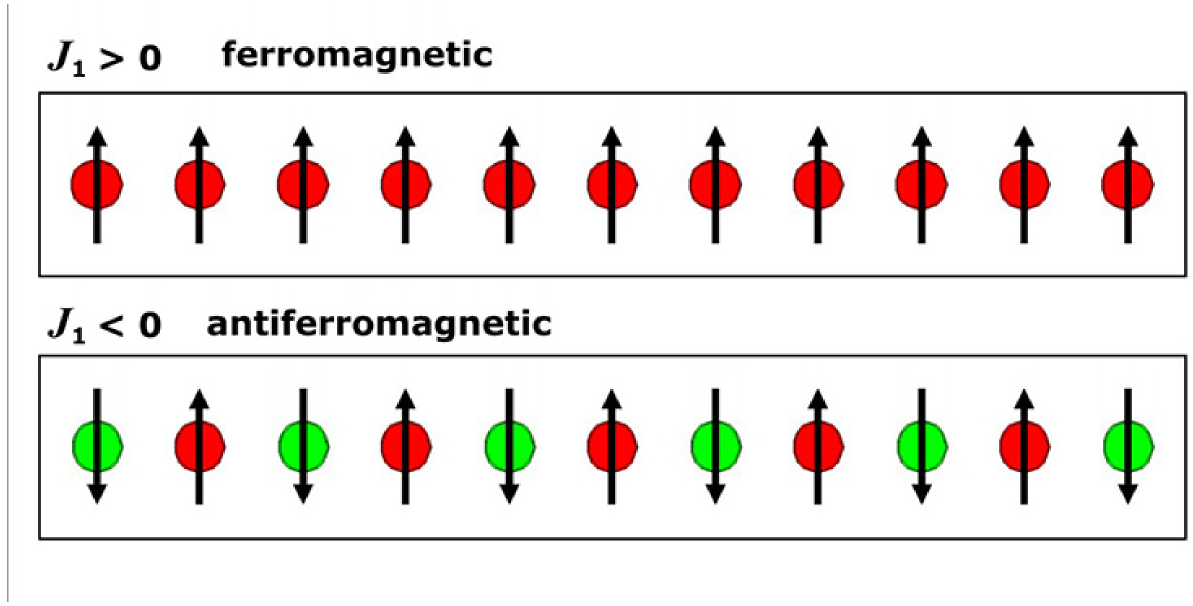


No magnet in $d = 2$

No T_c



Ferromagnetic order : MAGNET



Topology : no symmetry of the spins

(even at T = 0)

No breaking of the symmetry of the spins

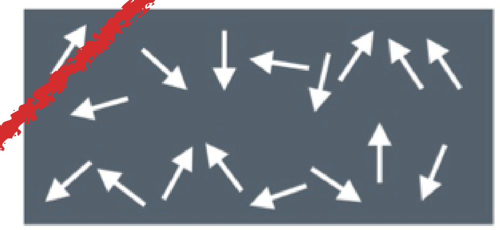
Heating a magnet destroys the magnetic order

B = ferromagnetic

$T \nearrow$

$T \geq T_c$

"c" for Curie
(Pierre not Marie)

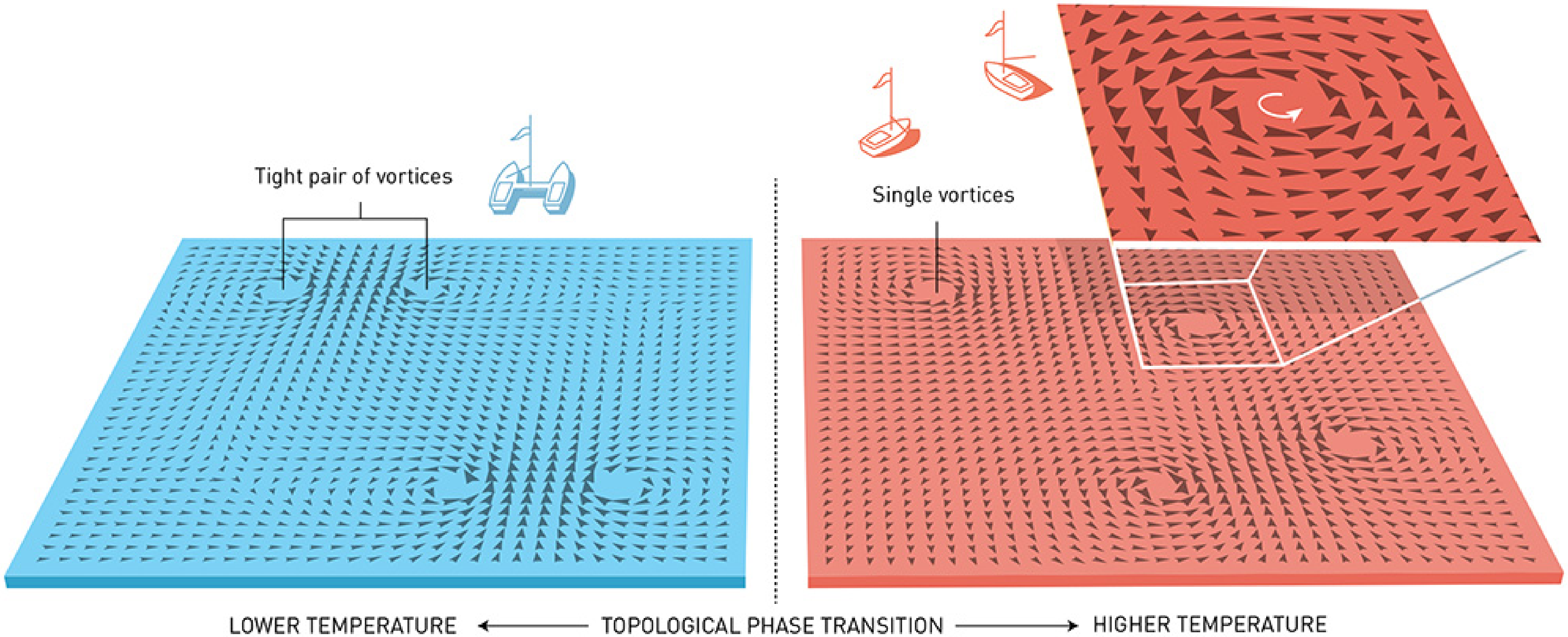


A = paramagnetic

Breaking the symmetry of the spins

Go again the consensus…

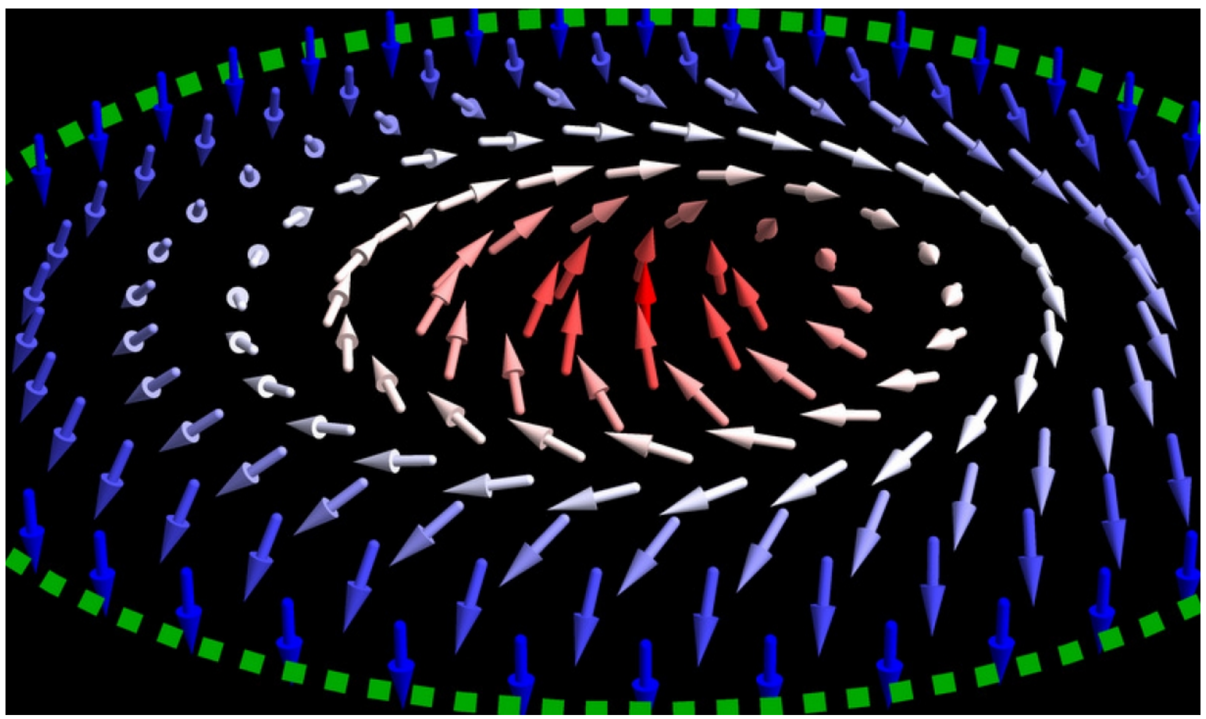
At large temperature T : topological phase transition

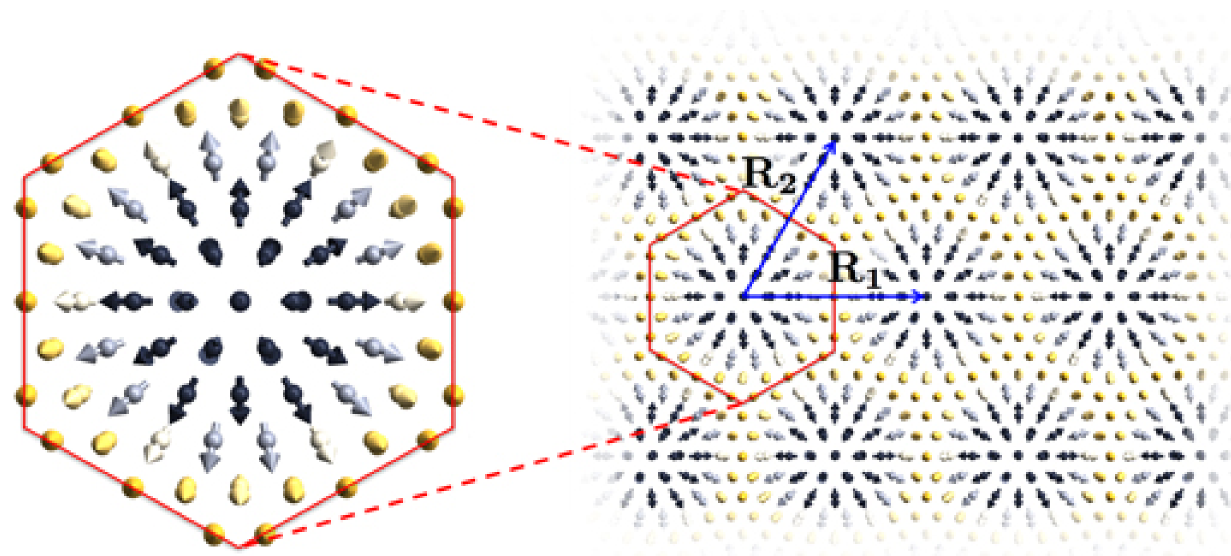


This topological phase transition has been observed ! $\Longrightarrow$ Nobel Prize

Quantum spin chains - The Haldane conjecture

Topology at its best





What did we learn about new states of matter Future ?

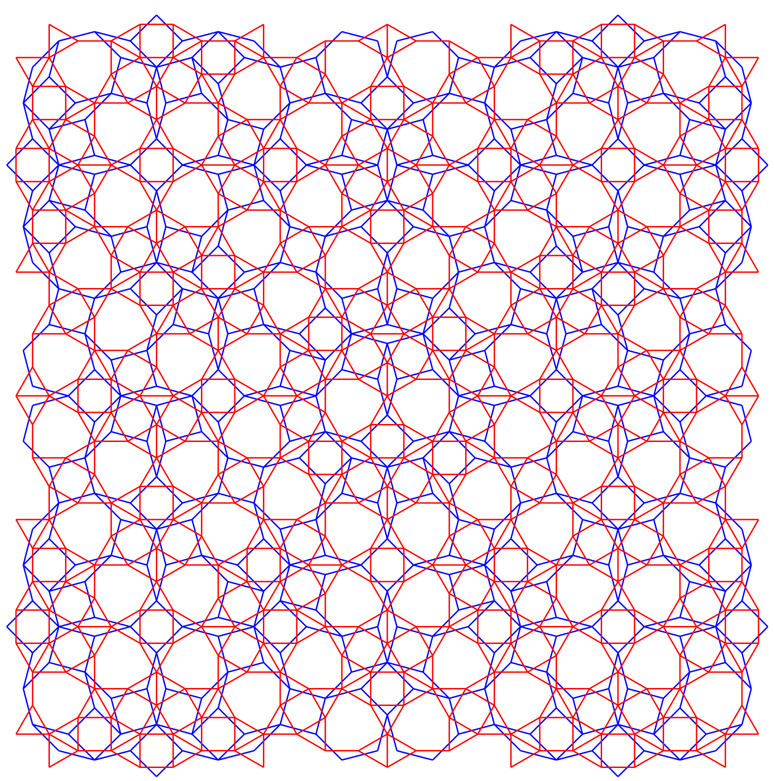
Topological features & phase transition plays now a very important role :

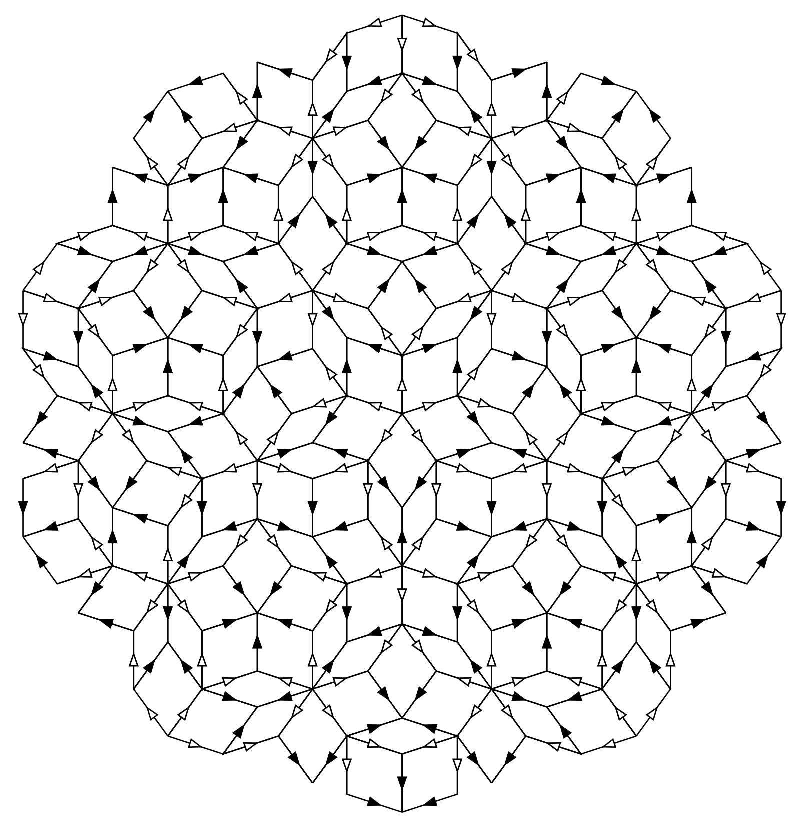
Superconductors - Superfluids - Liquid crystals - Magnets - Polymers - Gels, …

Electronic conduction in conductors, insulators, semiconductors.

What did we learn about new states of matter
Future ?

Topological features & phase transition plays now a very important role :





Some take-home messages

- Discoveries made already some time ago (1972) : It takes time to establish a new paradigm and to convince the community.

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- The processes by which research progresses are largely unknown and unpredictable (planification of research is, at least, dubious).

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- The processes by which research progresses are largely unknown and unpredictable (planification of research is, at least, dubious).

- Good research requires collaborations between excellent institutions for high education.

Thank you for your attention.