Cooperative effects and photon localization in atomic gases :
Phase transition in non Hermitian random matrices

PRL **101**, 103602 (2008),
EPL **101,** (2013)
PR **A88**, (2013),
PR A **90**, 063822 (2014)

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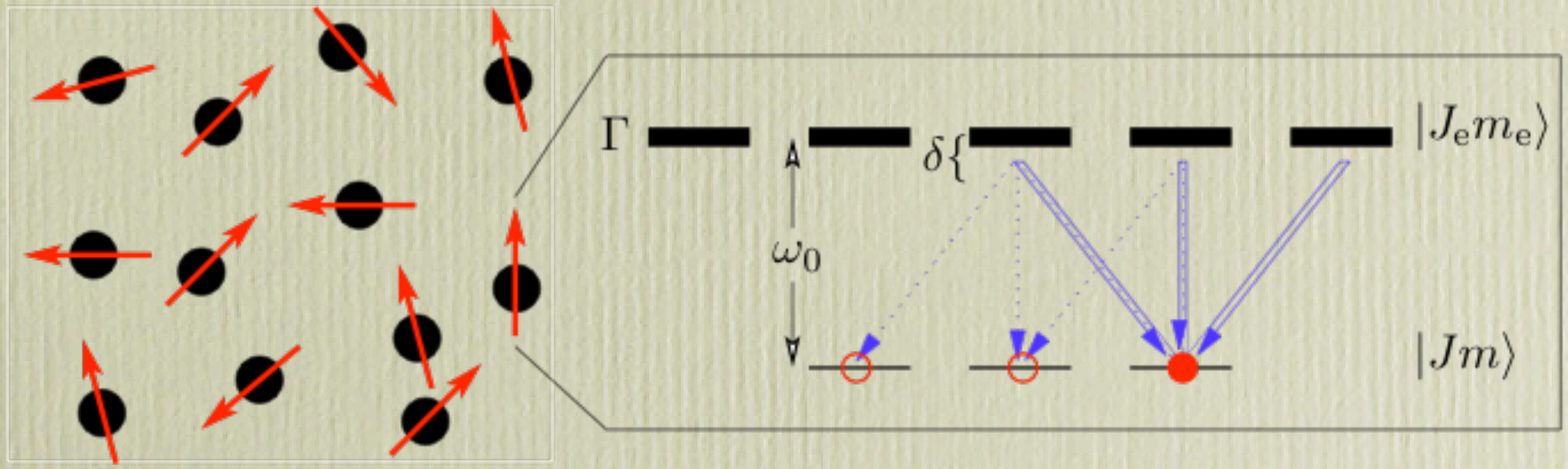
Cargese, September 28, 2016

What is it about ?

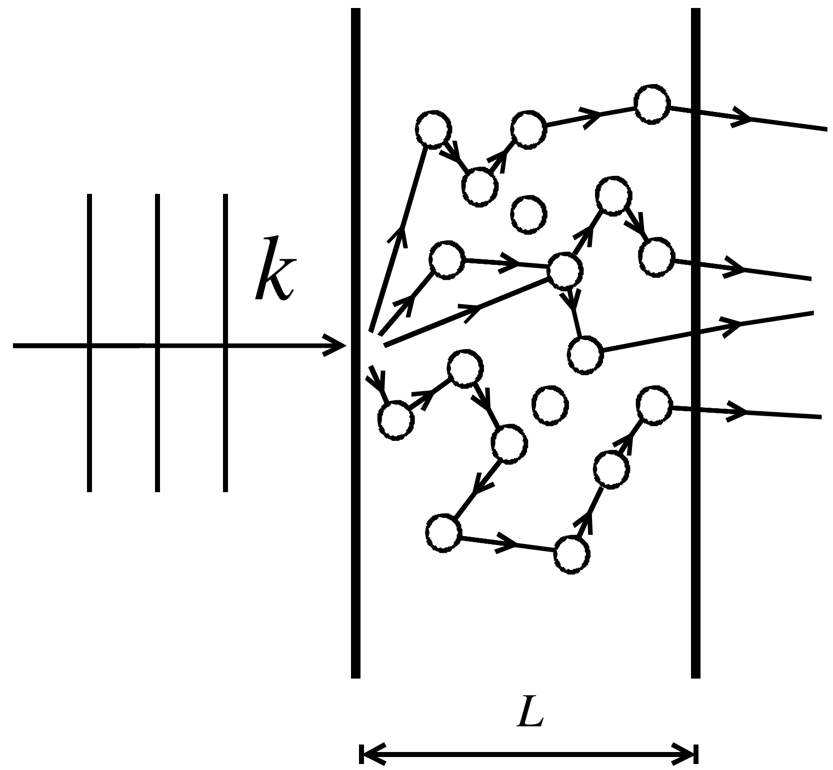
- Coherent multiple scattering of photons/waves
- Anderson photon localization : phase transition and scaling
- Cooperative effects and Dicke superradiance
- Photon escape rates : *Competition between Anderson and Dicke mechanisms*

Framework

Multiple scattering of photons/waves by a cold atomic gas.



Multiple scattering



2 characteristic lengths:

Wavelength: λ_0

Elastic mean free path: $l = \dfrac{1}{n_i \sigma} \gg n_i^{-1/d}$

density of scatterers

scattering cross section $\sigma \propto \lambda^2$

Disorder strength :

$$\boxed{W = \frac{1}{k_0 l} = \frac{\pi}{2} \frac{\lambda}{L} \frac{N}{N_\perp}}$$

$$\lambda = 2\pi / k_0$$

Elastic mean free path

$$l = \frac{1}{n\sigma} = \frac{L^3}{N\lambda^2}$$

Number of transverse channels $(d = 3)$

$$N_\perp = (k_0 L)^2 / 4$$

Weak disorder limit $W \ll 1$

Numerical calculations on the Anderson Hamiltonian

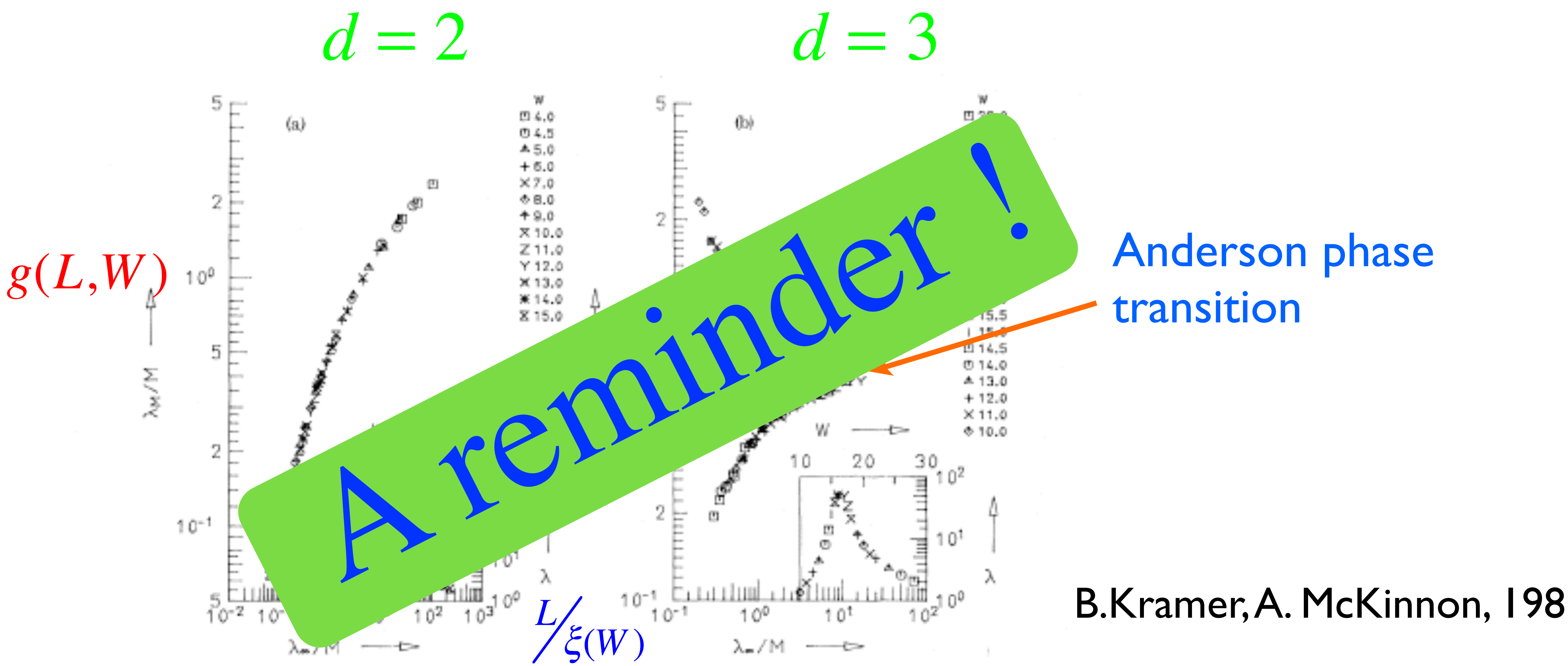


FIG. 1. Scaling function λ_M/M vs λ_∞/M for the localization length λ_M of a system of thickness M for (a) $d=2$ ($M \geq 4$) and (b) $d=3$ ($M \geq 3$). Insets show the scaling parameter λ_∞ as a function of the disorder W.

Anderson localisation phase transition occurs in $d>2$

Numerical calculations on the Anderson Hamiltonian

$d = 2$ $\qquad$ $d = 3$

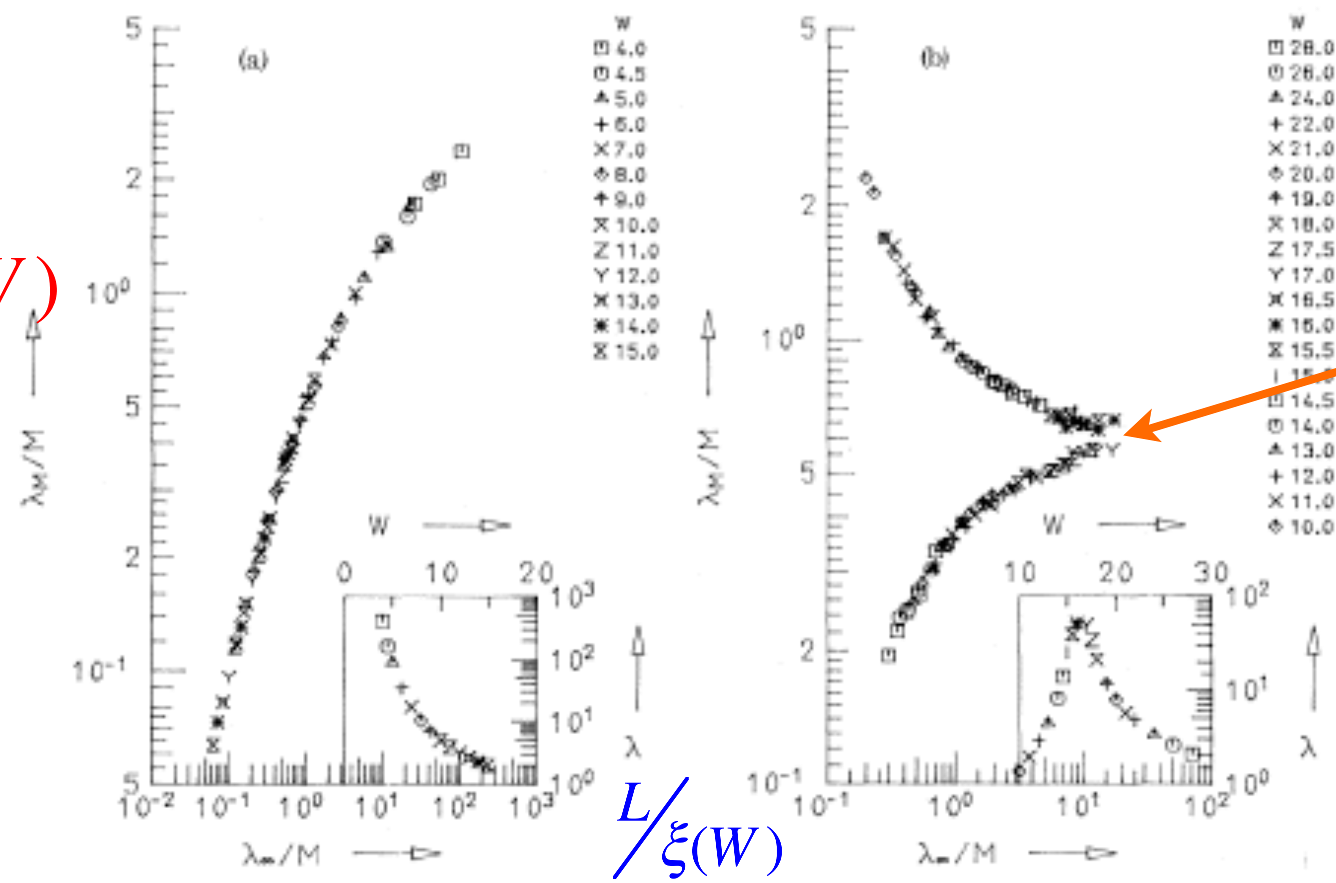


FIG. 1. Scaling function λ_M/M vs λ_∞/M for the localization length λ_M of a system of thickness M for (a) $d=2$ ($M \geq 4$) and (b) $d=3$ ($M \geq 3$). Insets show the scaling parameter λ_∞ as a function of the disorder W.

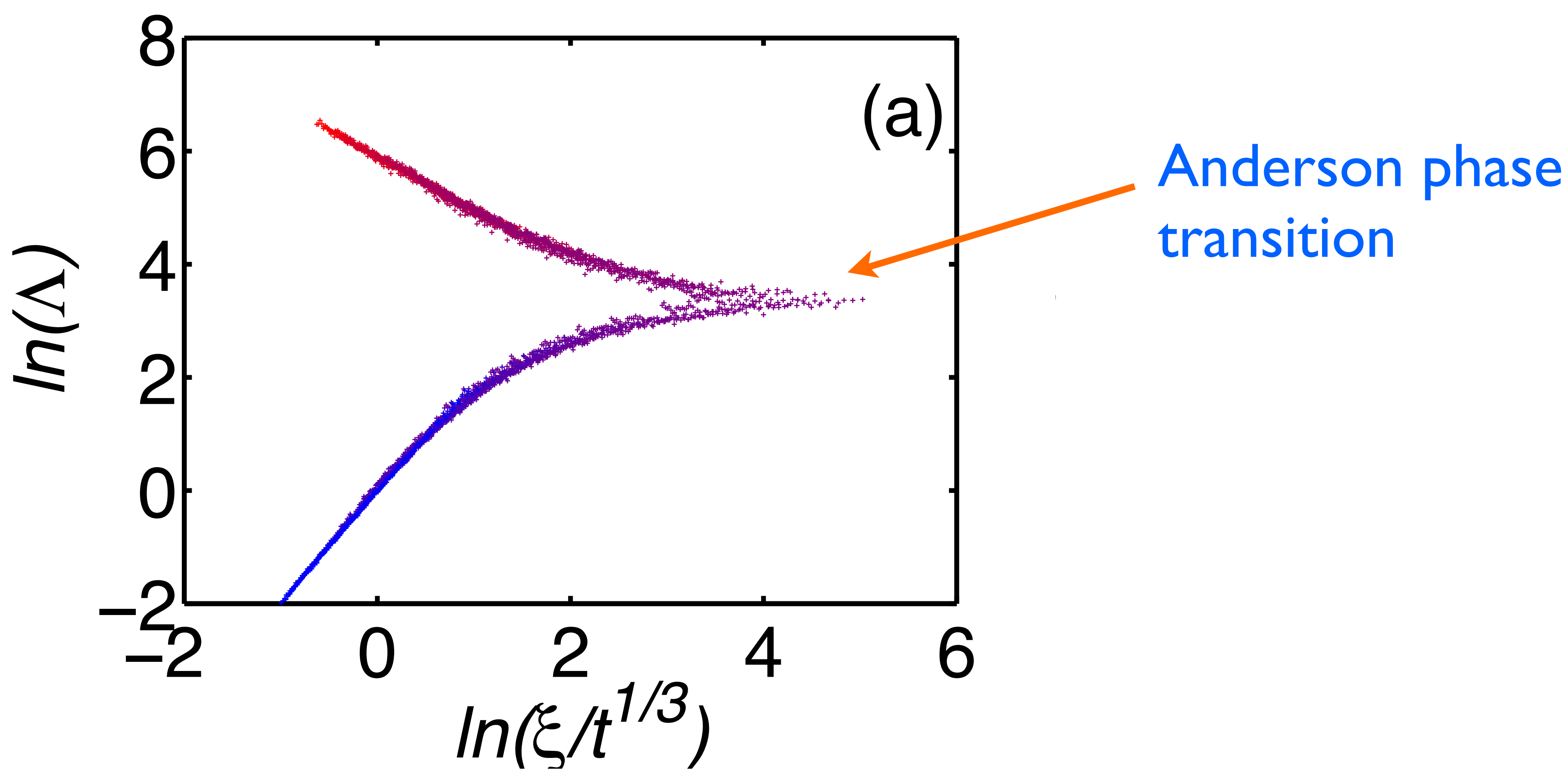
B.Kramer, A. McKinnon, 1981

Anderson localisation phase transition occurs in $d > 2$

Realisations of the Anderson Hamiltonian

Quantum evolution of the atomic kicked rotor

(localisation of the momentum in phase space (d=3))

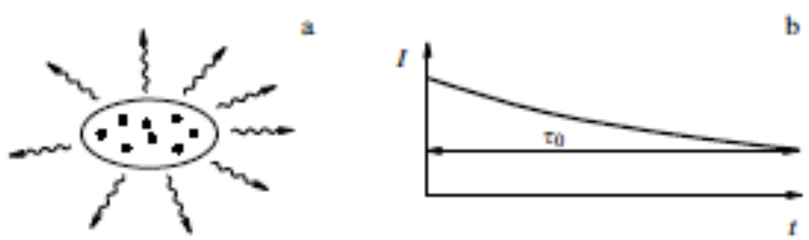


(P. Szriftgiser et al. 2010, for the experiment, theory : Casati, Chirikov, ('79) Fishman, Grempel, Prange, ('84), Guarneri et al. ('89),

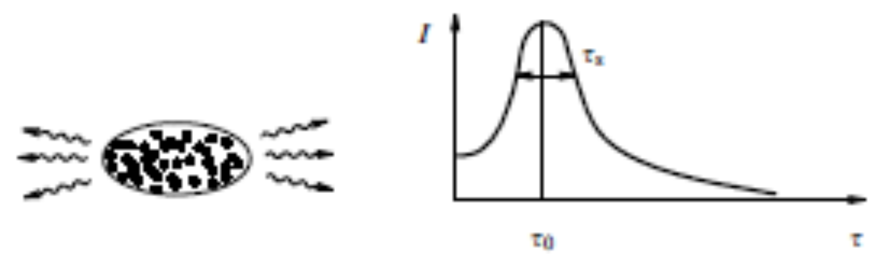
Cooperative effects
(superradiance-subradiance)

Cooperative spontaneous radiation (Superradiance) results from *quantum phase correlations* induced between atoms by dipole-dipole interactions.

Superradiant emission can be summarised by



$I \approx N$



$I \approx N^2$

But the dependence $I \approx N^2$ *does not constitute the main distinguishing feature of superradiance*.

It is rather the mechanism leading to *coherent phasing of atoms*.

Superradiant emission : all atoms must see (in phase) the same electromagnetic field.

→ small volumes (*Dicke limit*)

→ large systems : *Anderson localization* may play a role :

photon modes are spatially localized in volumes ξ^d

only a fraction $N\left(\xi/L\right)^d$ of atoms are coherent so that the emission time τ_S becomes large:

$$\tau_s \approx \left(\frac{L}{\xi}\right)^d \frac{\ln N}{N} \gg \frac{\ln N}{N}$$

Superradiant emission : all atoms must see (in phase) the same

electromagnetic field.

- small volumes (*Dicke limit*)
- large systems : *Anderson loc* ... a role :

photon modes are s ... volumes ξ^d

only a fra ... ms are coherent so that the

emi ... ecomes large:

Anderson localization and cooperative spontaneous emission are competing effects.

$$\tau_s \approx \left(\frac{L}{\xi}\right)^d \frac{\ln N}{N} \gg \frac{\ln N}{N}$$

Model

N identical two-level atoms located at random positions $\vec{r}_i$ (uniform distribution) with electric dipole moments $\vec{d}_i$ in the quantum radiation field $\vec{E}$

- Total Hamiltonian

$$H = H_0 + U$$

- Non-interacting Hamiltonian

$$H_0 = \hbar\omega_0 \sum_{i=1}^{N} |e_i\rangle\langle e_i| + \sum_{\vec{k}\varepsilon} \hbar\omega_k a^+_{\vec{k}\varepsilon} a_{\vec{k}\varepsilon}$$

- Electric dipole representation of the interaction

$$U = -\sum_{i=1}^{N} \vec{d}_i \cdot \vec{E}(\vec{r}_i)$$

Model

- Effective Hamiltonian
  - Tracing over the EM field degrees of freedom

$$H_e = \left(\hbar\omega_0 - i\frac{\hbar\Gamma_0}{2}\right)\sum_{i=1}^{N}|e_i\rangle\langle e_i| + \frac{\hbar\Gamma_0}{2}\sum_{i\neq j}V_{ij}\Delta_i^+\Delta_j^-$$

  - Atomic raising and lowering operators

$$\Delta_i^+ = |e_i\rangle\langle g_i| \qquad \Delta_j^- = |g_j\rangle\langle e_j|$$

Model

- Effective Hamiltonian
  - Tracing over the EM field degrees of freedom

$$H_e = \left(\hbar\omega_0 - i\frac{\hbar\Gamma_0}{2}\right)\sum_{i=1}^{N}|e_i\rangle\langle e_i| + \frac{\hbar\Gamma_0}{2}\sum_{i\neq j}V_{ij}\Delta_i^+\Delta_j^-$$

  - Atomic raising and lowering operators

$V_{ij} = \beta_{ij} - i\gamma_{ij}$ is random and complex valued

- Real part : interaction potential

$$\beta_{ij} = \frac{3}{2}\left[-p \; \frac{\cos k_0 r_{ij}}{k_0 r_{ij}} + q \left(\frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^3} + \frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^2} \right) \right]$$

- Imaginary part : photon escape rate

$$\gamma_{ij} = \frac{3}{2}\left[p \; \frac{\sin k_0 r_{ij}}{k_0 r_{ij}} - q \left(\frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^3} - \frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^2} \right) \right]$$

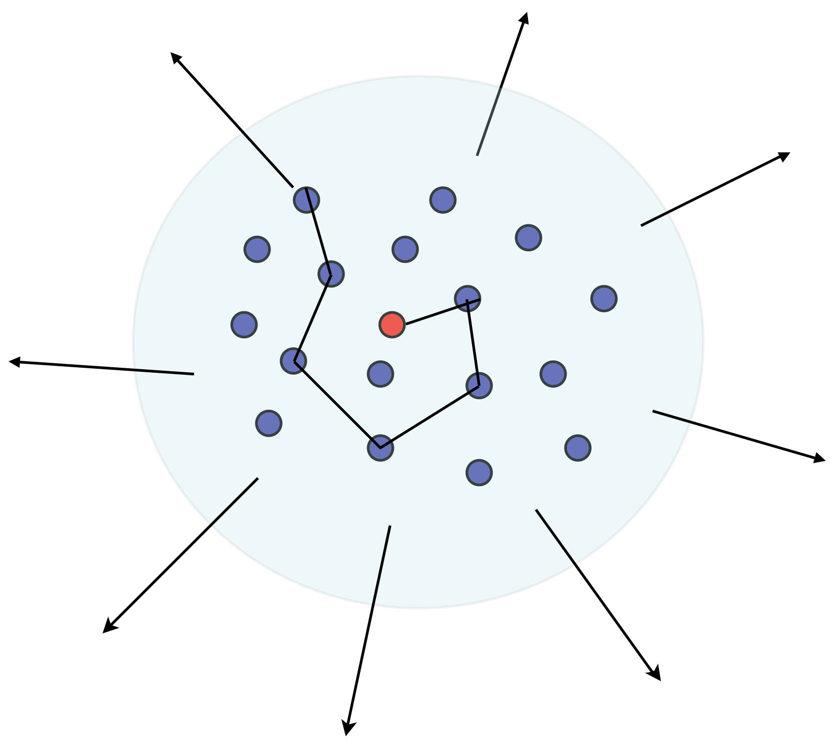
- For a scalar wave:

$$\beta_{ij} = -\frac{\cos\left(k_0 r_{ij}\right)}{k_0 r_{ij}}$$

$$\gamma_{ij} = \frac{\sin k_0 r_{ij}}{k_0 r_{ij}}$$

Which quantity to study ?

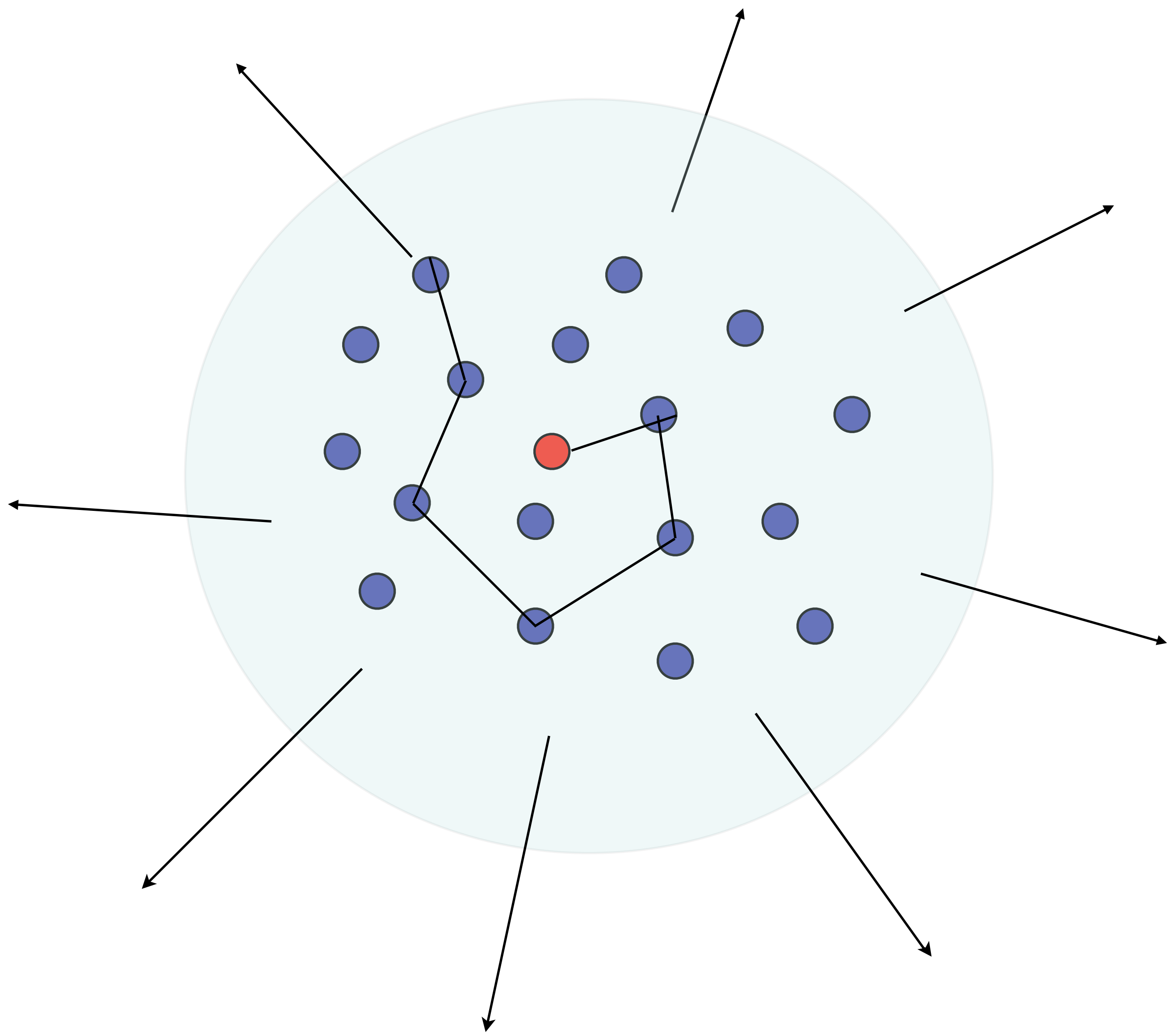
- The radiation pattern/intensity of the atomic cloud with a single excited atom



$$\Psi = \sum_{j=1}^{N} \beta_j(t)|b_1 b_2 \cdots a_j \cdots b_N\rangle|0\rangle + \sum_{\mathbf{k}} \gamma_{\mathbf{k}}(t)|b_1 b_2 \cdots b_N\rangle|1_{\mathbf{k}}\rangle.$$

Photon escape rates are a measure of localization and/or cooperative emission.

Escape rates are not a transport quantity.



More precisely : Photon escape rates

Evolution of the density matrix (Linblad form)

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}\left(H_e\rho - \rho H_e^{\dagger}\right) + \Gamma_0 \sum_{i\neq j} \gamma_{ij} \Delta_i^{+} \rho \Delta_j^{-}$$

$$H_e = \left(\hbar\omega_0 - i\frac{\hbar\Gamma_0}{2}\right)\sum_{i=1}^{N} |e_i\rangle\langle e_i| + \frac{\hbar\Gamma_0}{2}\sum_{i\neq j} V_{ij}\Delta_i^{+}\Delta_j^{-}$$

$$V_{ij} = \beta_{ij} - i\gamma_{ij}$$

M. Stephen (1964), R.H. Lehmberg (1970), E. Ressayre and A.Tallet (1976), Ellinger, Cooper and P. Zoller (1994)

Photon escape rates from the atomic gas are obtained from the eigenvalues of the euclidean random matrix γ_{ij}

Eigenvalue density $P(\Gamma)$ of the $N \times N$ random matrix γ_{ij}

(Scalar case) $$\gamma_{ij} = \frac{\sin k_0 r_{ij}}{k_0 r_{ij}} \qquad x_{ij} \equiv k_0 r_{ij}$$

Defining dimensionless quantities $L^d = (\lambda a)^d$ $\qquad \lambda = 2\pi / k_0$

$$\boxed{W = \frac{1}{k_0 l} = \frac{\pi}{2} \frac{\lambda}{L} \frac{N}{N_\perp}}$$

Elastic mean free path

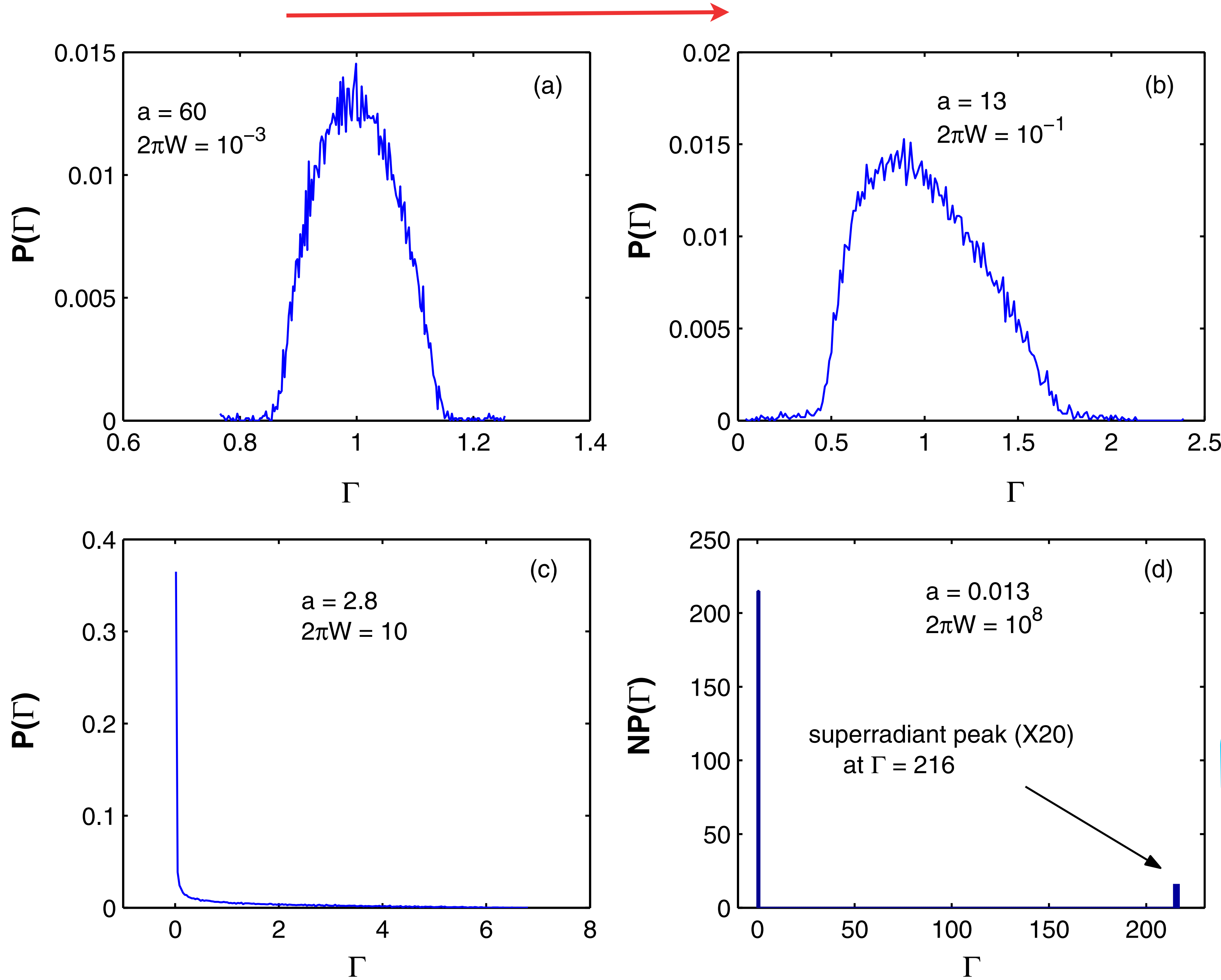
$$l = \frac{1}{n\sigma} = \frac{L^3}{N\lambda^2}$$

Number of transverse channels $\quad (d = 3)$

$$N_\perp = (k_0 L)^2 / 4$$

Eigenvalue density $P(\Gamma)$

increasing disorder W



Dicke limit

localized photons

Numerical results $N=216$

A. Gero, R. Kaiser, E.A PRL **101**, 103602 (2008),

Scaling ?

To characterize $P(\Gamma)$ we look for a scaling function $C(a,W)$

Relative number of localized states i.e. having a vanishing escape rate :

$$C(a,W) = 1 - 2\int_1^{\infty} d\Gamma\, P(\Gamma)$$

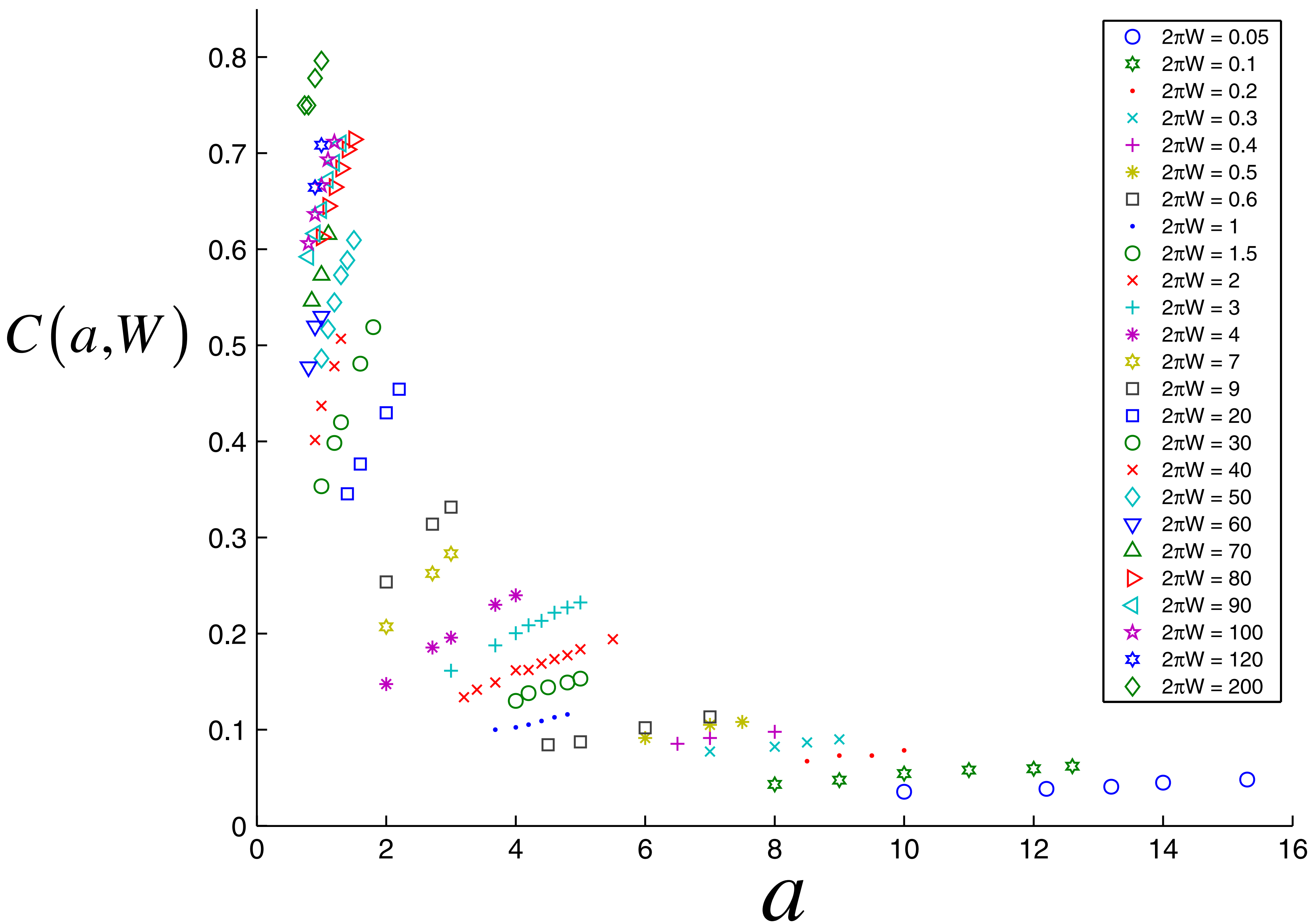
$C(a,W)$ is defined between 0 and 1. At finite size, we expect the scaling form:

$$C(a,W) = f\left(a/\xi(W) \right)$$

A. Gero, R. Kaiser, E.A PRL **101**, 103602 (2008),

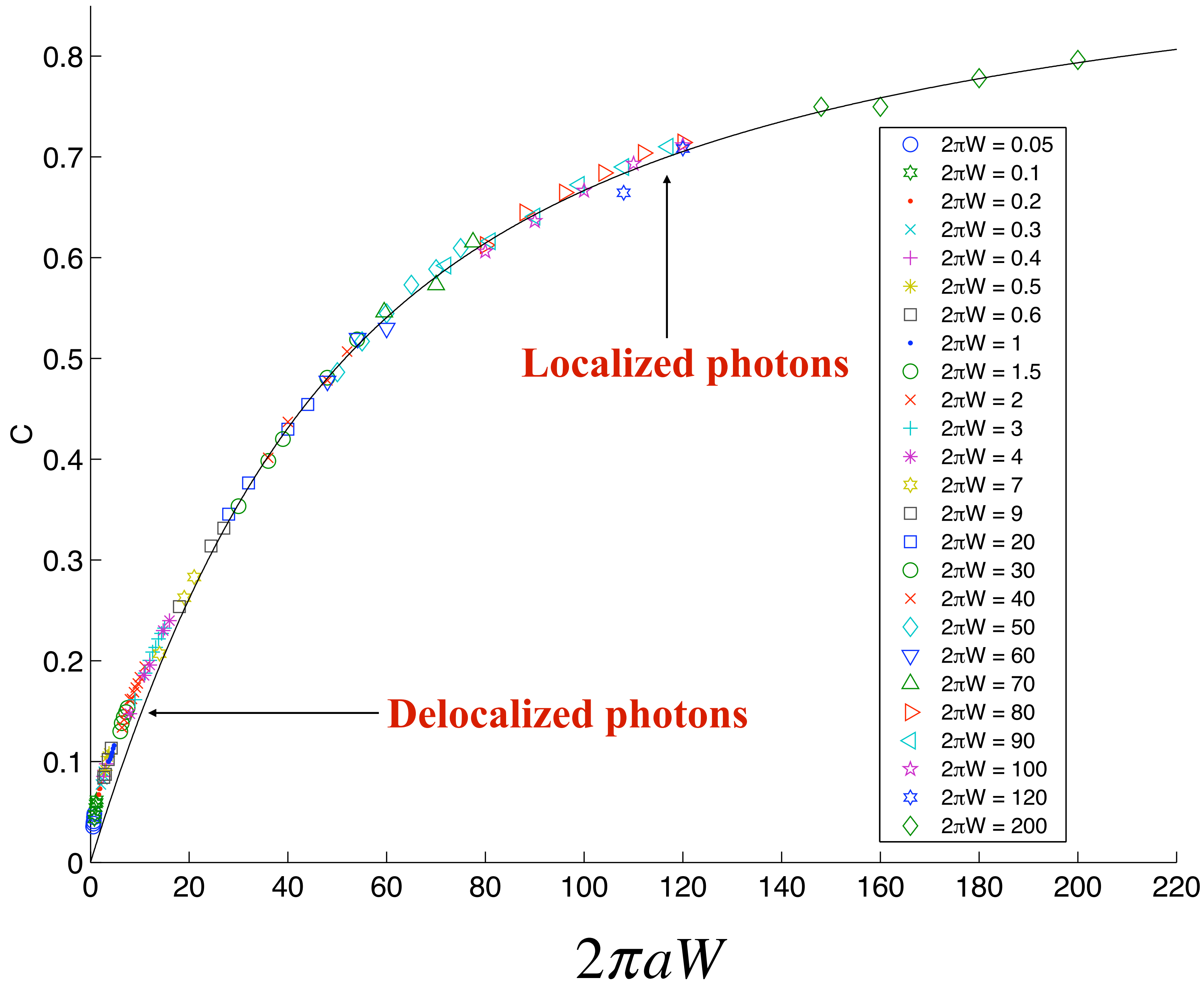
Scaling behaviour

Large sample limit ($a \geq 1$)



A. Gero, R. Kaiser, E.A PRL **101**, 103602 (2008),

Scaling behavior *(large sample limit)*



Is there a localisation phase transition ?

$C(a,W)$ depends on $2\pi a W = \pi^2 N / N_\perp$

Is there a localisation phase transition ?

- Microscopic QED approach

Large disorder limit $N \gg N_{\perp}$

- Phenomenological Markov process (Small world networks)

For the whole range of disorder

Microscopic QED approach

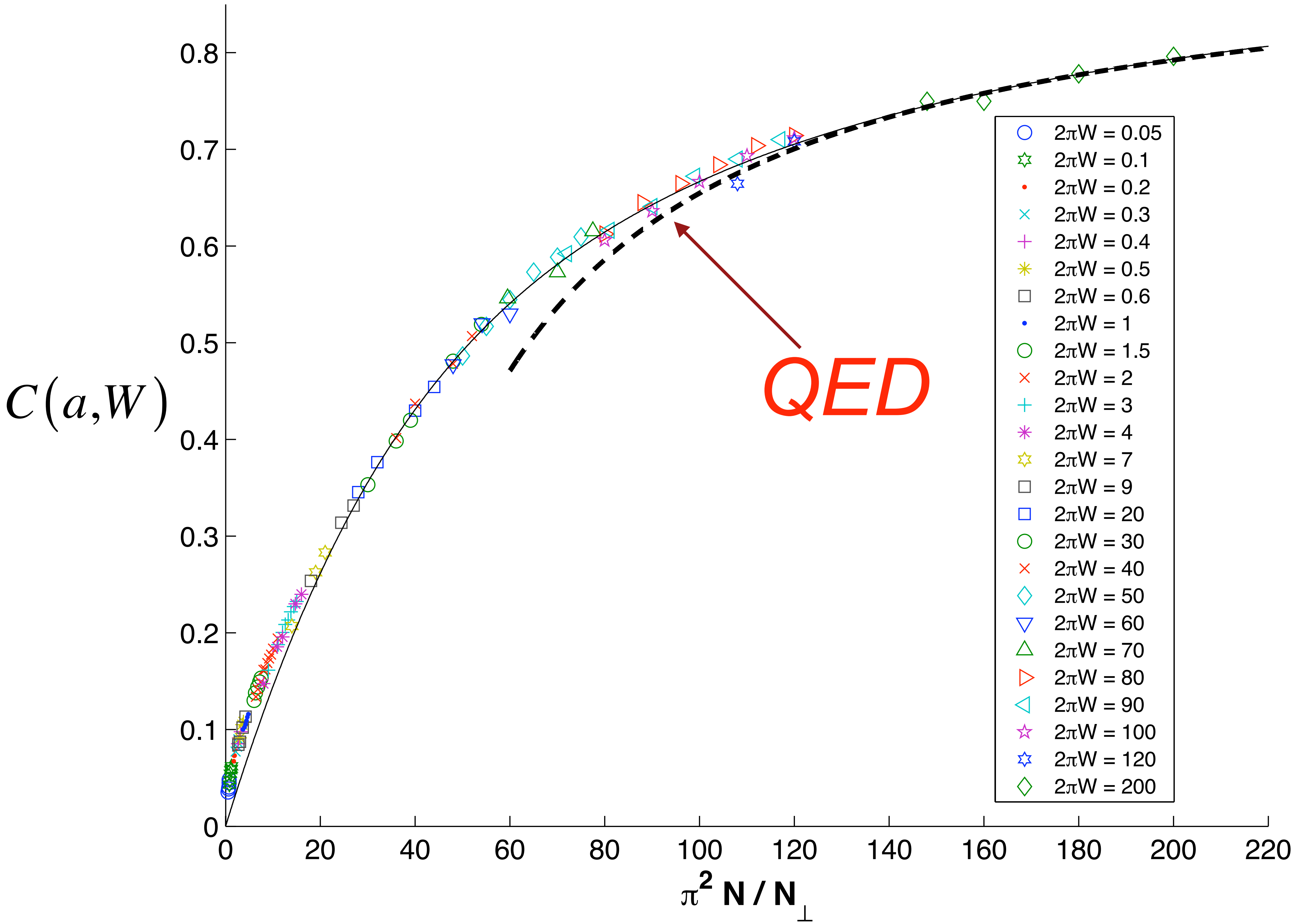
Large disorder limit $N \gg N_\perp$

Resummation of the cumulants of $P(\Gamma)$ leads to the asymptotic behavior

$$P(\Gamma) = \left(1 - \frac{3N_\perp}{2N}\right)\delta(\Gamma) + 3\Gamma\left(\frac{N_\perp}{N}\right)^3 \quad for \;\; \Gamma \leq N/N_\perp$$

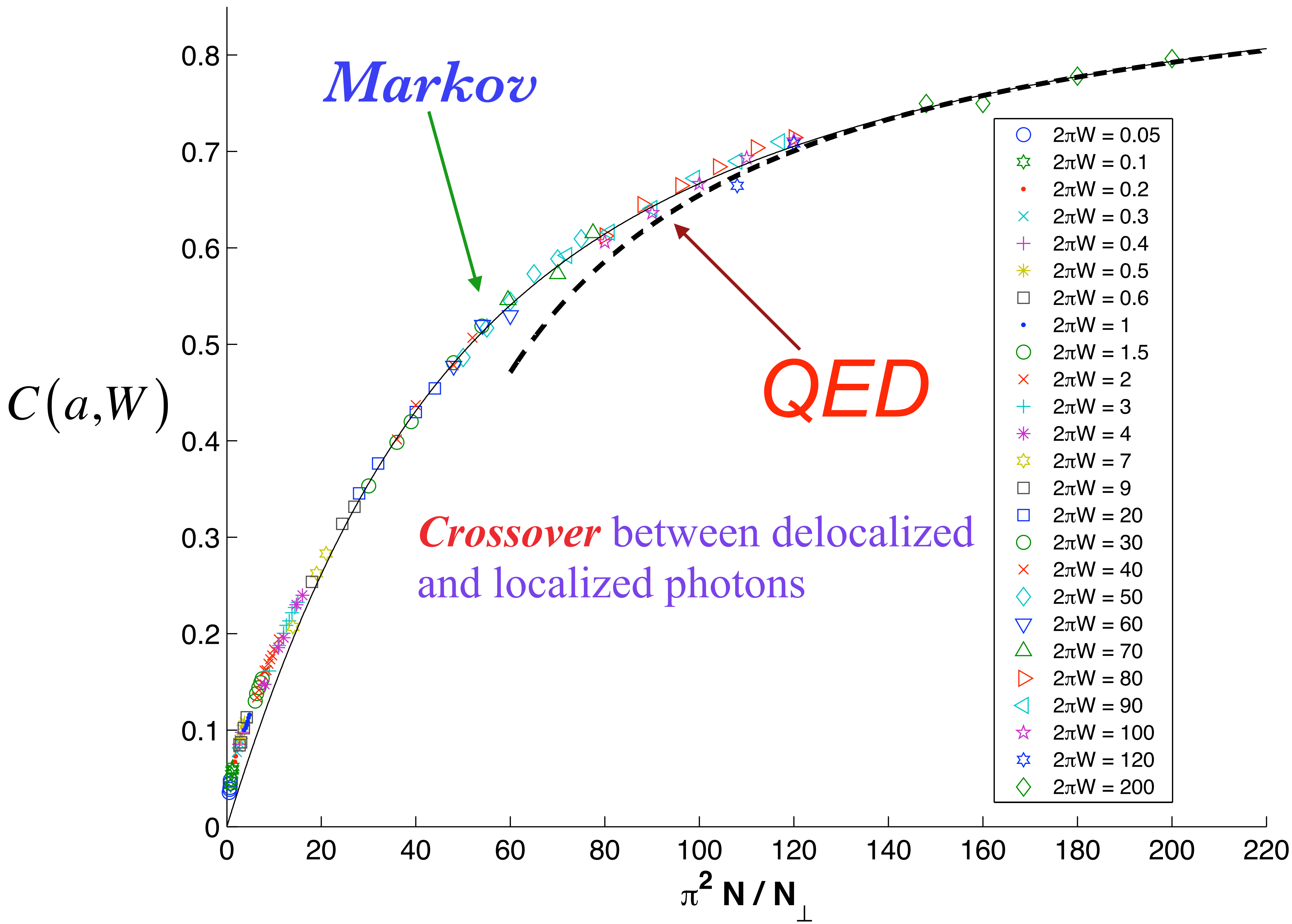
$$P(\Gamma) = 0 \quad otherwise$$

so that $$\boxed{C\left(N/N_\perp\right) = 1 - 3\frac{N_\perp}{N}}$$



$C(a,W)$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
140
160
180
200
220
π^2 N / N$_\perp$
QED
2πW = 0.05
2πW = 0.1
2πW = 0.2
2πW = 0.3
2πW = 0.4
2πW = 0.5
2πW = 0.6
2πW = 1
2πW = 1.5
2πW = 2
2πW = 3
2πW = 4
2πW = 7
2πW = 9
2πW = 20
2πW = 30
2πW = 40
2πW = 50
2πW = 60
2πW = 70
2πW = 80
2πW = 90
2πW = 100
2πW = 120
2πW = 200

Phenomenological Markov process
(Small world networks)


Markov
QED
Crossover between delocalized and localized photons
C(a,W)
π² N / N⊥
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 20 40 60 80 100 120 140 160 180 200 220
2πW = 0.05
2πW = 0.1
2πW = 0.2
2πW = 0.3
2πW = 0.4
2πW = 0.5
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2πW = 30
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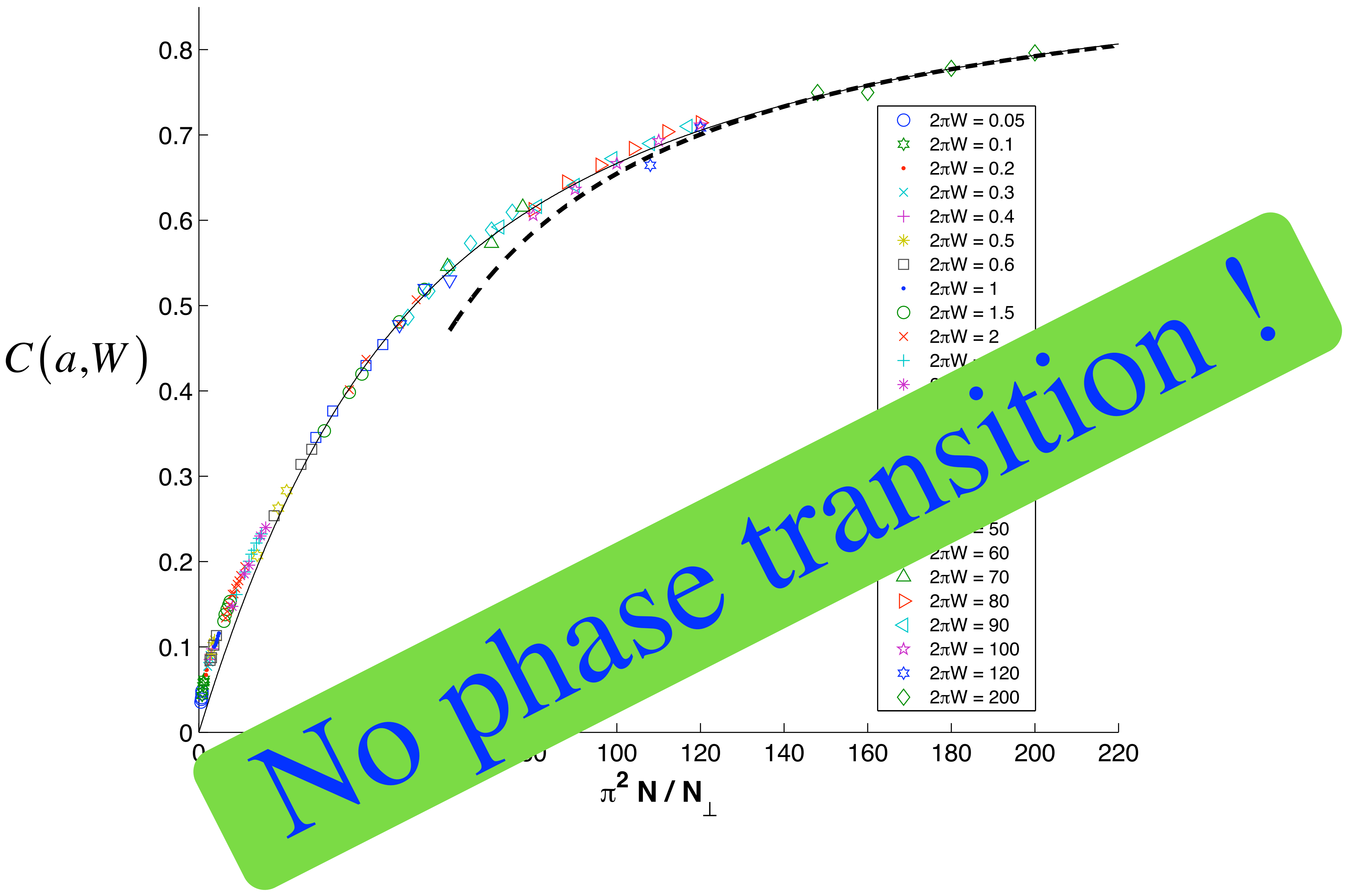

$C(a,W)$
π^2 N / N$_\perp$
2πW = 0.05
2πW = 0.1
2πW = 0.2
2πW = 0.3
2πW = 0.4
2πW = 0.5
2πW = 0.6
2πW = 1
2πW = 1.5
2πW = 2
2πW = 60
2πW = 70
2πW = 80
2πW = 90
2πW = 100
2πW = 120
2πW = 200
No phase transition !

Dependence upon the space dimension ?

disorder driven localisation transition
(Anderson)

One-dimensional random atomic gas : Absence of single atom limit (Wigner-Weisskopf)

$d=1$: Same expression of the effective atomic Hamiltonian H_e ,

$$H_e = \left(\hbar\omega_0 - i\frac{\hbar\Gamma_0}{2}\right)\sum_{i=1}^{N}|e_i\rangle\langle e_i| + \frac{\hbar\Gamma_0}{2}\sum_{i\neq j}V_{ij}\Delta_i^+\Delta_j^-$$

with $V_{ij}=\beta_{ij}-i\gamma_{ij}$ but $\gamma_{ij}=\cos k_0 r_{ij}$ instead of $\cancel{\gamma_{ij}=\frac{\sin k_0 r_{ij}}{k_0 r_{ij}}}$

A. Gero, E.A. ,EPL **101,** 2013

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with $V_{ij} = \beta_{ij} - i\gamma_{ij}$ but $\gamma_{ij} = \cos k_0 r_{ij}$ instead of $\gamma_{ij} = \cancel{\frac{\sin k_0 r_{ij}}{k_0 r_{ij}}}$

Two limits :

$a = L/\lambda \gg 1$ dilute large sample limit (Wigner-Weisskopf + disorder effects)

A. Gero, E.A. ,EPL **101,** 2013

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Two limits :

$a = L/\lambda \gg 1$ dilute large sample limit (Wigner-Weisskopf + disorder effects)

$a \ll 1$ Dicke limit (cooperative effects are expected) $\gamma_{ij} = \cos k_0 r_{ij} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}$

so that $P(\Gamma) = \frac{1}{N}[(N-1)\delta(\Gamma) + \delta(\Gamma - N)]$

A. Gero, E.A. ,EPL **101**, 2013

Method : Decomposition into a product of matrices

$N \times N$ matrix $U_{ij} = \cos k_0 r_{ij}$ can be written $U = \frac{1}{2} A^\dagger A$

with A is the $2 \times N$ matrix defined by $A_{0j} = e^{ik_0 r_j}$ *and* $A_{1j} = e^{-ik_0 r_j}$

U real symmetric matrix, its non vanishing eigenvalues are obtained from those of the 2×2 matrix $U^\dagger$

$$U^\dagger = \frac{1}{2} \begin{pmatrix} N & M \\ M^* & N \end{pmatrix}.$$

where $M = \sum_{k=1}^{N} e^{2ik_0 r_k}$ is a random variable.

The two eigenvalues of $U^\dagger$ are $\lambda_\pm = \frac{N \pm |M|}{2}$,

and the spectrum of U is

$$P(\Gamma) = \frac{1}{N} \left[(N-2)\delta(\Gamma) + \delta(\Gamma - \lambda_+) + \delta(\Gamma - \lambda_-) \right].$$

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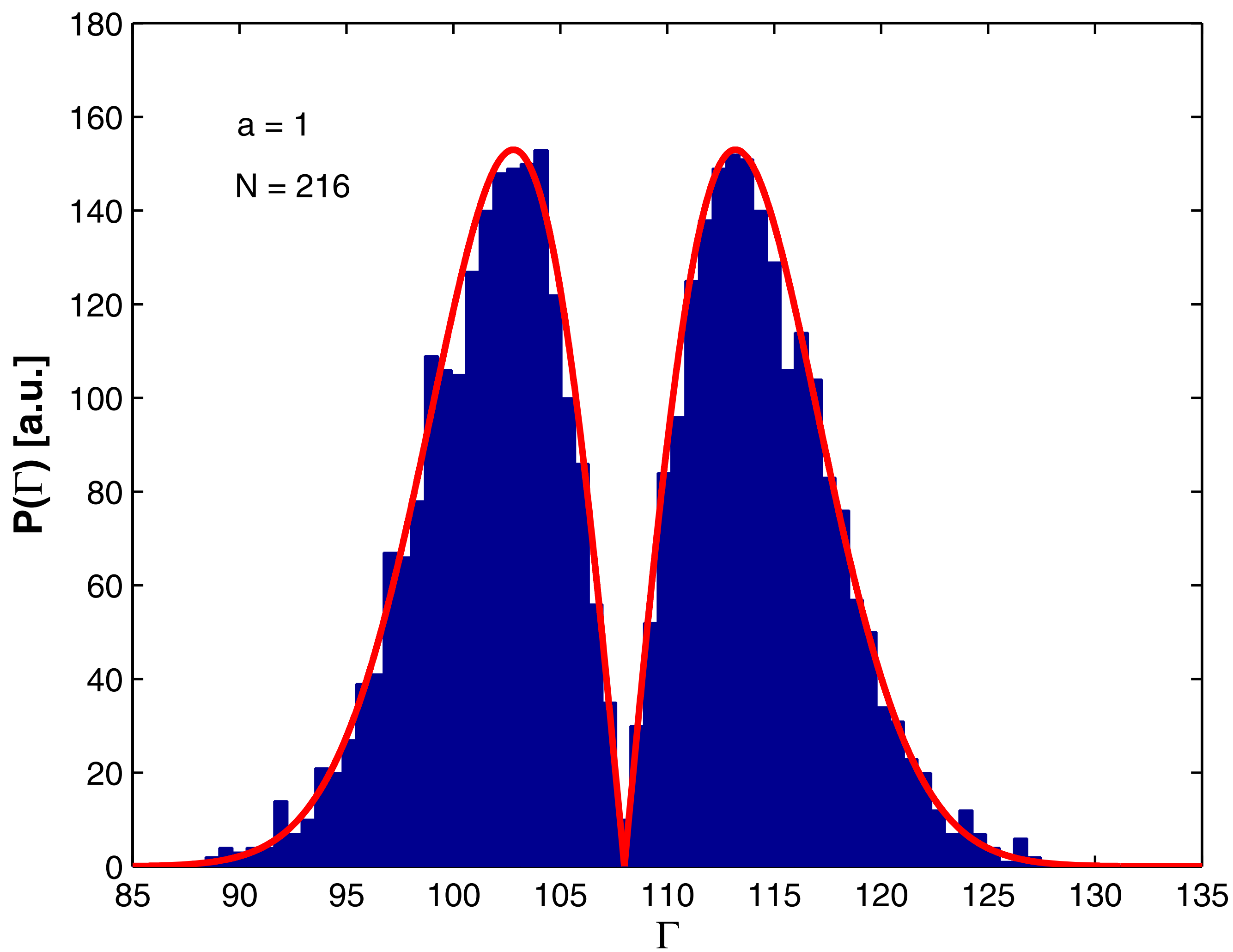
One-dimensional random atomic gas : Absence of single atom limit



Subradiant mode is not represented

$$|M|^2 = N + \sum_{p \neq q} e^{2ik_0(r_p - r_q)}$$

Rayleigh distribution $P(|M|) = \frac{2|M|}{N} e^{-\frac{|M|^2}{N}}$

One-dimensional random atomic gas

- d=1 : no crossover between localised and delocalised photons.
- Single atom (Wigner-Weisskopf) limit is never reached.
- Results in d=1 are valid for both **ordered** and **disordered** media (M is not a random variable)
- Cooperative effects (not disorder) is the mechanism underlying photon localisation in d=1.

Two-dimensional random atomic gas : Marchenko-Pastur distribution

In $d = 2$ the same expression of the effective atomic Hamiltonian H_e holds,

with $V_{ij} = \beta_{ij} - i\gamma_{ij}$ but $\gamma_{ij} = J_0\left(k_0 r_{ij}\right)$ instead of $\cancel{\gamma_{ij} = \frac{\sin k_0 r_{ij}}{k_0 r_{ij}}}$

A. Gero, E.A. ,PRA 88, 2013

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The $d=1$ trick does not work.

Instead we use the general decomposition: $U = H\,T\,H^\dagger$,

where $T(M\times M), H(N\times M)$

(S. Skipetrov and also "Free probability theory" (Voiculescu), "Wireless communications" (Debbah, Tulino, Verdu), "spin glasses" I. Kanter et al.

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Two-dimensional random atomic gas : Marchenko-Pastur distribution

Spectrum of $\gamma_{ij} = J_0\left(k_0 r_{ij}\right)$

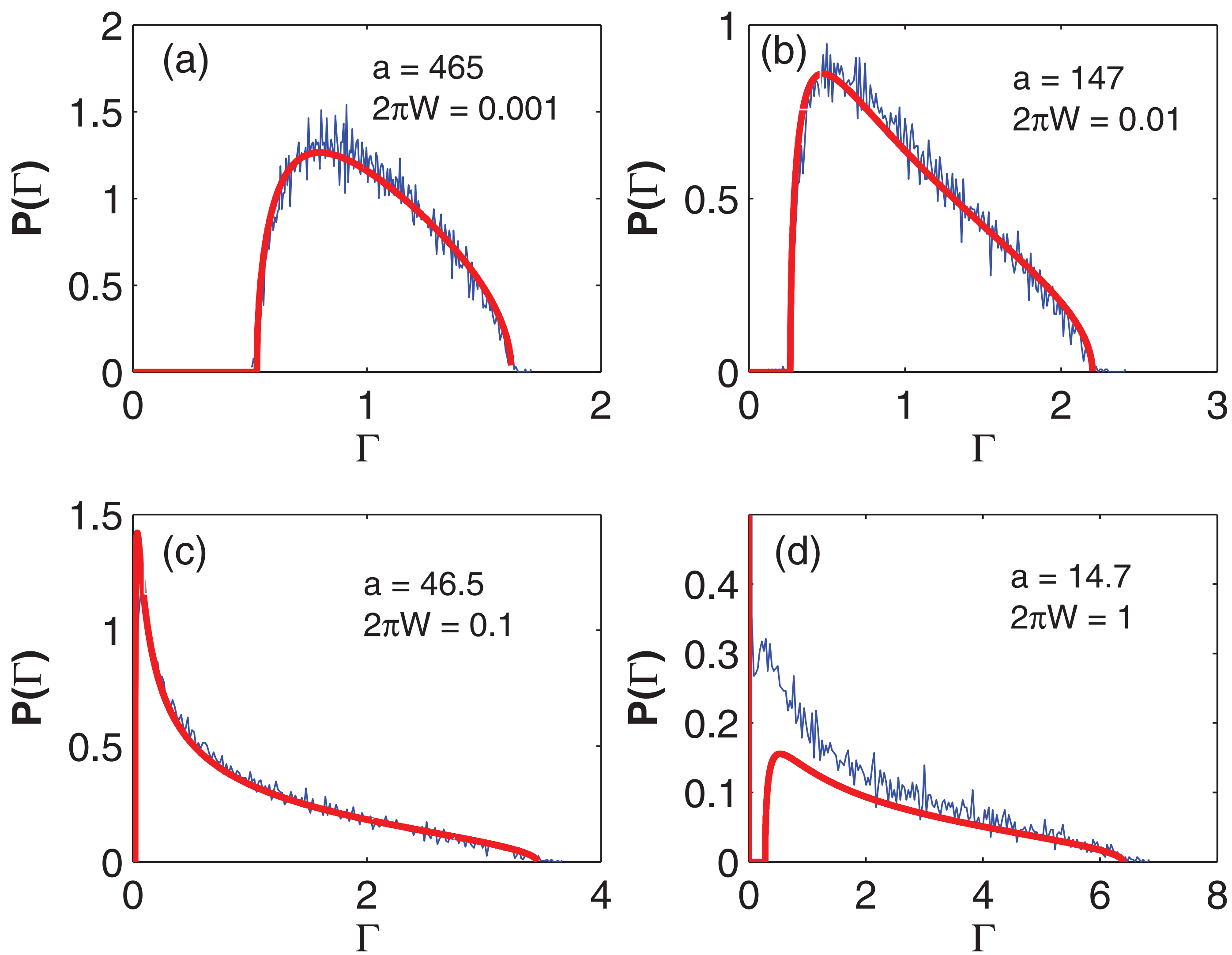
For $a \gg 1$, dilute limit,

$$P(\Gamma) \simeq \left(1 - \frac{M}{N}\right)^{+} \delta(\Gamma) + \frac{\sqrt{(\Gamma - \Gamma_-)^+(\Gamma_+ - \Gamma)^+}}{2\pi \frac{N}{M} \Gamma},$$

$$x^+ = \max(0, x)$$

$$\Gamma_\pm = (1 \pm \sqrt{N/M})^2$$

$$M = 2\pi a$$



$$L^d = (\lambda a)^d$$

Disorder parameter

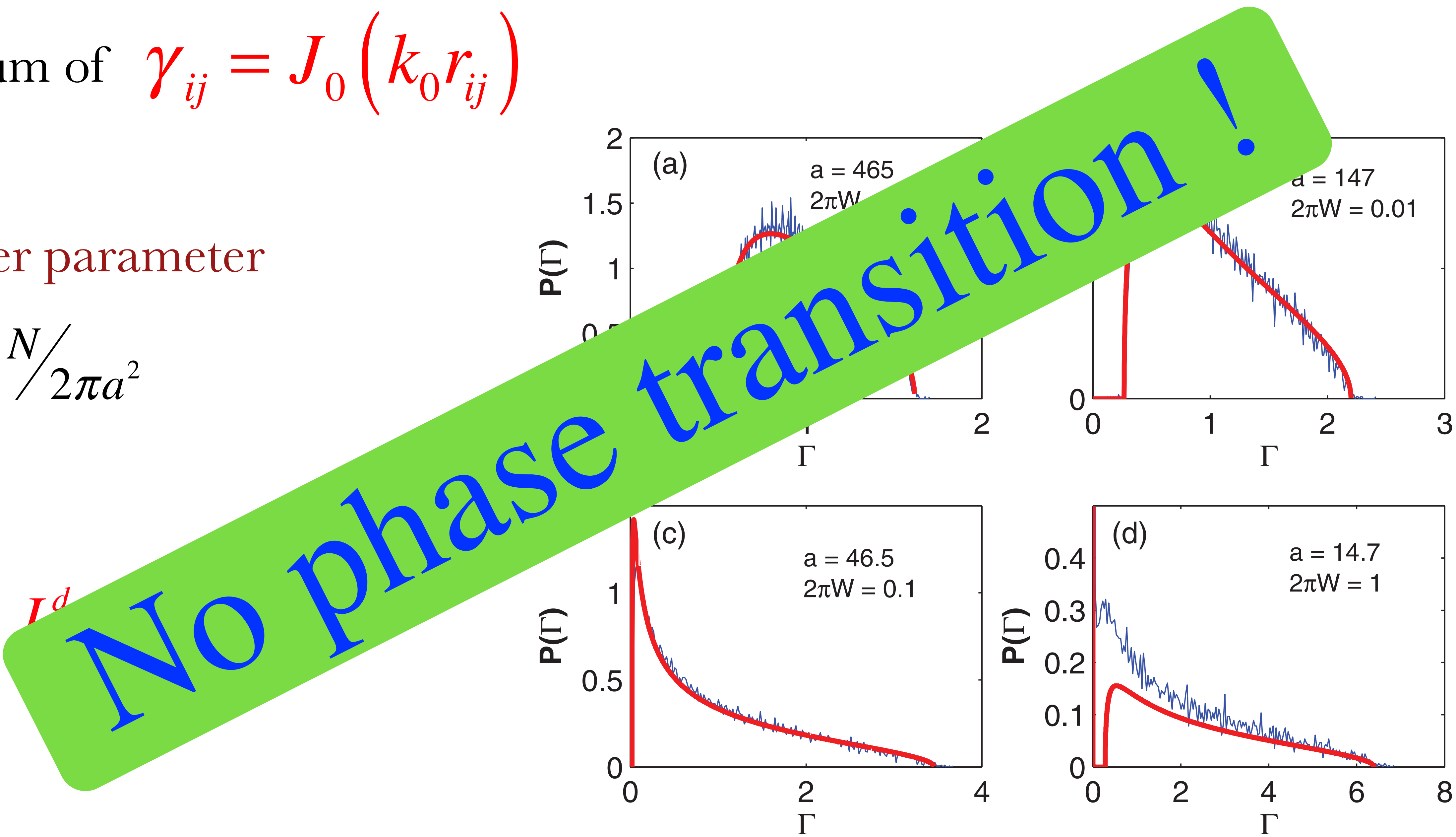
$$W = N / 2\pi a^2$$

Two-dimensional random atomic gas : Marchenko-Pastur distribution

Spectrum of $\gamma_{ij} = J_0\left(k_0 r_{ij}\right)$

Disorder parameter

$$W = {N}/{2\pi a^2}$$



Numerical results $N=216$

A. Gero, E.A. ,PRA 88, 2013

Eigenvalues of the non Hermitian random Hamiltonian

Time evolution of the ground state population is driven by the eigenvalues of the random matrix γ_{ij}

while the effective Hamiltonian is

$$H_e = \left(\hbar\omega_0 - i\frac{\hbar\Gamma_0}{2}\right)\sum_{i=1}^{N}|e_i\rangle\langle e_i| + \frac{\hbar\Gamma_0}{2}\sum_{i\neq j} V_{ij}\Delta_i^+\Delta_j^-$$

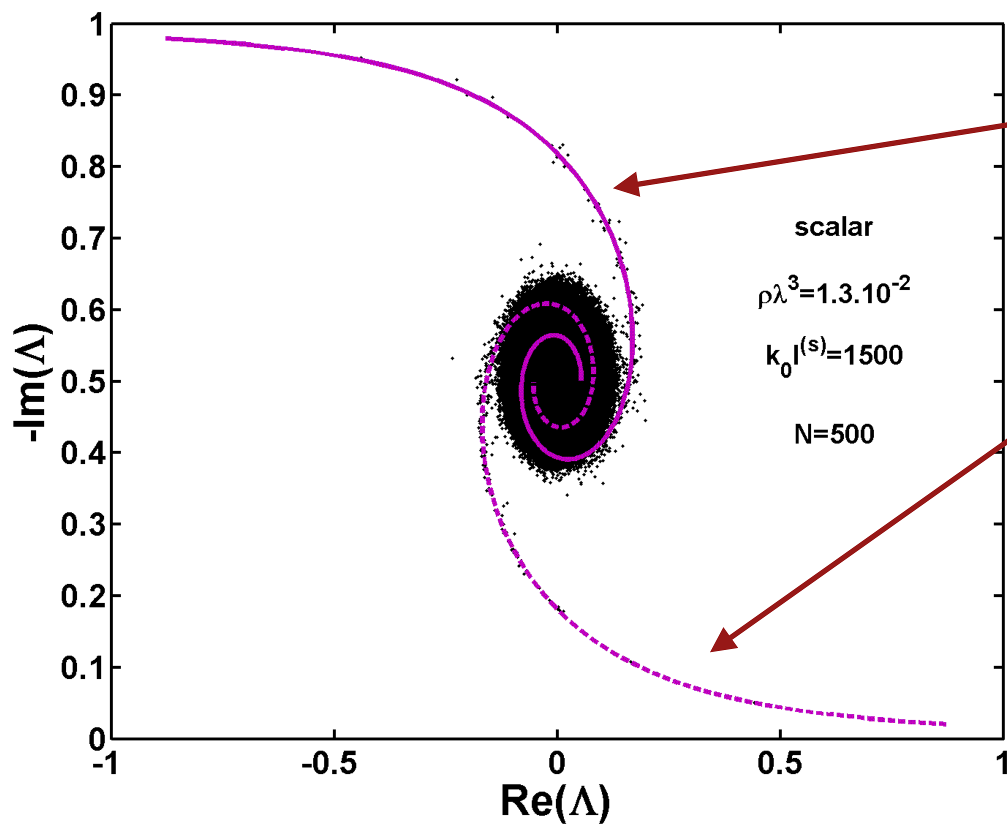
$$V_{ij} = \beta_{ij} - i\gamma_{ij}$$

Study the complex eigenvalues of H_e

$$\boxed{E_n - i\hbar\,{\Gamma_n}/{2} \equiv \hbar\omega_0 + \hbar\Gamma_0\Lambda_n}$$

Eigenvalues of the non Hermitian random Hamiltonian

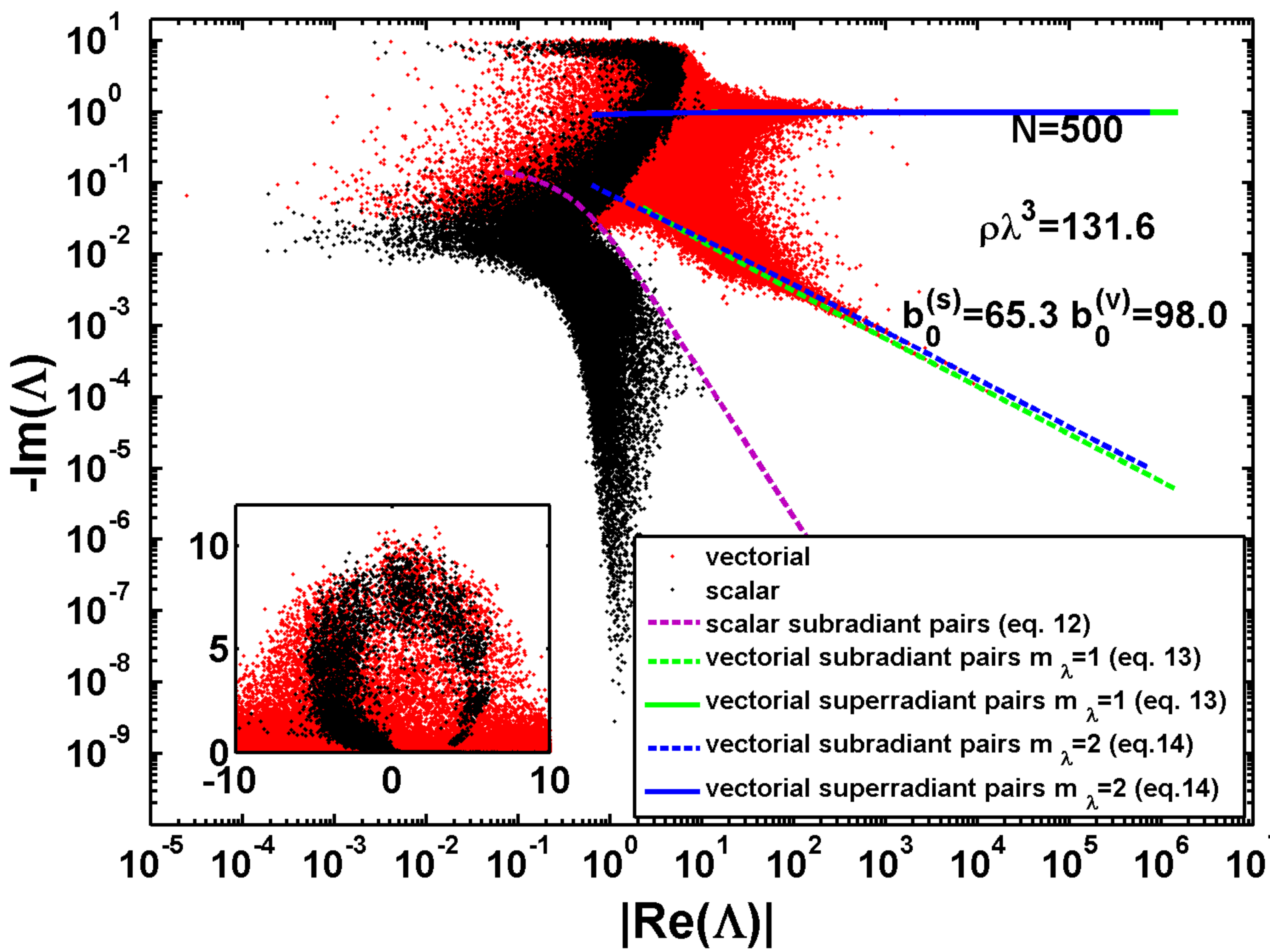
N=2 atoms case : The spectrum of H_e can be obtained explicitly (for both scalar and vectorial case)



Cooperative pairs

$$E_n - i\hbar \Gamma_n/2 \equiv \hbar\omega_0 + \hbar\Gamma_0 \Lambda_n$$

For N large and in the dense limit



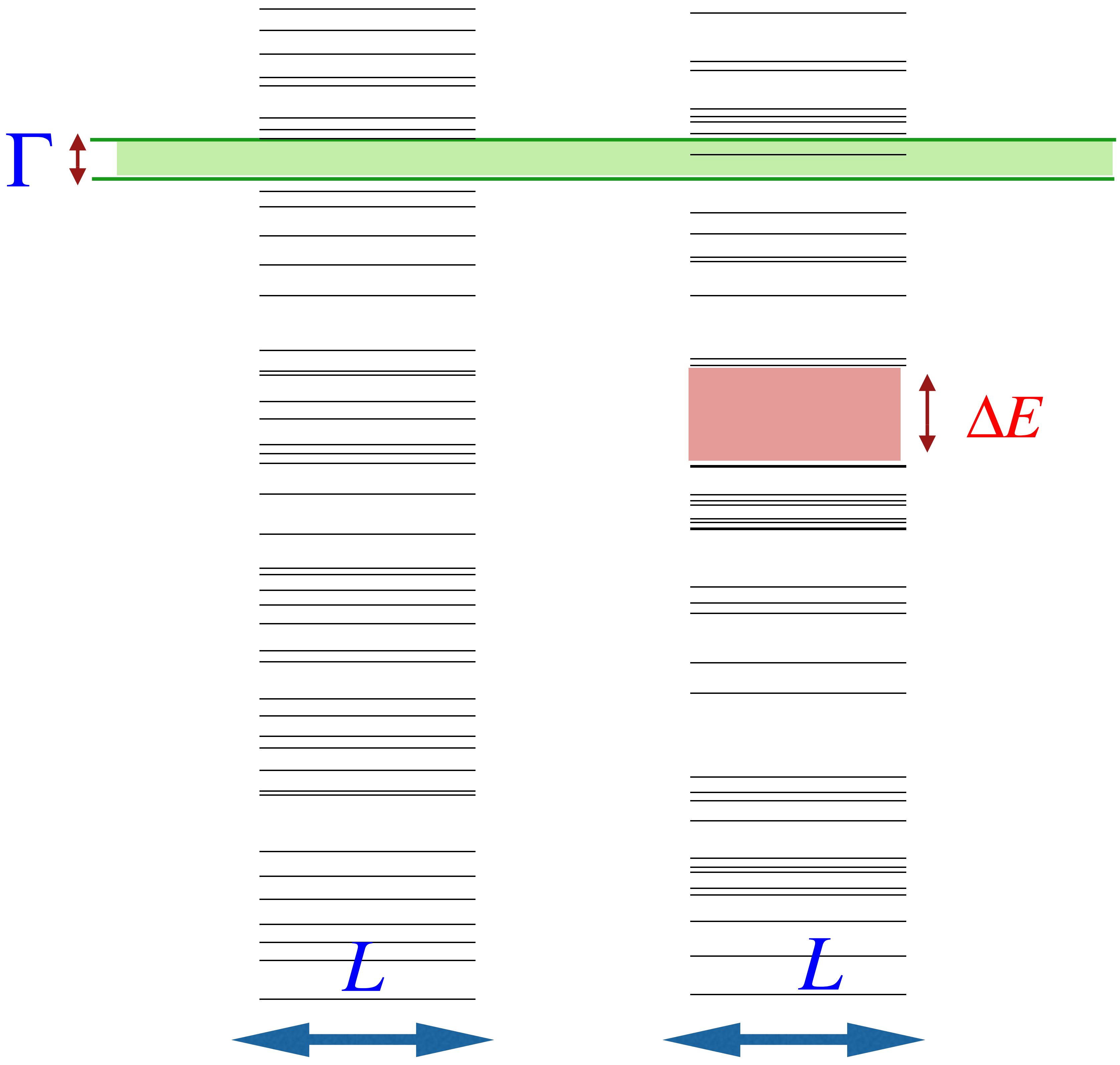
Thouless parameter : localisation phase transition

Edwards & Thouless ('72), Thouless ('77)

Coupling between open quantum systems
Transport (conductance)

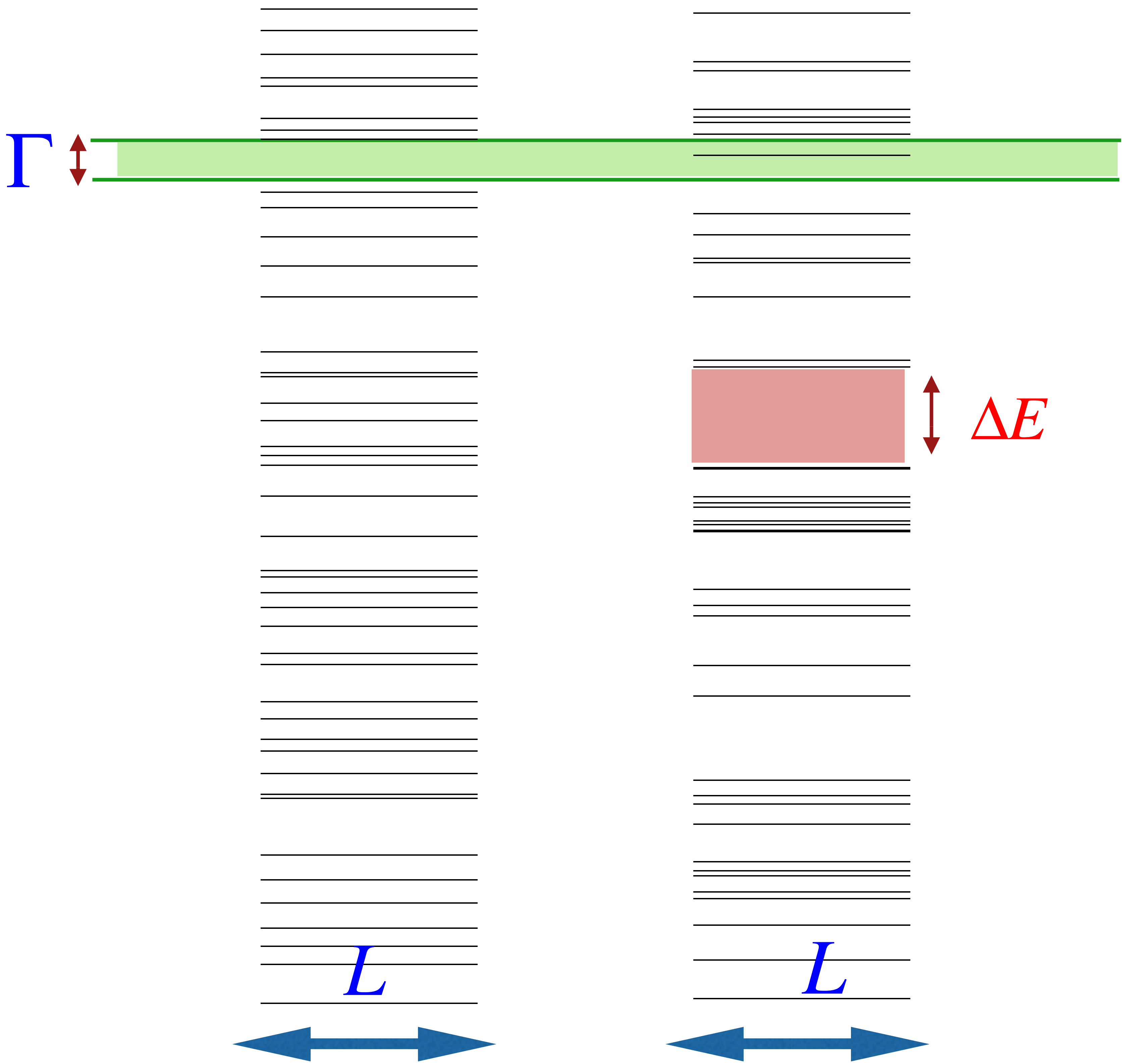
Using Random matrix theory :
G. Montambaux, E.A., (1992), I. Guarneri et al. (1994)

Thouless parameter - Resonance overlap



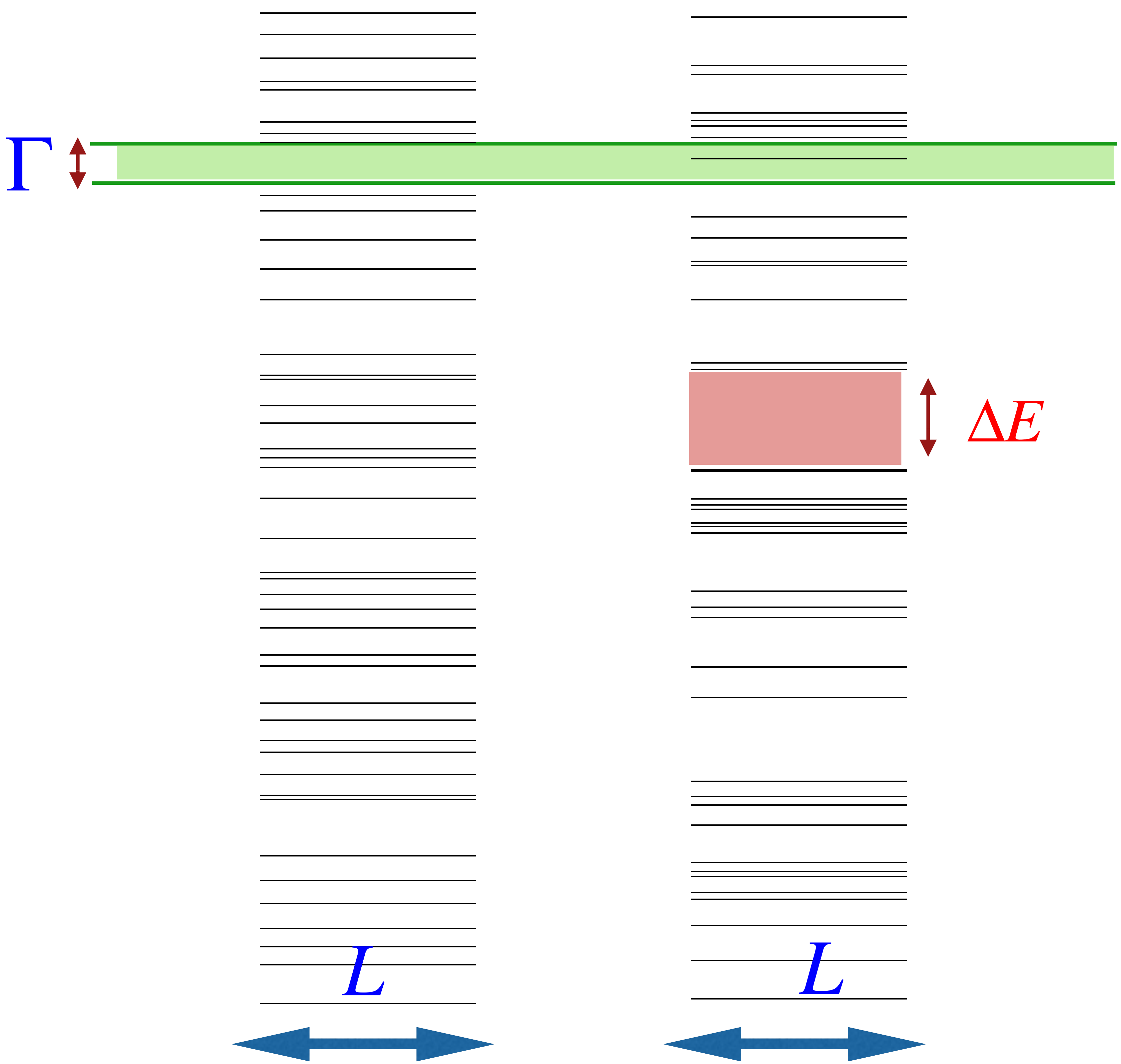
Energy spectrum of two random quantum systems

Thouless parameter - Resonance overlap



$$g(L) = \left\langle \frac{\langle \Gamma \rangle_i}{\langle \Delta E \rangle_i} \right\rangle$$

Thouless parameter - Resonance overlap



$$g(L) = \left\langle \frac{\langle \Gamma \rangle_i}{\langle \Delta E \rangle_i} \right\rangle$$

$$g(L \to \infty) \gg 1$$

large overlap : delocalised states - conductor

$$g \ll 1$$

Small overlap : localised states - insulator

Thouless parameter - localisation phase transition

Scaling and its meaning : (P.W. Anderson *et al*.,1979)

If we know $g(L)$, we know it at any scale :

$$g\big(L(1+\varepsilon)\big)=f\big(g(L),\varepsilon\big)$$

Scaling behavior :

$$g(L,W)=f\left(L/\xi(W)\right)$$

$\xi(W)$ is the localization length

Thouless parameter - localisation phase transition

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$$\beta(g)=\frac{d\ln g}{d\ln L}$$

is a function of g only.

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Thouless parameter - localisation phase transition

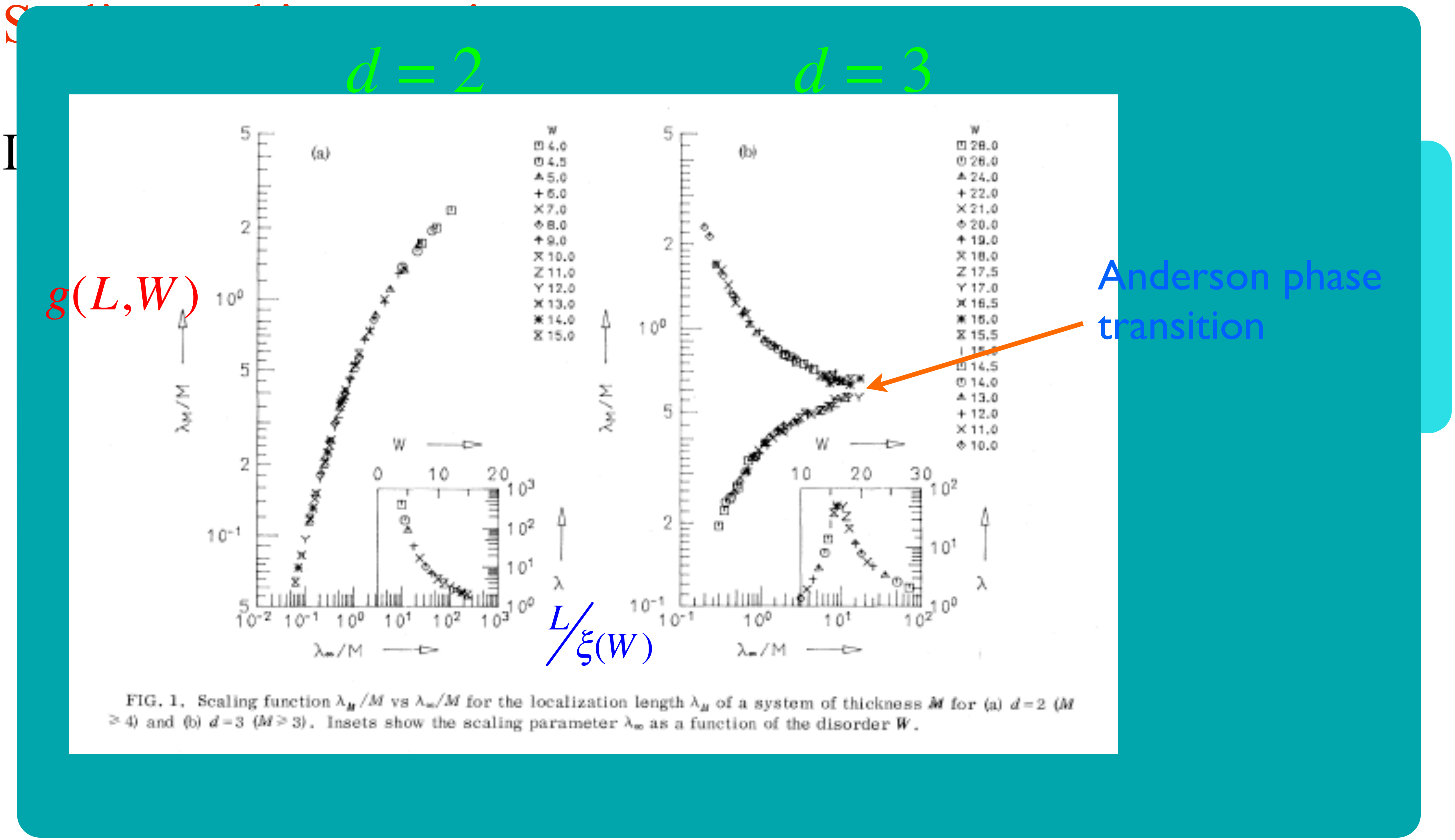

FIG. 1. Scaling function λ_M/M vs λ_∞/M for the localization length λ_M of a system of thickness M for (a) $d=2$ ($M \geq 4$) and (b) $d=3$ ($M \geq 3$). Insets show the scaling parameter λ_∞ as a function of the disorder W.

Scaling behaviour - phase transition

Thouless scaling parameter (conductance)

$$g(L) = \left\langle \frac{\langle \Gamma \rangle_i}{\langle \Delta E \rangle_i} \right\rangle$$

$$E_n - i\hbar \, {}^{\Gamma_n}\!/_{2} \equiv \hbar\omega_0 + \hbar\Gamma_0 \Lambda_n$$

S. Skipetrov and Sokolov (2014), Bellando, Gero, Kaiser, E.A. , PRA, 2014

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Instead we define :

$$\boxed{g \equiv \left\langle \frac{1}{\langle 1/\Gamma \rangle_i \langle \Delta E \rangle_i} \right\rangle}$$

S. Skipetrov and Sokolov (2014), Bellando, Gero, Kaiser, E.A. , PRA, 2014

Scaling behaviour - phase transition

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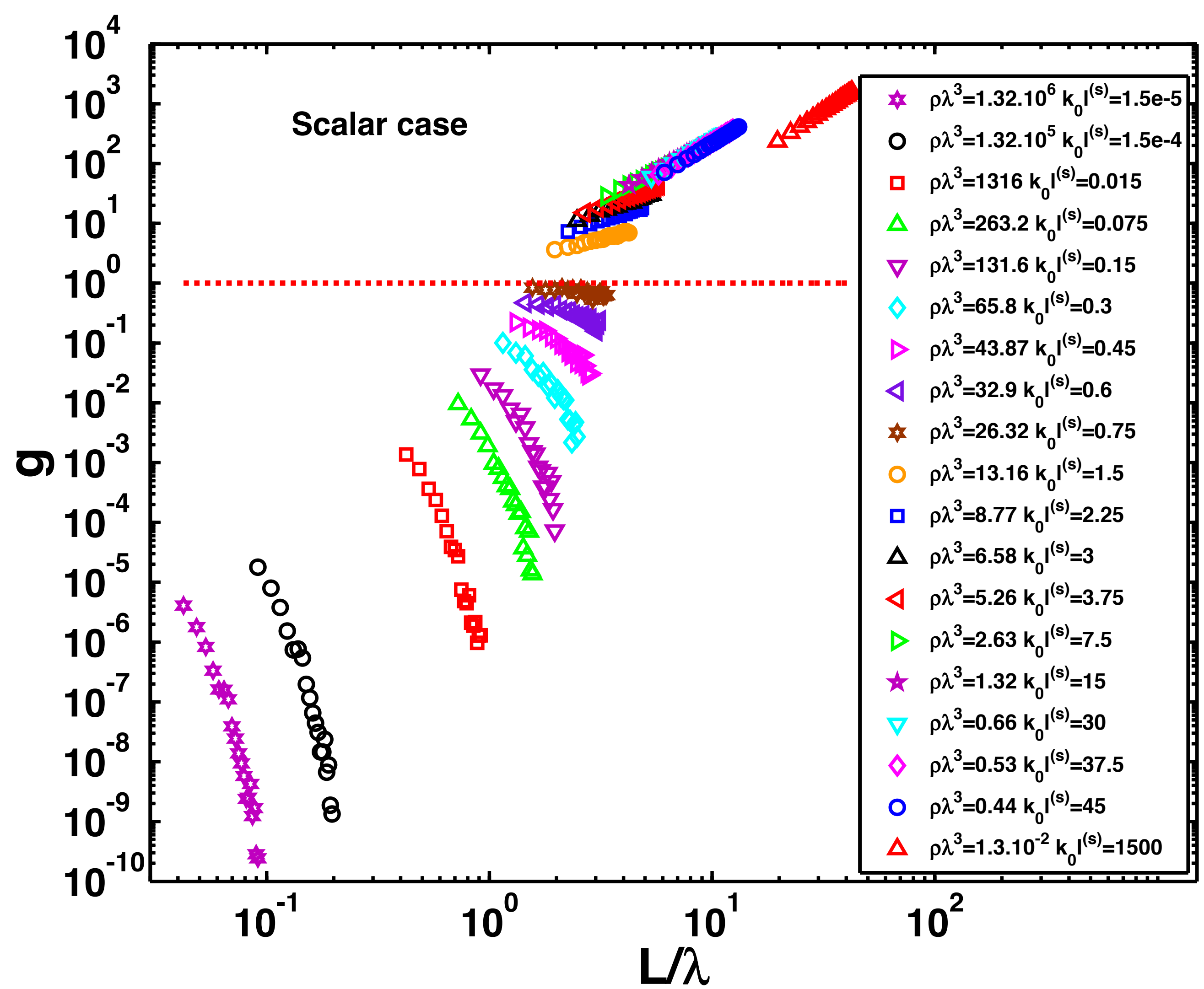
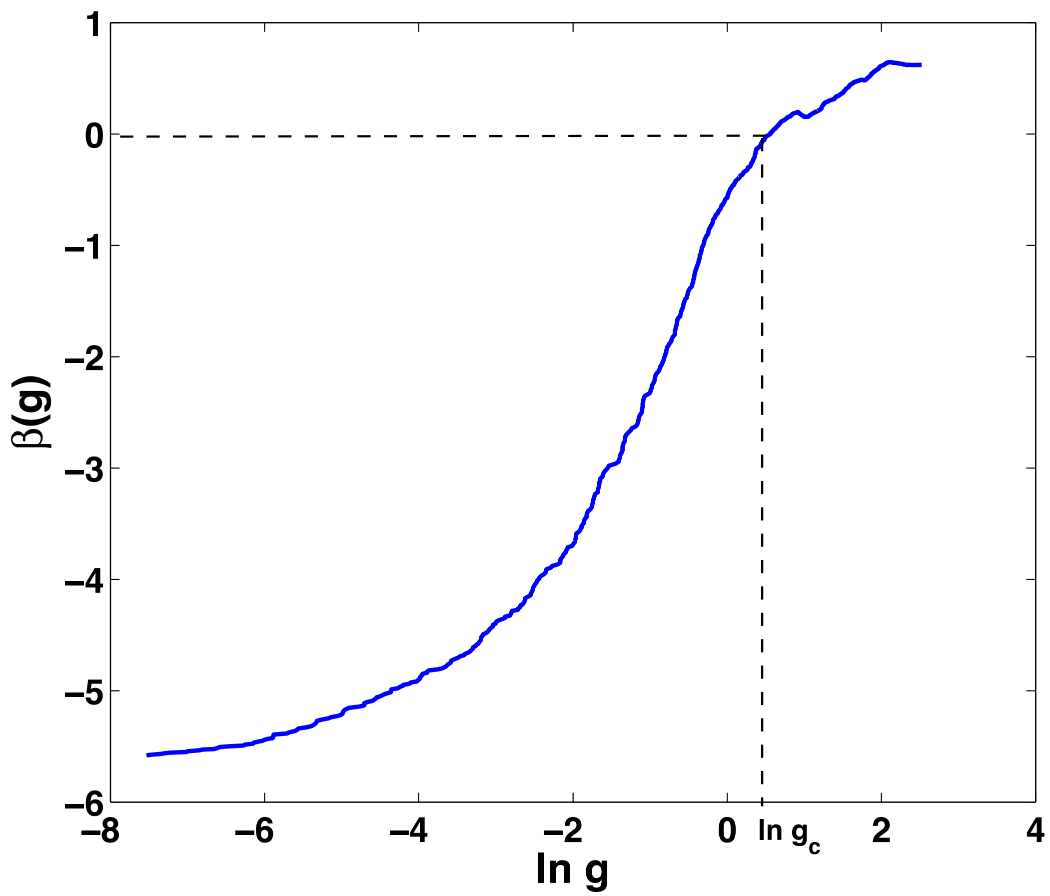
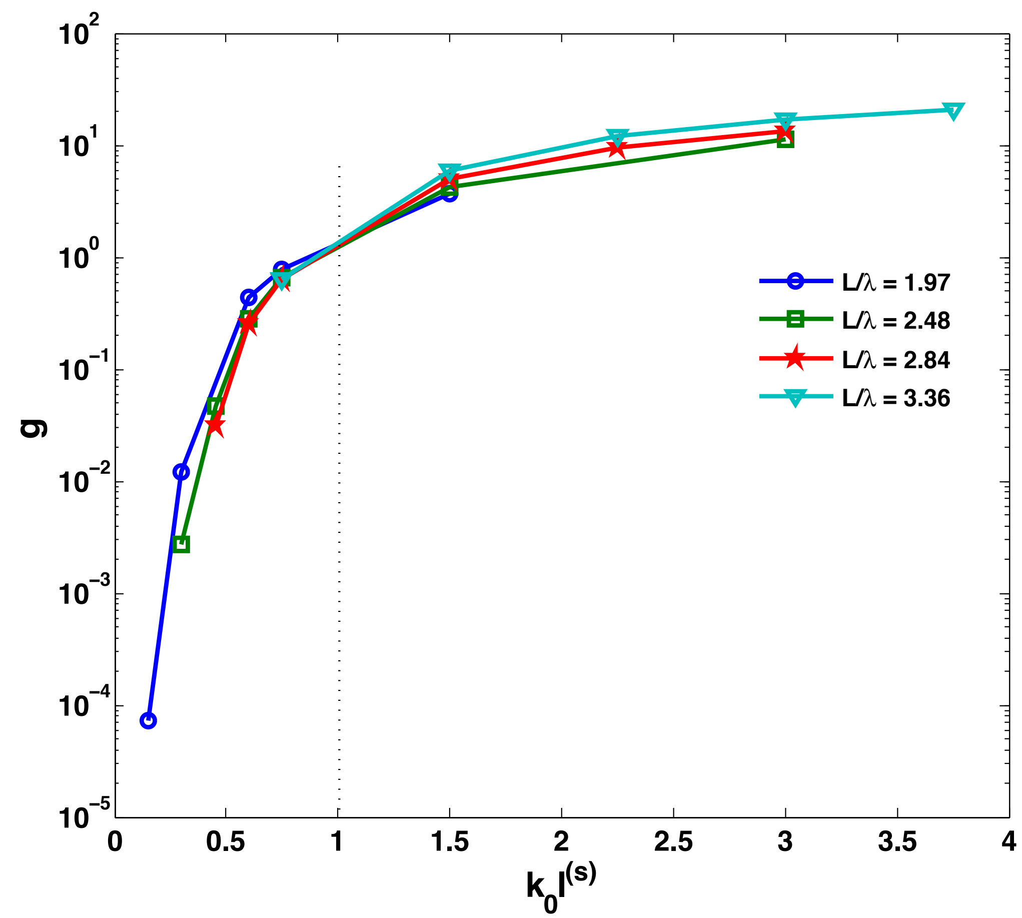


Critical point for a phase transition :

$g(L)$ becomes L- independent i.e. $\beta(g) = \dfrac{d \ln g}{d \ln L} = 0$





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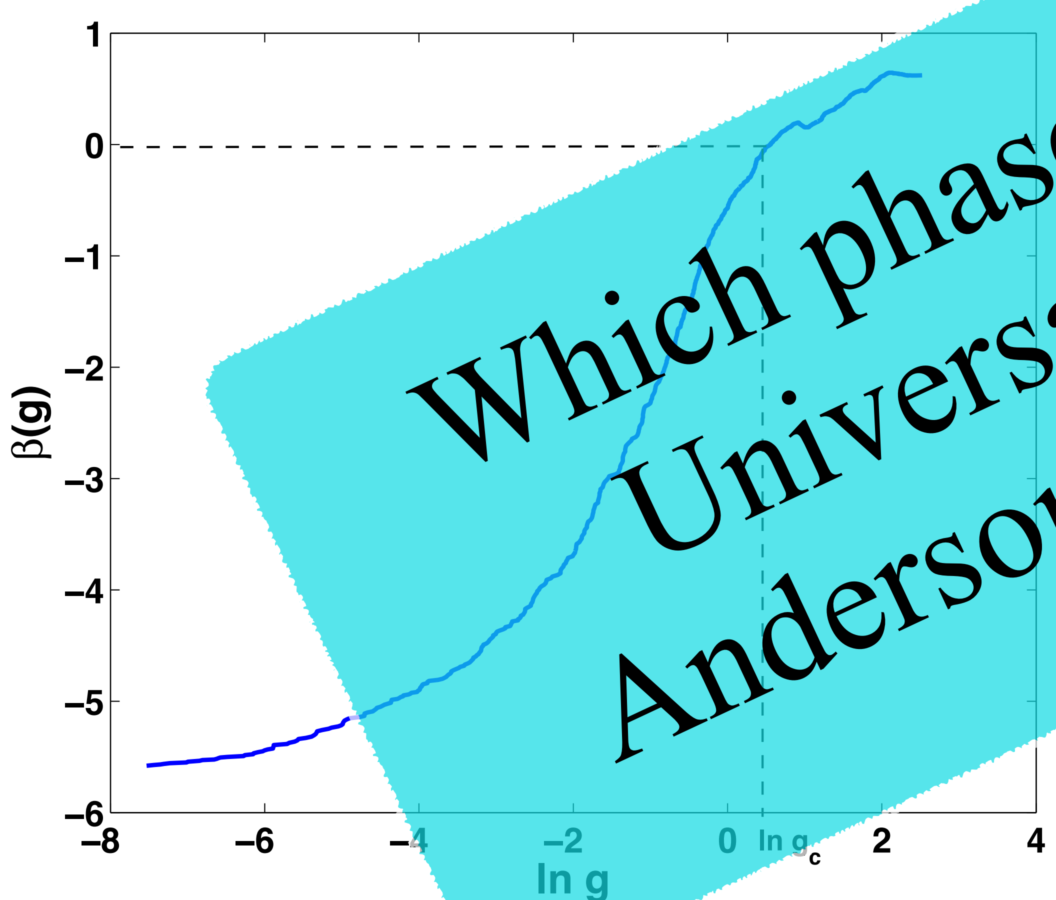
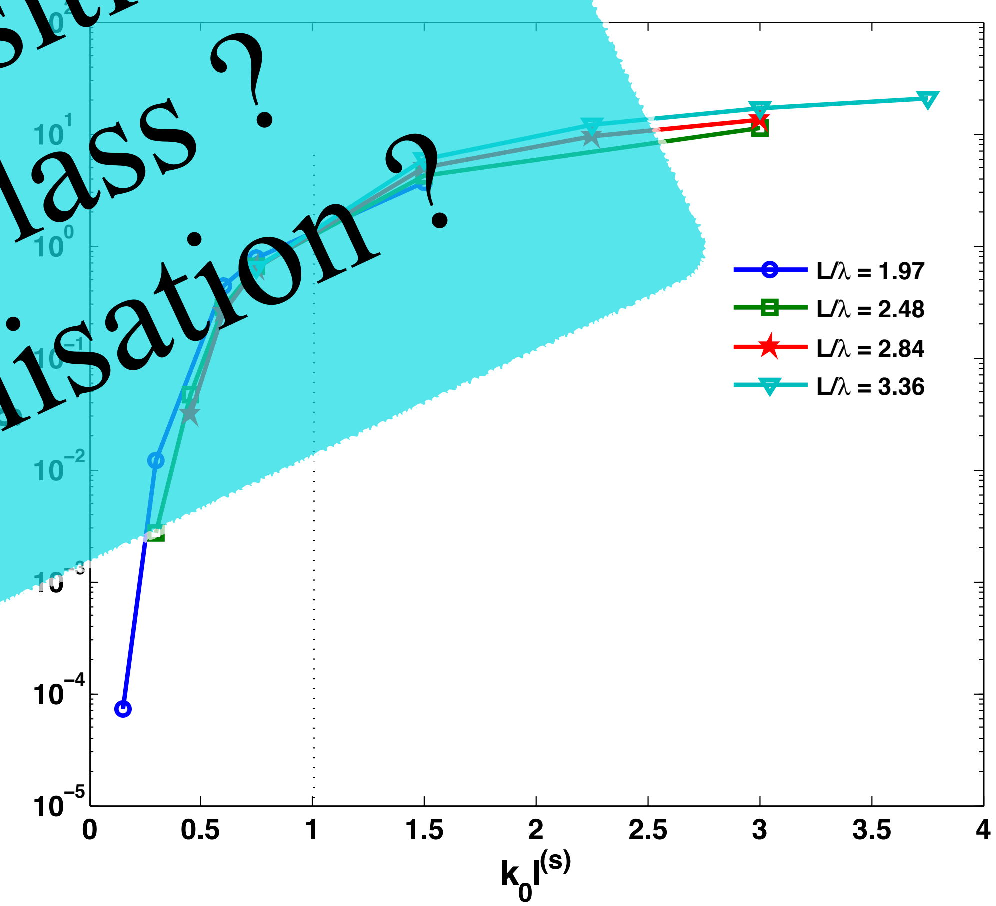
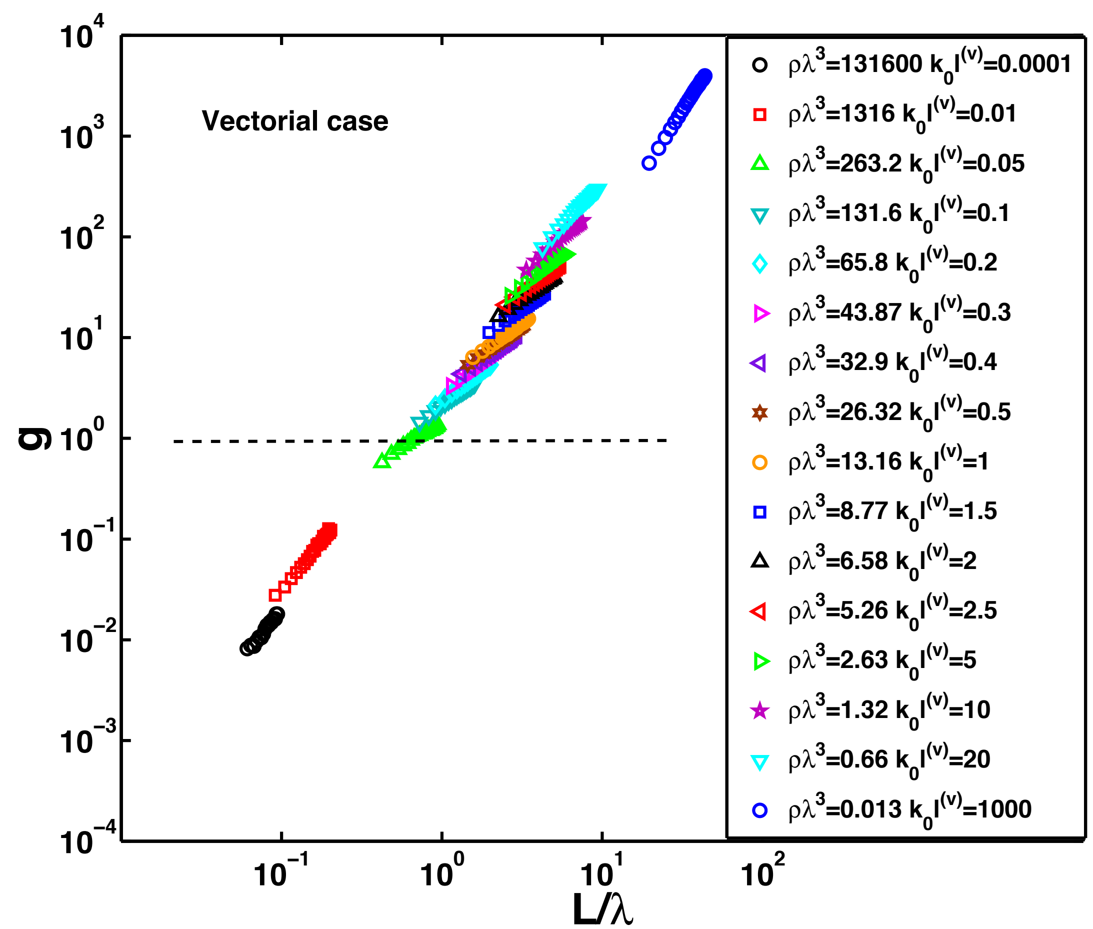


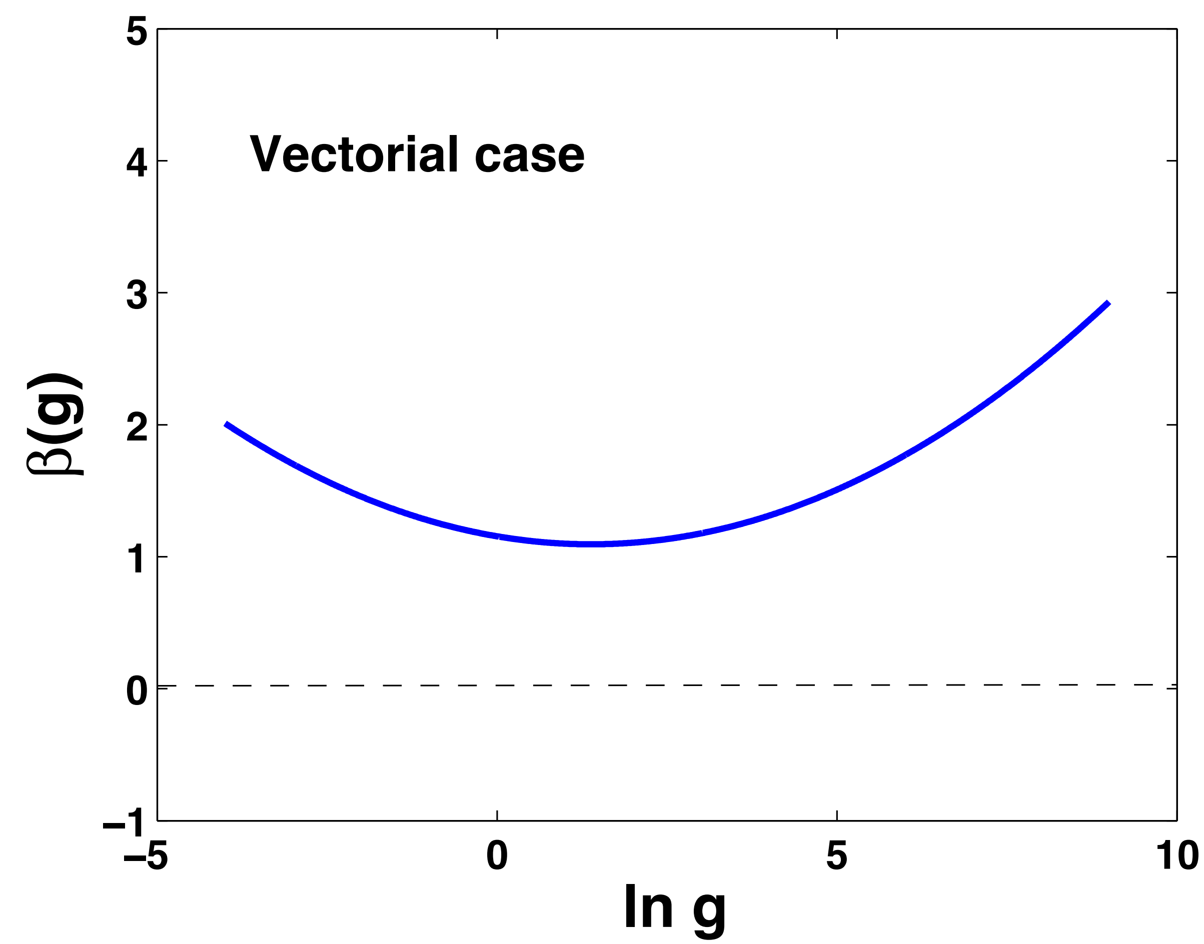
Which phase transition ?
Universality class ?
Anderson localisation ?



Scaling behaviour - phase transition
to make things more complicated ….

Vector case - polarised waves





Scaling behaviour - phase transition
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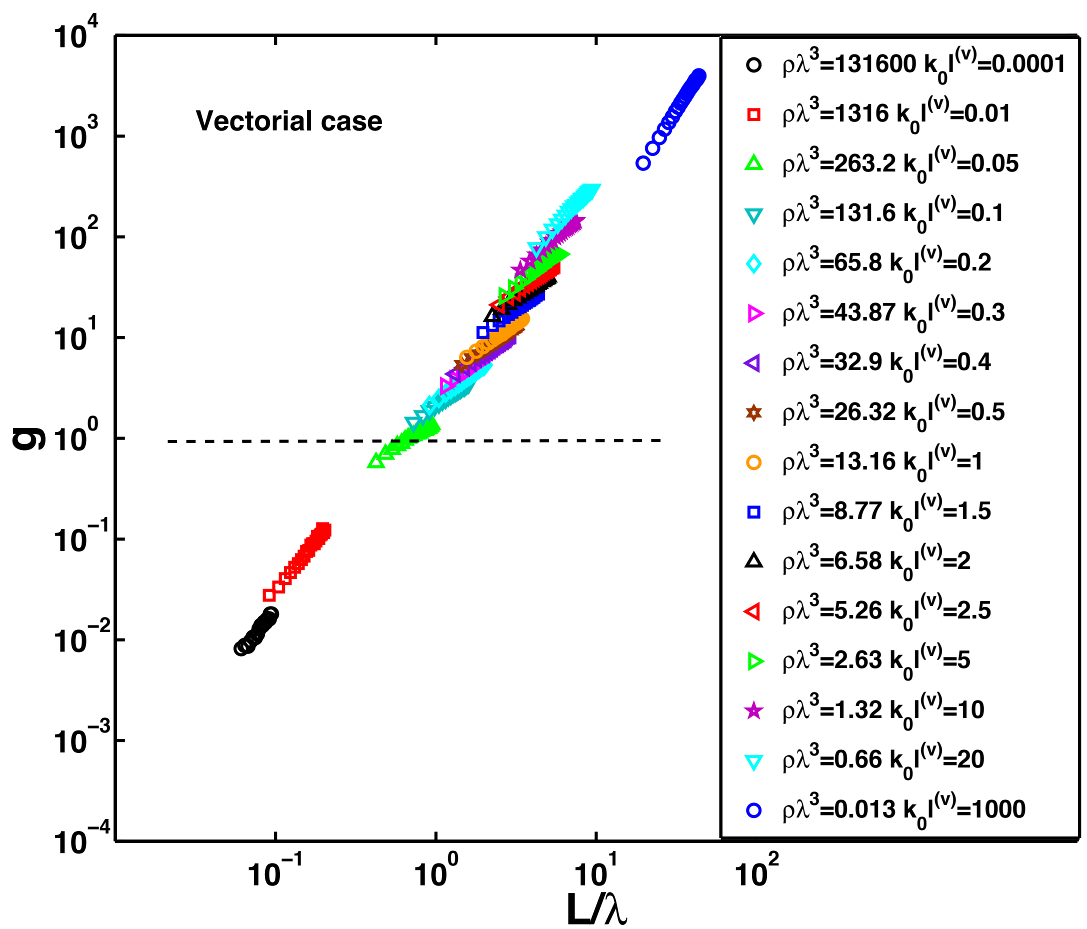

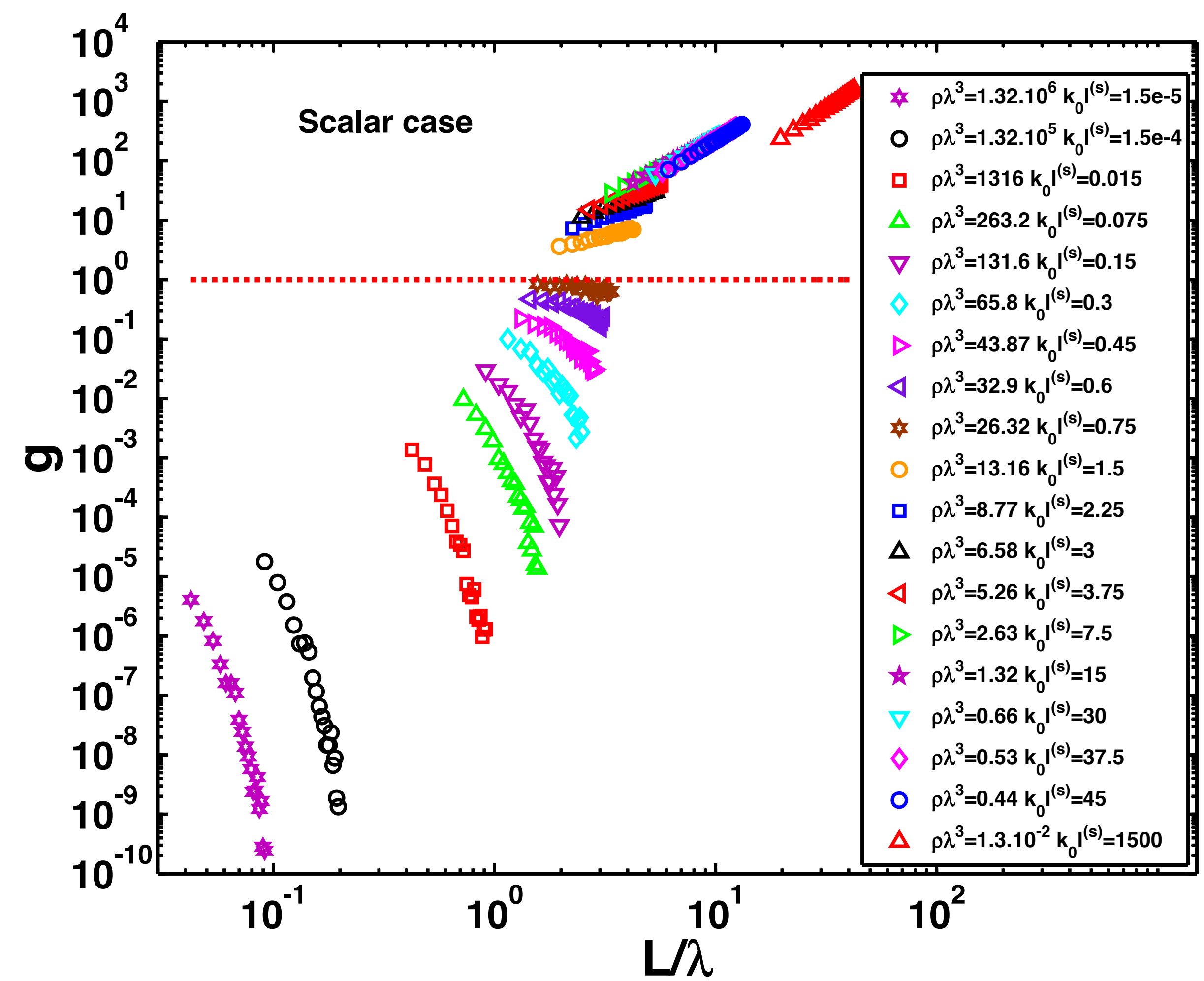

Scaling behaviour - phase transition
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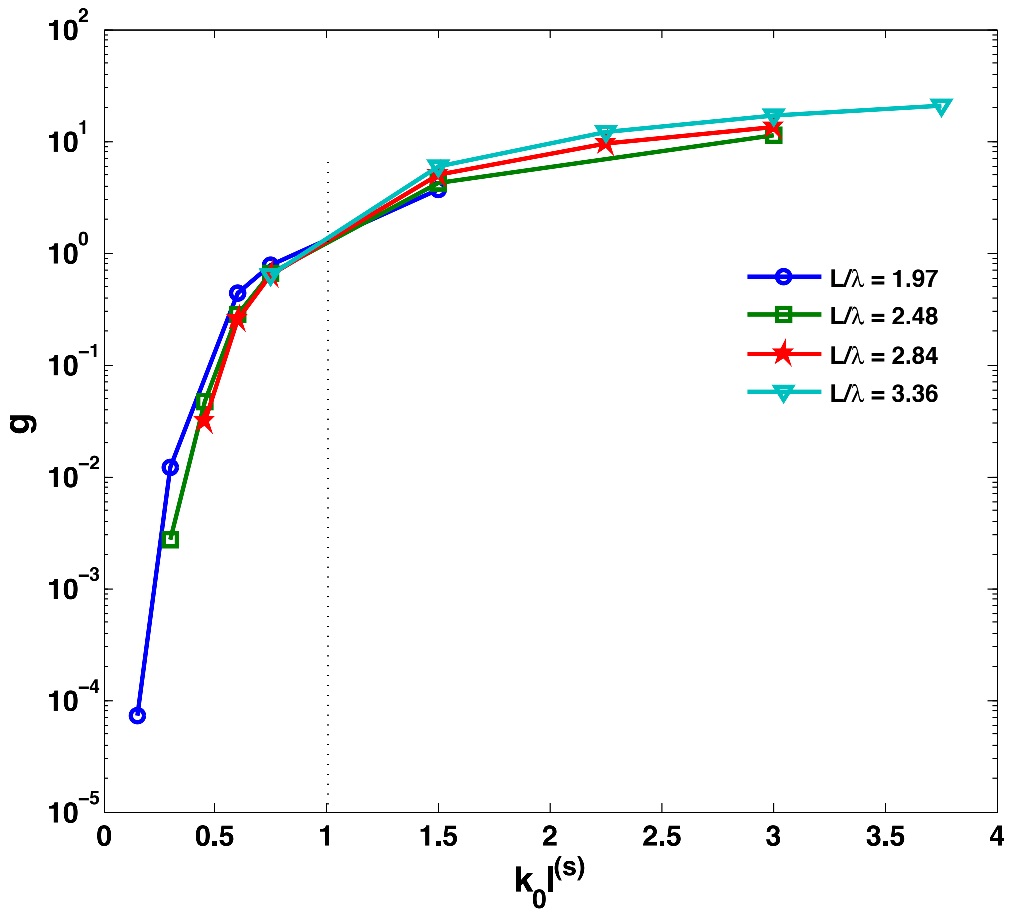

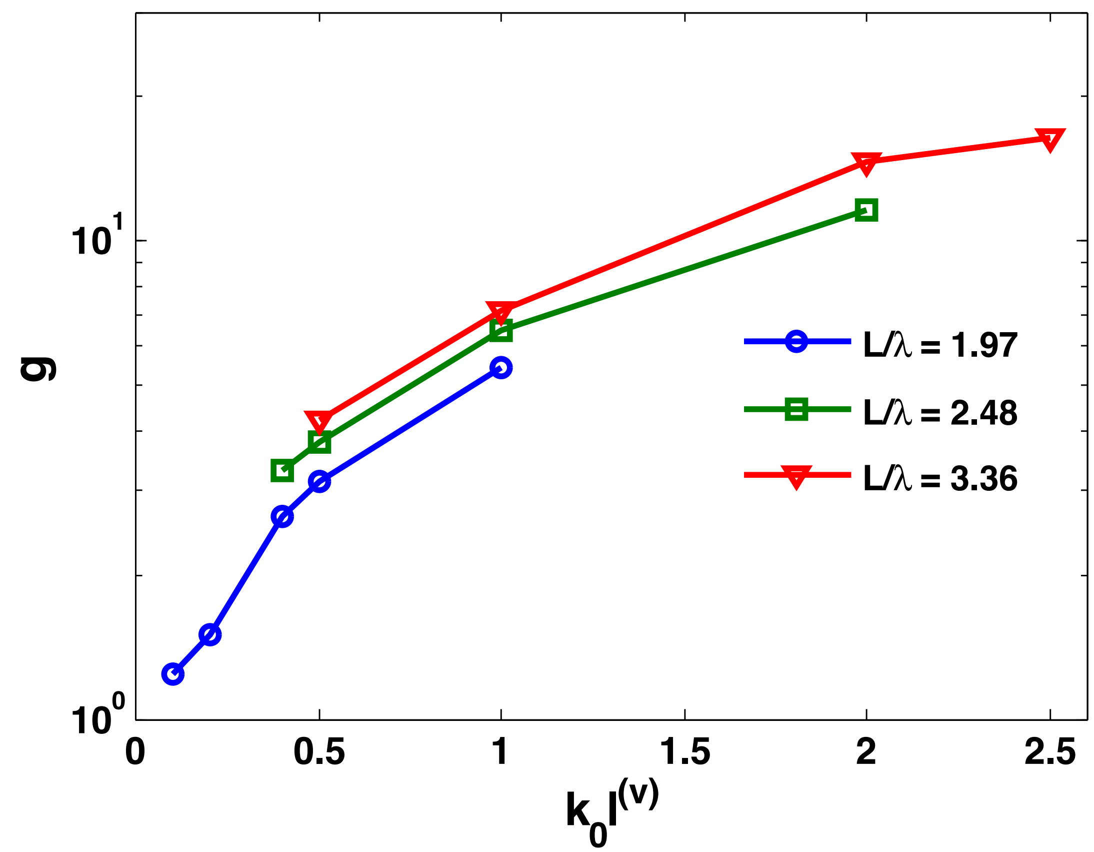

Scaling behaviour - phase transition
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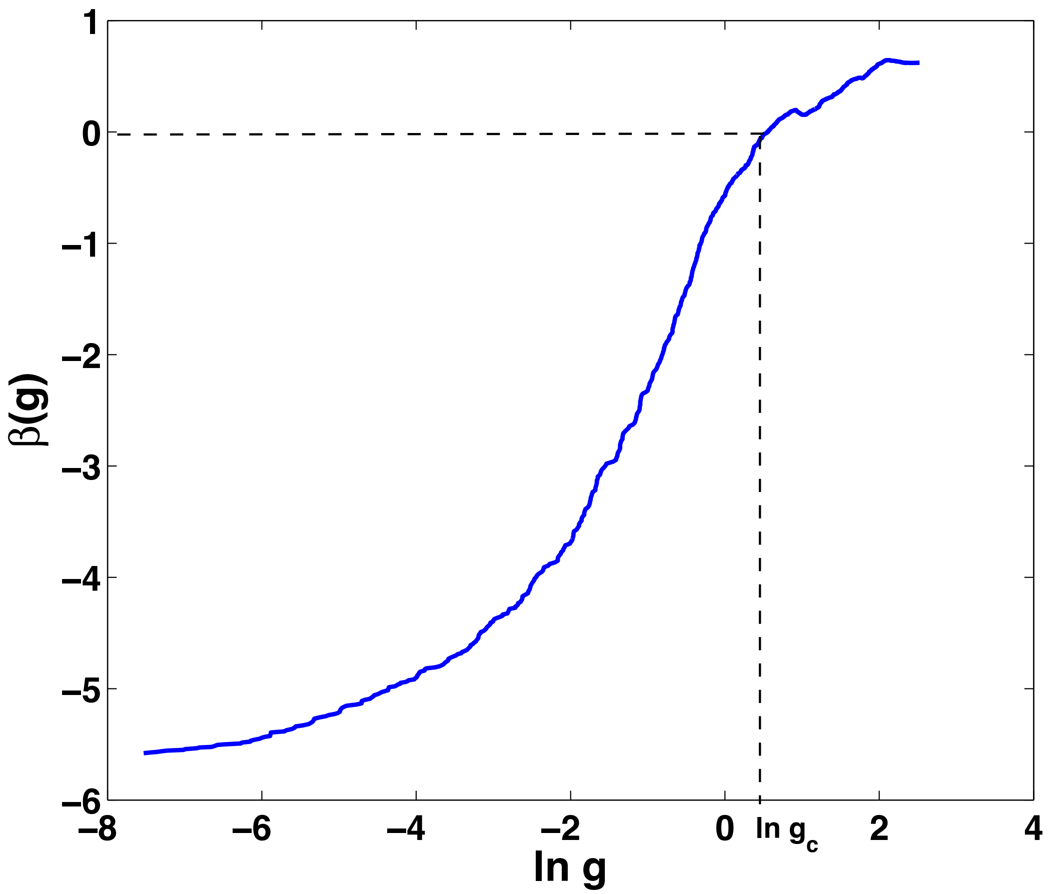
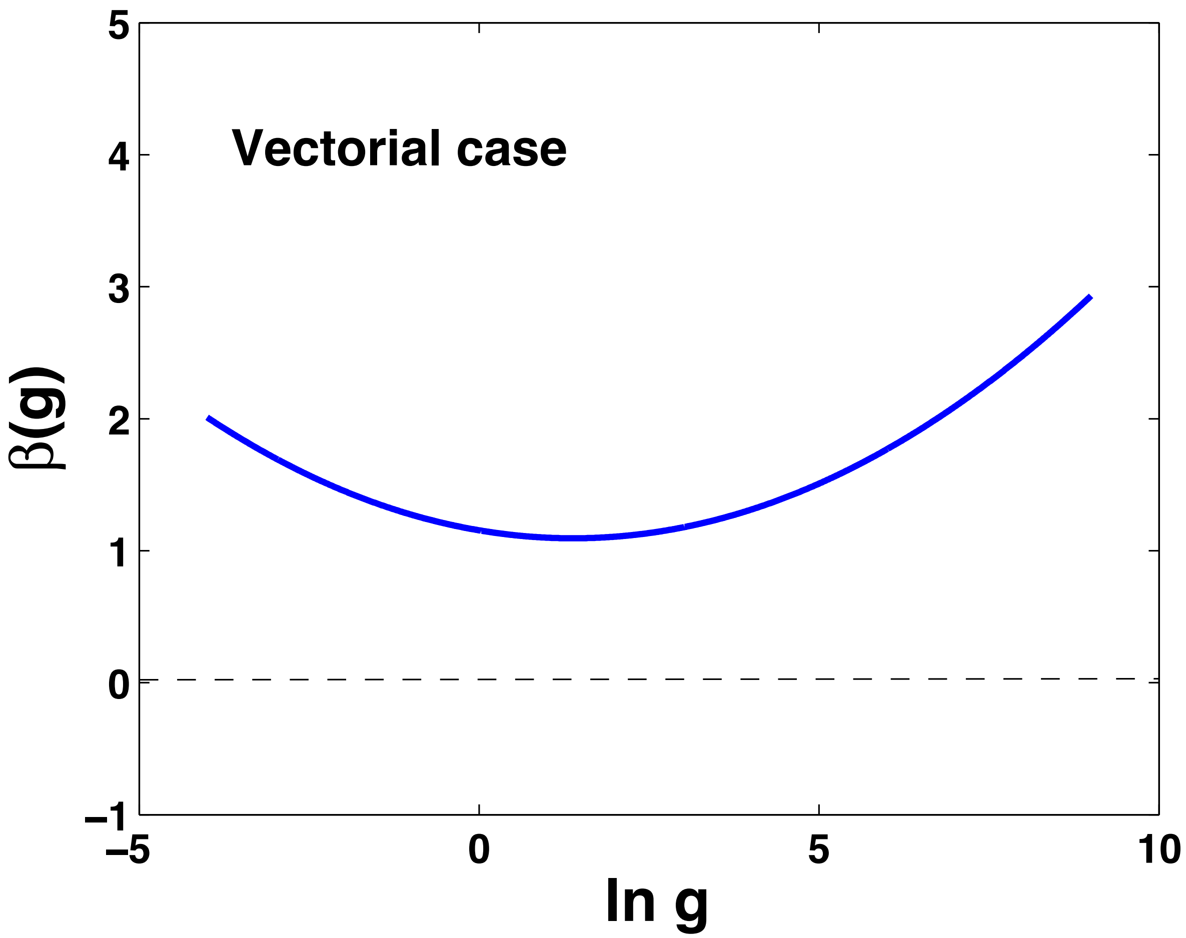
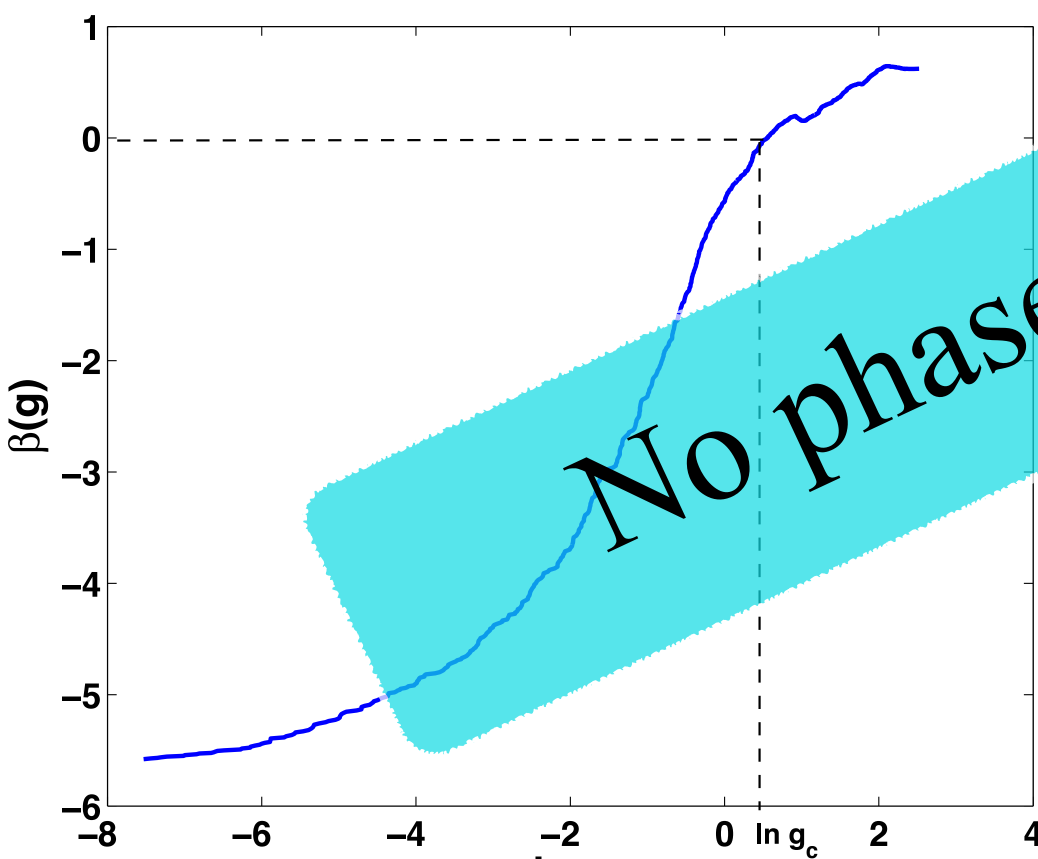


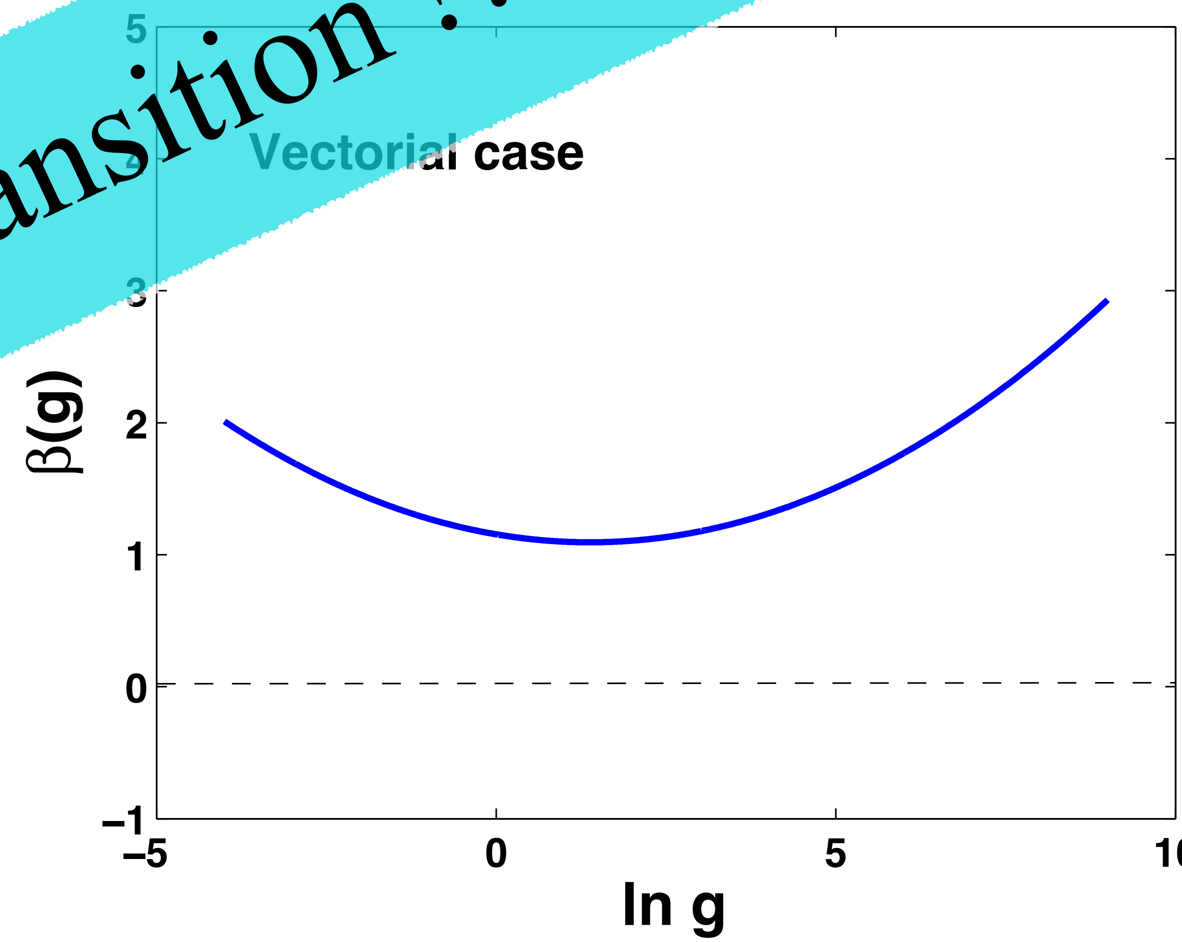


Scaling behaviour - phase transition
to make things more complicated ….

Vector case - polarised waves

No phase transition !?





Conclusion - Summary

- Study of the scaling properties of the Non Hermitian Euclidean random Hamiltonian

$$H_e = \left(\hbar\omega_0 - i\frac{\hbar\Gamma_0}{2}\right)\sum_{i=1}^{N}|e_i\rangle\langle e_i| + \frac{\hbar\Gamma_0}{2}\sum_{i\neq j}V_{ij}\Delta_i^+\Delta_j^-$$

with $V_{ij} = \beta_{ij} - i\gamma_{ij}$

- H_e accounts for **cooperative** properties of the atomic gas (Super- and Sub-radiance). It also depends on the disorder.
- The radiation pattern is well accounted by the part γ_{ij} of the interaction.
- The distribution of eigenvalues of γ_{ij} exhibits scaling properties but there is ***no indication of the existence of a phase transition*** driven either by disorder or interactions.

- The interplay between disorder and cooperative effects depend upon the space dimensionality.
- For $d = 2,3$, there is a ***crossover*** between a delocalised (Wigner-Weisskopf) regime and a behaviour driven by cooperative effects (eventually Dicke regime)
- For $d = 1$, there is no single atom limit.
- The eigenvalue distribution of the whole Hamiltonian H_e exhibits also scaling properties. ***A critical behaviour is obtained for scalar waves*** using a conveniently defined Thouless conductance for that problem.
- ***The critical behaviour disappears for vector waves.***
- The nature and universality of this transition is still unclear.
- Set of new experimental efforts to probe the interplay of disorder and cooperative effects (R. Kaiser, A. Browaeys, M. Havey,…)

Thank you for your attention.