Mesoscopic Physics of Electrons and Photons

Based on *Mesoscopic physics of electrons and photons*, by Eric Akkermans and Gilles Montambaux, Cambridge University Press, 2007

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2017 AMO Summer School, IAMS, Academia Sinica, National Taiwan University

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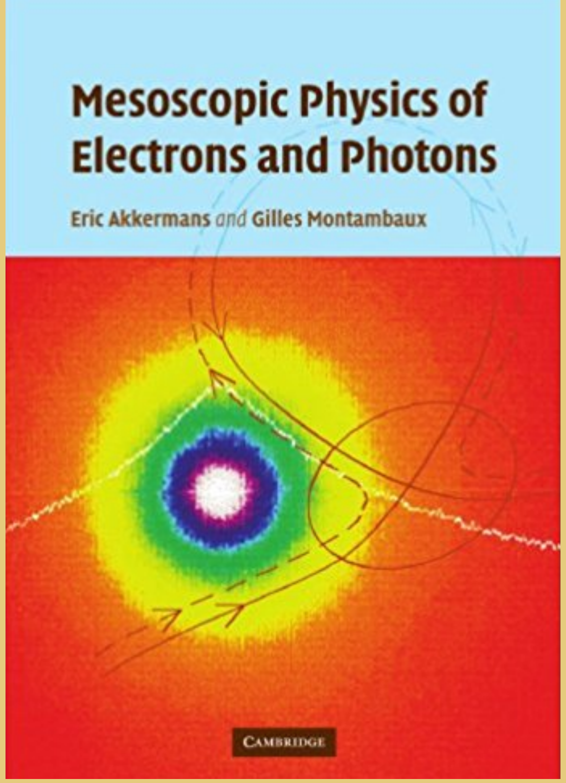

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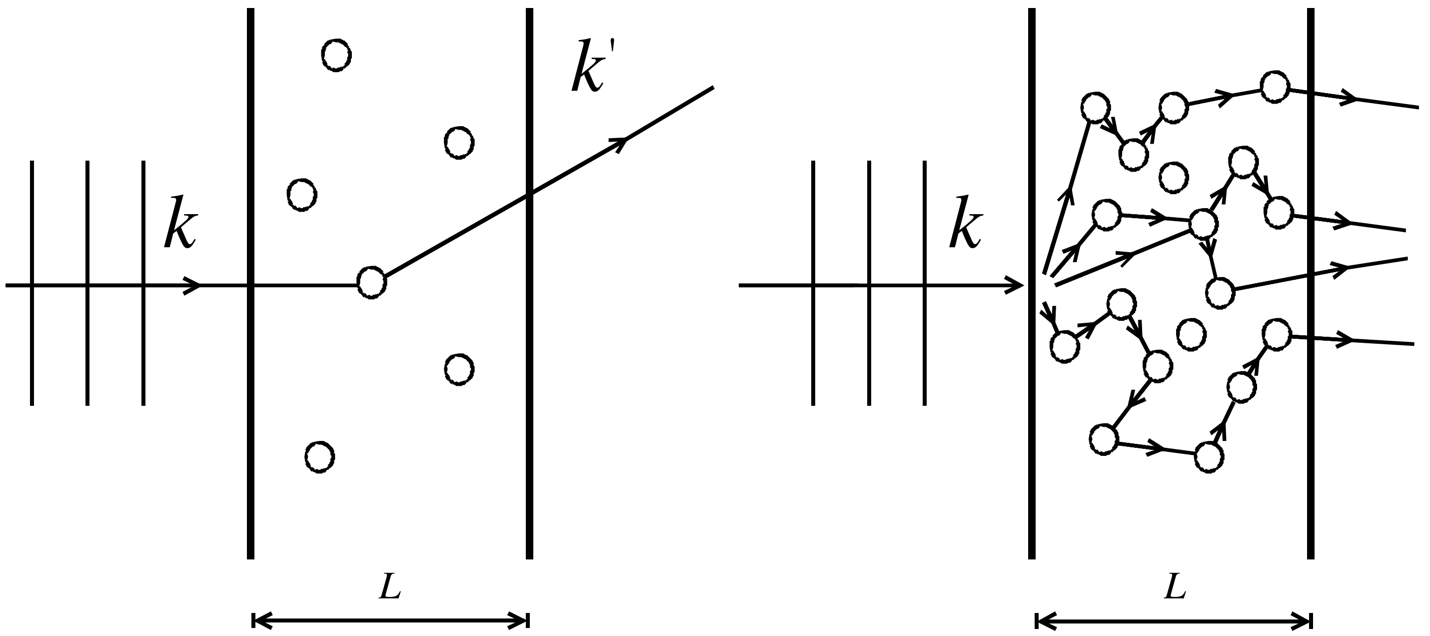
Part 1

Introduction to mesoscopic physics

- The Aharonov-Bohm effect in disordered conductors.
- Phase coherence and effect of disorder.
- Average coherence: Sharvin^2 effect and coherent backscattering.
- Phase coherence and self-averaging: universal fluctuations.
- Classical probability and quantum crossings.

Multiple scattering of waves



2 characteristic lengths:

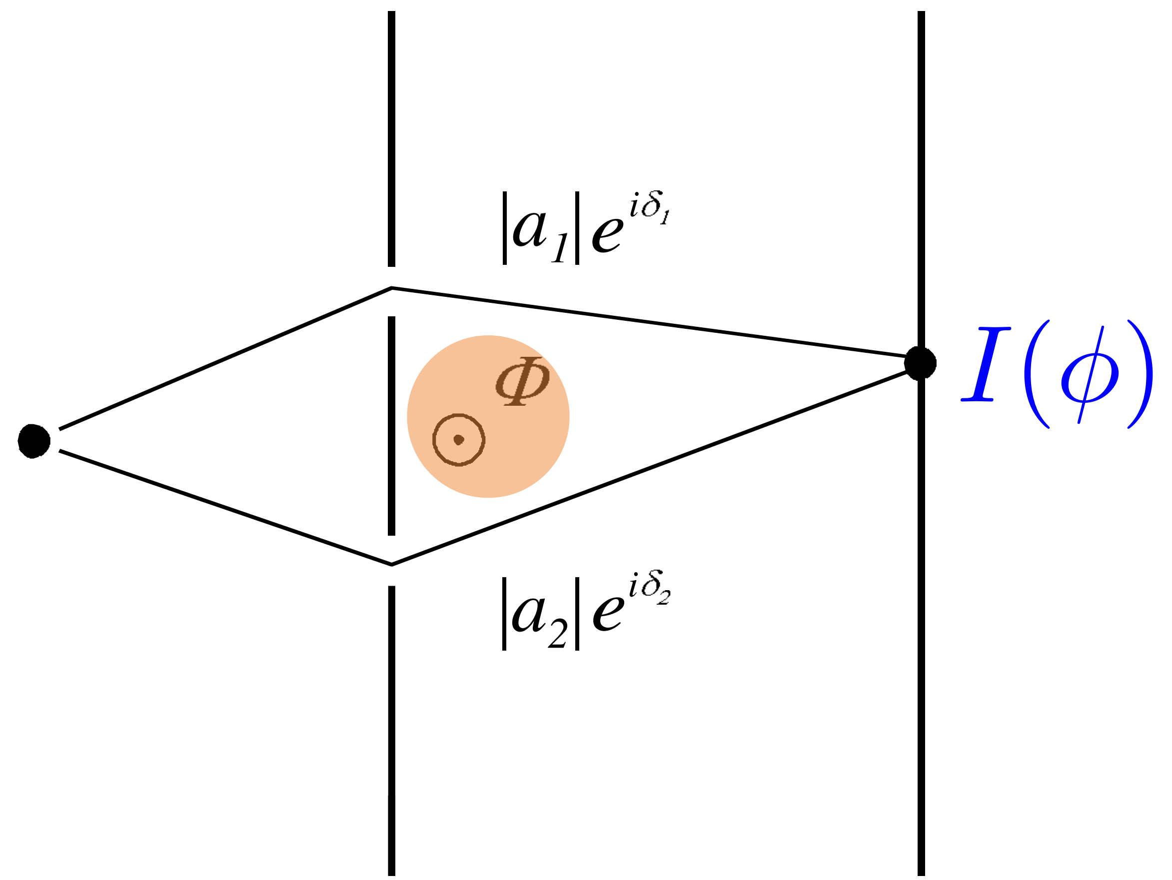
Wavelength: $\lambda_F = k_F^{-1}$

Elastic mean free path: l

Weak disorder $\lambda_F \ll l$: independent scattering events

Aharonov-Bohm effect in disordered metals

No magnetic field on the electrons : no Lorentz force and no orbital motion.



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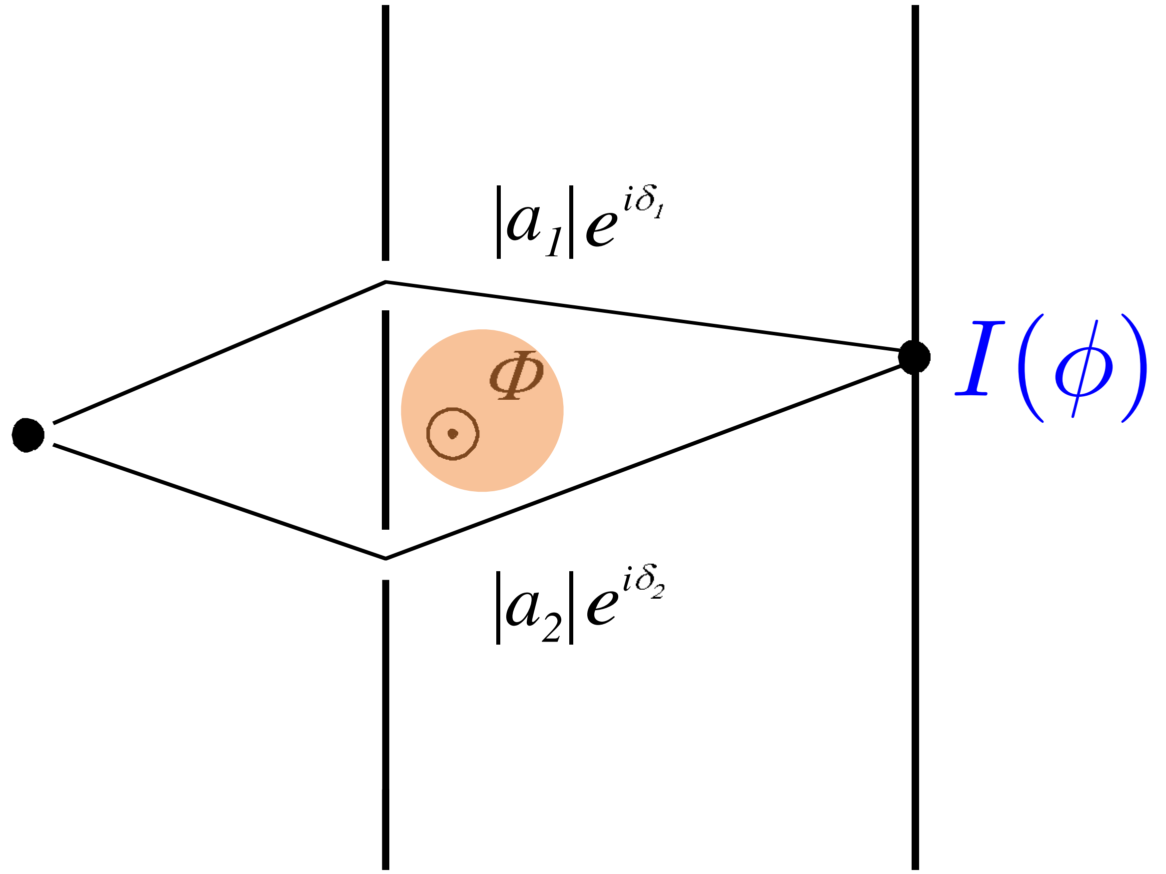

The quantum amplitudes $a_{1,2} = |a_{1,2}|e^{i\delta_{1,2}}$ have phases:

$$\delta_1 = \delta_1^{(0)} - \frac{e}{\hbar}\int_1 \mathbf{A}.d\mathbf{l} \quad \text{and} \quad \delta_2 = \delta_2^{(0)} - \frac{e}{\hbar}\int_2 \mathbf{A}.d\mathbf{l}$$

The intensity $I(\phi)$ is given by

$$I(\phi) = |a_1 + a_2|^2 = |a_1|^2 + |a_2|^2 + 2|a_1 a_2| \cos(\delta_1 - \delta_2)$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\delta_1 - \delta_2)$$

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The phase difference $\Delta\delta(\phi) = \delta_1 - \delta_2$ is modulated by the magnetic flux ϕ :

$$\Delta\delta(\phi) = \delta_1^{(0)} - \delta_2^{(0)} + \frac{e}{\hbar} \oint \mathbf{A}.d\mathbf{l} = \Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0}$$

where $\phi_0 = h/e$ is the quantum of magnetic flux.

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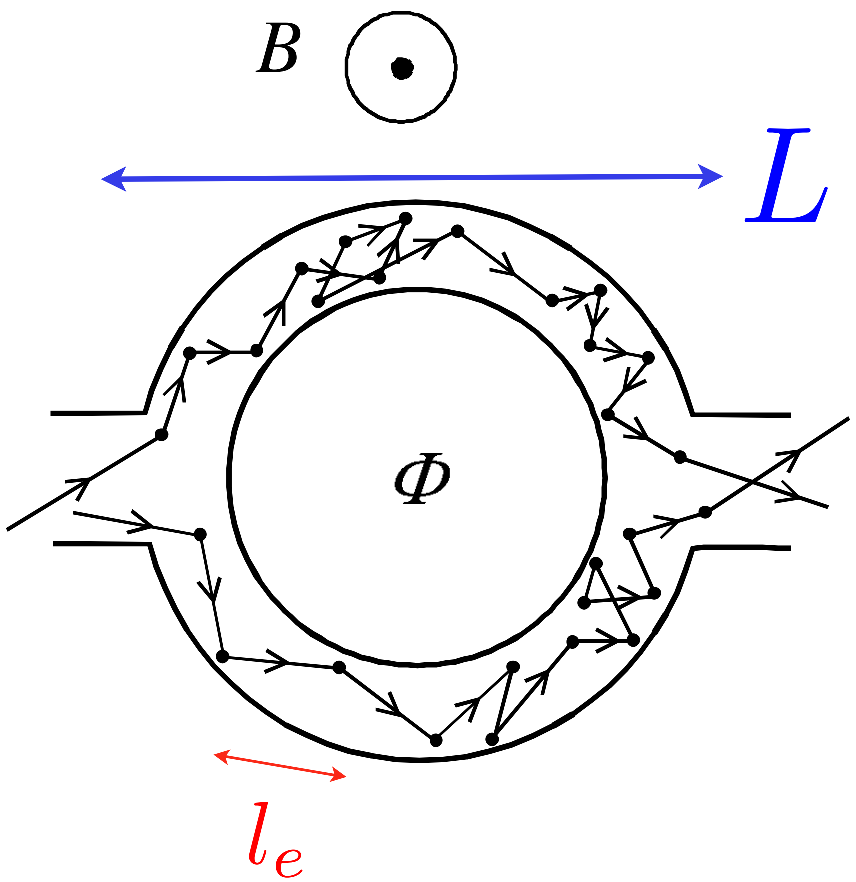
There is a continuous change of the state of interference:

Aharonov-Bohm effect (1959).

Implementation in metals : the conductance $G(\phi)$ is the analog of the intensity.



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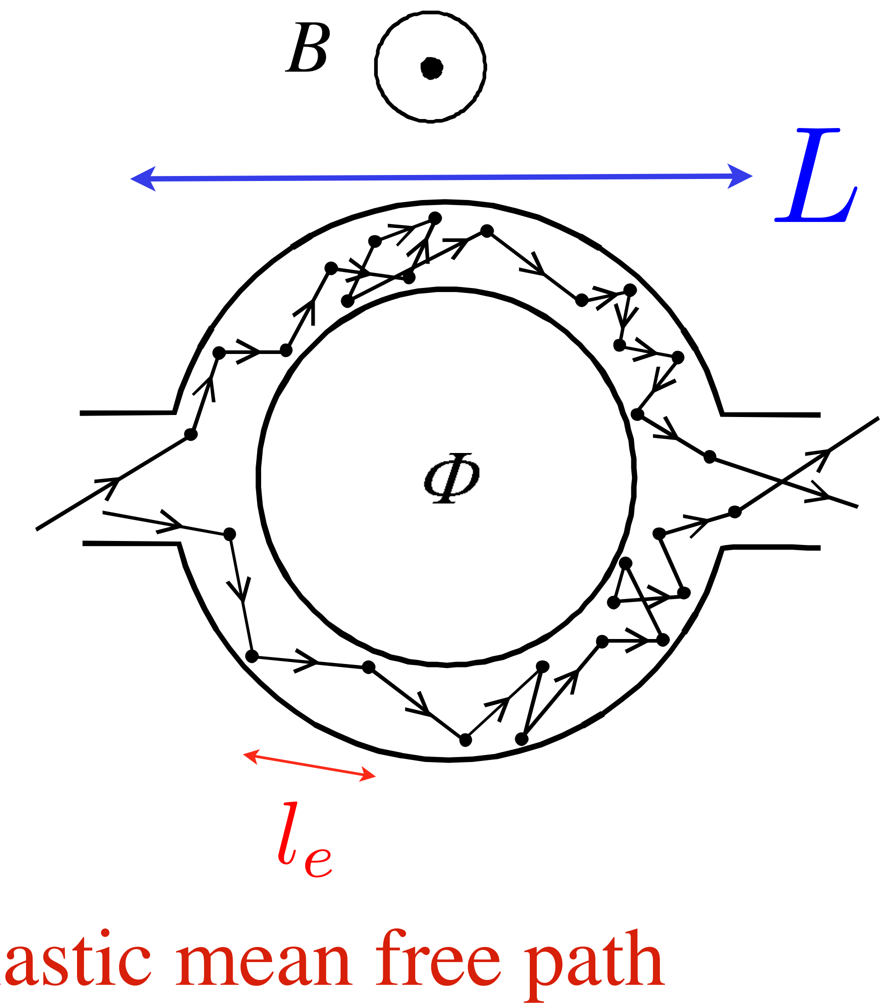

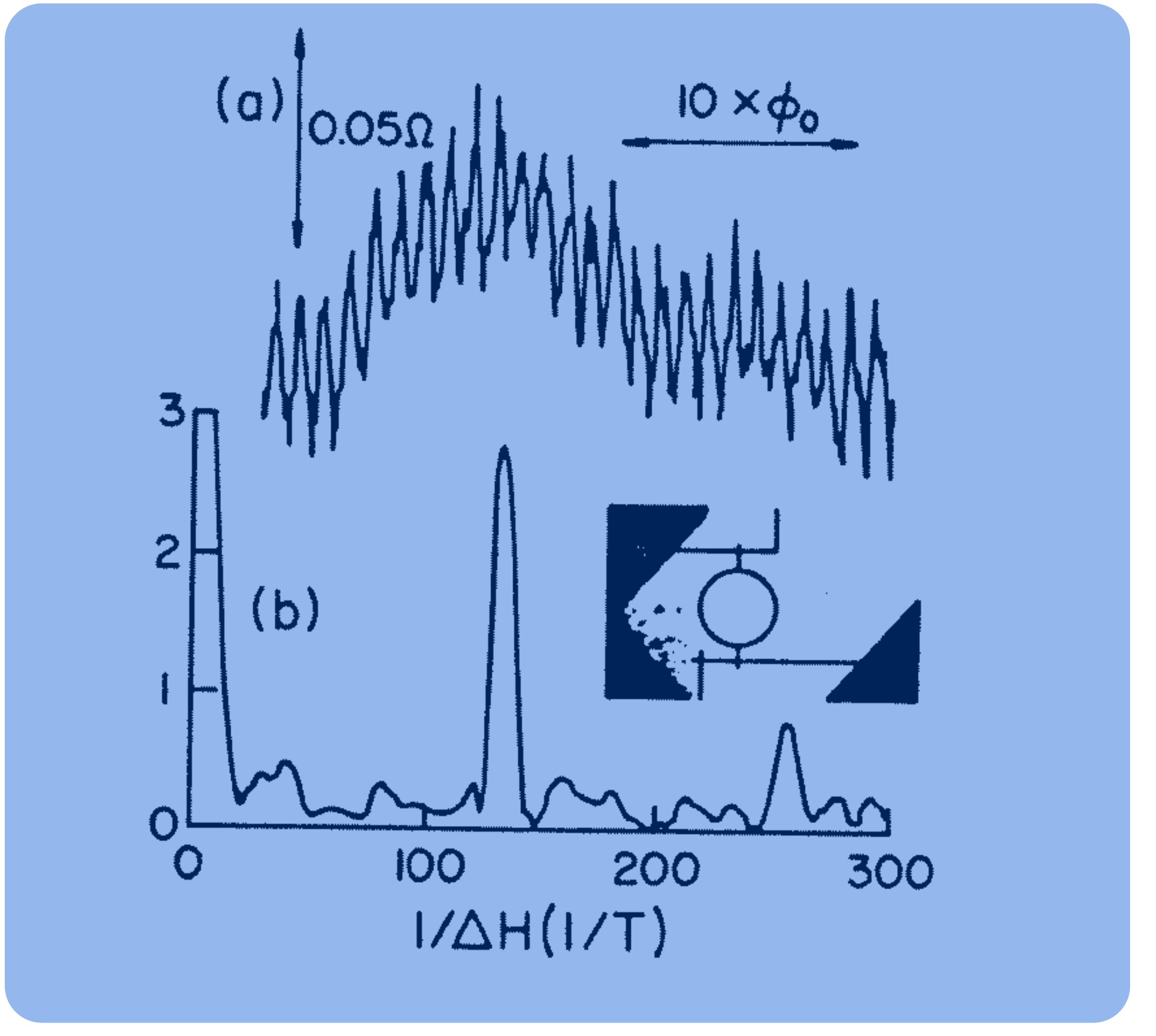

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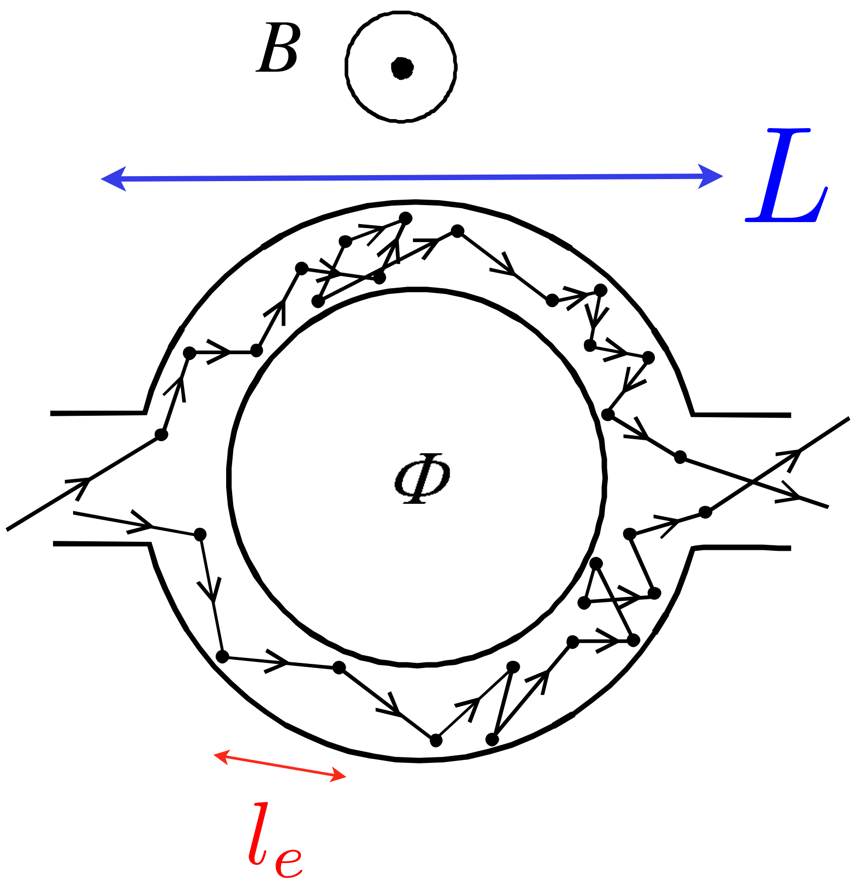
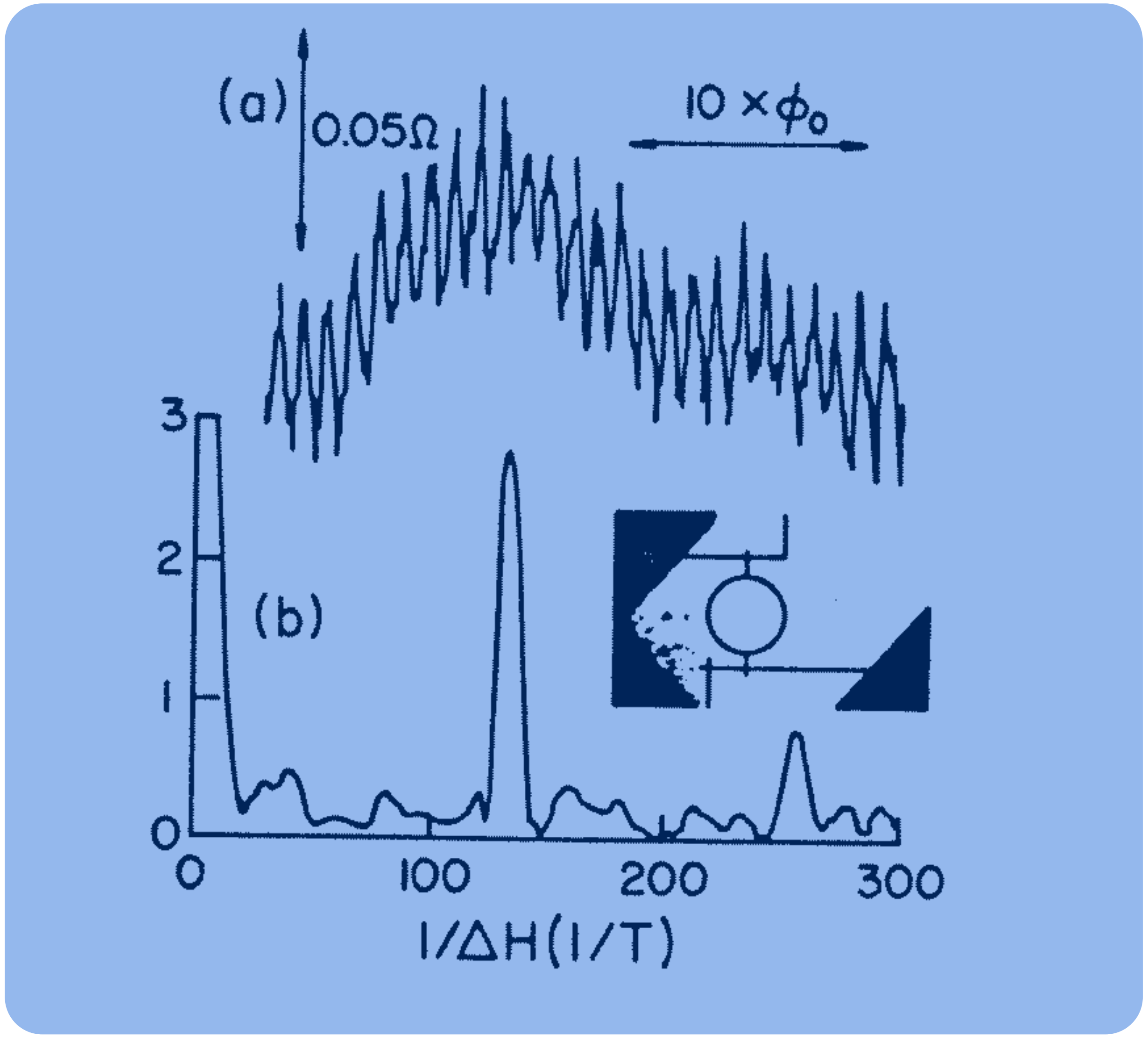


elastic mean free path



Webb et al. 1985

$$G(\phi) = G_0 + \delta G \cos(\Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0})$$

Phase coherent effects subsist in disordered metals.

Reconsider the Drude theory.

Phase coherence and effect of disorder

The *Webb* experiment has been realized on a ring of size $1\,\mu m$.

Phase coherence and effect of disorder

The *Webb* experiment has been realized on a ring of size $1\,\mu m$.

For a macroscopic normal metal, coherent effects are washed out.

⇨ It must exist a characteristic length L_ϕ called phase coherence length beyond which all coherent effects disappear.

Vanishing of quantum coherence results from the existence of incoherent and irreversible processes associated to the coupling of electrons to their surrounding (additional degrees of freedom) :

Coupling to a bath of excitations: thermal excitations of the lattice (phonons)

Chaotic dynamical systems (large recurrence times, Feynman chain)

Impurities with internal degrees of freedom (magnetic impurities)

Electron-electron interactions,....

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The understanding of decoherence is difficult. It is a great challenge in quantum mesoscopic physics. The phase coherence length L_ϕ accounts in a generic way for decoherence processes.

The observation of coherent effects requires

$$L << L_\phi$$

Average coherence and multiple scattering

What is the role of elastic disorder ? Does it erase coherent effects ?

Phase coherence leads to interference effects for a *given realization of disorder.*

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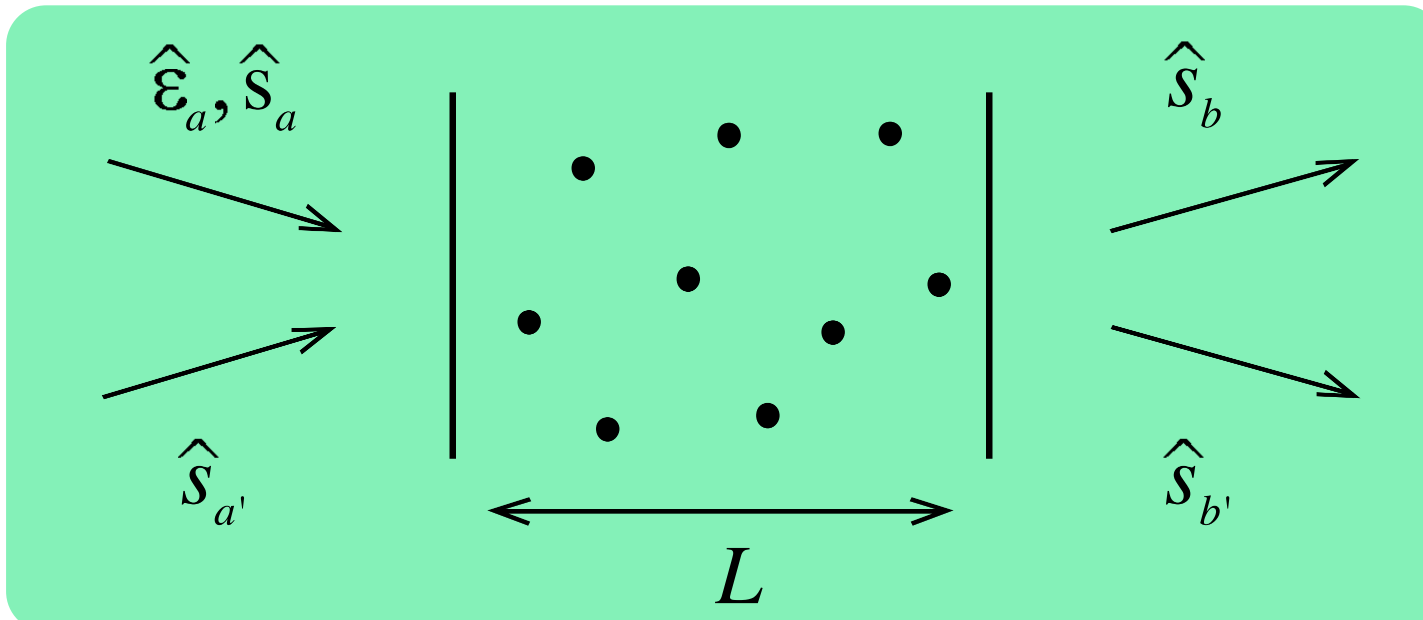
$$G(\phi) = G_0 + \delta G \cos(\Delta\delta^{(0)} + 2\pi \frac{\phi}{\phi_0})$$

$$\Rightarrow \quad \langle G(\phi) \rangle = G_0$$

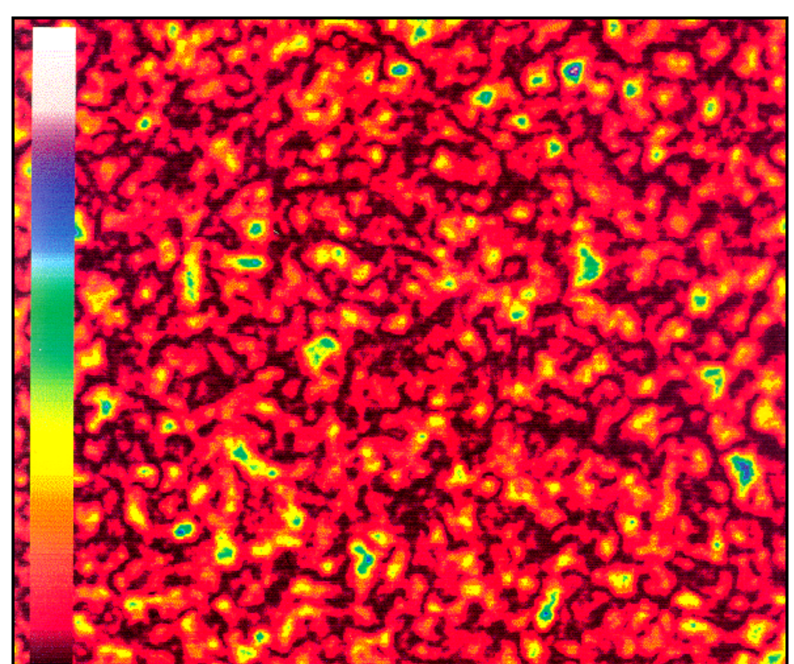
Disorder seems to erase coherent effects....

An analogous problem: *Speckle patterns in optics*

Consider the elastic multiple scattering of light transmitted through a fixed disorder configuration.

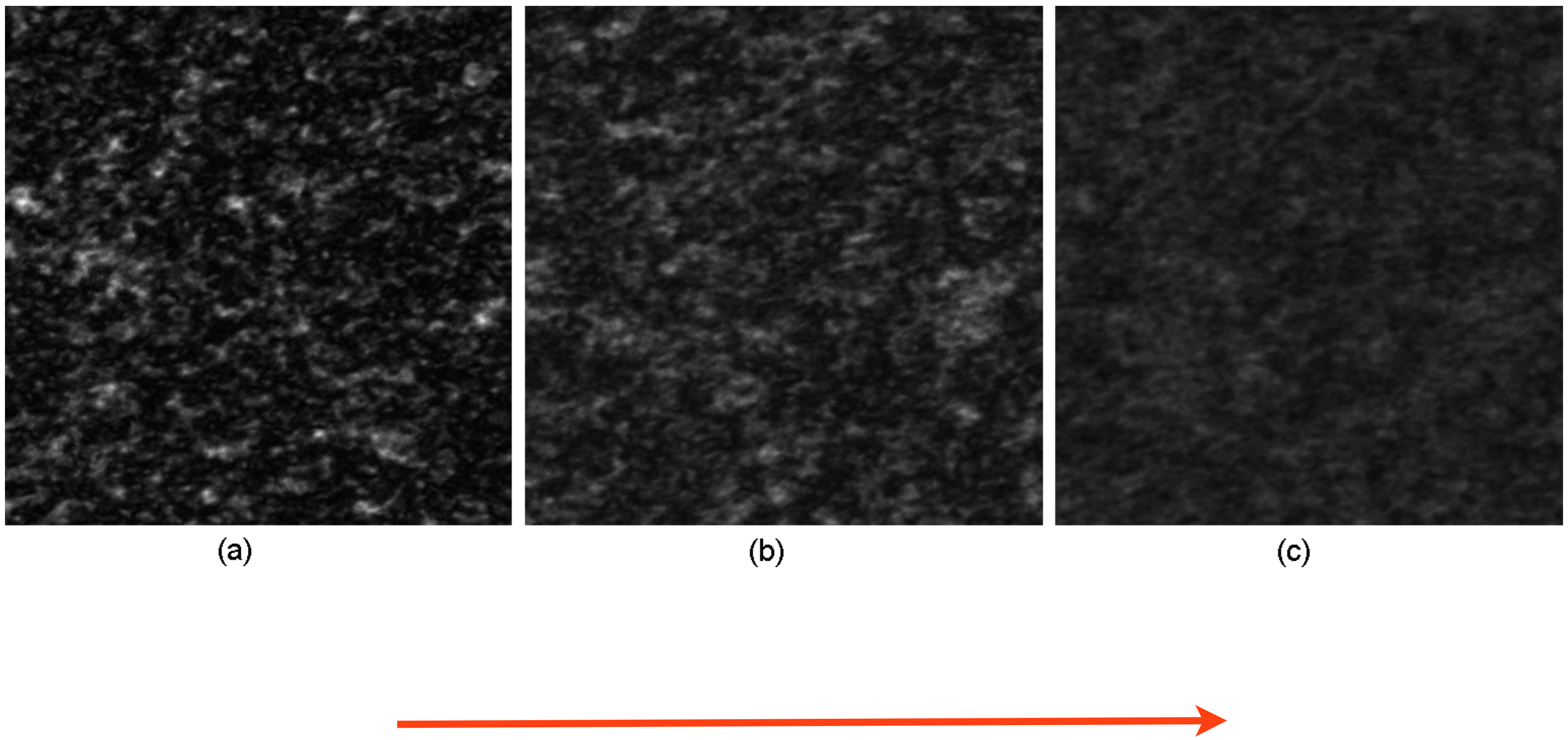


Outgoing light builds a speckle pattern *i.e.*, an interference picture:



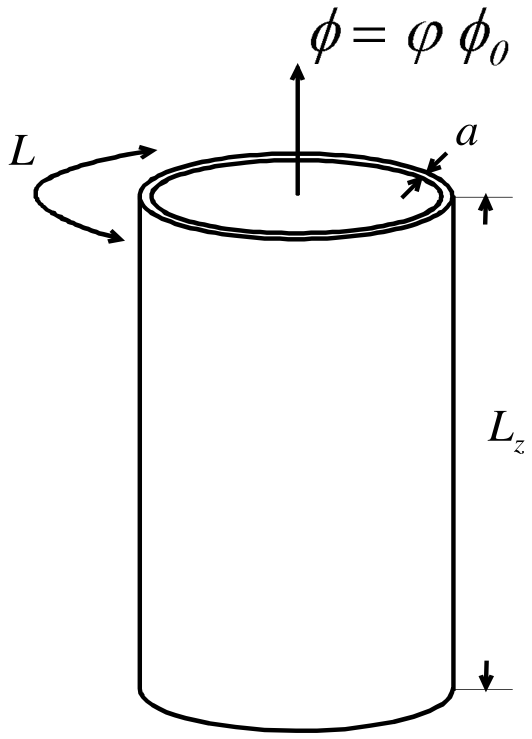
Averaging over disorder erases the speckle pattern:

Integration over the motion of the scatterers leads to self-averaging



Time averaging

The Sharvin^2 experiment



Experiment analogous to that of *Webb* but performed on a hollow cylinder of height larger than L_ϕ pierced by a Aharonov-Bohm flux.

Ensemble of rings identical to those of *Webb* but incoherent between themselves.

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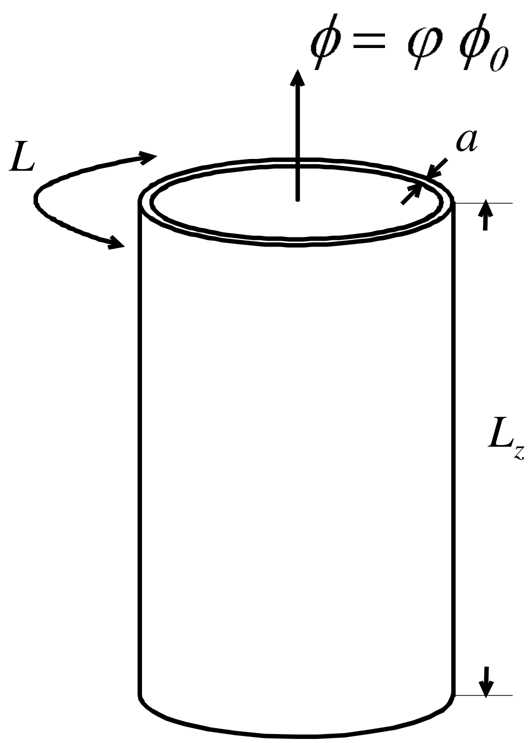

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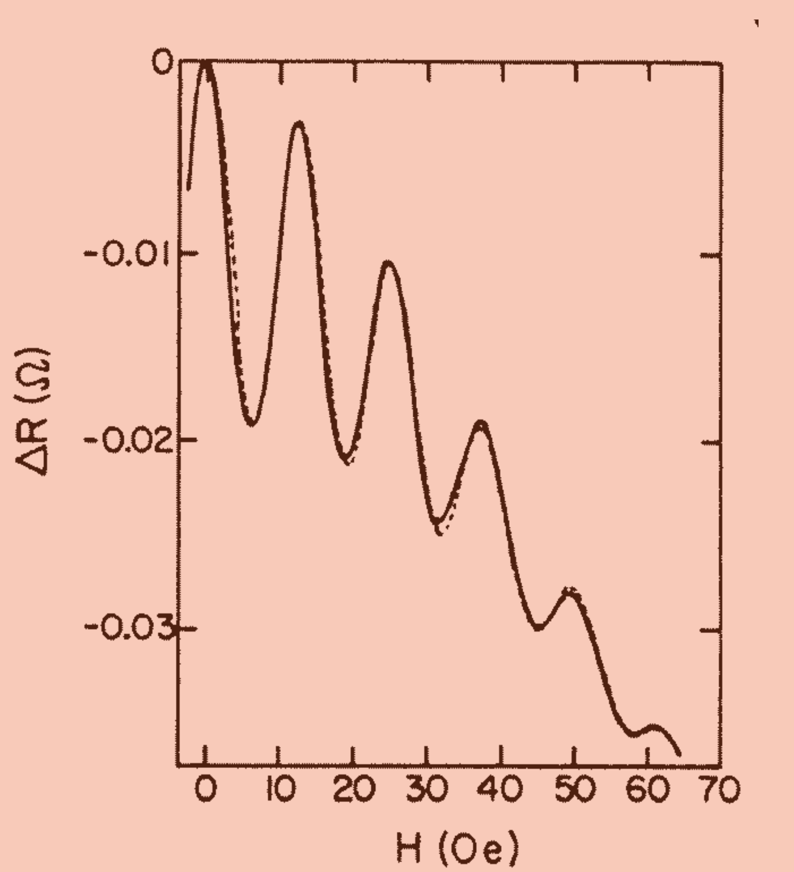

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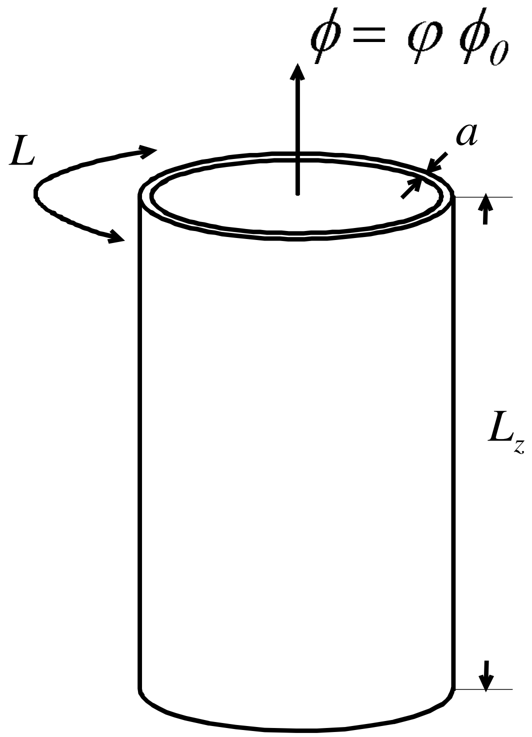

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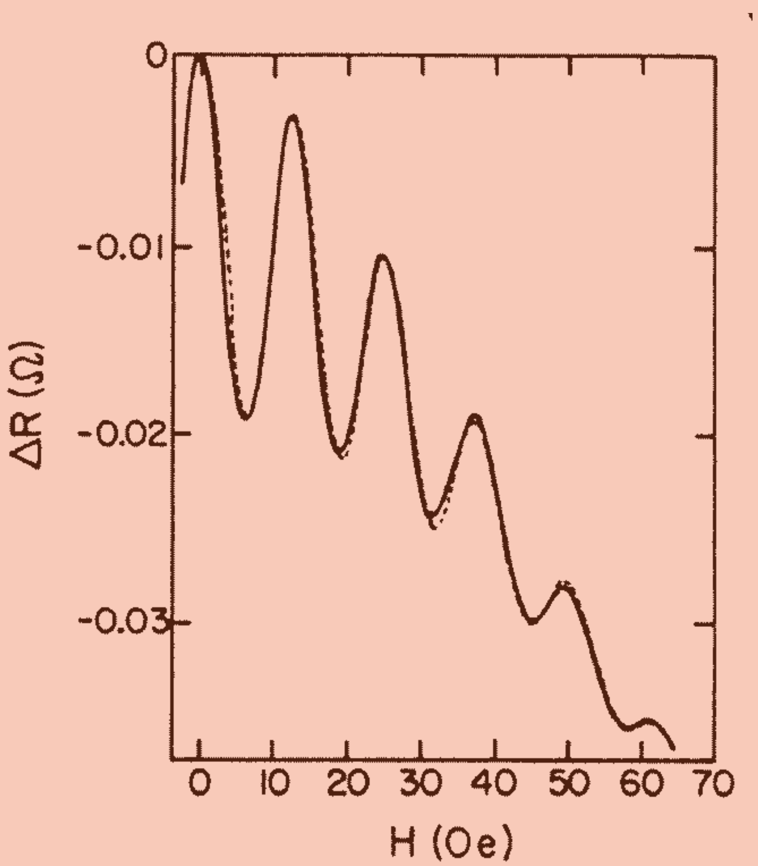

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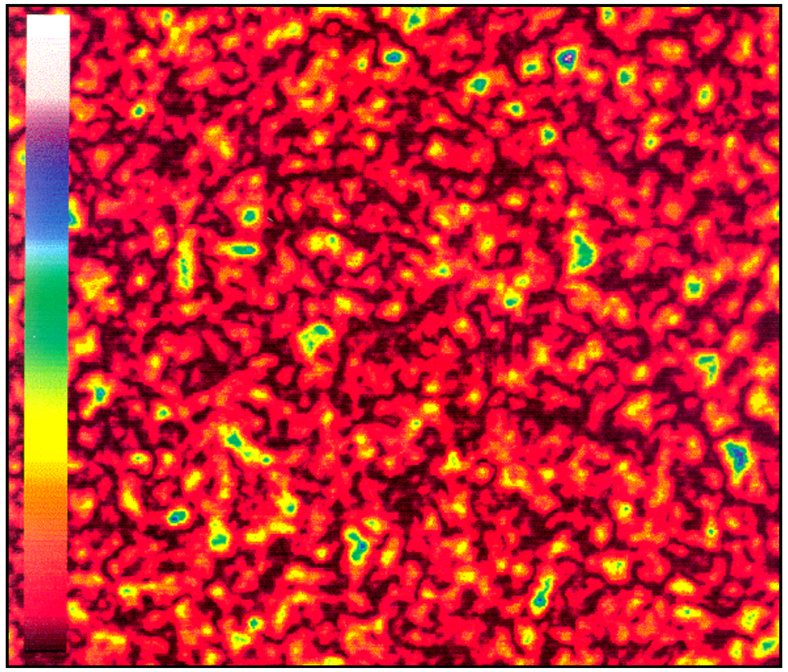
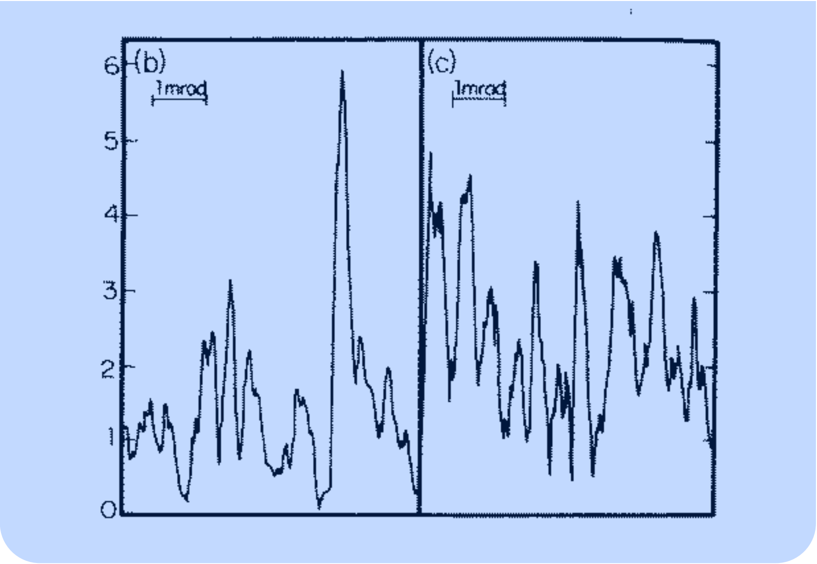
After all, disorder does not seem to erase coherent effects, but to modify them....

What about speckle patterns ?





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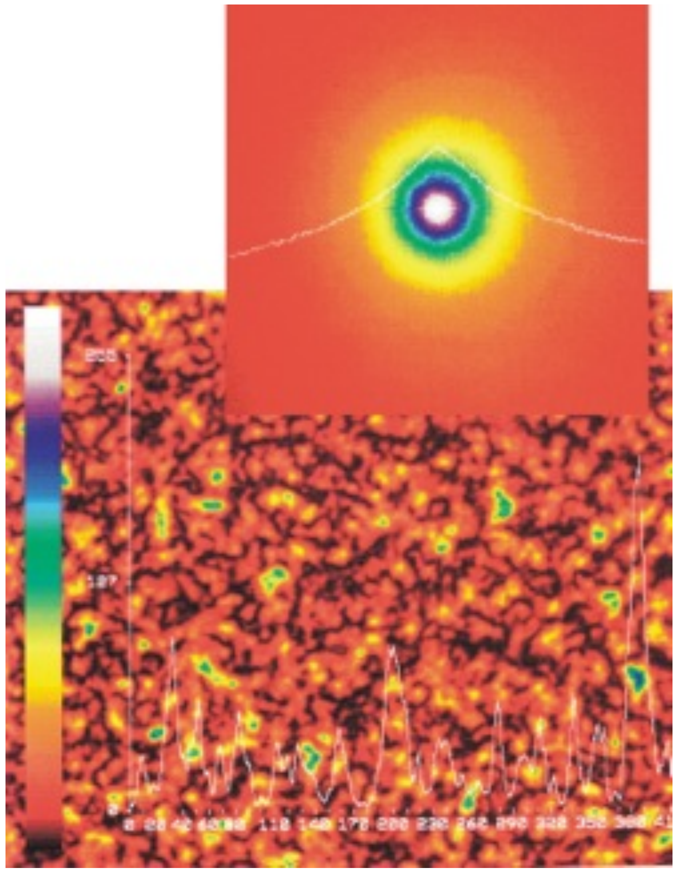
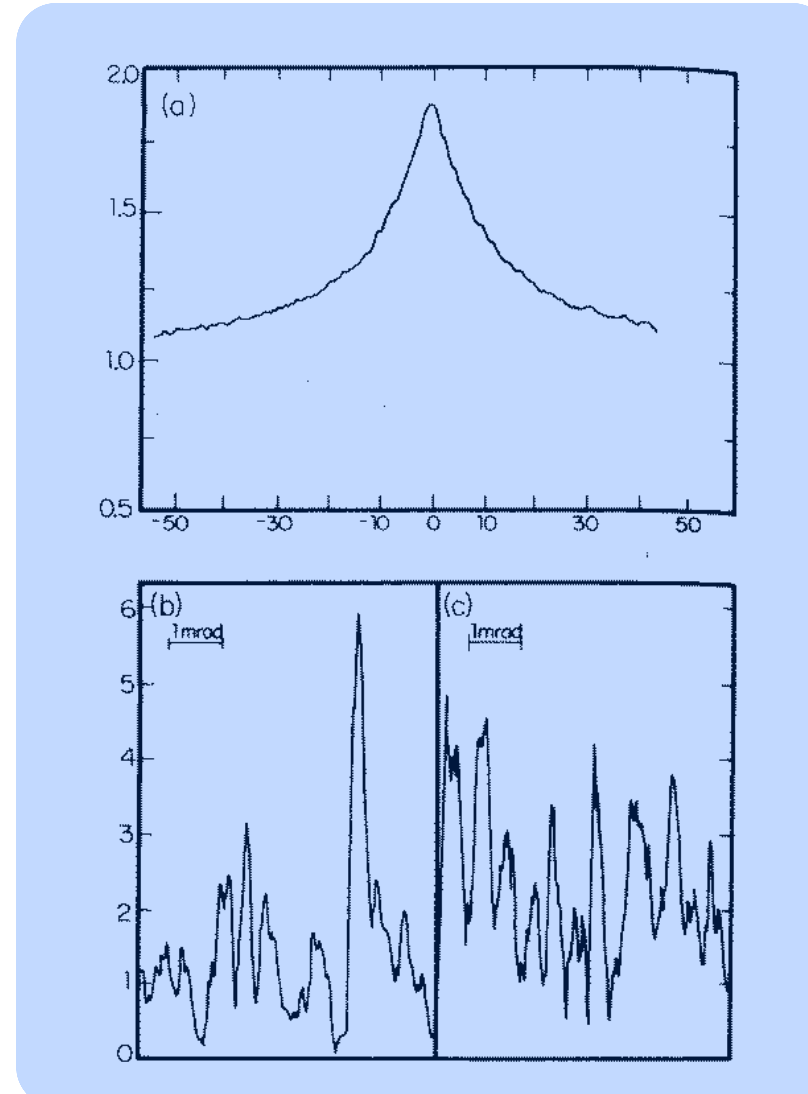
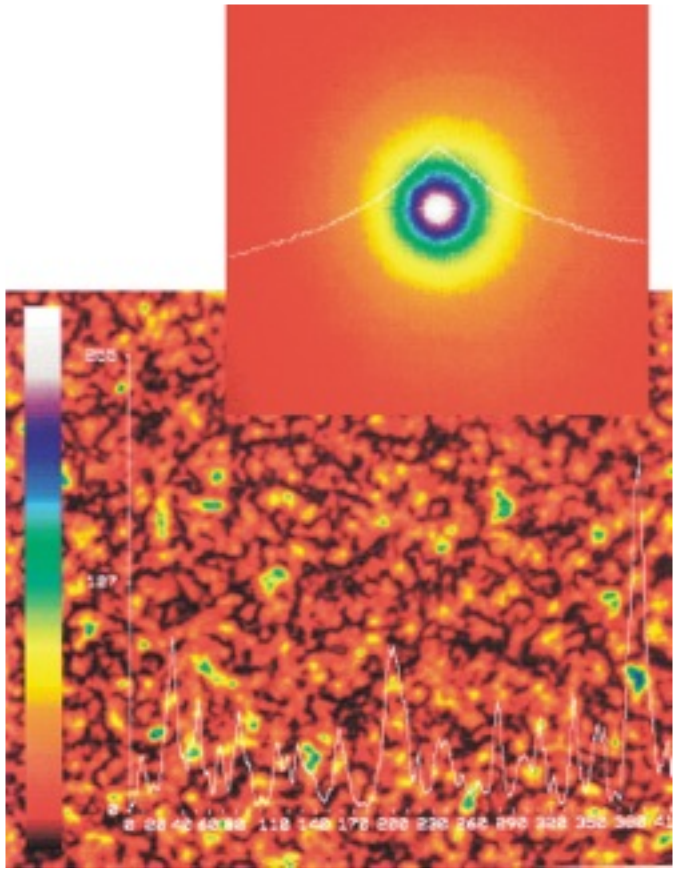
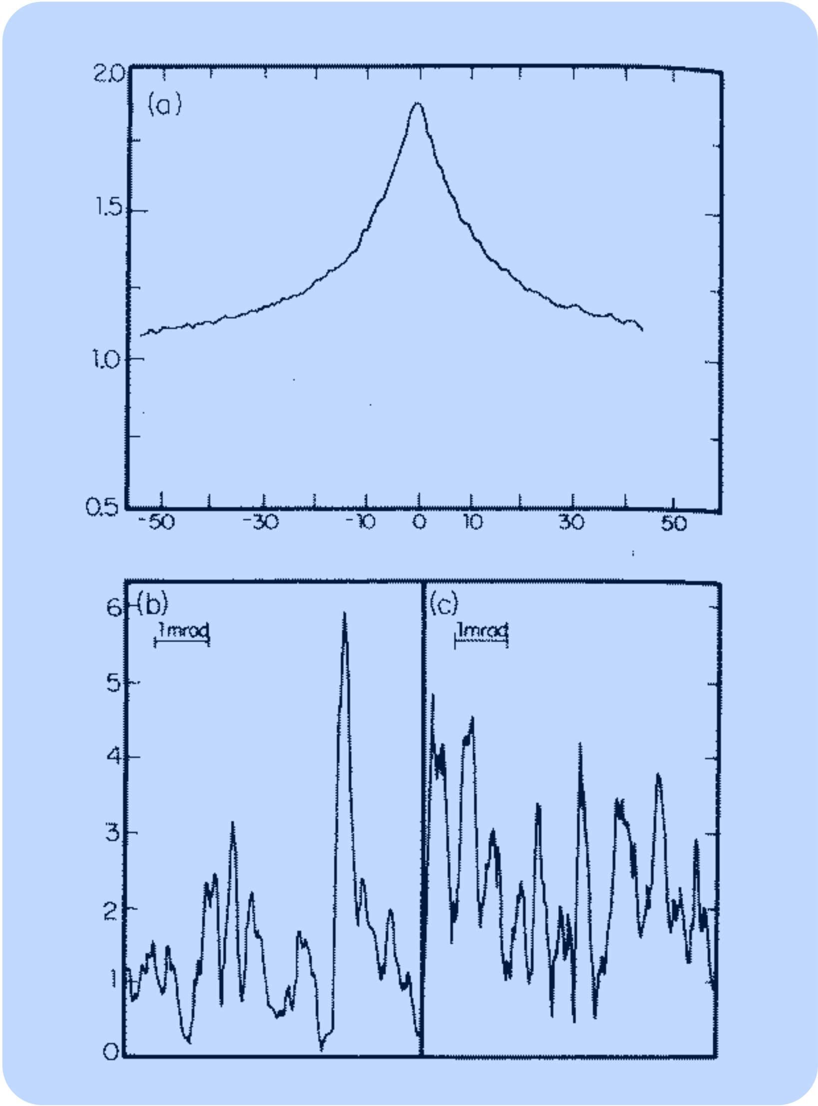




Averaging over disorder does not produce incoherent intensity only, but also an angular dependent part, the coherent backscattering, which is a coherence effect. We may conclude:

What about speckle patterns ?

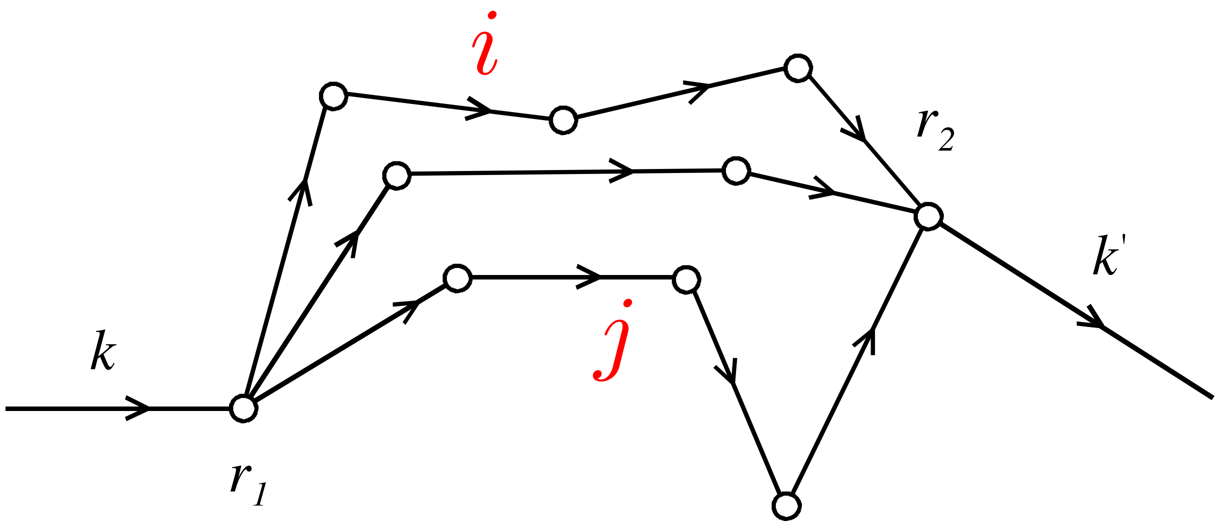




Averaging over disorder does not produce incoherent intensity only, but also an angular dependent part, the <u>coherent backscattering</u>, which is a coherence effect. We may conclude:

Elastic disorder is not related to decoherence : disorder does not destroy phase coherence and does not introduce irreversibility.

How to understand average coherent effects ?



Complex amplitude $A(\mathbf{k},\mathbf{k}')$ associated to the multiple scattering of a wave (electron or photon) incident with a wave vector $\mathbf{k}$ and outgoing with $\mathbf{k}'$

$$A(\mathbf{k},\mathbf{k}') = \sum_{\mathbf{r_1},\mathbf{r_2}} f(\mathbf{r_1},\mathbf{r_2}) e^{i(\mathbf{k}.\mathbf{r_1}-\mathbf{k}'.\mathbf{r_2})}$$

the complex amplitude $f(\mathbf{r_1},\mathbf{r_2}) = \sum_j |a_j| e^{i\delta_j}$ describes the propagation of the wave between $\mathbf{r_1}$ and $\mathbf{r_2}$.

The corresponding intensity is

$$|A(\mathbf{k},\mathbf{k}')|^2 = \sum_{\mathbf{r_1},\mathbf{r_2}} \sum_{\mathbf{r_3},\mathbf{r_4}} f(\mathbf{r_1},\mathbf{r_2}) f^*(\mathbf{r_3},\mathbf{r_4}) e^{i(\mathbf{k}.\mathbf{r_1}-\mathbf{k}'.\mathbf{r_2})} e^{-i(\mathbf{k}.\mathbf{r_3}-\mathbf{k}'.\mathbf{r_4})}$$

with

$$f(\mathbf{r_1},\mathbf{r_2}) f^*(\mathbf{r_3},\mathbf{r_4}) = \sum_{j,j'} a_j(\mathbf{r_1},\mathbf{r_2}) a^*_{j'}(\mathbf{r_3},\mathbf{r_4}) = \sum_{j,j'} |a_j||a_{j'}| e^{i(\delta_j - \delta_{j'})}$$

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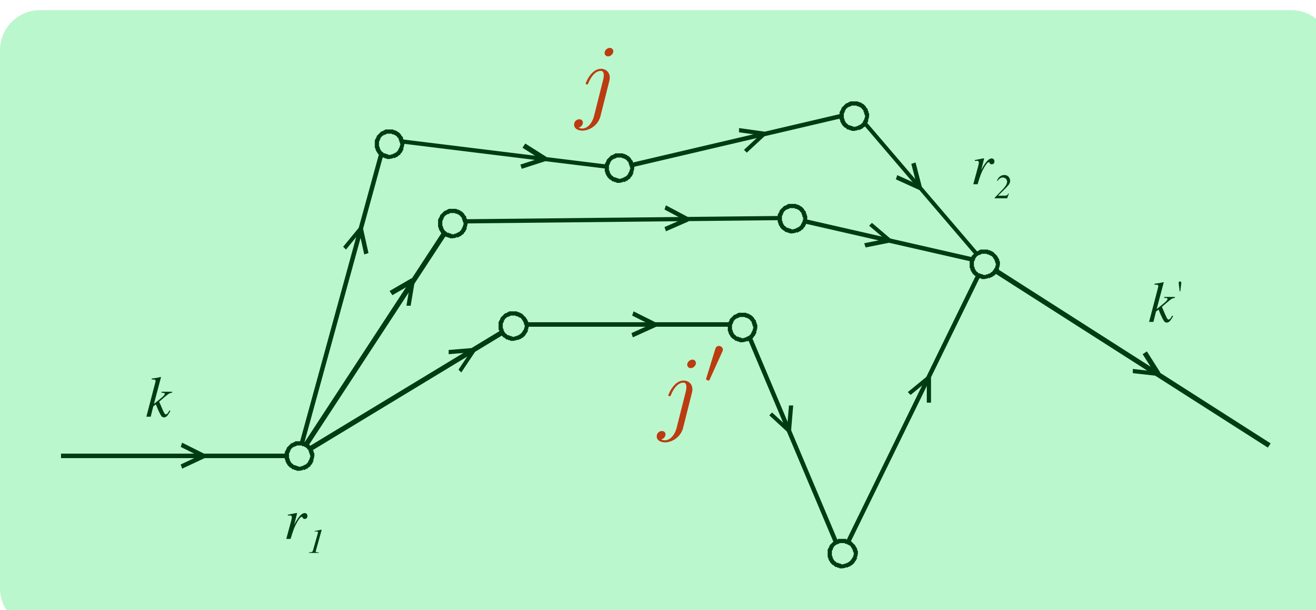
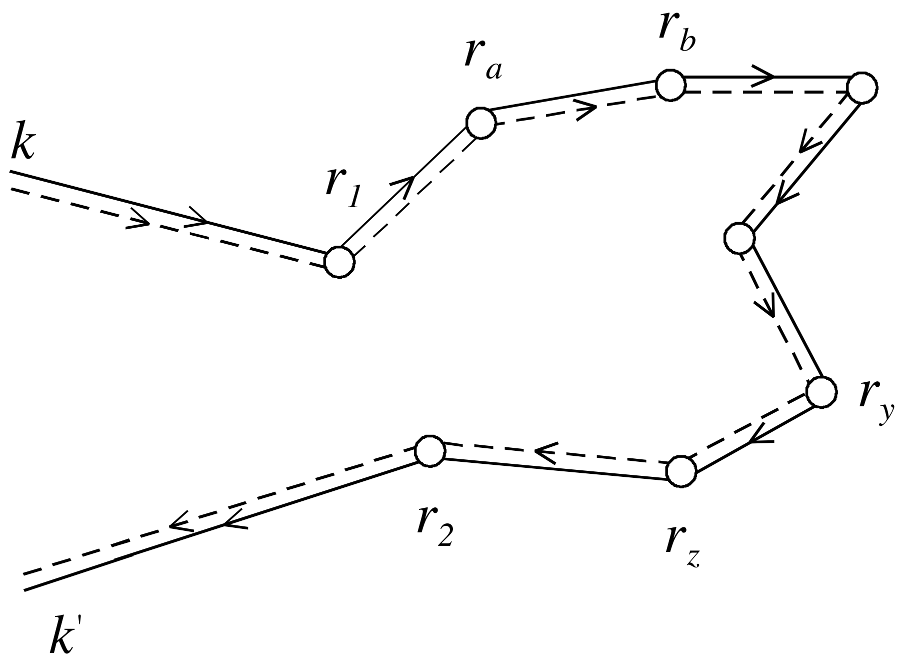


On average over disorder, most contributions to $f\,f^*$ disappear since the dephasing $\delta_j - \delta_{j'} \gg 1$

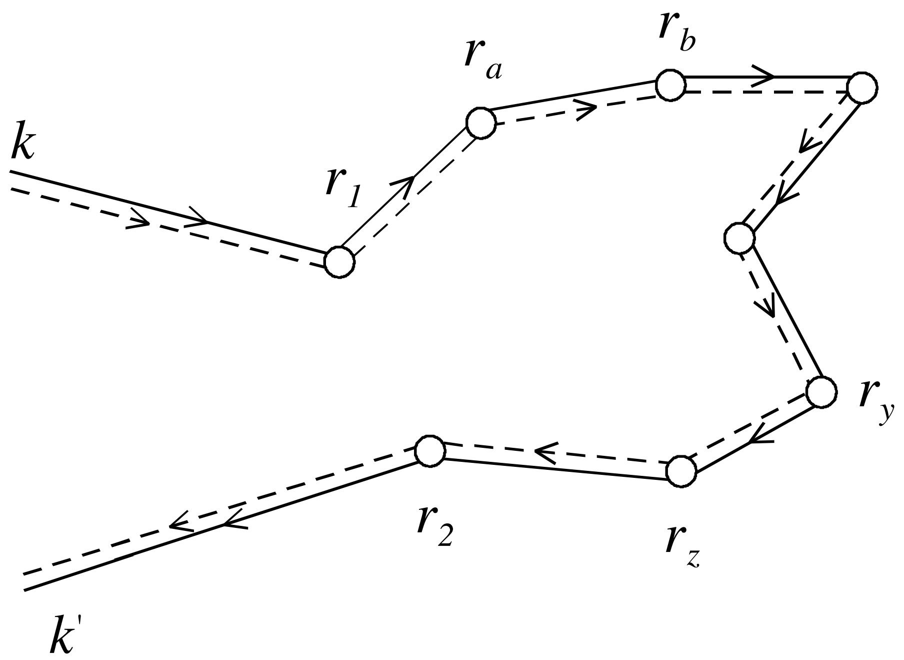
The only remaining contributions to the intensity correspond to terms with zero dephasing, *i.e.*, to identical trajectories.

(a)



$$\mathbf{r_1} \longrightarrow \mathbf{r_a} \longrightarrow \mathbf{r_b} \cdots \longrightarrow \mathbf{r_y} \longrightarrow \mathbf{r_z} \longrightarrow \mathbf{r_2}$$

(a)

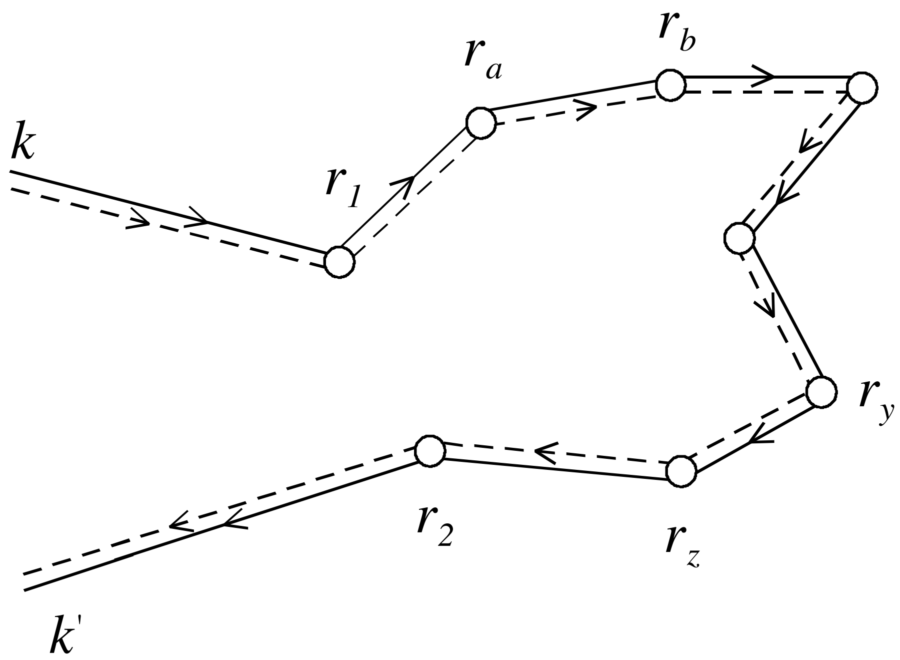


$$\mathbf{r_1} \longrightarrow \mathbf{r_a} \longrightarrow \mathbf{r_b} \cdots \longrightarrow \mathbf{r_y} \longrightarrow \mathbf{r_z} \longrightarrow \mathbf{r_2}$$

Reciprocity theorem:

If I see you, then you see me.

(a)



$\mathbf{r_1 \to r_a \to r_b \cdots \to r_y \to r_z \to r_2}$

Reciprocity theorem:

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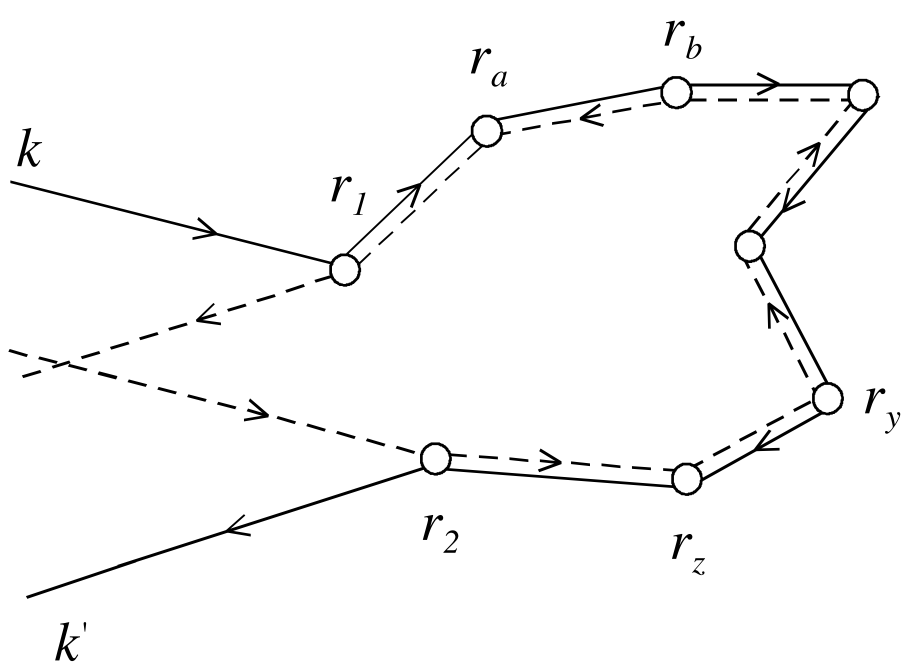
(b)



$\mathbf{r_2 \to r_z \to r_y \cdots \to r_b \to r_a \to r_1}$

incoherent classical term

interference term

The total average intensity is:

$$\overline{|A(\mathbf{k},\mathbf{k'})|^2} = \overline{\sum_{\mathbf{r_1},\mathbf{r_2}} |f(\mathbf{r_1},\mathbf{r_2})|^2 \left[1 + e^{i(\mathbf{k}+\mathbf{k'}).(\mathbf{r_1}-\mathbf{r_2})}\right]}$$

$$\overline{|A(\mathbf{k},\mathbf{k}')|^2} = \overline{\sum_{\mathbf{r_1},\mathbf{r_2}} |f(\mathbf{r_1},\mathbf{r_2})|^2 \left[1 + e^{i(\mathbf{k}+\mathbf{k}').(\mathbf{r_1}-\mathbf{r_2})}\right]}$$

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$\mathbf{r_1}-\mathbf{r_2} \simeq 0$: closed loops, weak localization and $\phi_0/2$ periodicity of the Sharvin effect.

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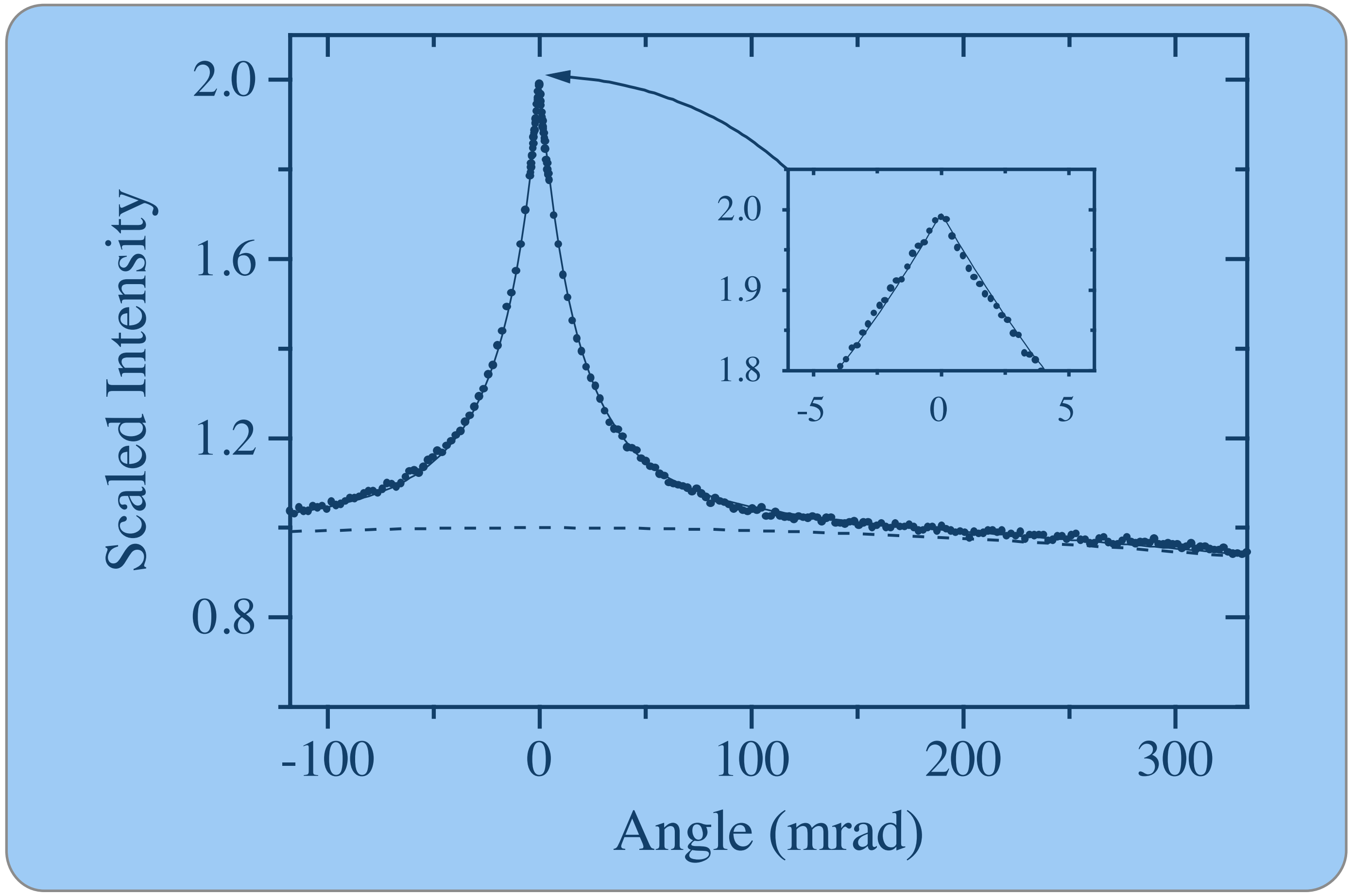


Coherent backscattering

Random quantum systems (quantum complexity)

Disorder does not break phase coherence and it does not introduce irreversibility

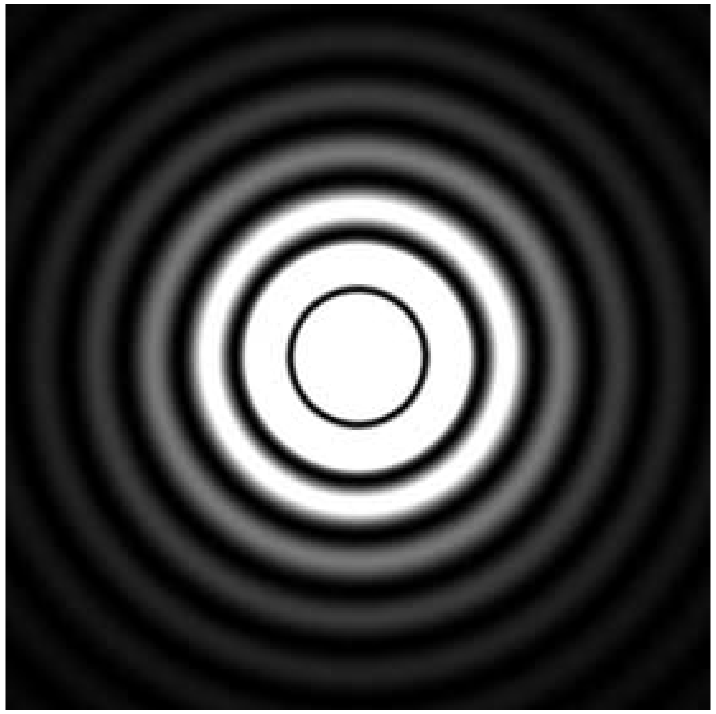
Random quantum systems (quantum complexity)

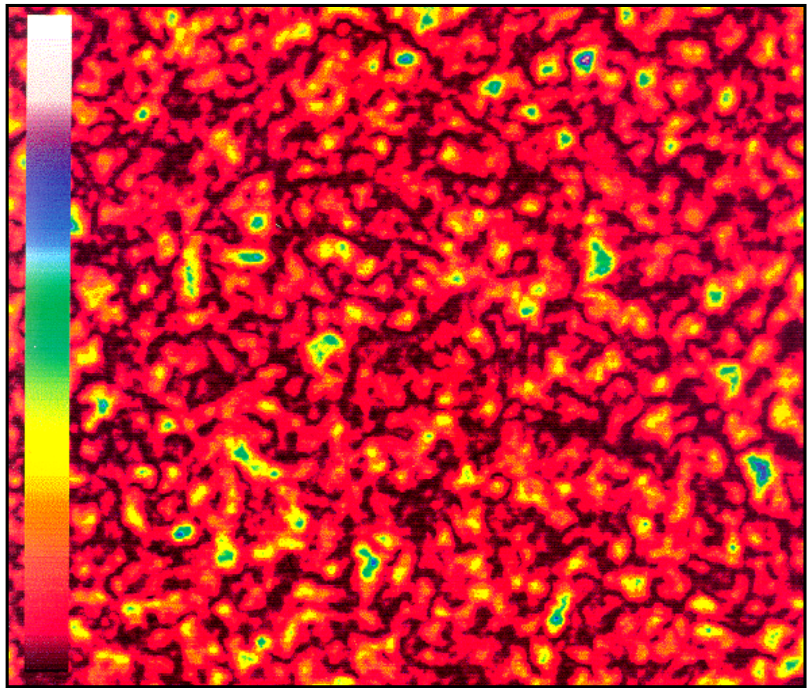
Disorder does not break phase coherence and it does not introduce irreversibility

It introduces randomness and complexity: all symmetries are lost, there are no good quantum numbers.

Exemple: speckle patterns in optics

Diffraction through a circular aperture: order in interference





Transmission of light through a disordered suspension: complex system

Mesoscopic quantum systems

- Most (all ?) quantum systems are complex
- Complexity (randomness) and decoherence are separate and independent notions.
- Complexity
  quantum numbers)
- Decoherence
  coherence $L \gg L_\varphi$

A mesoscopic quantum system is a coherent complex quantum system with $L \leq L_\varphi$

An Exemple

Phase coherence and self-averaging: universal fluctuations.

Classical limit : $L \gg L_\varphi$

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Classical limit : $L \gg L_\varphi$

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A macroscopic observable defined in each subsystem takes independent random values in each of the N pieces.

Law of large numbers: any macroscopic observable is equal with probability one to its average value.

The system performs an average over realizations of the disorder.

For $L \ll L_\varphi$, we expect deviations from self-averaging which reflect the underlying quantum coherence.

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- a good understanding of the phase coherence length L_φ
- a description of fluctuations and coherence in a quantum complex system.
- If disorder is strong enough, the system may undergo a *quantum phase transition*

Exemple: electrical conductance of a metal

A metal can be modeled as a quantum gas of electrons scattered by an elastic disorder.

At T=0 and in the absence of decoherence, it is a complex quantum system.

Due to disorder there is a finite conductance which is a quantum observable.

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Classically, the conductance of a cubic sample of volume L^d is given by Ohm's law: $G = \sigma L^{d-2}$ where σ is the conductivity.

Quantum conductance fluctuations

Classical self-averaging limit : $\frac{\delta G}{\overline{G}} = \frac{1}{N} = \left(\frac{L_\varphi}{L}\right)^{d/2}$

where $\delta G = \sqrt{\overline{G^2} - \overline{G}^2}$ and $\overline{G} = \sigma L^{d-2}$

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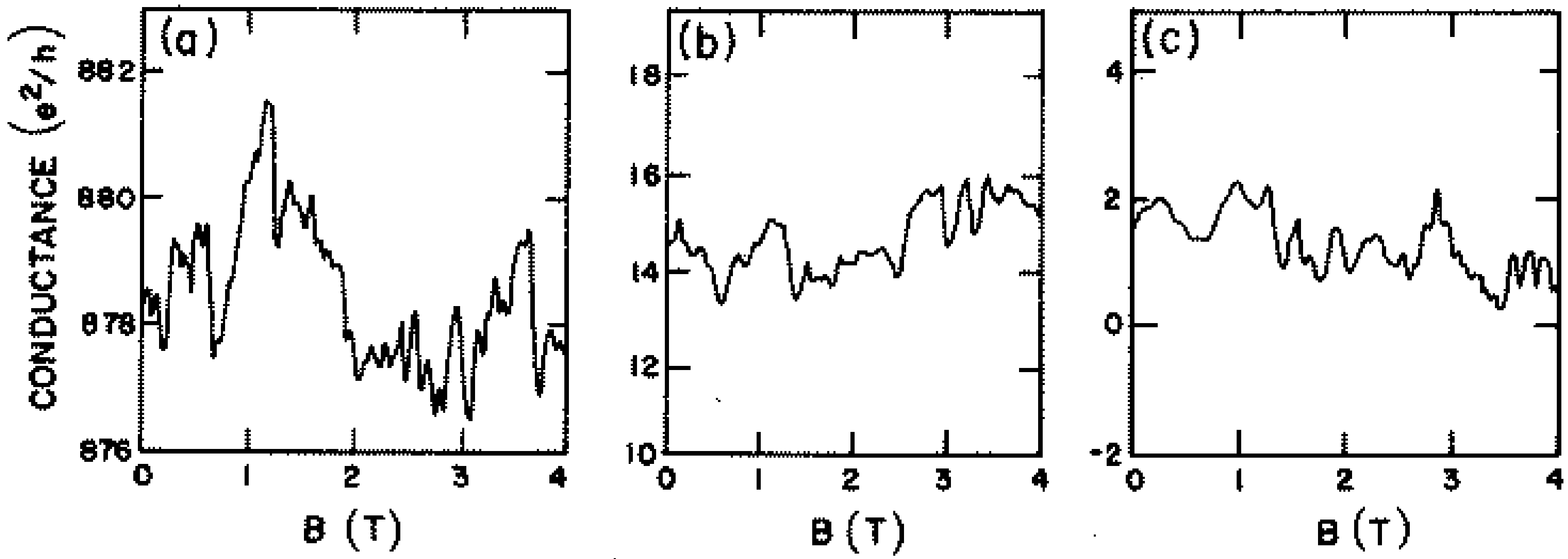
In the mesoscopic limit, the electrical conductance is not self-averaging.



Gold ring

Si-MOSFET

NUMERICS ON
THE ANDERSON MODEL